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SYSTEMATIC CORRECTIONS TO OBSERVED NORTH-POLAR DISTANCES DEDUCED FROM REFLECTION OBSERVATIONS AT GREENWICH.

By J. R. EASTMAN.

From the installation of the Transit-Circle at Greenwich in 1851, down to the present time, the Astronomers Royal have continued, with little variation, the method of dealing with Reflection observations which was adopted by AIRY while he was in charge of the observatory in Cambridge.

This method, together with the frequent comments of Sir GEORGE B. AIRY on its unsatisfactory results, has already been alluded to in a paper on the "Discordances between the North-Polar Distances of Stars derived from Direct and Reflection Observations," in the *Astronomical Journal*, No. 454, 1899 Jan. 25.

The process of obtaining the individual values of $R-D$ is practically the same in all observatories, but the methods of combining results and of determining the final systematic corrections to the observed north-polar distances, vary materially at different observatories.

This paper is designed to give the results of an examination of the observed positions of reflected stars, and of that part of the adopted systematic corrections depending on the values of $R-D$, at Greenwich, from 1851 to 1861 and from 1866 to 1893.

The plan adopted at Greenwich was to collect the $R-D$ results from stars arranged in groups covering, approximately, the same number of degrees in N.P.D.

The mean value of $R-D$ for each group was then taken by weights.

In nearly every year the values of $R-D$ in the groups *north* of the zenith differed appreciably from those *south* of the zenith; and, generally, the sign of the result changed at the zenith. The assumption was then made that the correction to be deduced from the means of these groups was variable, but continuous from the northern to the southern limit of observation, and could be represented by an expression of the form $a+b$ sin zen.dist. or $a+b$ sin zen.dist. $\times$ cos² zen.dist. The first form of this expression was used from 1851 to 1861 and from 1881 to 1893. The second form was used from 1862 to 1880. Of course these formulas were intended to represent the data from the means of the different groups.

To show how the astronomer's aptitude for solving puzzles was tested in this connection, I give below the mean data by groups, for several years, which exhibit an average range in the variation of values north and south of the zenith.

1852		1853		1855		1860		1866		1880	
N.	S.	N.	S.	N.	S.	N.	S.	N.	S.	N.	S.
″	″	″	″	″	″	″	″	″	″	″	″
+0.45	+0.62	+0.57	−0.02	+0.85	−0.71	+0.44	0.00	−0.55	+0.16	−0.48	+0.32
−0.02	+0.06	+0.08	−0.33	−0.03	+0.22	+0.11	+0.03	−0.29	+0.99	−0.26	+0.56
−0.61	−0.17	+0.35	−0.19	+0.45	−1.08	+0.31	−0.56	−0.52	+0.47	−0.14	+0.66
+1.08	−0.96	–	−0.66	−0.10	−0.04	+0.10	−0.39	−0.05	+0.15	−0.16	+0.15
+0.41	−0.29	–	−0.32	+0.23	+0.22	+0.02	−0.09	−0.53	+0.78	.00	+0.20
–	+0.10									+0.08	+0.31
	−0.10									−0.29	+0.82
										−0.03	+0.92

From these data it became necessary to construct formulas to represent the supposed observed errors; and it is easy to believe that the task was not always successfully performed.

In examining the work at the Greenwich Observatory the values of $R-D$, for each star, as given in the annual volumes, were collected into *fourteen* groups, each covering 5° of N.P.D., from 358° to 33°, and from 41° to 79°; thus using an equal number of groups north and south of the zenith.

Some uniform system of weights was desirable, and since the systems used at Greenwich were not uniform during the period under consideration, it was finally decided to employ as weights simply the number of reflection observations combined in the mean $R-D$ for each star.

In 1865 the Transit Circle was dismounted, the principal parts separated, and the central cube of the telescope was perforated in order to allow one collimator to be seen from the other without raising the pivots out of the Y's. As the work was thus interrupted, the results for 1865 were omitted from the investigation.

For the purpose of comparing results it was deemed desirable to collect the annual results into groups. Therefore the periods from 1851 to 1864 and from 1866 to 1893 were divided into *two* and *four* groups, respectively, of seven years each.

In considering observations of this character it should be borne in mind that the accuracy and agreement attained in direct observations alone are not to be expected.

In the series of observations under examination a range of more than 5″ is frequently found in a single group covering a space of 5° in N.P.D.

To illustrate the wide range in the observations the following observed mean values of $R-D$, in a 5° group, are copied from the Greenwich volumes. The subscript-figures show the number of reflection observations in each case.

1854		1860		1864		1881	
64° to 69° N.P.D.		8° to 13° N.P.D.		59° to 64° N.P.D.		18° to 23° N.P.D.	
″	″	″	″	″	″	″	″
$+0.43_5$	-1.60_1	$+0.11_5$	-0.30_3	$+1.05_1$	-1.06_3	$+0.35_2$	-0.96_2
2.37_1	2.88_1	0.11_3	2.00_1	1.53_1	0.37_2	0.50_1	0.57_5
1.22_1	0.23_1	0.01_2	0.59_2	1.54_1	0.53_1	0.17_2	2.16_1
$+0.05_1$	0.87_1	1.30_1	-1.13_1	0.19_4	0.20_1	3.25_1	0.27_2
–	2.14_1	3.11_1		0.38_1	0.16_4	0.36_1	0.32_4
	0.16_1	$+0.42_1$		0.62_2	-1.97_1	$+0.22_2$	1.46_1
	0.60_2			0.25_1	–	–	0.15_1
–	-1.98_3			$+0.30_3$	–	–	0.17_1
						–	0.18_1
						–	0.24_1
						–	0.16_1
						–	1.65_1
						–	0.72_7
						–	1.78_1
						–	-0.32_{11}

The following table shows the *maximum* annual range in a 5° period, together with the position of the group.

Year	Group	Max. range of values of $R-D$ in a group of 5° N.P.D.	Year	Group	Max. range of values of $R-D$ in a group of 5° N.P.D.	Year	Group	Max. range of values of $R-D$ in a group of 5° N.P.D.	Year	Group	Max. range of values of $R-D$ in a group of 5° N.P.D.
	° °	″		° °	″		° °	″		° °	″
1851	64 to 69	4.94	1862	13 to 18	5.80	1874	59 to 64	4.71	1884	18 to 23	5.01
52	23 : 28	5.87	63	8 : 13	5.25	75	59 : 64	5.57	85	64 : 69	4.64
53	59 : 64	4.16	64	59 : 64	6.51	76	49 : 54	4.49	86	28 : 33	6.38
54	64 : 69	5.25	66	18 : 23	5.93	77	49 : 54	4.87	87	28 : 33	6.52
55	13 : 18	6.05	67	3 : 8	4.51	78	8 : 13	5.25	88	23 : 28	5.48
56	64 : 69	5.74	68	23 : 28	3.88	79	18 : 23	3.56	89	74 : 79	6.65
57	8 : 13	3.57	69	18 : 23	4.44	80	28 : 33	5.13	90	18 : 23	5.30
58	74 : 79	4.58	70	69 : 74	5.20	81	18 : 23	7.90	91	8 : 13	4.46
59	18 : 23	5.85	71	54 : 59	4.66	82	69 : 74	6.16	92	74 : 79	4.57
60	8 : 13	7.24	72	69 : 74	5.93	83	69 : 74	5.08	93	69 : 74	6.12
61	64 : 69	7.03	73	49 : 54	4.29						

The mean value of the above maximum annual ranges in a 5° group is 5″.35. These maxima occur in an equal number of groups on each side of the zenith. If the maximum range for each year for the entire range of reflected observations on both sides of the zenith be examined, it will be found that the *mean* value of that annual range for the 42 years is, in the group 358° to 33°, N.P.D., 5″.40; and in the group 44° to 79° it is 5″.45.

These examples of the range of observed values will serve to make it plain that, except in the case of a very large number of observations made under similar and uniform conditions, apparently abnormal results may be expected.

So strongly marked is this peculiarity in these observations that it is believed that a careful examination of the observations and results will convince the astronomer that the attempt to draw definite conclusions concerning the stability of his instruments, from the fortuitous and irregular annual variations in the observed values of $R-D$, except with a very large number of observations, may conduct to misleading and unsatisfactory results.

In the examination of the Greenwich reflection observations the means of all the $R-D$ results in each group, in each year, were taken by using the weights already mentioned. Then the means of the separate groups in N.P.D. were collected for each series of seven years, and the mean for each seven years obtained; finally the mean by groups of 5° N.P.D. was taken for the period 1851 to 1864, two seven-year periods, and for the period 1866 to 1893, four seven-

year periods. As a sample of the results in each year, I give in the following table the mean results in each of the following years in each 5° group. The subscript-figures indicate the weights or number of observations.

Groups	1851	1852	1853	1854	1855
° °	″	″	″	″	″
358 to 3	$+0.67_{18}$	$+0.44_{31}$	$+0.44_{22}$	$+0.46_{17}$	$+0.93_{7}$
3 : 8	$+0.43_{21}$	-0.05_{13}	$+0.85_{5}$	-0.24_{5}	-0.55_{5}
8 : 13	-0.29_{10}	$+0.05_{7}$	-0.65_{5}	-1.38_{4}	$+0.15_{9}$
13 : 18	$+0.51_{15}$	-0.53_{9}	$+0.28_{15}$	$+0.34_{7}$	$+0.45_{21}$
18 : 23	$+0.12_{18}$	$+0.67_{13}$	$+0.20_{12}$	$+0.08_{31}$	-0.54_{18}
23 : 28	$+0.93_{7}$	$+0.81_{24}$	$+0.12_{20}$	$+0.31_{31}$	$+0.05_{11}$
28 : 33	$+0.42_{8}$	$+0.11_{6}$	$+0.98_{7}$	-0.53_{5}	$+0.74_{11}$
44 : 49	$+0.31_{4}$	$+0.40_{1}$	$+0.32_{1}$	-0.42_{2}	-0.43_{7}
49 : 54	-0.02_{7}	$+0.61_{8}$	-0.07_{3}	-0.72_{4}	-1.23_{4}
54 : 59	$+0.11_{14}$	$+0.05_{29}$	$+0.10_{14}$	$+0.35_{16}$	-0.16_{7}
59 : 64	$+0.13_{35}$	-0.23_{24}	-0.29_{22}	-0.31_{22}	$+0.02_{22}$
64 : 69	-0.29_{11}	-0.88_{21}	-0.62_{13}	-0.25_{19}	-1.12_{13}
69 : 74	-0.19_{15}	-0.23_{14}	-0.59_{21}	-0.68_{25}	-1.01_{7}
74 : 79	-0.32_{5}	$+0.09_{23}$	-0.21_{12}	-0.43_{19}	$+0.12_{18}$

The following table exhibits the *mean* results for each 5° group in N.P.D., and for each series of seven years. The subscript figures represent the number of observations by reflection, or the weights.

TABLE A.

Limits for each group in N.P.D.	1851 to 1857	1858 to 1864	1866 to 1872	1873 to 1879	1880 to 1886	1887 to 1893
° °	″	″	″	″	″	″
358 to 3	$+0.52_{99}$	$+0.64_{36}$	-0.32_{11}	-0.27_{167}	-0.31_{112}	-0.39_{295}
3 : 8	$+0.21_{59}$	$+0.54_{48}$	-0.57_{36}	-0.41_{108}	-0.39_{77}	-0.46_{190}
8 : 13	-0.04_{57}	$+0.17_{125}$	-0.67_{112}	-0.25_{90}	-0.47_{117}	-0.36_{147}
13 : 18	$+0.30_{105}$	$+0.39_{133}$	-0.29_{201}	-0.32_{227}	-0.47_{148}	-0.37_{138}
18 : 23	$+0.02_{139}$	$+0.35_{276}$	-0.21_{287}	-0.31_{296}	-0.29_{205}	-0.41_{82}
23 : 28	$+0.40_{150}$	$+0.43_{214}$	-0.16_{273}	-0.26_{371}	-0.04_{284}	-0.22_{81}
28 : 33	$+0.26_{76}$	$+0.14_{50}$	-0.30_{57}	-0.09_{127}	$+0.13_{96}$	$+0.12_{23}$
44 : 49	-0.31_{31}	-0.42_{17}	$+0.30_{13}$	$+0.16_{11}$	$+0.32_{14}$	$+0.78_{6}$
49 : 54	$+0.14_{37}$	-0.05_{100}	$+0.40_{173}$	$+0.31_{235}$	$+0.36_{138}$	$+0.63_{54}$
54 : 59	$+0.13_{97}$	-0.26_{121}	$+0.72_{189}$	$+0.37_{205}$	$+0.21_{104}$	$+0.51_{31}$
59 : 64	-0.11_{167}	-0.22_{228}	$+0.60_{279}$	$+0.30_{366}$	$+0.41_{224}$	$+0.32_{187}$
64 : 69	-0.63_{134}	-0.46_{203}	$+0.48_{270}$	$+0.25_{371}$	$+0.25_{268}$	$+0.35_{212}$
69 : 74	-0.59_{105}	-0.67_{116}	$+0.69_{57}$	$+0.24_{224}$	$+0.44_{222}$	$+0.30_{230}$
74 : 79	-0.20_{124}	-0.07_{90}	$+1.15_{5}$	$+0.31_{145}$	$+0.49_{207}$	$+0.57_{298}$

The following table shows the mean result for each group for the two periods, together with the mean $R-D$ for each period and for both sides of the zenith. The subscript figures indicate the number of observations or weights

TABLE B.

Group of Years	358° to 3°	3° to 8°	8° to 13°	13° to 18°	18° to 23°	23° to 28°	28° to 33°	358° to 33°
	″	″	″	″	″	″	″	″
1851–57	$+0.52_{99}$	$+0.21_{59}$	-0.04_{57}	$+0.30_{105}$	$+0.02_{139}$	$+0.40_{150}$	$+0.26_{76}$	
1858–64	$+0.64_{36}$	$+0.54_{48}$	$+0.17_{125}$	$+0.39_{133}$	$+0.35_{276}$	$+0.13_{214}$	$+0.14_{50}$	
1851–64	$+0.55_{135}$	$+0.36_{107}$	$+0.11_{182}$	$+0.35_{238}$	$+0.24_{415}$	$+0.42_{364}$	$+0.21_{126}$	$+0.31_{1567}$
1866–72	-0.32_{11}	-0.59_{36}	-0.67_{112}	-0.29_{201}	-0.21_{287}	-0.16_{273}	-0.30_{57}	
1873–79	0.27_{167}	0.41_{108}	0.25_{90}	0.32_{227}	0.31_{296}	0.28_{371}	-0.09_{127}	
1880–86	0.31_{112}	0.39_{77}	0.47_{117}	0.47_{148}	0.29_{205}	0.04_{284}	$+0.13_{96}$	
1887–93	-0.39_{295}	-0.16_{190}	-0.36_{147}	-0.37_{138}	-0.41_{82}	-0.22_{81}	$+0.12_{23}$	
1866–93	-0.34_{585}	-0.14_{411}	-0.44_{466}	-0.36_{714}	-0.28_{870}	-0.17_{1009}	-0.04_{303}	-0.29_{1358}

TABLE B — Continued.

Group of Years	44° to 49°	49° to 54°	54° to 59°	59° to 64°	64° to 69°	69° to 74°	74° to 79°	44° to 79°
	″	″	″	″	″	″	″	″
1851–57	0.31_{3}	$+0.11$	$+0.13$	-0.11_{167}	-0.63_{134}	-0.59_{105}	-0.20_{124}	
1858–64	0.42_{17}	-0.05_{100}	0.26_{121}	0.22	-0.16_{203}	-0.67_{116}	-0.02_{90}	
1851–64	0.35_{48}	0.00_{137}	-0.08_{218}	-0.17_{105}	-0.53_{337}	-0.63_{221}	-0.13_{214}	-0.22_{1580}
1866–72	$+0.30_{3}$	$+0.10_{173}$	$+0.72_{180}$	$+0.60$	$+0.18_{270}$	$+0.69_{57}$	$+1.15_{25}$	
1873–79	0.16_{11}	0.31	0.37_{205}	0.30	0.25_{371}	0.21_{224}	0.31_{145}	
1880–86	0.32_{14}	0.36	0.21_{101}	0.41_{224}	0.25_{268}	0.11_{222}	0.19_{207}	
1887–93	$+0.78$	$+0.63_{54}$	$+0.51$	$+0.32_{187}$	$+0.35_{212}$	$+0.30$	$+0.57_{293}$	
1866–93	$+0.34_{44}$	$+0.37_{600}$	$+0.48_{529}$	$+0.11_{1056}$	$+0.32_{1121}$	$+0.37_{763}$	$+0.52_{680}$	$+0.40_{4793}$

In the following table are given, for easy inspection, the mean values of $R-D$ for each group in N.P.D., and for the two periods, before and after 1865.

TABLE C.

N.P.D. of Groups	1851–64	1866–93	N.P.D. of Groups	1851–64	1866–93
°	″	″	°	″	″
358 to 3	$+0.55_{135}$	-0.31_{585}	44 to 49	-0.35_{48}	$+0.34_{44}$
3 : 8	0.36_{107}	0.11_{411}	49 : 54	0.00_{137}	0.37_{600}
8 : 13	0.11_{182}	0.11_{866}	54 : 59	-0.08_{218}	0.48_{529}
13 : 18	0.35_{238}	0.36_{714}	59 : 64	0.17_{105}	0.11_{1056}
18 : 23	0.24_{415}	0.28_{870}	64 : 69	0.53_{337}	0.32_{1121}
23 : 28	0.41_{364}	0.17_{1000}	69 : 74	0.63_{221}	0.37_{763}
28 : 33	$+0.21_{126}$	-0.01_{303}	74 : 79	-0.13_{214}	$+0.52_{680}$

A careful inspection of the above table will show the apparent difficulty in representing the mean values of $R-D$ for the period from 1851 to 1864 on either side of the zenith by any simple function of the zenith-distance.

From 1866 to 1893 the values north of the zenith show no change likely to be due to zenith-distance, except in the last two groups, and there the magnitude of the values is due to some peculiar data in 1882 and 1883. From 1866 to 1893 the mean values of $R-D$, south of the zenith, completely disprove, so far as they go, any theory of change of value due to zenith-distance.

If one considers the wide range of data from the observations, the most manifest determination of the corrections on each side of the zenith would seem to be the *mean* of the results from the seven 5° groups on either side.

The most obvious inference from these results is, that if the systematic corrections which have been used at Greenwich for many years are correct, a comparison of the Greenwich results with a thoroughly good system of star places, free from systematic errors, should give a set of residuals, if any, of nearly constant value from the northern to the southern limit of observation. On the other hand, if these systematic corrections are faulty, then the residuals, derived from a comparison with the best systems of star places, would take the general form of the corrections.

It has been mentioned already that the expression from which the systematic corrections depending on the values of $R-D$ were computed, had been used in two forms. From 1851 to 1861, and from 1881 to 1893, the formula used was, $a+b$ sin zen. dist., which for convenience I will call formula No. 1. From 1862 to 1880 the formula was $a+b$ sin zen. dist. $\times$ cos² zen. dist., which I will call formula No. 2.

Since the Transit-Circle was mounted at Greenwich, five star catalogues, depending on the work of that instrument, have been issued by the observatory.

The results of the comparison of these *five* catalogues with Prof. NEWCOMB's "Absolute System" are given in the following table. The quantities in the table are *corrections* to the Greenwich catalogues.

TABLE D.

N.P.D.	Greenwich Catalogues 1860	1864	1872	1880	1890
°	"	"	"	"	"
1	+0.01	+0.00	−0.01	+0.03	0.00
6	.06	.03	.08	.03	.00
11	.11	.04	.14	.06	.00
16	.17	.06	.17	.14	.00
21	.22	.08	.20	.28	.00
26	.29	.10	.22	.40	.00
31	.35	.13	.25	.37	.00
36	.36	.15	.26	.31	+ .01
41	.30	.17	.28	.27	.06
46	.27	.19	.30	.24	.11
51	.27	.21	.32	.25	.16
56	.32	.23	.35	.26	.19
61	.37	.25	.38	.27	.22
66	.44	.26	.42	.29	.25
71	.46	.26	.47	.32	.28
76	.47	.24	.55	.35	.31
81	.43	.18	.61	.39	.34
86	.38	.11	.70	.42	.40
91	.31	.06	.80	.45	.43
96	.30	.08	.91	.50	.46
101	.36	.18	1.07	.58	.56
106	.40	.34	1.15	.63	.61
111	.42	.51	1.35	.72	.72
116	.42	.68	1.70	.84	.81
121	+0.42	+0.89	−2.22	+1.00	+0.92

If the residuals in the above table were nearly constant it would indicate the relative accuracy of the standard system and the observed places, and that the form of the correction was satisfactory. Assuming the accuracy of the standard with which the observed results were compared, we should expect to find in the residuals a clue to the validity of the formulas used in computing that portion of the systematic corrections depending on the values of $R-D$.

In the data from which the catalogue for 1860 was derived the systematic corrections derived from the value of $R-D$ were computed for the years 1854 to 1860, from the formula $a+b$ sin zen. dist. which I have called formula No. 1.

An inspection of the residuals for the catalogue of 1860, in table D, shows the consecutive maxima and minima +0″.01, +0″.36, +0″.27, +0″.47, +0″.30 and +0″.42.

For the catalogue for 1864, formula No. 1 was used in 1861 and formula No. 2 was used in 1862 to 1867. The residuals vary between the limits 0″.00, +0″.26, +0″.06 and +0″.89. For the catalogue for 1872 the same formula was used and similar maxima and minima are found in the residuals, but of smaller amplitude. In the catalogue for 1880 both formulas were used with a complex result the maximum and minimum residuals, in order, being +0″.03, +0″.40, +0″.24 and +1″.00.

In the catalogue for 1890, formula No. 2 was used and the simpler form of residuals is found in table D.

It seems, therefore, that the residuals found by comparing the different catalogues with a standard system correspond very closely with the form of the systematic corrections used in preparing the annual data from which the catalogues were prepared. This appears to indicate that the portion of the systematic correction which was derived from the observed values of $R-D$, *introduced* errors instead of *eliminating* them.

In order to facilitate the comparison between the values of $\frac{R-D}{2}$ as finally used in the annual data from which the Greenwich Star Catalogues were derived, and the values computed, as already mentioned, from the data published in the Greenwich annual volumes, the following tables are given to show the results as *used* and as *computed* for each year, and also the mean values for the years used in each catalogue.

In the case of the computed results the mean of the values in each group is assumed to refer, in position, to the mean of the limiting N.P.D.'s of that group.

The limits of the reflection observations, for more than thirty years, were from about 358° to 80° N.P.D.; and, though this range was afterward considerably extended, it was deemed best for the sake of uniformity to confine the investigation to those limits throughout the whole period, from 1851 to 1895.

TABLE E. CATALOGUE FOR 1872.0.

Year	0°	10°	20°	30	40°	50°	60°	70	80	90°	100°	110	120°
	"	"	"	"	"	"	"	"	"	"	"	"	"
1868	−0.30	−0.29	−0.21	−0.08	+0.07	+0.22	+0.34	+0.40	+0.39	+0.33	+0.23	+0.11	+0.07
1869	.10	.09	.03	− .08	.24	.37	.48	.53	.52	.47	.38	.29	.23
1870	10	.09	.03	+ .08	.20	.32	.41	.46	.45	.40	.33	.25	.20
1871	.25	.24	.19	− .08	+ .04	.16	.25	.30	.29	.24	.17	.09	+ .04
1872	.25	.24	.20	.12	− .03	.07	.13	.17	.17	.13	.07	.01	− .03
1873	.15	.15	.10	.01	+ .10	+ .20	.27	.31	.31	.26	.20	+ .11	+ .09
1874	.33	.32	.28	.20	− .10	− .01	.06	.10	.09	.06	.00	− .07	− .10
1875	.13	.12	.09	.05	+ .01	+ .07	.11	.13	.13	.11	.07	+ .04	+ .01
1876	−0.14	−0.14	−0.11	−0.05	+0.01	+0.08	+0.13	+0.15	+0.15	+0.12	+0.08	+0.04	+0.01
Mean	−0.19	−0.19	−0.14	−0.05	+0.06	+0.16	+0.24	+0.28	+0.28	+0.24	+0.17	+0.10	+0.06

In table E are given the *annual* values of the correction $\frac{R-D}{2}$ as computed and used for the data from which the Greenwich 9-Year Catalogue for 1872.0 was derived. The mean of these values for *nine* years is also shown. In table E_1 are given the *annual* values of $\frac{R-D}{2}$ as I have computed them for 5° groups and also the *mean* values for the *nine* years. The subscript figures indicate the number of observations used.

Table E_1.

Year	358° to 3°	3° to 8°	8° to 13°	13° to 18°	18° to 23°	23° to 28°	28° to 33°
	″	″	″	″	″	″	″
1868	0.96_{1}	-0.69_{1}	-1.13_{11}	0.17_{25}	-0.36_{41}	-0.06_{36}	-0.64_{8}
69	$.95_{1}$	1.76_{2}	0.43_{6}	$-.06_{29}$	$+.05_{29}$	$+.01_{27}$	$+.52_{1}$
70		1.08_{4}	$.68_{26}$	$+.05_{44}$	$.00_{37}$	$+.21_{48}$	$+.20_{7}$
71	$+.62_{1}$	0.18_{4}	$.56_{21}$	$-.31_{29}$	$-.37_{50}$	$-.30_{38}$	$-.83_{13}$
72	$-.20_{2}$	$.60_{1}$	$.63_{21}$	$.53_{36}$	$.35_{5}$	$.25_{5}$	$.17_{30}$
73	$+.35_{3}$	0.03	$.17_{12}$	$.14_{25}$	$.45_{46}$	$.27_{47}$	$.29_{17}$
74	$-.70_{11}$	1.09	$.61_{11}$	$.45_{53}$	$.57_{51}$	$.46_{48}$	$-.56_{20}$
75	$.01_{19}$	0.44	$-.35_{19}$	$.39_{30}$	$.07_{4}$	$.33_{32}$	$+.71_{10}$
76	-0.07_{30}	-0.51	$+0.13_{14}$	-0.31_{38}	-0.15_{45}	-0.25_{77}	-0.15_{19}
Mean	-0.21_{119}	-0.71_{56}	-0.49_{145}	-0.33_{309}	-0.25_{424}	-0.19_{448}	-0.13_{120}

Year	44° to 49°	49° to 54°	54° to 59°	59° to 64°	64° to 69°	69° to 74°	74° to 79°
	″	″	″	″	″	″	″
1868	$+0.12_{1}$	$+0.69_{20}$	$+0.78_{21}$	$+0.38_{31}$	$+0.68_{29}$	$+0.74_{15}$	$+1.27_{9}$
69		$.92_{14}$	$.85_{11}$	$.93_{30}$	$.79_{28}$	1.35_{6}	1.76_{4}
70		$.53_{42}$	$.91_{40}$	$.95_{52}$	$.18_{40}$	1.17_{18}	1.08_{7}
71	$.29_{1}$	$.17_{37}$	$.69_{41}$	$.52_{49}$	$.59_{54}$	0.31_{8}	0.76_{6}
72	$+.68$	$.18_{36}$	$.27_{33}$	$.49_{60}$	$.12_{61}$	$.09_{17}$	1.07_{5}
73		$+.58_{27}$	$.67_{33}$	$.18_{46}$	$.53_{63}$	$.55_{27}$	0.38_{16}
74	$-.63_{1}$	$-.21_{56}$	$.03_{35}$	$.14_{54}$	$.26_{73}$	$.12_{36}$	$.33_{24}$
75		$+.47_{15}$	$.53_{22}$	$.25_{53}$	$.08_{52}$	$.08_{41}$	$.30_{27}$
76	$+1.61_{2}$	$+0.48_{41}$	$+0.16_{37}$	$+0.10_{64}$	$+0.12_{67}$	$+0.37_{32}$	$+0.33_{20}$
Mean	$+0.12_{1}$	$+0.42_{288}$	$+0.58_{278}$	$+0.17_{448}$	$+0.41_{457}$	$+0.53_{200}$	$+0.81_{118}$

Table F shows the corrections as computed and applied at Greenwich for the years used in the Catalogue for 1880.

Table F_1 shows the corrections as computed from the data published in the Greenwich annual volumes.

Table F. Catalogue for 1880.0.

Year	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
	″	″	″	″	″	″	″	″	″	″	″	″	″
1877	−0.09	0.09	0.07	−0.03	+0.02	+0.07	+0.10	+0.12	+0.12	+0.10	+0.07	+0.04	+0.02
1878	.25	.25	.19	.10	.01	.11	.19	.23	.23	.18	.12	.05	.00
1879	.11	.11	.08	.02	.01	.11	.16	.18	.18	.15	.11	.07	.04
1880	.11	.13	.09	.02	.08	.17	.24	.27	.27	.23	.18	.12	.08
1881	.33	.24	.16	.06	.03	.13	.22	.31	.38	.45	.50	.54	.56
1882	.22	.16	.09	.01	+ .06	.14	.21	.28	.31	.39	.44	.47	.48
1883	.45	.35	.24	.12	.00	.12	.24	.31	.41	.52	.59	.64	.67
1884	.43	.34	.23	.12	.01	.11	.22	.32	.41	.49	.55	.60	.62
1885	.45	.35	.24	.12	.00	.12	.23	.34	.41	.52	.59	.63	.66
1886	0.43	0.33	0.22	−0.10	+0.02	+0.14	+0.25	+0.36	+0.46	+0.54	+0.61	+0.66	+0.68
Mean	−0.29	0.24	0.16	−0.07	+0.02	+0.12	+0.21	+0.28	+0.33	+0.36	+0.38	+0.38	+0.38

TABLE F_1.

Year	358° to 3°	3° to 8	8° to 13°	13° to 18°	18° to 23°	23° to 28°	28° to 33
	″	″	″	″	″	″	″
1877	-0.37_{31}	-0.34_{19}	-0.13_{11}	$+0.11_{26}$	-0.27_{35}	$+0.02_{52}$	-0.08_{19}
78	$.62_{7}$	$-\ .64_{12}$	$.32_{18}$	$-\ .65_{26}$	$.24_{32}$	$-\ .44_{42}$	$+\ .04$
79	$.34_{17}$	$+\ .03_{7}$	$.25_{3}$	01_{29}	$.46_{22}$	$.11_{53}$	$+\ .05_{19}$
80	$.59_{26}$	$+\ .11_{13}$	$.43_{18}$	$.18_{32}$	$.13_{40}$	$.08_{58}$	$-\ .03_{25}$
81	$-\ .42_{19}$	$+\ .24_{9}$	$.34_{22}$	$.31_{24}$	$-\ .41_{39}$	$.35_{75}$	$+\ .07_{18}$
82	$+\ .14_{26}$	$+0.45_{12}$	0.43_{17}	$.33_{17}$	$+\ .15_{33}$	$.35_{50}$	$+\ .07_{14}$
83	$-\ .35_{23}$	-1.47_{8}	1.04_{10}	$.73_{18}$	$-\ .17_{18}$	$-\ .21_{25}$	$+\ .80_{16}$
84	$-\ .10_{21}$	0.83_{6}	-0.62_{19}	$.57_{15}$	$.23_{24}$	$+\ .16_{25}$	$-\ .01_{9}$
85	$+\ .46_{11}$	$.94_{19}$	$+\ .05_{16}$	$.64_{23}$	$.94_{20}$	$+\ .13_{20}$	$+\ .08_{6}$
86	-0.52_{12}	-0.28_{10}	-0.71_{15}	-0.75_{19}	-0.62_{22}	$+0.02_{31}$	-0.31_{8}
Mean	-0.27_{196}	-0.37_{115}	-0.42_{149}	-0.41_{229}	-0.34_{305}	-0.12_{431}	$+0.07_{157}$

Year	44° to 49°	49° to 54°	54° to 59°	59° to 64°	64° to 69°	69° to 74°	74° to 79
	″	″	″	″	″	″	″
1877	-0.02_{5}	$+0.19_{33}$	$+0.42_{20}$	$+0.24_{52}$	$+0.25_{48}$	$+0.14_{30}$	$+0.11_{30}$
78	$+\ .44_{2}$	$.39_{37}$	$.13_{28}$	$.64_{50}$	$.12_{33}$	$.45_{31}$	$.35_{18}$
79	$+\ .01_{1}$	$.28_{26}$	$.43_{30}$	$.37_{47}$	$.35_{45}$	$.06_{27}$	0.56_{10}
80	$-\ .40_{3}$	$.52_{41}$	$+\ .56_{22}$	$.35_{69}$	$.26_{61}$	$.59_{34}$	1.05_{35}
81	$+0.45_{3}$	$.15_{39}$	$-\ .02_{29}$	$.60_{36}$	$.43_{64}$	$.83_{39}$	0.72_{28}
82	$+1.16_{2}$	$.25_{17}$	$-\ .01_{28}$	$.46_{30}$	$.13_{35}$	$.31_{44}$	$.39_{35}$
83	-0.60_{2}	$+\ .10_{10}$	$+\ .27_{7}$	$.58_{13}$	$.20_{20}$	$.07_{26}$	$.24_{21}$
84	$+2.00_{1}$	$-\ .24_{11}$	$-\ .02_{4}$	$.15_{24}$	$.16_{38}$	$.22_{24}$	$.27$
85	$+1.52_{1}$	$+\ .78_{5}$	$+\ .70_{7}$	$.35_{33}$	$.22_{26}$	$.29_{28}$	$+0.47_{38}$
86	-0.16_{2}	$+1.07_{15}$	$+0.57_{7}$	$+0.56_{19}$	$+0.12_{24}$	$+0.58_{27}$	0.00_{22}
Mean	$+0.14_{22}$	$+0.35_{234}$	$+0.30_{182}$	$+0.43_{373}$	$+0.22_{394}$	$+0.35_{310}$	$+0.42_{265}$

Table G shows the corrections as *computed* and *applied* at Greenwich, for the years used in the catalogue for 1890.

Table G_1 shows the corrections as *computed* from the data published in the Greenwich annual volumes.

TABLE G.

Year	0°	10°	20	30°	40	50°	60°	70	80°	90°	100	110°	120
	″	″	″	″	″	″	″	″	″	″	″		″
1887	−0.44	−0.34	−0.23	−0.12	0.00	+0.12	+0.24	+0.34	+0.44	+0.52	+0.59	+0.63	+0.66
88	.46	.35	.23	.10	+0.03	.16	.28	.40	.50	.59	.67	.72	.75
89	.43	.33	.23	.11	+ .01	.12	.24	.34	.44	.52	.58	.63	.66
90	.36	.28	.19	.10	.00	.10	.19	.28	.35	.42	.47	.51	.54
91	−0.35	−0.27	−0.18	−0.09	0.00	+0.09	+0.18	+0.27	+0.34	+0.41	+0.46	+0.50	+0.52
Mean	−0.41	−0.31	−0.21	−0.10	+0.01	+0.12	+0.23	+0.33	+0.11	+0.49	+0.55	+0.60	+0.63

TABLE G_1.

Year	358° to 3	3° to 8°	8 to 13	13 to 18	18 to 23	23° to 28	28 to 33°
	″	″	″	″	″	″	″
1887	-0.54_{32}	-0.59_{36}	-0.53_{26}	-0.69_{27}	-1.10_{22}	-0.53_{20}	$+0.20_{9}$
88	$.45_{27}$	$.78_{12}$	$.59_{17}$	$.52_{20}$	-0.07_{23}	$+\ .21_{25}$	$-\ .22_{5}$
89	$.27_{28}$	$.66_{30}$	$-\ .41_{37}$	$.56_{23}$	$+\ .07_{17}$	$-\ .35_{26}$	$+\ .66_{6}$
90	$.49_{37}$	$.51_{24}$	$+\ .23_{16}$	$.10_{20}$	$-\ .43_{18}$	-0.29_{10}	-0.60_{3}
91	-0.22_{28}	-0.25_{13}	-0.50_{15}	-0.34_{14}	-0.48_{2}	–	
Mean	-0.39_{152}	-0.56_{115}	-0.36_{111}	-0.44_{104}	-0.40_{82}	-0.19_{81}?	$+0.01_{23}$?

Year	44° to 49	49° to 54	54 to 59	59° to 64	64° to 69°	69° to 74	74 to 79°
	″	″	″	″	″	″	″
1887	$+0.55_{4}$	$+0.51_{12}$	$+0.48_{8}$	$+0.41_{32}$	$+0.37_{31}$	$+0.26_{44}$	$+0.76_{50}$
88	-1.69_{1}	1.11_{13}	$.58_{10}$	$-\ .01_{28}$	$.40_{30}$	$.20_{25}$	$.75_{37}$
89	$+0.83_{1}$	0.54_{21}	$.21_{8}$	$+\ .42_{27}$	$.16_{39}$	$.23_{31}$	$.34_{34}$
90	–	$+0.25_{8}$	$+0.87_{5}$	$.35_{24}$	$.04_{22}$	$.21_{30}$	$.34_{7}$
91	–	–	–	$+0.21_{18}$	$+0.04_{23}$	$+0.20_{27}$	$+0.34_{38}$
Mean	-0.05_{6}?	$+0.60_{54}$?	$+0.54_{31}$?	$+0.28_{129}$	$+0.20_{145}$	$+0.22_{157}$	$+0.51_{196}$

After measures had been taken at Greenwich to make reflected observations at greater distances from the zenith, more attention was given to reflected work at the greater zenith distances, and few or no observations were made of stars in the two or three groups near the zenith. This change in the plan of observing makes the comparison of data in tables G and G_1 less satisfactory.

A study of all the above results derived from a re-computation of the $R-D$ data in the Greenwich volumes from 1851 to 1893, leads to the conclusion that, considering the range of variation in the individual observations, the variations in the results in each group are, generally, not greater than would be expected if we were seeking a uniform value of $R-D$ on each side of the zenith.

A comparison of the annual results from the $D-R$ observations with the Washington Transit-Circle, arranged in 5° groups, gave no evidence whatever that the values of $D-R$ varied with the zenith-distance, and in the final reductions of observations with that instrument these values were assumed to be *constant* on each side of the zenith.

Further investigation led to the belief that the *possible* tilting or sliding of the objective, the *probable* motion of the sliding tube carrying the eye-piece of the telescope, and the almost *certain* existence of errors in the flexure constants of the telescope, would, altogether, account for the difference in the mean values of $D-R$ *north* and *south* of the zenith.

This conclusion was strengthened by the fact that these annual differences between the *north* and *south* values were almost constant. The actual mean value of these differences, for the Washington instrument, for 21 years was, 0″.73 ±0″.01.

If, in discussing the Greenwich observations of $R-D$, we assume that the results, in the foregoing tables E_1, F_1 and G_1, do not vary with the zenith distance, then we may use the methods employed with the Washington observations and take the mean of the results on each side of the zenith.

This will give a constant value *north* of the zenith, $R-D_n$, and one *south* of the zenith, $R-D_s$, in each year.

These annual values, together with their differences, from 1851 to 1893, are given in table H.

TABLE H.

Year	Mean $R-D_n$	Mean $R-D_s$	$(R-D_n)-(R-D_s)$	Year	Mean $R-D_n$	Mean $R-D_s$	$(R-D_n)-(R-D_s)$	Year	Mean $R-D_n$	Mean $R-D_s$	$(R-D)_n-(R-D_s)$	Year	Mean $R-D_n$	Mean $R-D_s$	$(R-D_n)-(R-D_s)$
	″				″	″	″		″	″	″		″	″	″
1851	+0.26	+0.06	+0.20	1862	+0.26	−0.14	+0.10	1874	−0.23	+0.12	−0.35	1884	−0.06	+0.14	−0.20
1852	.24	− .02	.26	1863	.22	.24	.16	1875	.04	.18	.22	1885	.12	.25	.37
1853	.21	.10	.31	1864	+ .22	− .06	+ .28	1876	.04	.19	.23	1886	.15	.27	.42
1854	.12	.10	.22	1866	− .16	+ .36	− .52	1877	.01	.17	.18	1887	.24	.30	.54
1855	.14	.12	.26	1867	.11	.28	.39	1878	.12	.25	.37	1888	.09	.28	.38
1856	.20	.08	.28	1868	− .11	.40	.51	1879	.02	.22	.24	1889	.12	.22	.34
1857	.15	.13	.28	1869	+ .03	.53	.50	1880	.03	.30	.33	1890	.10	.20	.30
1858	.24	.01	.25	1870	+ .04	.45	.41	1881	− .10	.29	.39	1891	.10	.17	.27
1859	.37	.00	.37	1871	.13	.31	.44	1882	+ .12	.20	.08	1892	.06	.33	.39
1860	.20	.02	.22	1872	.12	.21	.33	1883	−0.10	+0.17	−0.27	1893	−0.12	+0.33	−0.15
1861	+0.20	0.20	+0.40	1873	−0.05	+0.33	−0.38								

The mean of $(R-D_n)-(R-D_s)$ in each of the above 14 year-periods is, from 1851 to 1864, +0″.30; from 1866 to 1879, −0″.36; and from 1880 to 1893, −0″.34.

As already stated, the Transit Circle was dismounted and the cube perforated in 1865, after which the *signs* of the mean values of $R-D$ *north* and *south* of the zenith were changed but the magnitude of these values remained, apparently, the same as before. Consequently the differences $(R-D_n)-(R-D_s)$ for 42 years may be discussed together without regard to the signs of the quantities; and we find the mean value of $(R-D_n)-(R-D_s)$, from 1851 to 1893, is 0″.334 ±0″.011.

The above treatment of the data in the Greenwich volumes from 1851 to 1893 leads to the following conclusions:—

1. There seems to be no evidence that the values of $R-D$, either *north* or *south* of the zenith, vary according to the zenith distance.

2. The range in the mean values of $R-D$, in the various 5° groups, is apparently no greater than would be expected from the character of the observations.

3. The small range in the annual values of $(R-D_n)-(R-D_s)$ as shown in Table H, and also by the probable error of the mean, indicates the *probability* of a constant error in the observations or reductions, or in both, on both sides of the zenith.

The non-reversible pattern of the Greenwich Transit Circle prevents an exhaustive discussion of some of the points raised in this paper.

An incomplete discussion of the latitude of Greenwich, using the data derived in this paper for the correction of the observed north-polar distances of circumpolar stars, gives, for the years 1877 to 1891, more accordant corrections to the assumed latitude than those published in the Greenwich volumes for that period.

Ashburn, N.H., 1900, *March* 12.

CONTENTS.

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NOTES ON VARIABLE STARS,—No. 32,

By HENRY M. PARKHURST.

T Sagittae. During seven months of observation, the fluctuation from the mean magnitude continued small and irregular, while there were nearly equal fluctuations in the comparison-stars. I found no tendency to a regular period, or to a decided maximum or minimum.

RZ Cygni. From the observations of J. A. PARKHURST I deduce the provisional epoch, 4342, with a period of 563 days, with which the comparisons are made, although there appeared to be intermediate fainter maxima.

To the following new variables I assign provisional numbers for typographic convenience:

[7040] ANDERSON's variable in *Aquila*: *A.N.* Dec. 1897.
[7244] DM. +15°1082; *A.J.* 464.
[7597] BARNARD's variable; *A.J.* 456, 474.
[7657] ANDERSON's variable in *Pegasus*; *A.J.* 457, 473.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors			Remarks
				1899							d	
6905	*R Sagittarii*	Max.	4896	Aug. 29	45	−13	2	7.4	–	–	–	Possibly earlier
6921	*S Sagittarii*	Max.	4838	July 2	52	−23	2	9.8	–	–	–	Perhaps earlier
6923	*Z Sagittarii*	Max.	4929	Oct. 1	9	− 4	4	9.94	0.37	0.67	8	A faint maximum
6943	*T Sagittae*	Max.	4527	Aug. 25	28	–	E	–	–	–	–	1898; see note above
"	"	Max.	4857	July 21	30	–	E	–	–	–	–	1899
[7040]	— *Aquilae*	Max.	4835	June 29	11	−15	3	6.7	–	–	–	Mean of two bright obsns.
7118	*X Aquilae*	Min.	4952	Oct. 24	7	–	E	–	–	–	–	
7155	*RR Aquilae*	Max.	4905	Sept. 7	4	−83	6	–	–	–	–	
7162	*RS Aquilae*	Min.	4800:	June	–	–	–	–	–	–	–	Period probably 400^d +
7234	*R Capricorni*	Max.	4847	July 11	42	−33	2	9.0	–	–	–	Earlier? unusually bright
[7244]	— *Aquilae*	Max.	4837.6	July 1	32	−2.8	2	8.32	–	–	–	Elements, *A.J.* 464
"	"	Max.	4846.6	July 10	33	−1.7	5	8.27	–	–	–	4587.60 + 7.90 E
"	"	Max.	4870.6	Aug. 3	36	−1.4	7	8.57	0.28	0.31	6	Period 7.87^d
"	"	Max.	4918.7	Sept. 20	42	−0.7	8	8.35	0.21	0.20	6	
7260	*Z Aquilae*	Max.	4911	Sept. 13	14	−26	5	8.72	–	–	–	
7261	*R Delphini*	Min.	4875	Aug. 8	44	−12	2	–	–	–	–	Probably earlier
7435	*Y Aquarii*	Max.	4849	July 13	–	–	7	9.1	–	–	–	370^d ?
7448	*W Aquarii*	Max.	4848	July 12	10	+91	6	10.27	0.56	1.26	33	
7455	*U Capricorni*	Max.	4950	Oct. 22	76	+ 3	2	–	–	–	–	
7458	*V Delphini*	Max.	4927	Sept. 29	–	–	7	–	–	–	–	Period probably 540^d
7492	*RZ Cygni*	Max.	3791	Aug. 19	−1	+12	2	–	–	–	–	1896; see note above
"	"	Min.	3918	Dec. 24	–	–	1	–	–	–	–	1896
"	"	Min.	4230	Nov. 1	–	–	1	–	–	–	–	1897
"	"	Max.	4883	Aug. 16	+1	−22	1	–	–	–	–	1899
7502	*X Delphini*	Max.	4837	July 1	1	0	8	8.47	0.64	1.07	13	My approx. period of 282^d
7577	*X Capricorni*	Max.	4973	Nov. 14	54	–	E	–	–	–	–	
7590	*Z Capricorni*	Max.	4952	Oct. 24	4	−142	3	–	–	–	–	—22, period *A.J.* 372
[7597]	— *Aquarii*	Max.	4896	Aug. 29	–	–	7	–	–	–	–	Uncertain. *A.J.* 457
[7657]	— *Pegasi*	Max.	4918	Sept. 20	1	+ 3	8	8.75	0.90	1.18	26	Comp. with period 204^d
7896	*V Pegasi*	Max.	4872	Aug. 5	5	−11	5	8.6	–	–	–	Elements, *A.J.* 464

Individual Observations.

Including Observations by Arthur C. Perry.

6905 *R Sagittarii.*

(Continued from 456.)

Julian	Calendar	Mag.
	1899	
4896.6	Aug. 29	7.4
4897.5	30	7.78
4905.5	Sept. 7	8.01_2
4910.5	12	7.86_2
4913.5	15	7.80_2
4918.5	20	7.92_2
4932.5	Oct. 4	8.55_2

6924 *S Sagittarii.*

(Continued from 425.)

Julian	Calendar	Mag.
	1899	
4837.6	July 1	10.0
4838.6	2	9.83_2
4843.6	7	9.94_2
4846.6	10	10.51_2
4866.6	30	10.49_2

6923 *Z Sagittarii.*

(Cont. from 456. Comp. Stars 311.)

Julian	Calendar	Mag.
	1899	
4926.5	Sept. 28	10.0
4927.5	29	10.08_2
4929.5	Oct. 1	9.94_2
4932.5	4	10.15_2
4935.5	7	10.20_2
4949.5	21	10.39_2

6943 *T Sagittae.*

Julian	Calendar	Mag.
	1898	
4481.6	July 10	8.5P
4547.6	Sept. 14	8.7P
4573.5	Oct. 10	9.0P
4597.5	Nov. 3	8.4P
4609.5	15	9.0P
4610.5	16	8.70
	1899	
4838.6	July 2	8.9P
4844.6	8	8.7:P
4867.7	31	8.9P
4873.6	Aug. 6	8.9P
4892.5	25	9.8
4897.6	30	8.9P
4911.5	Sept. 13	8.72_2
4912.5	14	8.84_2
4913.5	15	9.02_2
4914.5	16	8.95_2
4915.5	17	9.17_2:
4918.5	20	8.83_2
4919.5	21	8.64_2
4921.5	23	8.34_2
4922.5	24	8.37_2
4924.5	26	8.85_2
4925.5	27	8.41_2
4926.5	28	8.57_2
4927.5	29	8.51_2
4928.5	30	8.41_2
4929.5	Oct. 1	8.57_2
4930.5	2	8.42_2
4931.5	3	8.78_2:
4932.5	4	9.15_2

6943 *T Sagittae.*—Cont.

Julian	Calendar	Mag.
	1899	
4933.5	Oct. 5	8.97_2
4935.5	7	8.58_2
4948.5	20	8.98_2

[7040] —*Aquilae.*

Julian	Calendar	Mag.
	1897	
4278.5	Dec. 19	9.4
4278.5	19	9.8P
	1898	
4545.5	Sept. 12	8.8
4572.5	Oct. 9	10.0
4605.5	Nov. 11	9.3
	1899	
4827.6	June 21	7.27_2
4829.6	23	7.02_4
4833.6	27	7.85_2
4837.6	July 1	7.63_2
4838.6	2	7.95_2
4839.6	3	7.69_2
4840.6	4	6.70_2
4843.6	7	7.98_2
4854.6	18	8.25_2

7118 *X Aquilae.*

(Cont. from 425. Comp. Stars 339.)

Julian	Calendar	Mag.
	1899	
4882.6	Aug. 15	12.7
4910.6	Sept. 12 to	
4956.5	Oct. 28	12.7]
	4 dates	

7155 *RR Aquilae.*

(Cont. from 464. Comp. Stars 393.)

Julian	Calendar	Mag.
	1899	
4854.6	July 18	10.0]
4880.6	Aug. 13	9.63_2
4895.5	28	9.56_2
4910.5	Sept. 12	9.37_2
4913.5	15	9.70_2
4919.5	21	9.33_2
4924.5	26	10.00_2
4925.5	27	10.14_2
4933.5	Oct. 5	10.22_2
4949.5	21	10.18_2
4953.5	25	9.90_2

7162 *RS Aquilae.*

(Cont. from 464. Comp. Stars 464.)

Julian	Calendar	Mag.
	1899	
4817.7	June 11 to	
4949.5	Oct. 21	13]
	11 dates	

7234 *R Capricorni.*

(Cont. from 464. Comp. Stars 464.)

Julian	Calendar	Mag.
	1899	
4847.6	July 11	9.0
4850.6	14	10.80_2
4867.6	31	11.13_2
4867.7	31	11.1P
4873.6	Aug. 6	11.4P
4882.6	15	11.10_2
4897.6	30	11.1P
4902.6	Sept. 4	11.21_2

[7241] —*Aquilae.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4837.6	July 1	8.32_3
4838.6	2	8.47_3
4838.6	2	8.6P
4839.6	3	8.68_2
4839.6	3	8.7P
4840.6	4	8.6P
4843.6	July 7	9.01_4:
4844.6	8	8.8P
4845.6	9	8.57_2
4846.6	10	8.27_3
4847.6	11	8.43_2
4850.7	14	8.66_3
4867.6	July 31	8.93_2
4868.6	Aug. 1	8.96_2
4870.6	3	8.57_2
4871.6	4	8.78_2
4872.6	5	8.9
4873.6	6	9.05_2
4874.6	7	9.05_2
4875.6	8	9.0P
4883.6	Aug. 16	9.24_2
4884.5	17	8.67_3
4914.5	Sept. 16	9.24_2
4915.5	17	9.12_2
4918.5	20	8.35_2
4919.5	21	8.45_2
4920.6	22	8.64
4965.5	Nov. 6	8.9P

7260 *Z Aquilae.*

(Cont. from 464. Comp. Stars 393.)

Julian	Calendar	Mag.
	1899	
4837.6	July 1	12]
4840.6	4	12]
4880.6	Aug. 13	11:
4903.6	Sept. 5	9.2
4908.5	10	8.82_2
4911.5	13	8.72_2
4914.5	16	8.73_2
4919.6	21	8.91_2
4924.5	26	9.19
4928.5	30	9.99_2
4928.6	30	9.1
4933.5	Oct. 5	9.91
4940.5	12	10.0]

7261 *R Delphini.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4840.6	July 4	13]
4868.6	Aug. 1	13]
4910.6	Sept. 12	12.1
4920.6	22	11.80_2

7335 *Y Aquarii.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4840.6	July 4	9.3
4843.6	7	9.13_2
4846.6	10	9.14_2
4850.7	14	8.85_2
4867.6	31	9.46_2
4882.6	Aug. 15	9.92_2
4910.5	Sept. 12	9.96_2
4921.6	23	10.39_2
4931.5	Oct. 3	10.63_2
4949.5	21	11.1

7418 *W Aquarii.*

(Continued from 425.)

Julian	Calendar	Mag.
	1898	
4481.6	July 10 to	
4605.5	Nov. 11	13]
	4 dates	
	1899	
4840.6	July 4	10.9
4843.6	7	10.61_2
4846.6	10	10.06_2
4850.7	14	10.22_2
4867.6	Aug. 31	10.84_2
4882.6	15	11.20
4910.5	Sept. 12	12]

7455 *U Capricorni.*

(Continued from 425.)

Julian	Calendar	Mag.
	1899	
4882.6	Aug. 15	11.7]
4926.5	Sept. 28	11.7
4928.6	30	11.72_2
4938.5	Oct. 10	11.7
4947.5	19	11.07_2
4956.5	28	11.14_2

7458 *U Delphini.*

(Cont. from 393. Comp. Stars 311.)

Julian	Calendar	Mag.
	1899	
4882.6	Aug. 15	11]
4505.6	Sept. 7	9.5
4911.6	13	8.24_2
4912.6	14	8.30_2
4915.5	17	7.59_2
4919.5	21	7.95_2
4925.5	27	7.87_2
4928.5	30	7.66_2
4928.6	30	7.8P
4935.5	Oct. 7	8.00_2
4949.5	21	7.92_2
4965.5	Nov. 6	8.3P

7492 *RZ Cygni.*

(Continued from 372.)

Julian	Calendar	Mag.
	1896	
3711.7	May 31	12.0
3712.6	July 1	11.3
3755.6	14	11.2
3759.6	18	10.7
3777.6	Aug. 4	10.9
3810.6	Sept. 7	10.9

7492 *RZ Cygni.*—Cont.

Julian	Calendar	Mag.
	1896	
3859.5	Oct. 26	10.9
3897.5	Dec. 3	11.2
3918.5	24	11.2]
	1897	
3930.5	Jan. 5	11.5
3948.5	23	11.3
4052.6	May 7	11.0
4074.6	29	11.0
4103.7	June 27	11.0
4138.7	Aug. 1	12.0
4165.6	28	12.5
4215.5	Oct. 17	13.0]
4253.5	Nov. 24	13.0]
4283.5	Dec. 24	11.7
	1898	
4579.6	Oct. 16	12.2
4605.5	Nov. 11	11.5
4639.5	Dec. 15	12.0
	1899	
4864.6	July 28	10.63_2
4875.6	Aug. 8	10.28_2
4883.6	16	10.43_2
4931.5	Oct. 3	10.95_2
4947.6	19	11.01_2

7502 *X Delphini.*

(Cont. from 464. Comp. Stars 438.)

Julian	Calendar	Mag.
	1899	
4829.6	June 23	8.80_2
4833.6	27	8.85_2
4837.6	July 1	8.15_2
4838.6	2	8.66_2
4839.6	3	8.41_2
4843.6	7	9.35_3
4845.6	9	8.65_2
4854.6	18	8.97_2
4867.7	31	8.51_2
4873.6	Aug. 6	9.9P
4882.6	15	9.9
4897.6	30	11.5P
4965.5	Nov. 6	12]P

7577 *X Capricorni.*

(Continued from 400.)

Julian	Calendar	Mag.
	1899	
4931.5	Oct. 3	13]
4949.5	21	13]

7590 *Z Capricorni.*

(Cont. from 464. Comp. Stars 372.)

Julian	Calendar	Mag.
	1899	
4882.6	Aug. 15	12.8]
4926.5	Sept. 28	11.9
4927.5	29	10.51_3
4929.5	Oct. 1	10.61_2
4935.5	7	10.52_2
4938.5	10	10.16_2
4947.5	19	9.90_3
4948.5	20	9.42_2
4953.5	25	9.10_2

[7597] — *Aquarii.*

Julian	Calendar 1899	Mag.
4835.7	June 29	11.0
4846.6	July 10	10.2
4850.7	14	10.90_4
4867.6	31	10.46
4882.6	Aug. 15	9.89
4910.5	Sept. 12	9.88_2
4921.6	23	10.46_2
4931.5	Oct. 3	10.62_2
4949.5	21	12]

[7657] — *Pegasi.*

Julian	Calendar 1899	Mag.
4911.5	Sept. 13	9.03_2
4912.6	14	8.92_2
4915.5	17	8.75_2
4919.5	21	8.71_2
4925.5	27	8.70_2
4931.5	Oct. 3	9.18_2
4938.5	10	9.25_2
4947.5	19	9.86_2

7896 *U Pegasi.*

(Continued from 464.)

Julian	Calendar 1899	Mag.
4866.7	July 30	8.87
4867.7	July 31	8.64_2
4883.6	Aug. 16	9.16_2
4892.6	25	9.46_2
4895.5	28	9.92_2
4905.6	Sept. 7	9.8r
4984.6	Nov. 25	11.1:r

COMPARISON-STARS. 1893–1899.

[7040] — *Aquilae.*

Star	DM.	Mag.	n
1*D*	+11°3906	5.72	5
I	+11°3901	7.35	6
J	+11°3893	7.34	5
S	+11°3916	8.29	12
W	+11°3922	9.20	10
1*W*	+11°3923	8.91	2
2*W*	+11°3918	8.71	2
Y	+11°3921	9.44	12
Z	+11°3919	9.24	2
a	2*n*6*f* *Z*	10.27	2

7435 *Y Aquarii.*

Star	DM.	Mag.	n
Q	−1°5236	8.48	3
1*Q*	−5°5361	8.26	12
1*R*	−5°5355	8.62	15
S	−5°5258	8.63	16
2*T*	−4°5242	8.76	6
W	−5°5360	9.52	10
X	−5°5363	9.93	12
Y	−5°5357	9.89	20
a	3*s*4*p* *S*	10.69	3
f	10*s*2*f* *S*	11.53	7

7455 *U Capricorni.*

Star	DM.	Mag.	n
N	−15°5796	8.29	1
Q	−15°5801	9.03	1
X	−15°5797	10.69	23
b	1*n*4*f* *X*	10.90	32
e	2*n*8*p* *X*	11.71	10
g	2*n*2*f* *X*	11.74	14
h	1*p* *e*	12.04	3
j	2*p* *e*	12.61	2
k	2*n*4*p* *X*	12.27	2
l	5*n* *X*	12.43	1

7458 *U Delphini.*

Star	DM.	Mag.	n
R	+18°4611	8.44	12
S	+18°4619	8.60	19
T	+18°4618	9.03	25
2*T*	+18°4610	8.63	4
V	+18°4614	9.54	10
W	+18°4617	8.95	29
e	1*n*2*f* *h*	10.73	2
f	12*f* *T*	10.75	2
g	4*n*2*f* *S*	11.00	3
h	1*n*1*f* *g*	11.28	3

MAXIMA AND MINIMA OF LONG-PERIOD VARIABLES,

By J. A. PARKHURST.

The observations giving the following results were made, unless otherwise stated, with the 6-inch reflector; also, except as stated to the contrary, the stars are continuously followed through the year.

267. *V Andromedae.*

After the maximum reported in *A.J.* 458 this star fell to 11ᴹ.1, 1899 Mar. 13, then was not seen till July 29, at 10ᴹ.5, after which it rose to 9ᴹ.1 in September, and fell to 12ᴹ.5 the last of December. A minimum is roughly indicated early in June, and a well-defined maximum, at 9ᴹ.1, is fixed for 1899 Sept. 20. There are eighteen observations between the above limits.

678. *U Persei.*

Following the series in *A.J.* 465, there was the usual halt in the rise a month before maximum, then a rise to 6ᴹ.9 1899 July 26. If the curve is smoothed out, the maximum becomes July 21, at 7ᴹ.1. Then followed a steady decline to a minimum at 11ᴹ.6, Dec. 23, and a rise to 8ᴹ.7 by 1900 March 12. These changes are covered by 23 observations.

2376. *S Lyncis.*

Since the minimum reported in *A.J.* 465 I have 19 observations, giving a well-defined maximum at 9ᴹ.4, 1899 July 18, and a minimum, at about 13ᴹ, suggested for the middle of December 1899. At the last comparison, 1900 March 13, it had risen to 10ᴹ.6. The variable was indistinguishable from its 12ᴹ.5 companion from 1899 Oct. 23 to 1900 Jan. 8.

5601. *S Ursae minoris.*

I have 23 observations of this star since the report in *A.J.* 465, covering the time from 1899 June 12 to 1900 March 13. They give a maximum at 7ᴹ.9, 1899 Aug. 15, and a minimum at 11ᴹ.3, 1900 Jan. 13. Indications of a "stand-still" about two months before the highest point, are found in this and in two previous maxima, but are wanting in two other maxima.

5798. *RU Herculis.*

The maximum of 1899 is covered by 26 observations, between 1899 March 19 and 1900 Jan. 8. The rise was rapid, with some fluctuations, from 11ᴹ to the maximum at 7ᴹ.0 June 21, followed by a slower and more regular decline to 11ᴹ.8 at the last mentioned date. At maximum it was 0ᴹ.1 brighter, to my eye, than +25°3039, which is 7ᴹ.1 in the DM. and in GRAHAM's Cambridge (Eng.) *Astr. Gesell. Catalogue.*

6894. *S Lyrae.*

The present series began 1899 March 22, with the variable not seen, limit 12ᴹ.5. It was found April 21 at 12ᴹ.3, and has been followed till the present time. The result of 23 observations gives a well determined maximum 1899 July 7, at about 10ᴹ.5. The decline was slow and steady till the last comparison, 1900 March 22, made with the 40-inch Yerkes refractor, showed it to be about 15ᴹ. The only break in the series since May 1897 was for the three months between 1899 Nov. 22 and 1900 Feb. 24. No intermediate maxima have been observed.

7502. *X Delphini.*

After the maximum reported in *A.J.* 458 there was a break in the series from 1899 Jan. 27 to April 16; on both these dates the variable was invisible, limit 11ᴹ.5. May 10 it was found at 10ᴹ.8; then follow 16 observations, ending Nov. 27. The curve suggests a minimum, below 13ᴹ within 15 days of 1899 March 1, and shows a well determined maximum, 8ᴹ.5, 1899 June 23. The decline was steady, slower than the rise, reaching 12ᴹ.8 at the last comparison.

CERASKI'S VARIABLE IN *Cepheus.*

R.A. = $21^h\ 3^m\ 38^s.5$; Decl. = +82° 39′ 50″.3 (1900).

In continuation of the report in *A.J.* 475, this star slowly faded from a maximum 1899 Oct. 1 (or a week or two earlier) at 9ᴹ.7, to about 14ᴹ at the last comparison, 1900 March 22. I have 20 observations between these dates, those since 1900 Jan. 4 being made with the 12 and 40-inch Yerkes telescopes. The DM. numbers of the comparison-stars given in *A.J.* 475 should evidently be +82°635-6, instead of +32 635-6.

A comparison with the light-curve observed in 1898 gives 186 days as a first approximation to the period. This is derived from the intervals between the times of passing 12ᴹ and 13ᴹ on the decline.

7896. *V Pegasi.*

In continuation of the series reported in *A.J.* 458, this variable was not seen, limit 12ᴹ.0, 1899 Jan. 27 and April 16; and was below 12ᴹ.5 May 10. It then rose rapidly to a fairly well-defined maximum at 7ᴹ.8, 1899 July 16, and then fell more slowly to about 13ᴹ when last seen, Nov. 27. The curve suggests a minimum fainter than 13ᴹ within 20 days of 1899 April 1. There are 18 observations.

(5481). — *Librae.* (II Catalogue.)

R.A. = $15^h\ 11^m\ 15^s$; Decl. = +2° 36′.9 (1855).

This star is No. 42 on Professor PICKERING's photograph of the cluster *Messier* 5. I have 24 observations between June and October 1898 and 28 between June and September 1899, giving

Maxima Date	Mag.	Wt.	Minima Date	Mag.	Wt.
1898 July 21	9.8	6	1898 July 17	11.8±	1
Aug. 16	9.8	8	Aug. 12	12.0	4
1899 June 21	9.4	7	1899 June 14	11.8	3
July 18	9.6	2	July 12	11.8	5
Aug. 11	9.6	3	Aug. 7	11.8	3
			Sept. 2	11.6	5

The characteristics of the curve were found to be — a quick rise to maximum, the value of $M-m$ being 5.2 days, 20% of the period; a halt in the decline amounting to nearly or quite a secondary maximum, about 9 days after the principal, then a slower decline to a sharply defined minimum. It seems, therefore, to be either of the *δ Cephei* or *η Aquilae* type, and distinct from the fainter variables in the cluster, which remain constant in light at minimum for the greater part of their period.

The mean interval between the successive maxima and successive minima of the two years, taken separately, is 26.2 days. From the two best determined maxima of the different years, 1898 Aug. 16 and 1899 June 21, assuming 12 periods to have elapsed, the value of the period becomes 25.8 days. From the two best minima, 1898 Aug. 12 and 1899 Sept. 2, (15P) the value is 25.7 days. The observations in 1898 were made with 12 and those of 1899 with 6 inches aperture. The comparisons with 6-inches were difficult on account of the background formed by the outliers of the group; those with the 12-inch were uncertain near maximum from the distance of the comparison-stars.

FILAR-MICROMETER MEASURES OF THE POSITION OF THE STAR DM.+37°4131 FOR PARALLAX,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

In measuring the parallax of 61 *Cygni* with the Göttingen heliometer, Professor SCHUR found one of his four comparison-stars (star *a*) had a larger parallax than 61 *Cygni*—about 0″.6. This star he identified as DM. +37°4131, which is of the 7ᴹ.8, and whose position for

$$1855.0 = 20^h\ 53^m\ 31^s.7 \quad +37°\ 22'.1$$
$$1900.0 = 20\ 55\ 20 \quad +37\ 32.5$$

Professor SCHUR's paper on the subject will be found in *Astr. Nach.* 3590. In a review of this article, in *The Observatory* for October, 1899, Mr. CROMMELIN shows that this DM. +37°4131 has no sensible proper motion, from observations extending over nearly three-fourths of a century.

The small magnitude of the star, and the absence of proper motion, would rather militate against its being near to us.

Examining this object last fall with the 40-inch, it was seen to be excellently placed for micrometrical measurement with reference to four small stars, whose position-angles were 36°, 137°, 155° and 265°. A series of measures was made of these stars for the parallax of +37°4131.

An absence of three months from the observatory pre-

vented the objects being kept continuously under observation. Three nights' measures have been made this spring, however, and they seem to indicate that the star certainly has no such large parallax, and probably has no sensible parallax at all.

Professor SCHUR calls attention to the fact that the line of maximum parallax lies in position-angle 62°.5. In the observations made with the 40-inch it will be seen that the stars *A* and *D* should show a large displacement in their distances (something like 1″), and that stars *B* and *C* should measurably change their position-angles (by a couple of degrees) in the two sets of measures. From the observations it is apparent that no such change has occurred in the relative positions of any of the stars.

Following are the measures so far obtained. The results are such that it would seem useless to continue the observations.

DM. +37°4131 and Star *A* (12M).

		°	°	″	″
1899 Oct.	21	35.60	+0.02	99.03	−0.14
	22	35.80	−0.18	98.84	+0.05
	23	35.53	+0.09	99.00	−0.11
	28	35.73	−0.11	98.91	−0.02
	29	35.74	−0.12	98.85	+0.04
	30	35.55	+0.07	98.83	+0.06
Nov.	4	35.57	+0.05	98.83	+0.06
	5	35.49	+0.13	98.88	+0.01
	6	35.66	−0.04	98.96	−0.07
	19	35.83	−0.21	98.87	+0.02
	20	35.83	−0.21	98.95	−0.06
	25	35.55	+0.07	99.06	−0.17
	26	35.75	−0.13	98.82	+0.07
	27	35.45	+0.17	98.88	+0.01
1900 Mar.	23	35.49	+0.13	98.71	+0.18
	30	35.66	−0.04	98.78	+0.11
Apr.	6	35.35	+0.27	98.95	−0.06
		35.62		98.89	

DM. +37°4131 and Star *B* (12M.3).

		°	°	″	″
1899 Oct.	21	137.50	+0.17	25.60	−0.02
	22	137.13	+0.20	25.50	+0.08
	23	137.27	+0.06	25.41	+0.17
	28	137.11	+0.22	25.46	+0.12
	29	137.35	−0.02	25.46	+0.12
	30	137.25	+0.08	25.68	−0.10
Nov.	4	137.18	+0.15	25.53	+0.05
	5	137.13	+0.20	25.53	+0.05
	6	137.17	+0.16	25.59	−0.01
	19	137.50	−0.17	25.70	−0.12
	20	137.15	+0.18	25.81	−0.23
	25	137.53	−0.20	25.66	−0.08
	26	137.36	−0.03	25.59	−0.01
	27	137.22	+0.11	25.71	−0.13
1900 Mar.	23	137.92	−0.59	25.65	−0.07
	30	137.63	−0.30	25.37	+0.21
Apr.	5	137.24	+0.09	25.54	+0.04
		137.33		25.58	

DM. +37°4131 and Star *C* (12M.2).

		°	°	″	″
1899 Oct.	21	155.33	+0.12	62.44	+0.07
	22	155.42	+0.03	62.52	−0.01
	23	155.46	−0.01	62.37	+0.14
	28	155.58	−0.13	62.47	+0.04
	29	155.68	−0.23	62.46	+0.05
	30	155.49	−0.04	62.55	−0.04
Nov.	4	155.52	−0.07	62.52	−0.01
	5	155.39	+0.06	62.61	−0.10
	6	155.59	−0.14	62.67	−0.16
	19	155.32	+0.13	62.57	−0.06
	20	155.30	+0.15	62.61	−0.10
	25	155.65	−0.20	62.50	+0.01
	26	155.56	−0.11	62.54	−0.03
	27	155.31	+0.14	62.49	+0.02
1900 Mar.	23	155.50	−0.05	62.46	+0.05
	30	155.44	+0.01	62.57	−0.06
Apr.	6	155.16	+0.29	62.36	+0.15
		155.45		62.51	

DM. +37°4131 and Star *D* (12M.3).

		°	°	″	″
1899 Oct.	21	264.87	+0.06	50.25	+0.50
	22	264.91	+0.02	50.75	0.00
	23	264.82	+0.11	50.53	+0.22
	28	265.03	−0.10	50.74	+0.01
	29	265.16	−0.23	50.77	−0.02
	30	264.91	+0.02	50.79	−0.04
Nov.	4	264.78	+0.15	50.87	−0.12
	5	265.00	−0.07	50.98	−0.23
	6	265.01	−0.08	50.88	−0.13
	19	264.88	+0.05	50.67	+0.08
	20	264.85	+0.08	50.52	+0.23
	25	265.00	−0.07	50.88	−0.13
	26	265.08	−0.15	50.73	+0.02
	27	264.78	+0.15	50.74	+0.01
1900 Mar.	23	264.88	+0.05	50.90	−0.15
	30	264.87	+0.06	50.71	+0.04
Apr.	6	264.94	−0.01	51.12	−0.37
		264.93		50.75	

The third and fifth columns contain the deviations from the mean. The method of double distances has been used throughout. Five settings were made for the position-angles, and four settings on each side of the fixed wire for distance. The measures have been corrected for refraction.

The identity of the star has been carefully established by comparison with other DM. stars near, by charting and by reference to the DM. charts. It is of a strong yellow color.

On every night on which these observations were made, measures were also made of the *Aδ* of *Atlas* and *Pleione*, and *Electra* and *Celaeno* of the *Pleiades*, as a check on any changes in the telescope.

During these observations, the 9M star, DM. +37°4133, whose place for 1855.0 is $20^h\ 54^m\ 2^s.5$, $+37°\ 18'.7$ was found to be a new double, or rather triple star. The following measures were obtained:

		A and *B*.		
1899.812	247.7	0.77	$9^M.5$ $9^M.8$	very bad seeing
.826	251.1	0.87		" " "
.829	249.5	1.11		better seeing
1899.822	250.4	0.92		
		(*A* + *B*) and *C*.		
1899.812	112°.0	7″.75	15^M	difficult from
.826	111.5	7.99		bad seeing
1899.819	111.7	7.87		

For identification of this star, which is south following DM. +37°4131, two transits were obtained, and one measure for $\Delta\delta$.

Triple star, DM. +37°4131.

$$\Delta\alpha = +0^m\,27^s.90\,(2) \quad , \quad \Delta\delta = -3'\,23''.7\,(1)$$

which gives a position in accord with that in the DM.

Yerkes Observatory, Williams Bay, Wis., 1900 *April* 7.

NOTE ON THE DETERMINATION OF THE *SUN'S* DIAMETER DURING THE TOTAL SOLAR ECLIPSE OF MAY 28, 1900,

By WILLIAM HARKNESS.

As it is not customary to observe the position-angles of the points of contact in solar eclipses, it may not be inopportune to invite the attention of astronomers to the importance of observing them during the approaching total solar eclipse of May 28.

On account of the difficulty of getting rid of irradiation, the diameter of the sun is not yet accurately known, and almost the only possible way of measuring it seems to be by first determining the diameter of the moon from occultations, and then finding the difference between the diameters of the sun and moon from observations of total solar eclipses. The latter determination can best be made from observations of the second and third contacts, and if both the times and position-angles are observed, each such set of observations will give a complete determination of the difference of diameters of the bodies in question. If the position-angles are not observed, then the difference of diameters can only be got by combining observations made at different stations, and in very many cases the geographical positions of stations along the line of totality are not sufficiently well known to admit of that procedure.

Washington, D.C., 1900 *April* 17.

OBSERVED MAXIMA OF LONG-PERIOD VARIABLES, 1897–1899,

By PAUL S. YENDELL.

103. *T Andromedae.*

I observed this star on 1897 Nov. 21, when its light was estimated at $9^M.4$; from this date to 1898 Feb. 12, twelve observations indicate a maximum of $7^M.9$ on 1897 Dec. 27; at the last observation the star had decreased to $9^M.8$.

678. *U Persei.*

I observed *U Persei* thirteen times from 1897 Sept. 26 to Dec. 31. At the first observation its light was estimated at $9^M.4$. It rose to a well-defined maximum of $8^M.3$ on Nov. 28, from which it decreased sharply, and on Dec. 11 was observed at $9^M.4$; after this it again increased to $8^M.7$, which it had reached at the last observation.

806. *o Ceti.*

From 1897 Sept. 18 to 1898 Jan. 24, I observed *o Ceti* thirteen times; the observations are evenly enough spaced to indicate that the star passed a maximum of $3^M.4$ on 1897 Nov. 30. At the last observation it had decreased to $4^M.4$.

5758. *X Herculis.*

Nine observations of *X Herculis*, from 1899 May 30 to Aug. 26, show a maximum of about $6^M.4$ on Aug. 3.

6225. *RS Herculis.*

Eleven observations of *RS Herculis*, from 1899 July 30 to Oct. 12, indicate that a maximum of $8^M.2$ was passed on Sept. 2. At the first and last observations, the star's light was about $9^M.2$.

6733. *R Scuti.*

This star was observed twenty-nine times in 1899. At the first observation, June 29, it was about $5^M.1$; from this it decreased to a minimum of $6^M.1$, which occurred Aug. 3, and was followed on Sept. 2 by a maximum of $5^M.0$, from which the star again decreased till the last observation on Oct. 10, when it was apparently at or near another minimum of $6^M.7$.

7085. *RT Cygni.*

Eight observations, from 1896 Dec. 6 to 1897 Jan. 23, indicate a maximum of $7^M.4$ on 1897 Jan. 1; the light at the last observation was $7^M.9$.

From 1899 Aug. 1 to Oct. 4, nine observations show a well-defined maximum of $7^M.5$ on Aug. 26. The light at the first observation was $8^M.3$, and at the last, $8^M.6$.

7792. *SS Cygni.*

This star was observed on thirty-four nights, from 1899 June 29 to Nov. 5.

The following observations were obtained when its brightness was above the normal:

Local M.T.	Mag.	Local M.T.	Mag.
1899 Aug. 25.357	8.03	1899 Sept. 5.365	9.00
26.380	8.06	6.365	9.42
30.361	8.22	Oct. 26.417	8.72
Sept. 4.333	8.36	Nov. 3.337	9.18
.538	8.87		

Dorchester, 1900 *May* 1.

OBSERVATIONS OF COMETS 1899 III AND 1899 I,

MADE WITH THE 18-INCH EQUATORIAL, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA,

By HENRY B. EVANS.

1899 Greenwich M.T.	*	No. Comp.	☌ — * $\Delta\alpha$	☌ — * $\Delta\delta$	☌'s apparent α	☌'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			Comet 1899 III.					
April 5 13h 22m 26s	1	7, 10	+3m 27s.00	+4′ 18″.7	3h 14m 32s.52	+22° 32′ 35″.0	9.689	0.722
5 13 22 26	2	7, 10	+2 34.75	+4 45.2	3 14 32.48	+22 32 35.7	. .	. .
16 13 35 18	3	7	+1 24.90	. . . .	3 54 11.01	. . . .	9.674	. .
16 13 24 46	3	4	. . . .	+4 53.0	. . . .	+18 8 13.3	. .	0.740
			Comet 1899 I.					
May 21 18 40 10	5	5, 10	+0 53.66	−2 16.4	21 54 42.49	+49 6 29.4	n9.788	0.168
23 15 31 35	6	8, 12	+4 10.97	+2 31.1	21 24 55.97	+52 3 17.2	n9.850	0.658
23 15 31 35	7	8, 12	+3 7.83	+1 2.3	21 24 55.86	+52 3 17.8	. .	. .
24 16 18 17	8	8, 12	+2 7.61	−0 57.7	21 5 51.52	+53 31 43.1	n9.873	0.451
25 16 13 4	9	12, 12	+0 31.82	+1 34.9	20 45 36.99	+54 47 18.0	n9.881	0.357
26 15 23 8	10	12	−0 10.76	−2 56.1	20 24 57.44	+55 48 52.5	n9.898	0.445
28 17 17 4	11	12	−0 16.81	+3 35.8	19 34 12.39	+57 10 50.6	n9.748	n0.025
30 17 53 12	12	12, 16	+0 5.61	+1 54.1	18 42 54.83	+57 12 16.1	n9.431	n0.356
June 2 15 21 31	13	12	+0 27.58	+2 1.2	17 35 2.26	+55 3 52.2	n9.673	n0.061
3 16 44 19	14	12	+0 13.32	+1 37.0	17 13 32.47	+53 46 59.0	n9.124	n0.300
4 14 53 17	15	12	−0 10.05	+3 14.0	16 56 29.14	+52 30 47.7	n9.597	n0.021
14 14 5 56	16	8, 10	−1 9.35	−5 37.0	15 10 42.90	+36 58 38.6	n8.907	9.695
19 16 55 51	17	8, 10	−2 53.93	+4 3.2	14 47 28.79	+30 32 24.1	9.562	0.384
21 14 55 32	18	8, 12	−1 39.92	−0 13.2	14 41 19.51	+28 29 16.3	9.202	0.277
22 14 48 47	19	10, 8	−2 5.94	−0 26.8	14 38 31.77	+27 29 28.1	9.198	0.309
23 14 18 23	20	12	−0 33.98	+0 32.8	14 36 0.78	+26 33 20.4	9.026	0.319

Mean Places for 1899.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
1	3h 11m 4s.64	+0s.88	+22° 27′ 41″.0	+5″.3	Becker, Berlin A.G. Catal., No. 966
2	3 11 56.84	+0.89	+22 27 45.2	+5.3	" " " " No. 972
3	3 53 15.15	+0.96	+18 3 17.1	+3.2	Compared with *4
4	3 54 59.29	+0.96	+17 54 32.7	+3.1	Auwers, Berlin A.G. Catal., No. 1063
5	21 53 47.07	+1.76	+49 8 51.5	−5.7	Deichmüller, Bonn A.G. Catal., No. 16174
6	21 20 43.04	+1.96	+52 0 52.7	−6.6	Rogers, Cambridge A.G. Catal., No. 7058
7	21 21 46.08	+1.95	+52 2 22.1	−6.6	" " " " No. 7069
8	21 3 41.84	+2.07	+53 32 47.8	−7.0	" " " " No. 6907
9	20 45 2.97	+2.20	+54 45 50.3	−7.2	" " " " No. 6775
10	20 24 59.87	+2.33	+55 51 56.0	−7.4	Krueger, Hels.-Gotha A.G. Catal., No. 11388
11	19 34 56.55	+2.65	+57 7 22.2	−7.4	" " " " No. 10643
12	18 42 46.28	+2.91	+57 10 29.0	−7.0	" " " " No. 9959
13	17 34 31.47	+3.21	+55 1 56.8	−5.8	" " " " No. 9367
14	17 13 15.90	+3.25	+53 45 27.3	−5.3	Rogers, Cambridge A.G. Catal., No. 5207
15	16 56 36.22	+3.27	+52 27 38.6	−4.9	" " " " No. 5130
16	15 11 49.04	+3.21	+37 1 19.4	−3.8	Lund. A.G. Zones. 4 obs.
17	14 50 19.59	+3.13	+30 28 25.1	−4.2	Paris, No. 18394
18	14 42 56.33	+3.10	+28 29 31.0	−4.5	Graham, Cambridge A.G. Catal. No. 6967
19	14 40 34.61	+3.10	+27 29 59.5	−4.6	" " " " No. 6957
20	14 36 31.67	+3.09	+26 32 52.4	−4.8	" " " " No. 6933

The observations of May 25 to June 4 inclusive, and of June 23, $\Delta\alpha$ were measured directly by micrometer.

Flower Observatory, 1900 *April* 7.

OBSERVATIONS OF COMET a 1900 (GIACOBINI),

MADE AT THE LICK OBSERVATORY OF THE UNIVERSITY OF CALIFORNIA,

BY C. D. PERRINE.

1900 Mt. Hamilton M.T.	*	No. Comp.	☌ – * $\Delta\alpha$	☌ – * $\Delta\delta$	☌'s apparent α	☌'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ	Telescope
March 14 7 9 16	1	10 , 8	–0 4.33	+2 12.0	1 57 12.51	+5 33 52.5	9.639	0.703	36
20 7 36 51	3	8 , 6	+0 32.09	–2 48.6	1 51 38.17	+7 58 35.6	9.674	0.715	12
23 7 33 47	4	10 , 8	+0 20.80	+2 50.1	1 49 53.76	+8 44 54.7	9.677	0.719	12

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
1	1 57 46.11	+0.73	+5 31 38.8	+1.7	* 9m.3 compared with *2
2	1 54 44.03	+0.72	+5 33 2.3	+1.8	Leipzig A.G. 759
3	1 51 5.75	+0.63	+8 1 22.2	+2.0	" " 737
4	1 49 32.33	+0.63	+8 12 2.4	+2.0	" " 730

Mt. Hamilton, California, 1900 April 18.

MICROMETRICAL MEASURES OF THE COMPANIONS OF *PROCYON*,

BY E. E. BARNARD.

The distant companion was measured on four nights during the past season. The measures of this object are kept up from year to year, in the hope that they will be useful in the investigation of the proper motion of *Procyon*. It has long ago been shown that this small star has no physical connection with the large star; the changes in its relative position are due solely to the proper motion of *Procyon*.

THE DISTANT COMPANION.

	°	"
1899.771	343.3	59.04
.826	342.7	59.35
.883	343.3	59.28
.905	343.4	59.46
1899.846	343.19	59.28

The following observations were obtained of the close companion, which was difficult from poor seeing:

	°	"
1899.829	335.5	4.99
.853	333.5	4.68*
.892	335.5	5.12
1900.245	335.4	4.97
.253	338.9	5.31
.258	337.4	5.48
1900.055	336.03	5.09

* Single distances.

The measures are double distances, with the exception of the second set, which is from single distances. If we give half weight to the second distance measures, the mean will be 5".13.

My previous measures of the close companion with the 40-inch, are —

	°		"	
1898.213	326.0	7n	4.83	6n
1899.073	330.6	5n	4.91	3n

From these measures the distance seems to be increasing at the rate of about 0".14 a year, while the increase in the angle is about 5° annually.

Yerkes Observatory, Williams Bay, Wis., 1900 May 6.

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A METEORIC THEORY OF THE *GEGENSCHEIN*.

By F. R. MOULTON.

While the writer was making a short stay at the Yerkes Observatory last September, Professor BARNARD pointed out to him the *Gegenschein*, and had the kindness to refer him to the theories that have been advanced by SCHAEBERLE, ANDERSON, EVERSHED, SEARLE and others for its explanation. A number of them ascribe the light to meteors or asteroids. There are some objections to all of them as is evidenced by the criticisms and discussions that have followed their publication. Doubtless many of them contain elements of truth, but so far they have not been so conclusive quantitatively as to secure general acceptance.

The hypothesis advanced in this paper was developed during the few days of residence at Williams Bay, and the mathematical theory was extended in the next two months to a complete analytical discussion of the oscillating satellites, a few examples of which were given by DARWIN in his memoir in the *Acta Mathematica*, Tome 21. It will be seen that the theory of Professor WM. H. PICKERING, published in the January number of *Popular Astronomy*, is in many respects similar to this, although lacking something of the precision which results from a mathematical investigation.

The *Gegenschein* was first discovered by BRORSEN about the middle of this century,* but very few astronomers have made systematic observations of it. By far the most consistent and extensive locations of its position and determinations of its shape are due to BARNARD, who has followed it almost constantly for sixteen years. Although it is generally considered a difficult object, and one in regard to which it is easy to be mistaken, yet BARNARD's observations cover so great a period, and are so consistent, that there can be no doubt that his results represent very closely the actual appearances of the object. From an examination of all these observations, which he kindly placed at my disposal, I came to the conclusion, which he had already reached, that the *Gegenschein* is always exactly opposite to the sun as nearly as can be determined. Some other observers, notably DOUGLASS, seem to have found that it has a slight variation in both latitude and longitude. These conclusions must be regarded as being somewhat less certain than BARNARD's, because of the short time over which the observations extend. As an illustration of the uncertainty of the existence of these slight variations, I find that at the time of the greatest southern latitude, according to DOUGLASS, the mean of BARNARD's observations for sixteen years gives a slight northern latitude; and the observations for the same year in which those of DOUGLASS were made, the latitude, according to BARNARD, was north instead of south. From a discussion of all of BARNARD's observations it was not possible to discover any relation between the phase of the moon and the variations in latitude and longitude.

It is proposed to show in the following paragraphs that it is possible for meteors to move for long periods of time in the vicinity of a point in opposition to the sun, and about 900,000 miles distant from the earth. The suggestion will then be made, as being a reasonable hypothesis, that the meteors revolving around this point are sufficiently numerous to cause the faint glow of the *Gegenschein* by reflecting the light of the sun.

The space around the sun occupied by the planets seems to be teeming with meteors, at least as far out as the earth. They are indeed so numerous that the late Professor H. A. NEWTON calculated that ten millions strike into the earth's atmosphere daily. They seem to be the cause of the zodiacal light, and they doubtless crumpled and destroyed the tail of BROOKS's comet in the remarkable manner shown in BARNARD's excellent photographs of this object. From observations they have been found to move with widely different velocities, and in every direction, so that considering their countless numbers it does not strain our imagination seriously to suppose that a great multitude would get in the vicinity of the opposition point, and be

* For a more complete historical account see Prof. BARNARD's paper, in *Popular Astronomy*, No. 64.

near enough some of the triple infinity of possible initial conditions so that they would remain there for a long time. This hypothesis evidently explains qualitatively the established phenomena of the *Gegenschein*. Quantitatively the question seems to be beyond the power of mathematical treatment, and individual judgments regarding the efficiency of the theory will doubtless vary widely.

1. *The differential equations of motion.* As the meteors are exceedingly small compared to the earth, we shall treat them as infinitesimal bodies, disturbing neither the earth nor each other. Since we shall only prove that they may stay a finite, but long, time in opposition this procedure is perfectly allowable. For the sake of simplicity we shall neglect the eccentricity of the earth's orbit. Let us consider the circumstances of the motion of one meteor, referring the position of the system to rectangular axes with the origin at the center of gravity of the sun and the earth. Suppose the units are selected so that the Gaussian constant is equal to unity. Let the coördinates of the sun, earth and meteors be $\xi_1', \eta_1', \zeta_1'$; ξ_2', η_2', ζ_2, and ξ, η, ζ, respectively. Take the distance of the earth from the sun as unity. Let the distance from the meteor to the sun be r_1, and from the meteor to the earth r_2. Let S and E represent the masses of the sun and earth respectively. Then the differential equations of motion of the meteor are

$$(1)\qquad \left\{\begin{aligned}
\frac{d^2\xi'}{dt^2} &= -\frac{S(\xi'-\xi_1')}{r_1^3} - \frac{E(\xi'-\xi_2')}{r_2^3}\\
\frac{d^2\eta'}{dt^2} &= -\frac{S(\eta'-\eta_1')}{r_1^3} - \frac{E(\eta'-\eta_2')}{r_2^3}\\
\frac{d^2\zeta'}{dt^2} &= -\frac{S(\zeta'-\zeta_1')}{r_1^3} - \frac{E(\zeta'-\zeta_2')}{r_2^3}
\end{aligned}\right.$$

Suppose the sun and the earth revolve in the $\xi'\eta'$-plane with the angular velocity n. Let us choose the unit of mass so that $S+E=1$; then from this and our previous choice of units we find $n=1$. We may refer the motion of the system to axes rotating in the $\xi'\eta'$-plane with the uniform angular velocity unity by the substitution

$$(2)\qquad \left\{\begin{aligned}
\xi' &= \xi\cos t - \eta\sin t\\
\eta' &= \xi\sin t + \eta\cos t\\
\zeta' &= \zeta
\end{aligned}\right.$$

Substituting (2) in (1), and eliminating the trigonometrical functions, we obtain

$$(3)\qquad \left\{\begin{aligned}
\frac{d^2\xi}{dt^2} - 2\frac{d\eta}{dt} &= \xi - \frac{S(\xi-\xi_1)}{r_1^3} - \frac{E(\xi-\xi_2)}{r_2^3}\\
\frac{d^2\eta}{dt^2} + 2\frac{d\xi}{dt} &= \eta - \frac{S(\eta-\eta_1)}{r_1^3} - \frac{E(\eta-\eta_2)}{r_2^3}\\
\frac{d^2\zeta}{dt^2} &= \quad - \frac{S\zeta}{r_1^3} - \frac{E\zeta}{r_2^3}
\end{aligned}\right.$$

Let the coördinates be so chosen that at the origin of time $\eta_1 = \eta_2 = 0$; then equations (3) become

$$\left.\begin{aligned}
\frac{d^2\xi}{dt^2} - 2\frac{d\eta}{dt} &= \xi - \frac{S(\xi-\xi_1)}{r_1^3} - \frac{E(\xi-\xi_2)}{r_2^3}\\
\frac{d^2\eta}{dt^2} + 2\frac{d\xi}{dt} &= \eta - \frac{S\eta}{r_1^3} - \frac{E\eta}{r_2^3}\\
\frac{d^2\zeta}{dt^2} &= \quad - \frac{S\zeta}{r_1^3} - \frac{E\zeta}{r_2^3}
\end{aligned}\right\}\qquad (4)$$

These equations are identically verified if we put

$$\left.\begin{aligned}
\xi - \frac{S(\xi-\xi_1)}{r_1^3} - \frac{E(\xi-\xi_2)}{r_2^3} &= 0\\
\eta = 0\quad,\quad \zeta &= 0
\end{aligned}\right\}\qquad (5)$$

Since the origin is at the center of gravity of S and E we have, supposing E to be in the positive direction,

$$\left.\begin{aligned}
\xi_1 &= -E\\
\xi_2 &= S
\end{aligned}\right\}\qquad (6)$$

The first equation of (5) has one, and only one, variation of sign, and hence one, and only one, real root in each of the intervals $+\infty > \xi > \xi_2$, $\xi_2 > \xi > \xi_1$, $\xi_1 > \xi > -\infty$.

Let the three values of ξ satisfying the first of (5) be ξ_{01}, ξ_{02} and ξ_{03}. Then we have

$$\left.\begin{aligned}
\xi_{01} &= r_2 + \xi_2 = r_2 + S\\
\xi_{02} &= -r_2 + \xi_2 = -r_2 + S\\
\xi_{03} &= -r_1 + \xi_1 = -r_1 - E
\end{aligned}\right\}\qquad (7)$$

The first of (5) gives for the determination of ξ_{01}, ξ_{02} and ξ_{03}, through r_2, $-r_2$, and r_1, respectively, the following equations:

$$\left.\begin{aligned}
r_2^5 + (2+S)r_2^4 + (1+2S)r_2^3 - Er_2^2 - 2Er_2 - E &= 0\\
r_2^5 - (2+S)r_2^4 + (1+2S)r_2^3 - Er_2^2 + 2Er_2 - E &= 0\\
r_1^5 + (2+E)r_1^4 + (1+2E)r_1^3 - Sr_1^2 - 2Sr_1 - S &= 0
\end{aligned}\right\}\qquad (8)$$

If at the origin of time the infinitesimal body be placed at one of the points $\xi = \xi_{0i}$, $\eta = 0$, $\zeta = 0$, $(i = 1, 2, 3)$, it will forever remain there unless disturbed by some external force. These particular solutions are evidently those of LAGRANGE, in which one of the masses is infinitesimal.

Suppose now the initial conditions are slightly different from those of the particular solutions above. It is proposed to investigate the nature of the movement of the infinitesimal body with special reference to the circumstances under which it will make periodic oscillations around the points $\xi = \xi_{0i}$, $\eta = 0$, $\zeta = 0$, $(i = 1, 2, 3)$.

Since all the properties derived are similar for each of the three points, we shall henceforth omit the subscripts. We shall denote the point in opposition to the sun by (A), the one between the earth and sun by (B), and the one

beyond the sun in opposition to the earth by (C). Suppose the initial conditions, instead of exactly fulfilling the equations above are

$$(9)\quad \begin{cases} \xi = \xi_0 + x\ , & \eta = y\ , \quad \zeta = z \\ \dfrac{d\xi}{dt} = \dfrac{dx}{dt}\ , & \dfrac{d\eta}{dt} = \dfrac{dy}{dt}\ , \quad \dfrac{d\zeta}{dt} = \dfrac{dz}{dt} \end{cases}$$

where $x, y, z, \dfrac{dx}{dt}, \dfrac{dy}{dt}, \dfrac{dz}{dt}$ are small quantities. Substituting (9) in (4), and developing in powers of $x, y, z, \dfrac{dx}{dt}, \dfrac{dy}{dt}, \dfrac{dz}{dt}$ we have

(10)

$$\begin{aligned} \frac{d^2x}{dt^2} - 2\frac{dy}{dt} &= x + \frac{2Sx}{\{(\xi_0-\xi_1)^2\}^{\frac{3}{2}}} + \frac{2Ex}{\{(\xi_0-\xi_2)^2\}^{\frac{3}{2}}} + \text{higher terms} \\ \frac{d^2y}{dt^2} + 2\frac{dx}{dt} &= y - \frac{Sy}{\{(\xi_0-\xi_1)^2\}^{\frac{3}{2}}} - \frac{Ey}{\{(\xi_0-\xi_2)^2\}^{\frac{3}{2}}} + \text{higher terms} \\ \frac{d^2z}{dt^2} &= -\frac{Sz}{\{(\xi_0-\xi_1)^2\}^{\frac{3}{2}}} - \frac{Ez}{\{(\xi_0-\xi_2)^2\}^{\frac{3}{2}}} + \text{higher terms} \end{aligned}$$

2. *Integration of the differential equations neglecting all terms of order higher than the first.* In the problem under discussion it is permissible to limit ourselves to small quantities of the first order. Then we have

$$(11)\quad \begin{cases} \dfrac{d^2x}{dt^2} - 2\dfrac{dy}{dt} = (1+2A)x \\ \dfrac{d^2y}{dt^2} + 2\dfrac{dx}{dt} = (1-A)y \\ \dfrac{d^2z}{dt^2} = -Az \end{cases}$$

where we have put for abbreviation

$$A = \frac{S}{\{(\xi_0-\xi_1)^2\}^{\frac{3}{2}}} + \frac{E}{\{(\xi_0-\xi_2)^2\}^{\frac{3}{2}}} = \frac{S}{r_{10}^3} + \frac{E}{r_{20}^3}$$

The last equation of (11) is independent of the first two, and may be integrated separately; therefore z and $\dfrac{dz}{dt}$ depend only upon the values of these variables at the origin of time. The integrals are at once found to be

$$(12)\quad \begin{cases} z = c_1\cos(\sqrt{A}\,t) + c_2\sin(\sqrt{A}\,t) \\ \dfrac{dz}{dt} = -c_1\sqrt{A}\sin(\sqrt{A}\,t) + c_2\sqrt{A}\cos(\sqrt{A}\,t) \end{cases}$$

Since A is a positive quantity the component of motion perpendicular to the xy-plane is in all cases periodic with the period $\dfrac{2\pi}{\sqrt{A}}$. If $t_0 = 0$, the constants c_1 and c_2 are given by the equations

$$(13)\quad \begin{cases} c_1 = z_0 \\ c_2 = \dfrac{1}{\sqrt{A}}\dfrac{dz_0}{dt} \end{cases}$$

It remains to consider the simultaneous system

$$\left.\begin{aligned} \frac{d^2x}{dt^2} - 2\frac{dy}{dt} &= (1+2A)x \\ \frac{d^2y}{dt^2} + 2\frac{dx}{dt} &= (1-A)y \end{aligned}\right\} \quad (14)$$

The solution of this linear system is at once obtained by letting

$$\left.\begin{aligned} x &= Ke^{\lambda t} \\ y &= Le^{\lambda t} \end{aligned}\right\} \quad (15)$$

Substituting (15) in (14), and dividing out common factors, we have

$$\left.\begin{aligned} (\lambda^2-1-2A)K - 2\lambda L &= 0 \\ 2\lambda K + (\lambda^2-1+A)L &= 0 \end{aligned}\right\} \quad (16)$$

$K = L = 0$ is a solution of these equations, but furnishes the integrals $\xi = \xi_0$, $\eta = 0$ from which we started, and which we no longer desire to study. The equations cannot be satisfied for other values of K and L unless the determinant vanishes. The determinant set equal to zero, or the characteristic equation, is

$$\begin{vmatrix} \lambda^2-1-2A\ , & -2\lambda \\ 2\lambda\ , & \lambda^2-1+A \end{vmatrix} \equiv \lambda^4 + (2-A)\lambda^2 + (1+A-2A^2) = 0 \quad (17)$$

Let the four roots of this biquadratic be λ_1, λ_2, λ_3 and λ_4. Substituting these values in turn in (16), the ratios of K and L are determined in each case, each ratio involving one arbitrary constant. Representing the corresponding values of K and L by K_1, L_1; K_2, L_2; K_3, L_3; K_4, L_4, we obtain as the general solution of (14), involving four arbitrary constants of integration,

$$\left.\begin{aligned} x &= K_1e^{\lambda_1 t} + K_2e^{\lambda_2 t} + K_3e^{\lambda_3 t} + K_4e^{\lambda_4 t} \\ y &= L_1e^{\lambda_1 t} + L_2e^{\lambda_2 t} + L_3e^{\lambda_3 t} + L_4e^{\lambda_4 t} \end{aligned}\right\} \quad (18)$$

We must now determine the nature of the roots, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, of (17). From (8) we have approximately

$$r_{20}^3 = \frac{E}{1+2S}$$

$$r_{10}^3 = \frac{S}{1+2E}$$

Then we have for A, approximately

$$A_1 = A_2 = \frac{S}{r_{10}^3} + 1 + 2S$$

$$A_3 = \frac{E}{r_{20}^3} + 1 + 2E$$

Thus we see that A is positive in all three cases, and that $1-A-2A^2<0$. Therefore two of the roots of the biquadratic, (17), are real and two are pure imaginaries. Then it follows that for general initial conditions x and y

increase in numerical value without limit with t, instead of being periodic as were the z-coordinates. Under these circumstances the Lagrangian straight-line solutions are said to be unstable.

Suppose λ_1 and λ_2 are the conjugate imaginary roots, and λ_3 and λ_4 the real ones. Then, if the initial conditions are such that $K_3 = K_4 = L_3 = L_4 = 0$ the motion will be periodic. There will then remain only two arbitrary constants in the solutions. Suppose $\lambda_1 = \sqrt{-1}\,\sigma$. $\lambda_2 = -\sqrt{-1}\,\sigma$, where σ is a real number. Then in order that the solution may be real, as well as periodic, we must select the constants as conjugate complex quantities having the form

$$(19)\quad \left\{\begin{aligned} K_1 &= \alpha_1 + \sqrt{-1}\,\alpha_2 \\ K_2 &= \alpha_1 - \sqrt{-1}\,\alpha_2 \\ L_1 &= \beta_1 + \sqrt{-1}\,\beta_2 \\ L_2 &= \beta_1 - \sqrt{-1}\,\beta_2 \\ K_3 &= K_4 = L_3 = L_4 = 0 \end{aligned}\right.$$

where α_1, α_2, α_3 and α_4 are real numbers. Writing (18) in the trigonometrical form, we have

$$(20)\quad \left\{\begin{aligned} x &= 2\alpha_1 \cos(\sigma t) - 2\alpha_2 \sin(\sigma t) \\ y &= 2\beta_1 \cos(\sigma t) - 2\beta_2 \sin(\sigma t) \end{aligned}\right.$$

α_i and β_i are expressed in terms of the initial conditions by the equations

$$(21)\quad \left\{\begin{aligned} \alpha_1 &= \frac{x_0}{2}\ , \quad \alpha_2 = -\frac{1}{2\sigma}\frac{dx_0}{dt} \\ \beta_1 &= \frac{y_0}{2}\ , \quad \beta_2 = -\frac{1}{2\sigma}\frac{dy_0}{dt} \end{aligned}\right.$$

When the constants of integration are chosen as defined in (19) and (21) the x and y-coördinates and velocities in each of the three cases, (A), (B) and (C), are periodic functions of the time with the period $\frac{2\pi}{\sigma}$. Since there are two arbitrary constants involved there is a doubly infinite system of these solutions, all having the same period. In space of three dimensions there is a triple infinity of solutions, all having the same period of oscillation through the xy-plane.

Eliminating t from (20) we have as the equation of the projection of the orbit on the xy-plane, the ellipse whose center is at the point (A), (B), or (C),

$$(22)\quad (\beta_1^2+\beta_2^2)x^2 + (\alpha_1^2+\alpha_2^2)y^2 - 2(\alpha_1\beta_1+\alpha_2\beta_2)xy = 4(\alpha_1\beta_2-\alpha_2\beta_1)^2$$

It follows from the properties of central orbits that, under the initial conditions defined above, the resultants of all the forces, both attractive and centrifugal, up to terms of the second order of small quantities, are directed to the relatively stationary points (A), (B) and (C), and that the intensities are directly proportional to the distances from them. Since the forces depend upon the velocities as well as upon the coördinates, this is not true in general. Meteors passing near one of these points with approximately one of the triple infinity of circumstances of motion defined by (13) and (19) would be subject to forces directed nearly to this point, and would have a tendency to revolve around it. Doubtless it would frequently happen that they would make a few revolutions and then pass on. Since the orbits are unstable the meteors would always escape from this region and continue their way, and on the average they would be escaping as fast as they were being captured; nevertheless the result would be a condensation with respect to space, if not with respect to time. It is as though a certain number had to pass by a given point in a given time; the slower they moved the closer they would be together.

The difficult question is to determine whether a sufficient number would be temporarily captured to become visible. Without pretending to give a positive answer the following considerations may be advanced. It is known that there is a great disc of meteoric matter revolving around the sun, and extending somewhat beyond the earth's orbit. It is seen as the zodiacal band extending even up to 180° from the sun. The orbits of the individual meteors are doubtless somewhat nearly circular in form. Therefore they would be passing the opposition point with small velocities referred to the rotating axes, and generally in the negative direction. They would be crossing the x-axis nearly orthogonally, and the question is whether the velocity would ever be such as to fulfill the second equation of (21).

From (11) we at once obtain the integral

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+2A)x^2 + (1-A)y^2 + C \qquad (23)$$

When the meteor is crossing the x-axis orthogonally this becomes

$$\left(\frac{dx}{dt}\right)^2 = (1+2A)x^2 + C \qquad (24)$$

At the stationary points the velocity would necessarily be zero, but as the distance from them is increased the velocity is also increased continually. Therefore it is practically certain that the initial conditions would be approximately fulfilled. It seems to follow necessarily that there should be a condensation in the zodiacal band in opposition to the sun, and that from purely theoretical grounds we should be led to expect to find a *Gegenschein*.

If there is difficulty in perceiving the dynamical relations when the system is referred to rotating axes the question may be regarded in the following elementary fashion. Suppose a meteor is revolving around the sun in a circular orbit at a distance of about 900,000 miles greater than the mean distance of the earth. It will be moving a little more slowly than the earth, which will gradually overtake it in longitude. As it gets near to opposition to

the earth the perturbations of the earth will begin to become sensible. They will retard it in its orbit, and draw it in towards the sun. The secondary effect of the latter is to accelerate the motion. When it gets to opposition it will be subject to the attractions of both the sun and earth in the same direction. This is equivalent dynamically to increasing the mass of the sun, and it will accelerate the motion of the meteor. At the proper distance from the earth its angular velocity will equal that of the earth. Moreover the perturbations on the meteors at all distances will tend to bring them into the plane of the ecliptic, so that there would be a condensation from that cause aside from the other, and besides it would be the greatest in opposition.

These results agree with those found by the analysis, which showed that there is always a tendency to oscillation in latitude, while particular initial conditions are necessary for the tendency to oscillation in longitude. It follows from this that the *Gegenschein* should be somewhat elongated in longitude.

Since the conditions are similar for all three points (A), (B) and (C), it is seen that Professor PICKERING's remark that a meteor could not remain in a line with the earth and sun and between them is erroneous. Indeed, DARWIN found (*loc. cit.*) five such orbits by numerical processes. It seems also that Professor PICKERING's hypothesis regarding their origin is scarcely sufficient. He supposes that they were driven by tidal disturbances from orbits nearer the earth to the greater distance at which there would be a tendency to collect in opposition. In the first place, the point between the earth and the sun is nearer the earth than the one in opposition, and they would collect at the former instead of the latter. In the second place, their tidal evolution would go on much more slowly than that of the moon, and it cannot be supposed that their tides have been sufficient to have driven them even so far out as the moon. In the third place, the necessary initial conditions could not be exactly fulfilled by this process, and the meteors would soon be lost. Since the orbits are unstable a continual supply of material is required.

Instead of being opposite the earth the point (A) will be nearly opposite the center of gravity of the earth and moon, and consequently the *Gegenschein* will have a monthly oscillation in longitude of the nature of that indicated by the observations of DOUGLASS, but very much less in extent, as will be shown when we pass to the numerical applications. The oscillation in latitude would be monthly instead of yearly as the observations seem to indicate. It is not intended to imply that the observations are not correct because they do not agree with this theory, for it is necessary to make the theory fit the observations. The theory is advanced because it does not seem established beyond all question that the contradictory phenomena which have been mentioned exist. These phenomena are enumerated in order that they may receive more careful attention in the future by observers in favorable localities.

There is another phenomenon of a striking character which has been observed by BARNARD alone so far as I am aware. It is that the *Gegenschein* undergoes marked changes in form in a comparatively short time. He has observed many times that it is large and round in September and about the first of October, becoming slightly elongated by the 4th or 5th, and very much elongated by the 10th or 11th, and a mere swelling on the zodiacal band by the 18th. I do not pretend to assign a cause for this phenomenon. It does not, however, seem to be necessarily contradictory to the theory of this paper, since the shape of the *Gegenschein* would depend upon the thickness of the zodiacal disc, and if the opposition point should pass through a dense meteoric stream it might change its form very rapidly.

3. *Numerical applications.* The expansions (10) are only convergent for sufficiently small values of x and y. The expansions come from functions of the form

$$1/\{(\xi_0-\xi_2+x)^2+y^2+z^2\}^{\frac{3}{2}}$$

and it is easily seen that they are absolutely convergent so long as the absolute value of

$$2x/(\xi_0-\xi_2)+(x^2+y^2+z^2)/(\xi_0-\xi_2)^2$$

is less than unity. In order to get an idea of how great the values of x, y and z may be relatively, suppose the meteor is describing an ellipse whose major axis is orthogonal to the x-axis. Suppose it is at one end of this axis, and that the orbit is in the xy-plane. Then $x=z=0$, and we have $|y|<(\xi_0-\xi_2)$. Therefore in this case the limit of the value of y is the distance of the opposition point from the earth, and the limit of the elongation of the *Gegenschein* from the opposition point is 45°, giving a total length of 90°, which is more than three times as great as has been observed. Of course, when the values of the variables are so large that the series converge slowly the higher terms which have been neglected become important, and the results correspondingly inexact. We should not expect the *Gegenschein* to be visible more than 10° or 15° each side of the opposition point.

We have taken the unit of mass so that $S+E=1$, the unit of distance so that $\xi_2-\xi_1=1$, and the unit of time so that the Gaussian constant $k^2=1$. Let P represent the year expressed in mean solar days; then we may determine the unit of time in days by the equation

$$P=\frac{2\pi(\xi_2-\xi_1)^{\frac{3}{2}}}{k\sqrt{S+E}}=2\pi=365.25636$$

This gives as the unit of time 58.1324 mean solar days. From the first equation of (8) we find

$$r_{20}=\xi_0-\xi_2=.0100134=930{,}240 \text{ miles},$$

the distance of the opposition point from the earth. The equatorial horizontal parallax of this point is less than 15′. Since the center of gravity of the earth and moon is within the earth the parallactic displacement in longitude would be very much less than this. If it were possible to make the observations near the horizon the combined effect of the displacement of the earth and the distance of the observer from the earth's center could not cause the center of the *Gegenschein* to depart from the opposition point by so much as 30′. From (11) we find $A = 3.970566$. The period of oscillation perpendicular to the xy-plane is

$$\frac{2\pi}{\sqrt{A}} = 3.15322 = 183.301 \text{ days}$$

The University of Chicago, 1900 *March* 10.

Equation (17) becomes

$$\lambda^4 - 2.970566\lambda^2 - 26.510148$$

The modulus of the conjugate imaginary roots of this equation is $\sigma = 2.61665$. The period of oscillation in the xy-plane is

$$\frac{2\pi}{\sigma} = 2.40121 = 139.6 \text{ days}$$

The opposition point is about 70,000 miles beyond the apex of the earth's shadow, so that the central part of the *Gegenschein* is somewhat dimmed by an annular eclipse of the sun.

NOTE. Since the above paper was sent to the Editor of the *A.J.*, Mr. W. S. ADAMS, Fellow in Astronomy at the Yerkes Observatory, has called my attention to a memoir by M. HUGO GYLDÉN in the *Bulletin Astronomique*, Tome 1, entitled, "Sur un Cas Particulier du Problème des Trois Corps." M. GYLDÉN investigates the conditions of stability of the Lagrangian solutions of the Problem of Three Bodies, and derives results similar in essence to those given in my paper. He has used a complicated notation, making five transformations of variables, and obscuring the geometry of the problem. Besides, the approximations which he introduced are such that the results are the same whatever be the relative mass of the planet. The method of my paper gives the correct results to small quantities of the second order, the period being unchanged even when the terms of all orders are included.

The last paragraph in the memoir of M. GYLDÉN, consisting of the following reference to the *Gegenschein*, is of special interest:

"Comme on est porté à admettre que les espaces interplanétaires sont traversés par une multitude de corpuscules dont les directions de mouvement et les vitesses sont bien différentes, il ne paraît pas trop hardi d'en supposer des essaims retenus dans le voisinage des centres de libration. En effet, il n'est pas nécessaire que la direction et la vitesse du mouvement soient telles que l'exigent les equations (11), pour qu'il se forme un système de trois corps de la manière que nous avons envisagée dans ce qui précède. Peut-être n'est-il pas même trop exagéré de attribuer à de tels essaims l'aspect de la lueur très faible que nous voyons du côté du ciel opposé au Soliel, lueur que l'on a supposée être en connexion avec la lumière zodiacale, et qui est appelée par les Allemands le '*Gegenschein*.'"

It is seen that this is precisely the hypothesis of my paper, and it follows that M. GYLDÉN is entitled to all of the credit which may come from having suggested such a theory. The reason that it was overlooked is that the title of the memoir gives no indication that it mentions the *Gegenschein*, and that it was unknown to Professor BARNARD, who furnished me with the bibliography of the subject. It seems to have been overlooked also by all previous writers on the theory of the *Gegenschein*.

There is one fact worthy of mention given in M. GYLDÉN's memoir which had escaped my notice. It is that the eccentricities of all of the possible elliptic orbits of the infinitesimal body are the same. It follows equally from the results of my paper. It has been seen that in the periodic solutions $\lambda_1 = \sqrt{-1}\,\sigma$. Then substituting (19) in (16), we have

$$-(\sigma^2+1+2A)(\alpha_1+\sqrt{-1}\,\alpha_2)-2\sqrt{-1}\,\sigma(\beta_1+\sqrt{-1}\,\beta_2) = 0 \qquad (a)$$

Let
$$\epsilon = \frac{\sigma^2+1+2A}{\sigma}$$

then
$$\begin{aligned} \beta_2 &= \epsilon\alpha_1 \\ \beta_1 &= -\epsilon\alpha_2 \end{aligned} \qquad (b)$$

Substituting (*b*) in (22), it becomes

$$\frac{x^2}{4(\alpha_1^2+\alpha_2^2)} + \frac{y^2}{4\epsilon^2(\alpha_1^2+\alpha_2^2)} = 1 \qquad (c)$$

Then the eccentricity is given by the equation

$$e = \frac{\sqrt{\epsilon^2-1}}{\epsilon}$$

Referring to the numerical values of σ and A, it is found that $e = 0.91982$. M. GYLDÉN's less exact methods gave $e = 0.9502$. It is also noticed from (*c*) that the axes of the ellipse coincide with the x and y-axes.

The method of M. GYLDÉN's memoir gives as the period of revolution in the xy-plane 176.32 days, instead of the correct period of 176.98 days given in my paper. The deviation would be much more marked in the case of a large planet.

OBSERVED MAXIMA AND MINIMA OF SHORT-PERIOD VARIABLES,

By PAUL S. YENDELL.

The subjoined dates are the results of my observations of short-period stars, not heretofore published, up to the beginning of the year 1900.

2279. *T Monocerotis.*

Twenty-two observations of this star, from 1896 Dec. 4 to 1899 Nov. 4, yield the following dates of maxima and minima, by the use of a mean light-curve:

Maxima	Obs.	Minima	Obs.
1897 Jan. 4.71	3	1896 Dec. 26.10	3
Feb. 1.69	4	1897 Jan. 23.32	1
1898 Jan. 17.07	4		
1899 Oct. 4.58	1		

6404. *Y Ophiuchi.*

During the seasons of 1897 and 1899, I observed *Y Ophiuchi* twenty-five times. The following dates of maxima are indicated:

Maxima.

1897		1899		1899	
June 28.5	wt. 4	July 1.8	wt. 3	Aug. 6.2	wt. 3

6472. *W Sagittarii.*

Eight observations in 1897, and twenty-two in 1899, show maxima and minima as follows, by the use of a mean light-curve:

Maxima	Obs.	Minima	Obs.
1897 June 20.85	3	1897 June 25.54	3
28.23	1	1899 July 2.86	3
July 5.97	2	Aug. 1.36	1
1899 June 28.84	3	8.38	2
July 28.79	4	Sept. 7.30	2
Sept. 3.51	1		

6573. *Y Sagittarii.*

Eight observations in 1897, and nineteen in 1899. The following maxima and minima are found by the use of a mean light-curve:

Maxima	Obs.	Minima	Obs.
1897 June 25.71	2	1897 June 22.83	3
July 6.46	2	28.31	1
1899 July 27.10	2	1899 July 1.71	4
Aug. 1.70	1	30.54	2
8.47	3		
13.18	1		
30.34	1		
Sept. 5.93	3		

6636. *U Sagittarii.*

Seven observations in 1897, and seventeen in 1899. The application of a mean light-curve shows the following maxima and minima:

Maxima	Obs.	Minima	Obs.
1897 June 24.04	3	1897 June 20.05	1
July 8.63	2	28.40	1
1899 June 28.78	2	1899 July 2.71	2
July 4.36	1	29.00	1
31.57	3	Aug. 25.97	4
Aug. 7.57	3	Sept. 6.89	1
11.51	1		
Sept. 3.79	2		

6984. *U Aquilae.*

Seven observations in 1897, and thirteen in 1899. The following maxima and minima are shown by the mean light-curve:

Maxima	Obs.	Minima	Obs.
1897 June 18.90	3	1897 June 28.86	2
26.92	1	1899 July 31.07	1
July 4.42	1	Aug. 7.37	1
1899 Aug. 2.95	2	26.4	1
9.42	1		
23.35	1		
30.42	1		
Sept. 6.1*			
18.8	1		

* From single curve, wt. 4.

7149. *S Sagittae.*

I have eleven observations of this star in 1897, and sixty-four in 1899. The following maxima and minima are shown by the single curves:

Maxima	Wt.	Minima	Wt.
1897 June 21.2	4	1897 June 25.1	4
1899 June 12.5	3	1899 July 2.6	3
29.1	2	Sept. 8.7	3
July 31.4	3	15.1	2
Aug. 9.4	3	Nov. 6.7	2
Sept. 21.3	4		

7437. *X Cygni.*

I have fourteen observations of *X Cygni* in 1897, and sixty-five in 1899. The single curves show maxima and minima as follows:

Maxima	Wt.	Minima	Wt.
1899 July 6.6	2	1897 June 22.9	4
26.3	3	1899 July 17.2	3
Aug. 9.2	3	Aug. 4.2	4
25.0	3	16.4	3
Sept. 11.0	4	Sept. 4.6	4
28.1	4	22.4	4
		Oct. 7.0	4
		Nov. 9.0	2

7183. *T Vulpeculae.*

I have twelve observations of this star in 1897, and fifty-five in 1899. The application of a mean curve gives maxima and minima as follows:

Maxima	Obs.	Minima	Obs.
1897 June 17.65	1	1897 May 29.41	1
22.79	1	June 16.43	1
26.54	2	21.35	1
1899 June 10.35	1	25.40	1
28.67	2	July 8.42	1
July 2.92	2	1899 June 13.37	3
29.39	1	July 1.40	1
Aug. 25.67	2	31.98	2
29.92	1	Aug. 5.82	2
Sept. 7.31	1	14.69	2
11.51	1	Sept. 6.35	1
20.58	2	10.39	1
		23.33	1

8598. *U Pegasi.*

I have observations of *U Pegasi* as follows: in 1896, eighty-six; in 1897, twenty-seven; in 1899, one hundred and seventy. The observed dates of maxima and minima are as follows. The time given is Local Mean Time.

1896

Maxima		h m	Wt.	Mag.	Minima		h m	Wt.	Mag.
Aug.	8	10 31.5	3	8.92	Sept.	11	10 15	4	9.60
Sept.	4	9 18.5	1	8.95		26	9 16	4	9.46
Oct.	31	8 27.0	1	8.98		27	8 29	1	9.41
Nov.	2	6 33.0	3	8.82		30	9 8	4	9.42
					Oct.	31	6 36	3	9.46
					Nov.	2	8 11	1	9.56

1897

Oct.	2	8 41	2	8.95	Oct.	26	10 37	3	9.39
	26	8 11	5	8.89					

1899

Aug.	30	9 37	2	8.73	Sept.	1	13 1	5	9.57
Sept.	1	10 50	4	9.05		4	11 22.5	4	9.58
	4	12 6.5	4	8.97		9	11 25	3	9.66
	6	9 51	1	8.95		14	13 1	1	9.39
	9	13 40	4	8.98		23	8 30	3	9.60
	14	10 50	4	9.05		24	11 2	4	9.53
	21	13 37	3	8.98		28	9 35	4	9.40
	27	8 22	1	8.95		30	8 53	3	9.41
Oct.	1	7 52	3	8.96					

Dorchester, 1900 *May* 1.

THE GREAT INEQUALITY OF *EROS* AND THE EARTH.

By HENRY NORRIS RUSSELL.

The period of *Eros* is very nearly $1\frac{3}{4}$ years. The perturbation of its mean longitude by the *Earth* depending on the argument $7g-4g'$ (g being the mean anomaly of *Eros* and g' that of the *Earth*), is therefore of long period. A rough calculation shows that it is enormously large, sometimes displacing *Eros* by over a degree of geocentric longitude.

The part of this term depending on the direct action of the *Earth* is of the 3d order in the eccentricities, and that depending on the indirect action is of the 11th, and therefore may be neglected. The perturbative function reduces to the reciprocal of the distance of the planets, which was determined graphically, and the perturbative function expanded by LeVerrier's method of interpolation, the orbits being divided into 16 parts with reference to the mean anomalies.

The perihelion of *Eros* is 2° from its descending node, and that of the *Earth* about 20° from the same point. By assuming that all three were coincident, the length of the calculation was halved, on account of the resulting symmetry. The results obtained for this hypothetical case cannot be very different from those in the actual case.

The terms in the perturbative function on which long-period perturbations depend are

$$\begin{aligned}&+0.0893\ \cos 3g\ \cos 4(g'-g)\\&+0.0500\ \sin 3g\ \sin 4(g'-g)\\&+0.027\ \ \cos 6g\ \cos 8(g'-g)\end{aligned}$$

The first two of these are equivalent to

$$+0.0697\ \cos(7g-4g')+0.0197\ \cos(4g'-g)$$

The first of these gives rise to the long-period term. To find the perturbation $d\rho$ of the mean longitude we have

$$d\rho = -3a\,m'\,\frac{7\mu^2}{(7\mu-4\mu')^2}\left\{0.0697\ \cos(7g-4g')\right\}$$

where a is the major axis of *Eros*, μ its mean motion, μ' that of the *Earth*, and m' the *Earth's* mass.

Substituting

$$\log a = 0.16379,\quad \mu = 2015''.233,$$
$$\mu' = 3518''.193\quad m' = \tfrac{1}{322800}$$

we obtain
$$d\rho = +747''\ \sin(7g-4g')$$

The period of this term is 41.24 years — very short for so large a perturbation.

The term involving $14g-8g'$ is also large. Since the coefficient of $\sin 6g\ \sin 8(g'-g)$ has not been calculated, the coefficient of this term has not been determined, but it appears from the value of the coefficient of $\cos 6g \cos 8(g'-g)$ that it is probably not far from $+0.020$, which would give the perturbation

$$d\rho = +100''\ \sin(14g-8g')$$

The geocentric displacement of *Eros* may be much greater than 750″. When opposition occurs when the planets are about 40° past perihelion, $7g-4g'=90°$, and the displacement of *Eros* in mean longitude is a maximum. But in this position the geocentric distance of *Eros* is about 0.23, and the heliocentric distance 1.20, so that the geocentric displacement of *Eros* is five times the heliocentric. Since *Eros* is near perihelion, the change in its heliocentric longitude is 1.4 times that in its mean longitude. The displacement of *Eros* by 750″ in mean longitude thus produces one of about 5200″ in geocentric longitude, or about $1\frac{1}{2}$ degrees. When opposition occurs the same distance on the other side of perihelion, the displacement is equal and of opposite sign. The whole range of the effect of the great inequality on the geocentric place is nearly three degrees. This great perturbation of *Eros*, running its course in about forty years, will give eventually one of the best determinations of the *Earth's* mass, and thus a second determination of the solar parallax, quite independent of that derived by geometrical methods from the same planet.

Princeton University, 1900 *May* 12.

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THE GENERAL PERTURBATIONS OF THE MAJOR AXIS OF *EROS*, BY THE ACTION OF *MARS*,

By HENRY NORRIS RUSSELL.

The most interesting of the perturbations of *Eros*, from a theoretical standpoint, are those caused by *Mars*. The periods of the two are nearly equal, and their orbits interlock. The disturbing force consequently varies greatly in magnitude, and may have any direction whatever. Since *Eros* is sometimes nearer the sun than *Mars*, and sometimes more remote, the development of the perturbative function proceeding by powers of the ratio of their radii vectores gives rise to a divergent series. The magnitude of the eccentricities and inclinations makes development in ascending powers of these quantities undesirable. Methods depending on mechanical quadrature seem therefore best adapted to this case.

In the present investigation LeVerrier's method of interpolation* has been employed. It has been found to possess the following advantages:

(1) It can be applied equally well, however great the eccentricities and inclinations.

(2) It gives the coefficient of each term directly, and not by a more or less slowly convergent series.

(3) Accidental errors of calculation are almost sure to be detected, if of serious magnitude.

(4). The coefficients are obtained as fractions with a large denominator, and hence may safely be carried to at least one more place of decimals than the body of the work.

(5) The work can be carried on simultaneously by several computers.

This method has, however, two serious disadvantages:

(1) A large number of useless terms must usually be computed in order to get all the sensible ones.

(2) The derivative of the perturbative function with respect to the elements must all be computed and expanded into series separately, since this method gives only the numerical values of the coefficients, and these cannot be differentiated.

The present investigation was originally intended to include a determination of the general perturbations of all the elements of *Eros* by *Mars*. The great extent of the numerical developments, however, finally necessitated the limitation of the work to those of the major axis, and the corresponding terms in the mean longitude. It was desired to obtain all terms whose coefficients were greater than 0″.005. This has been done, except for a few small long-period terms of very high order, whose determination did not seem worth while till the elements of *Eros* are more accurately known.

The elements of *Mars* used as basis of the work are Newcomb's, from p. 182 of his "Fundamental Constants of Astronomy," brought up to 1900.0 by the secular variations on p. 186. The mass of *Mars* is Hall's (p. 99 of same work).

Those of *Eros* are from an original computation (published in *A.J.* 473) brought up to the equinox of 1900.0. They differ but little from the results of the more elaborate investigations of Millosevich* and Osten.†

Elements of *Mars*.

$$\begin{array}{rcl}
\text{Epoch} & = & \text{1900 Jan. 0, Gr. M.T.} \\
M & = & 319^\circ\ 47'\ 1''.37 \\
\Omega & = & 48\ \ 47\ \ \ 9.34 \\
i & = & 1\ \ 51\ \ \ 1.32 \\
\pi & = & 334\ \ 13\ \ \ 7.04 \\
\varphi & = & 5\ \ 21\ \ 14.39 \\
\mu & = & 1886''.518297 \\
\log a & = & 0.1828971 \\
m & = & \frac{1}{3093500}
\end{array}
\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} 1900.0 \ (\Omega, i, \pi)$$

Elements of *Eros*.

$$\begin{array}{rcl}
\text{Epoch} & = & \text{1900 Jan. 0, Gr. M.T.} \\
M & = & 133^\circ\ 57'\ 12''.7 \\
\Omega & = & 303\ \ 31\ \ 11.5 \\
i & = & 10\ \ 49\ \ 31.6 \\
\pi & = & 121\ \ 10\ \ 48.3 \\
\varphi & = & 12\ \ 52\ \ 14.2 \\
\mu & = & 2015''.2326 \\
\log a & = & 0.1637876 \\
m & = & 0
\end{array}
\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} 1900.0 \ (\Omega, i, \pi)$$

*Annales de l'Observatoire de Paris, Tome 1, pp. 109–119, 137–151.

* A.N. 3609. † A.N. 3577.

Dr. CHANDLER's value of the mean motion has been retained, since it depends on the observations of 1891 and 1896, as well as 1898–99.

In computing the values of the perturbative function the plane of *Mars's* orbit was chosen as fundamental plane, and the axes of x passed through the ascending node of *Eros* on this plane. The radii vectores and rectangular coordinates of the planets being computed for as many values of the mean anomaly as was necessary, the remaining work involved only logarithms of numbers, which secured a considerable saving of time. Four decimal places were sufficient.

The angles ζ and ζ', which were chosen as fundamental arguments, were respectively the mean anomaly g' of *Mars*, and the difference $g-g'$ of those of *Eros* and *Mars*.

The circumference was divided into 64 parts with respect to ζ', and sometimes into 16 parts and sometimes into 8 with respect to ζ. In all, 800 values of the perturbative function were computed. The final results contain 251 sensible terms in the perturbations of the major axis, and 169 in the mean longitude.

Some of the perturbations of the mean longitude are quite large. They are given in the following table:

Term		Period in Years
$+\ 1''.69$	$\cos(\ g-\ g'+\ 20°\ 50')$	27.40
$+\ 9''.12$	$\cos(\ 2g-\ 2g'+\ 28°\ 35')$	13.70
$+\ 1''.59$	$\cos(\ 3g-\ 3g'+176°\ 40')$	9.13
$+\ 2''.37$	$\cos(13g-14g'+195°\ 30')$	16.2
$+11''.59$	$\cos(14g-15g'+344°\ \ 6')$	40.8
$+22''.81$	$\cos(15g-16g'+136°\ 42')$	78.0
$+11''.85$	$\cos(29g-31g'+330°\ \ 5')$	85.5
$+35''$	$\cos(44g-47g'+\ 10°)$	890 ?

The greatest displacements of *Eros* in mean longitude by the combined effects of all but the last of these terms during the 20th century are $+38''$ in 1927, and $-53''$ in 1959.

The last term in the above table was obtained by an extensive extrapolation, and should only be considered as an indication of the order of magnitude of the true value. Its integrating divisor is uncertain, as a decrease of less than $0''.1$ in the adopted μ of *Eros* would make it zero. Whatever its value it may be shown that this term will not sensibly affect the planet's place as derived from elements based on observation for some 15 years to come. It is of the form $\frac{A}{n^2}\cos nt + \frac{B}{n^2}\sin nt$, where A and B are functions of the elements which change very little for relatively large changes of n, and may be expanded into the series

$$\frac{A}{n_2}+\frac{B}{n}t - A\frac{t^2}{2'} - Bn\frac{t^3}{3'} + \ldots$$

The first two terms are included in the epoch and mean motion derived from observation. The third is almost independent of n, and the rest vanish with n. With the value of n resulting from the assumed mean motion of *Eros* the numerical value of this series is

$$-21'' + 0''.191\,(t-1900) + 0''.00018\,(t-1900)^2 \\ -0''.0000015\,(t-1900)^3 - \ldots$$

t being expressed in years.

The third term amounts to only $0''.1$ in fifteen years.

The uncalculated terms previously referred to are those whose arguments are

$$32g-34g',\quad 33g-35g',\quad 43g-46g',\quad 45g-48g',$$
$$59g-63g',\quad 73g-78g',\quad \text{and}\quad 88g-94g'.$$

These coefficients are at most a few hundredths of a second of arc.

The following summary of the results of the present investigation may be of interest:

(1) It has been shown by actual computation that LEVERRIER's method of interpolation, which is theoretically capable of solving any case of general perturbations, will solve the case of *Mars* and *Eros* practically, that is, without a prohibitive amount of labor.

(2) It has been found that the "great inequality," of period about 1000 years, will not affect the place of *Eros* sensibly during the next dozen years, after which time it may be approximately determined.

(3) The perturbations of moderately long period are much the largest produced by *Mars* on any planet. They may displace *Eros* by $90''$ in mean longitude; and since at a perihelion-opposition any change in the mean longitude of *Eros* produces one ten times as great in its geocentric longitude, the measurement of this displacement will eventually lead to a valuable determination of the mass of *Mars*.

The numerical results obtained are given in the following table. The first column gives the coefficients of the two fundamental arguments, ζ (the mean anomaly of *Mars*), and ζ' (the mean anomaly of *Eros* minus that of *Mars*) in the argument of the terms whose coefficients are given on the same horizontal line. The second and third contain the coefficients of the cosine and sine of this argument in the expansion of the perturbative function, which are given for purposes of comparison with the results of future calculations. The fourth and fifth give the coefficients of the cosine and sine of the argument in the perturbations of $\log a$, and the sixth and seventh the same for the mean longitude.

Argument		Perturbative Function in units of 4th place		Pertbs. log a in units 7th pl.		Pertbs. mean long. in seconds of arc	
ζ	ζ'	cos	sin	cos	sin	cos	sin
0	0	+7364.87	0.00	0.0	0.0	0.00	0.00
0	+ 1	+ 84.07	+ 221.39	+0.5	+1.4	+1.58	−0.60
0	+ 2	+1257.56	+2311.15	+8.1	+14.8	+8.26	−4.50
0	+ 3	+ 36.68	− 665.89	+0.2	−4.3	−1.59	−0.09
0	+ 4	+ 171.77	− 207.43	+1.1	−1.3	−0.37	−0.31
0	+ 5	− 572.04	+ 200.84	−3.7	+1.3	+0.29	+0.82
0	+ 6	+ 550.48	+ 115.27	+3.5	+0.7	+0.14	−0.66
0	+ 7	− 261.39	− 245.37	−1.7	−1.6	−0.25	+0.26
0	+ 8	+ 37.58	+ 122.99	+0.2	+0.8	+0.11	−0.03
0	+ 9	− 16.52	+ 32.73	−0.1	+0.2	+0.03	+0.01
0	+10	+ 92.90	− 70.30	+0.6	−0.4	−0.05	−0.06
0	+11	− 125.00	+ 16.96	−0.8	+0.1	+0.01	+0.08
0	+12	+ 82.58	+ 21.21	+0.5	+0.1	+0.01	−0.05
0	+13	− 22.72	− 31.29	−0.1	−0.2	−0.02	+0.01
0	+14	+ 0.92	− 3.52	0.0	0.0	0.00	0.00
0	+15	− 15.40	+ 22.00	−0.1	+0.1	+0.01	+0.01
0	+16	+ 29.86	− 12.49	+0.2	−0.1	−0.01	−0.01
0	+17	− 25.79	− 4.65	−0.2	0.0	0.00	+0.01
0	+18	+ 10.62	+ 11.28	+0.1	+0.1	+0.01	−0.01
0	+19	− 0.24	− 4.28	0.0	0.0	0.00	0.00
0	+20	+ 1.14	− 1.47	0.0	0.0	0.00	0.00
0	+21	− 7.85	+ 4.17	0.0	0.0	0.00	0.00
0	+22	+ 10.60	+ 3.74	+0.1	0.0	0.00	0.00
0	+23	− 6.43	+ 3.09	0.0	0.0	0.00	0.00
1	−31	− 2.12	− 2.85	0.0	0.0	0.00	0.00
1	−30	+ 0.65	+ 5.01	0.0	+0.1	0.00	0.00
1	−29	+ 0.31	− 5.68	0.0	−0.1	+0.01	0.00
1	−28	− 1.86	+ 5.27	0.0	+0.1	−0.01	0.00
1	−27	+ 1.87	+ 0.65	0.0	0.0	0.00	0.00
1	−26	− 0.97	+ 3.27	0.0	0.0	0.00	0.00
1	−25	− 1.19	+ 4.60	0.0	+0.1	−0.01	0.00
1	−24	+ 4.34	− 5.28	+0.1	−0.1	+0.01	+0.01
1	−23	− 6.79	+ 3.37	−0.1	+0.1	−0.01	−0.02
1	−22	+ 3.84	− 0.31	+0.1	0.0	0.00	+0.01
1	−21	+ 2.87	+ 2.01	+0.1	0.0	−0.01	+0.01
1	−20	− 6.32	− 9.26	−0.2	−0.2	+0.05	−0.03
1	−19	+ 1.29	+ 17.00	0.0	+0.5	−0.12	+0.01
1	−18	− 14.02	− 17.33	−0.5	−0.5	+0.20	−0.16
1	−17	− 10.40	+ 5.60	−0.5	+0.3	−0.13	−0.26
1	−16	− 0.86	− 1.80	−0.1	−0.1	+0.12	−0.05
1	−15	+ 17.20	+ 17.82	+4.8	+5.0	−16.22	+15.65
1	−14	− 13.63	− 48.00	+1.9	+6.6	+11.14	−3.17
1	−13	− 18.51	+ 67.01	+0.9	−3.4	−2.28	−0.63
1	−12	+ 46.14	− 47.11	−1.3	+1.4	+0.57	+0.56
1	−11	− 22.63	+ 10.58	+0.4	−0.2	−0.06	−0.13
1	−10	− 52.99	− 29.60	+0.7	+0.4	+0.10	−0.18
1	− 9	+ 99.66	+ 142.99	−1.0	−1.4	−0.29	+0.20
1	− 8	− 26.05	− 275.31	+0.2	+2.1	+0.36	−0.03
1	− 7	− 123.87	+ 276.08	+0.7	−1.6	−0.21	−0.14
1	− 6	+ 135.21	− 114.11	−0.6	+0.5	+0.07	+0.08
1	− 5	+ 166.70	+ 38.65	−0.5	−0.1	−0.01	+0.06
1	− 4	− 608.52	− 123.06	+1.5	+1.0	+0.14	−0.45
1	− 3	+ 628.74	+1344.53	−1.0	−2.0	−0.21	+0.10
1	− 2	+ 223.90	−2201.60	−0.2	+2.2	+0.20	+0.02
1	− 1	− 550.88	+1459.76	+0.3	−0.7	−0.06	−0.02
1	+ 0	− 27.46	+ 273.10	0.0	0.0	0.00	0.00
1	+ 1	+ 618.52	+ 299.92	+0.3	+0.1	+0.01	−0.02
1	+ 2	− 784.04	− 967.20	−0.6	−0.8	−0.05	+0.01
1	+ 3	+453.44	+1781.17	+0.5	+1.9	+0.12	−0.03
1	+ 4	+419.52	−1243.72	+0.6	−1.7	−0.10	−0.03
1	+ 5	−561.82	+447.27	−0.9	−0.7	+0.04	+0.05
1	+ 6	−217.31	− 10.31	+0.4	0.0	0.00	−0.02
1	+ 7	+ 99.59	+ 28.40	+0.2	+0.1	0.00	−0.01
1	+ 8	−138.17	−192.81	−0.3	−0.4	−0.02	+0.01
1	+ 9	+ 6.90	+235.09	0.0	+0.6	+0.03	0.00
1	+10	+ 81.93	−136.18	+0.2	−0.3	−0.02	−0.01
1	+11	− 59.75	+ 25.30	−0.2	+0.1	0.00	+0.01
1	+12	− 12.96	+ 2.47	0.0	0.0	0.00	0.00
1	+13	+ 14.71	+ 30.93	+0.1	+0.1	0.00	−0.01
1	+14	− 21.21	− 55.96	−0.1	−0.2	−0.01	0.00
1	+15	− 12.54	+ 14.26	0.0	+0.1	+0.01	0.00
1	+16	+ 20.34	− 13.80	+0.1	0.0	0.00	0.00
1	+17	− 2.62	− 3.56	0.0	0.0	0.00	0.00
1	+18	+ 8.28	− 2.93	0.0	0.0	0.00	0.00
1	+19	+ 13.43	+ 14.00	0.0	0.0	0.00	0.00
1	+20	− 0.72	− 19.50	0.0	−0.1	0.00	0.00
1	+21	− 10.73	+ 11.29	0.0	0.0	0.00	0.00
1	+22	+ 9.92	− 0.06	0.0	0.0	0.00	0.00
2	−31	− 0.72	+ 1.46	−0.1	+0.2	−0.11	−0.06
2	−30	+ 1.85	+ 0.18	+0.5	+0.1	−0.08	+0.84
2	−29	− 2.926	− 4.856	+1.7	+2.9	+10.25	−5.90
2	−28	+ 0.66	− 0.42	−0.1	+0.1	+0.05	+0.08
2	−27	+ 0.09	+ 0.74	0.0	0.0	−0.02	0.00
2	−26	− 1.24	− 2.42	+0.1	+0.1	+0.01	−0.02
2	−25	+ 0.23	+ 8.70	0.0	−0.3	−0.08	0.00
2	−24	+ 5.54	− 2.01	−0.2	+0.1	+0.01	+0.03
2	−23	− 0.38	− 1.73	0.0	0.0	+0.01	0.00
2	−22	− 5.86	+ 3.21	+0.1	−0.1	−0.01	−0.02
2	−21	+ 12.38	− 2.13	−0.2	0.0	0.00	+0.03
2	−20	− 12.66	− 9.36	+0.2	+0.1	+0.02	−0.02
2	−19	+ 5.84	+ 13.16	−0.1	−0.2	−0.02	+0.01
2	−18	+ 0.00	+ 23.97	0.0	−0.2	−0.02	0.00
2	−17	+ 8.04	− 6.47	−0.1	+0.1	0.00	+0.01
2	−16	− 26.90	+ 8.15	+0.2	−0.1	−0.01	−0.02
2	−15	+ 39.82	+ 7.57	−0.3	−0.1	0.00	+0.02
2	−14	− 29.56	+ 1.74	+0.2	0.0	0.00	−0.01
2	−13	+ 9.84	+ 19.68	0.0	−0.1	−0.01	0.00
2	−12	− 15.67	+ 12.88	+0.1	−0.1	0.00	0.00
2	−11	+ 68.92	− 33.52	−0.3	+0.1	+0.01	+0.02
2	−10	−137.08	+ 1.48	+0.4	0.0	0.00	−0.03
2	− 9	+116.52	+ 77.33	−0.4	−0.2	−0.01	+0.02
2	− 8	− 81.53	−102.58	+0.2	+0.2	+0.01	−0.01
2	− 7	+ 35.32	− 5.66	−0.1	0.0	0.00	0.00
2	− 6	−174.23	+167.85	+0.3	−0.3	−0.01	−0.01
2	− 5	+519.82	−168.31	−0.7	+0.2	+0.01	+0.03
2	− 4	−819.58	−154.22	+0.8	+0.2	+0.01	−0.04
2	− 3	+776.66	+652.36	−0.6	−0.5	−0.02	+0.02
2	− 2	−359.59	+220.33	+0.2	−0.1	0.00	−0.04
2	− 1	+132.32	+510.39	0.0	−0.1	0.00	0.00
2	+ 0	−438.84	+ 16.97	0.0	0.0	0.00	0.00
2	+ 1	+621.14	+ 7.45	+0.1	0.0	0.00	0.00
2	+ 2	−275.09	+899.25	−0.1	+0.4	+0.01	0.00
2	+ 3	− 62.14	− 7.46	0.0	0.0	0.00	0.00
2	+ 4	+ 86.72	+494.20	+0.1	+0.4	+0.01	0.00
2	+ 5	+283.86	−570.49	+0.3	−0.5	−0.02	−0.04
2	+ 6	−524.93	+284.79	−0.6	+0.3	+0.01	+0.02

Argument		Perturbative Function in units of 4th place		Pertbs. log a in units 7th pl.		Pertbs. mean long. in seconds of arc	
ς	ς'	cos	sin	cos	sin	cos	sin
2	+ 7	+432.50	+ 41.38	+0.5	+0.1	0.00	−0.02
2	+ 8	−177.99	−157.90	−0.2	0.2	−0.01	+0.01
2	+ 9	+ 0.20	+ 61.41	0.0	+0.1	0.00	0.00
2	+10	+ 1.84	+ 57.56	0.0	+0.1	0.00	0.00
2	+11	+ 68.18	− 76.98	+0.1	−0.1	0.00	0.00
2	+12	−100.27	+ 22.62	−0.2	0.0	0.00	+0.01
2	+13	+ 63.22	+ 25.96	+0.1	+0.1	0.00	0.00
2	+14	− 11.52	+ 9.42	0.0	0.0	0.00	0.00
2	+15	− 6.14	− 10.91	0.0	0.0	0.00	0.00
2	+16	− 9.32	+ 25.67	0.0	+0.1	0.00	0.00
2	+17	+ 28.08	− 13.03	+0.1	0.0	0.00	0.00
2	+18	− 25.42	+ 21.95	−0.1	+0.1	0.00	0.00
2	+19	+ 10.18	+ 12.36	0.0	0.0	0.00	0.00
2	+20	+ 1.30	− 4.50	0.0	0.0	0.00	0.00
3	−14	+ 0.01	− 0.10	0.0	−0.1	+35.0	+3.5
3	−16	− 23.7	− 16.5	+0.1	+0.1	0.00	0.00
3	−15	+ 35.0	− 5.2	−0.1	0.0	0.00	0.00
3	−14	+ 2.8	− 5.2	0.0	0.0	0.00	0.00
3	−13	− 25.7	− 33.5	+0.1	+0.1	0.00	0.00
3	−12	− 4.2	+ 70.0	0.0	−0.2	−0.01	0.00
3	−11	+ 48.7	+269.9	−0.1	−0.6	−0.02	0.00
3	−10	− 80.9	+ 55.8	+0.2	−0.1	0.00	−0.01
3	− 9	+ 11.3	− 13.0	−0.1	0.0	0.00	0.00
3	− 8	+ 32.1	+ 64.6	0.0	−0.1	0.00	0.00
3	− 7	− 43.9	−199.5	+0.1	+0.3	+0.01	0.00
3	− 6	− 98.3	+341.0	+0.4	−0.3	−0.01	0.00
3	− 5	+357.2	−158.7	−0.3	+0.1	0.00	+0.01
3	− 4	−472.0	+151.1	+0.3	−0.1	0.00	−0.01
3	− 3	+421.7	− 59.3	−0.2	0.0	0.00	+0.01
3	− 2	−227.5	−109.5	+0.1	0.0	0.00	0.00
3	− 1	+140.1	− 29.1	0.0	0.0	0.00	0.00
3	+ 0	−219.7	+159.5	0.0	0.0	0.00	0.00
3	+ 1	+193.5	− 99.9	0.0	0.0	0.00	0.00
3	+ 2	− 32.5	+ 40.7	0.0	0.0	0.00	0.00
3	+ 3	− 30.3	−110.5	0.0	0.0	0.00	0.00
3	+ 4	− 44.0	+110.3	0.0	+0.1	0.00	0.00
3	+ 5	+118.6	−125.5	+0.1	−0.1	0.00	0.00
3	+ 6	+ 42.1	−165.6	0.0	−0.1	0.00	0.00
3	+ 7	−208.1	+112.9	−0.2	+0.1	0.00	0.00
3	+ 8	+255.1	+ 47.0	+0.3	0.0	0.00	−0.01

Argument		Perturbative Function in units of 4th place		Pertbs. log a in units 7th pl.		Pertbs. mean long. in seconds of arc	
ς	ς'	cos	sin	cos	sin	cos	sin
3	+ 9	−143.7	−173.6	−0.2	−0.2	0.00	0.00
3	+10	+ 1.6	+158.0	0.0	+0.2	0.00	0.00
3	+11	+ 56.3	+114.9	+0.1	+0.1	0.00	0.00
3	+12	− 18.4	− 11.4	0.0	0.0	0.00	0.00
3	+13	− 42.7	+ 8.9	−0.1	0.0	0.00	0.00
3	+14	+ 43.6	+ 43.6	+0.1	+0.1	0.00	0.00
3	+15	− 7.2	− 70.0	0.0	−0.1	0.00	0.00
3	+16	− 11.7	+ 30.9	0.0	0.0	0.00	0.00
4	−15	− 21	+ 51	0.0	−0.1	No further sensible perturbations of mean longitude.	
4	−14	+ 55	− 45	−0.1	+0.1		
4	−13	− 56	+ 25	+0.1	0.0		
4	−12	+ 24	− 18	0.0	0.0		
4	−11	+ 4	+ 56	0.0	−0.1		
4	−10	+103	−108	−0.1	+0.1		
4	− 9	− 80	+126	+0.1	−0.1		
4	− 8	+149	− 64	−0.2	+0.1		
4	− 7	−148	− 56	+0.1	+0.1		
4	− 6	− 33	+149	0.0	−0.1		
4	− 5	+ 65	−147	0.0	+0.1		
4	− 4	−118	+ 72	+0.1	0.0		
4	− 3	+ 82	+ 10	0.0	0.0		
4	− 2	− 13	+ 23	0.0	0.0		
4	− 1	− 7	− 74	0.0	0.0		
4	+ 0	− 12	+ 96	0.0	0.0		
4	+ 1	− 7	− 76	0.0	0.0		
4	+ 2	+ 49	+ 79	0.0	0.0		
4	+ 3	− 48	−106	0.0	0.0		
4	+ 4	+ 20	+104	0.0	0.0		
4	+ 5	− 29	− 89	0.0	0.0		
4	+ 6	− 33	+125	0.0	+0.1		
4	+ 7	− 26	−180	0.0	−0.1		
4	+ 8	− 49	+186	0.0	+0.1		
4	+ 9	+104	−118	+0.1	−0.1		
4	+10	+ 7	+ 24	0.0	0.0		
4	+11	+ 6	+ 18	0.0	0.0		
4	+12	+ 64	+ 10	+0.1	0.0		
4	+13	− 70	− 65	−0.1	−0.1		
4	+14	+ 27	+ 85	0.0	+0.1		
4	+15	+ 17	− 17	0.0	−0.1		

VISUAL EXAMINATION OF *CAPELLA*,

BY W. J. HUSSEY.

Some months ago Professor CAMPBELL discovered that *Capella* is a spectroscopic binary having a period of 104 days. From the spectroscopic observations it is known that the orbit is nearly circular, and that the components are not very unequal in magnitude. The variations of the velocities of the components in the line of sight furnish the minimum dimensions of the orbit. A combination of these dimensions with ELKIN's parallax gives a result from which it appeared not impossible that *Capella* might at times be seen as a visual double star, thus forming a connecting link between the visual and spectroscopic binaries. The recent most favorable dates for the examination of the star to this end were April 15, June 6 and July 28. At these times the apparent distance between the components would be a maximum. On account of the orbit's being nearly circular the apparent distance has nearly its maximum value for a few days on either side of the dates given.

Early in June, I examined *Capella* on several afternoons with the thirty-six inch telescope without obtaining any

visual evidence of its being a double star. The results then obtained, however, were not regarded as conclusive, for the seeing, though good, was not such as to be rated as excellent according to our standards. These examinations were made when the star was from three to four hours west of the meridian.

On the nights of August 2d and 5th, I made further examinations of the star with the same telescope, using powers 1000, 1500, 1900 and 2600. With all powers the star appeared round. On these dates the seeing was excellent and stood all these powers perfectly. On the last date the seeing was perhaps the best. Even at Mt. Hamilton we have few nights that are better in the course of a year. With such conditions as then prevailed an elongation of a tenth of a second would have been readily perceptible with the lowest power used, and a considerably smaller distance would have been noted with the higher ones. At the times of these observations the star was between three and four hours east of the meridian. On August 5th, color screens of various shades were used a part of the time to reduce the light. On this date Mr. Perrine was with me. He also made a very careful examination of the star with all powers, without detecting any elongation.

Lick Observatory, Mt. Hamilton, Cal., 1900 *August* 10.

OBSERVATIONS OF THE SOLAR ECLIPSE OF MAY 28.

[Communicated by the Superintendent of the Coast and Geodetic Survey.]

Mr. G. R. Putnam, Assistant, Coast and Geodetic Survey, has just furnished the Superintendent with the following results in connection with the observations recently made at the total eclipse station near Wadesboro, N.C.

The position of the central point in the grounds was determined by astronomical observations, and gave the result:

Latitude	34° 57′ 52″
Longitude	80 4 27
"	5h 20m 17s.8

The latitude is based upon observations by the Talcott method on two nights, using nine pairs. The longitude depends on the transit of stars on five nights for local time, and a comparison with the noon telegraph signals from the Naval Observatory, at Washington. Two chronometers were used, and determinations were made on five days. An azimuth was determined, by means of which the meridians and other lines necessary for the location of the different instruments, were laid out.

The times of contact, reduced to 75th meridian mean time, were as follows:

1	19h 36m 19s.7
2	20 15 15.5
3	20 46 14.1
4	22 5 37.3

The observation of the first contact is noted as being from two to three seconds late. The third contact depends on Mr. Hoxie's record of the appearance of the flash of light on the face of the chronometer. The observations were made with the meridian telescope, swung out of the meridian. They were recorded on a chronograph with an observing key, and the time was also noted by the eye-and-ear method by the observer. As an additional check the time of the key tap was also recorded by another observer.

OBSERVATIONS OF *EROS*,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

1900—90th Merid. M.T.	Comp. Stars	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ
Sept. 19 12h 2m 28s	6	. . .	+1′ 9″.2	. . . .	+40° 30′ 42″.1
12 15 35	6	−0m 19s.20	. . .	2h 39m 14s.83	. . . .
12 26 5	6	. . .	+1 32.4	. . . .	+40 31 5.3

The $\Delta\alpha$ was measured direct = 218″.9.

Mean Places for 1900.0 of Comparison-Star.

α	Red. to app. place	δ	Red. to app. place	Authority
2h 39m 28s.74	+5s.29	+40° 29′ 21″.2	+11″.7	Deichmüller, Bonn A.G.C. 2351

Yerkes Observatory, Williams Bay, Wis.

OBSERVATIONS OF *EROS*,

MADE AT THE CHAMBERLIN OBSERVATORY, UNIVERSITY PARK, COLORADO,

By HERBERT A. HOWE.

The following observations give as corrections to the ephemeris of the planet by J. B. Westhaver, published in *A.J.*, No. 479, the results +1ˢ.6 in α, and +28″ in δ. He estimated magnitude of the planet was 13.

1900 Univ. Park M.T.		*	No. Comp.	Planet — * $\Delta\alpha$	Planet — * $\Delta\delta$	Planet's Apparent α	Planet's Apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
	h m s			m s	′ ″	h m s	° ′ ″		
May 27	15 1 54	1	20 , 6	−3 12.50	+ 5 16.3	23 47 3.43	+2 46 27.3	n9.616	0.736
	15 18 10	2	9 , 3	−3 24.13	−10 42.8	23 47 4.37	+2 46 38.6	n9.601	0.735
29	15 5 33	3	18 , 6	−1 1.09	+ 9 35.3	23 50 39.27	+3 20 19.2	n9.609	0.733
	15 13 54	4	8 , 3	−7 1.92	+ 2 33.4	23 50 39.82	+3 20 56.4	n9.601	0.732

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	23 50 14.00	+1.93	+2 41 0.7	+10.3	Boss, Albany A.G. 8197
2	23 50 26.57	+1.93	+2 57 11.2	+10.2	Boss, Albany A.G. 8198
3	23 51 38.36	+2.00	+3 11 3.3	+10.6	Boss, Albany A.G. 8204
4	23 57 39.75	+1.99	+3 18 12.6	+10.4	Boss, Albany A.G. 8233

OBSERVATIONS OF COMET a 1900 (GIACOBINI),

By R. G. AITKEN.

1900 Mt. Hamilton M.T.		*	No. Comp.	☄ — * $\Delta\alpha$	☄ — * $\Delta\delta$	☄'s apparent α	☄'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
	h m s			s	′ ″	h m s	° ′ ″		
May 25	15 35 33	2	4	. .	+0 42.5	. . .	+25 53 18.7	. .	0.634
	15 39 12	2	6	− 7.52	. . .	1 12 53.14	. . .	n9.711	. .
27	15 19 49	4	8 , 8	+18.55	−4 44.0	1 10 43.06	+26 35 7.8	n9.711	0.638
31	15 2 11	6	10 , 10	− 3.18	−3 53.9	1 5 49.28	+28 2 31.7	n9.721	0.624
June 24	14 2 30	8	10 , 8	− 8.19	+1 1.8	0 10 45.51	+38 45 6.9	n9.713	0.281
26	14 11 38	9	1	. .	−0 26.8	. . .	+39 46 18.4	. .	0.124
	14 26 32	9	10	+51.08	. . .	0 2 43.73	. . .	n9.664	. .
28	12 51 27	10	10 , 8	+10.07	+6 41.7	23 54 15.71	+10 45 27.2	n9.762	0.376
July 3	12 21 51	11	10 , 6	−11.14	−1 10.5	23 28 29.55	+13 12 9.8	n9.759	0.232
5	14 19 31	13	10 , 8	−13.57	+7 42.9	23 15 48.35	+41 9 49.5	n9.515	n9.623
22	10 50 37	14	10 , 10	+30.46	−0 31.0	20 56 52.48	+46 25 20.0	n9.542	n9.851

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	1 12 40.13	+1.42	+25 55 29.0	+2.7	Graham, Cambridge A.G. Catal. 743
2	1 12 59.24	+1.42	+25 52 33.5	+2.7	Micrometer comparison with (1)
3	1 9 52.32	+1.48	+26 39 23.3	+2.7	Graham, Cambridge A.G. Catal. 726
4	1 10 23.03	+1.48	+26 39 19.1	+2.7	Micrometer comparison with (3)
5	1 1 32.15	+1.63	+28 10 6.1	+2.8	Graham, Cambridge A.G. Catal. 672
6	1 5 50.83	+1.63	+28 6 22.8	+2.8	Micrometer comparison with (5)
7	0 7 26.75	+2.75	+38 43 54.1	+3.6	Lund A.G. Zones. BD. +38°14
8	0 10 50.95	+2.75	+38 44 1.5	+3.6	Micrometer comparison with (7)
9	0 1 49.78	+2.87	+39 46 41.3	+3.9	Lund A.G. Zones. BD. +39°5222
10	23 54 2.68	+2.96	+10 38 41.2	+4.3	Deichmüller, Bonn A.G. Catal. 18345
11	23 28 37.93	+3.26	+13 13 15.3	+5.0	Micrometer comparison with (12)
12	23 29 6.50	+3.26	+13 21 1.6	+5.0	Deichmüller, Bonn A.G. Catal. 17918
13	23 15 58.54	+3.38	+41 2 0.9	+5.7	" " " " 17681
14	20 56 18.50	+3.52	+46 25 41.7	+6.3	" " " " 14952

The observations of May 25, 27, June 24, and July 22, were made with the 36-inch, the others with the 12 inch equatorial. In every case $\Delta\delta$ was measured directly with the micrometer. The comet has a well-marked central condensation, but no sharp nucleus.

It is hardly as bright now as it was a month ago, when it was seen without trouble in the 3¼ inch finder. At the last observation its brightness was estimated as about that of a 12th or 13th magnitude star.

The comparison-star used on June 28 (Bonn A.G. Catal. 18345), is double — apparently a new pair — having a 13th magnitude companion at a distance of 3″.30 in 1877.0.

Lick Observatory, University of California, 1900 *July* 24.

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY OBSERVATORY,

By F. L. ADAMS, A. R. CURL and E. F. SMITH, Students in Astronomy.

W.M.T. 1899–1900		Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Isol. Sps. N	Isol. Sps. S	Totals Grps.	Totals Spts.	Def.
1899 Oct. 16	23[h]	0	0	0	0	0	0	0	0	F
18	23	0	0	0	0	0	0	0	0	G
20	4	0	0	0	0	0	0	0	0	G
21	1	0	0	0	0	0	0	0	0	G
24	23	0	1	0	16	0	0	1	16	G
26	3	0	2	0	24	0	0	2	24	G
26	23	0	3	0	27	0	0	3	27	G
30	0	0	1	0	7	0	0	1	7	P
Nov. 1	23	0	1	0	4	0	0	1	4	F
2	23	0	0	0	0	0	0	0	0	G
4	1	0	0	0	0	0	0	0	0	G
5	21	0	0	0	0	0	0	0	0	G
6	1	0	0	0	0	0	0	0	0	E
6	22	0	0	0	0	0	0	0	0	E
7	23	0	0	0	0	0	0	0	0	P
8	22	0	0	0	0	0	0	0	0	G
9	2	0	0	0	0	0	0	0	0	E
9	22	0	0	0	0	0	0	0	0	G
12	23	0	1	0	7	0	0	1	7	G
13	2	0	1	0	5	0	0	1	5	G
13	23	0	1	0	5	0	0	1	5	P
15	23	0	1	0	10	0	0	1	10	P
16	22	1	1	7	8	0	0	2	15	G
20	0	0	1	0	4	0	0	1	4	G
20	23	0	0	0	0	0	0	0	0	F
22	22	0	0	0	0	0	0	0	0	G
24	0	0	1	0	5	0	0	1	5	G
28	23	0	0	0	0	0	1	0	1	G
Dec. 4	22	0	1	0	3	0	0	1	3	F
6	1	0	1	0	6	0	0	1	6	G
6	23	0	1	0	6	0	0	1	6	G
8	0	0	1	0	7	0	0	1	7	G
8	22	0	0	0	0	0	1	0	1	F
12	22	1	1	5	2	0	0	2	7	G
13	22	1	1	9	2	0	1	2	12	G
15	1	1	2	7	4	0	1	3	12	F
16	0	1	1	5	7	0	1	2	13	G
18	0	1	2	3	16	0	0	3	19	G
19	23	0	0	0	0	0	0	0	0	G
20	23	0	0	0	0	0	0	0	0	G
1900 Jan. 3	22	0	0	0	0	0	0	0	0	P
8	0	0	0	0	0	0	0	0	0	G
8	22	0	0	0	0	0	0	0	0	P
10	21	0	0	0	0	0	0	0	0	F
15	0	2	0	6	0	0	0	2	6	G
16	1	1	0	13	0	1	0	1	14	E
17	1	1	0	6	0	0	0	1	6	P
18	22	0	0	0	0	0	0	0	0	P
22	1	0	0	0	0	0	0	0	0	G
23	1	0	0	0	0	0	1	0	1	F
23	22	0	1	0	3	0	0	1	3	P
25	23	0	1	0	3	0	0	1	3	P
26	3	1	1	1	1	0	0	2	2	P
26	23	1	1	3	7	0	0	2	10	F
29	1	1	0	4	0	0	0	1	4	G

W.M.T. 1900		Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Isol. Sps. N	Isol. Sps. S	Totals Grps.	Totals Spts.	Def.
1900 Jan. 29	23[h]	1	0	7	0	0	0	1	7	F
31	22	1	0	7	0	0	0	1	7	F
Feb. 1	22	1	0	2	0	0	0	1	2	F
3	0	0	0	0	0	1	0	0	1	F
6	23	1	1	21	10	0	0	2	31	E
13	23	0	0	0	0	0	0	0	0	G
16	4	0	0	0	0	0	0	0	0	P
16	23	0	0	0	0	0	0	0	0	P
18	22	0	0	0	0	0	0	0	0	G
20	23	0	0	0	0	0	0	0	0	E
23	22	0	0	0	0	0	0	0	0	G
26	22	0	0	0	0	0	0	0	0	P
27	22	0	0	0	0	0	0	0	0	P
Mar. 2	3	0	1	0	2	0	0	1	2	E
2	22	0	1	0	7	0	0	1	7	G
6	23	0	1	0	9	0	0	1	9	E
7	22	0	1	0	2	0	0	1	2	G
8	2	0	1	0	2	0	0	1	2	E
9	0	0	1	0	6	0	0	1	6	E
10	0	0	1	0	4	0	0	1	4	G
12	0	0	0	0	0	0	0	0	0	F
13	1	0	0	0	0	0	0	0	0	G
14	2	0	0	0	0	0	0	0	0	E
17	1	0	0	0	0	0	0	0	0	G
20	22	0	0	0	0	0	0	0	0	G
21	22	0	0	0	0	0	0	0	0	P
23	22	0	0	0	0	0	0	0	0	G
25	23	0	1	0	2	0	0	1	2	P
26	23	0	1	0	2	0	0	1	2	G
27	23	0	1	0	4	0	0	1	4	F
28	23	1	1	3	2	0	0	2	5	G
29	22	1	1	4	3	0	0	2	7	G
Apr. 1	22	0	1	0	6	0	0	1	6	E
3	23	0	1	0	9	0	0	1	9	E
1	23	0	1	0	2	0	0	1	2	F
5	23	0	1	0	3	0	0	1	3	G
6	22	0	0	0	0	1	1	0	2	G
7	1	1	1	2	1	0	0	2	3	E
8	22	1	0	1	0	0	0	1	1	F
9	22	1	0	2	0	0	0	1	2	G
13	22	1	0	2	0	0	0	1	2	G
15	22	0	0	0	0	0	0	0	0	F
19	2	1	0	1	0	0	0	1	1	E
24	3	1	1	1	11	0	0	2	12	F
24	22	2	1	13	14	0	0	3	27	F
25	23	1	1	9	7	1	1	2	18	P
26	3	2	1	9	9	0	0	3	18	E
28	2	1	1	–	–	0	0	2		VP very cloudy
29	22	1	1	12	18	0	0	2	30	F
30	23	1	1	11	15	0	0	2	26	G
May 1	23	1	1	4	14	0	0	2	18	F
3	23	1	1	2	25	0	0	2	27	G
10	23	0	0	0	0	0	0	0	0	E
		35	57	182	378	4	8	92	572	

Explanations concerning the work may be found in *A.J.* 466.

The number of different groups observed was 26, containing 236 different spots. Eleven groups were north of the equator, containing 91 spots, while 15 were south, containing 145 spots; one group, entered as south, being, however, extremely near to or on the equator. Fourteen had a latitude of less than 10°, 9 more than 10°, and 3 were on the parallel of 10°. Only 2 groups had a latitude of more than 15°.

The determinations of latitude and longitude indicate but one reappearance with any high degree of certainty. This group was nearly central April first, on its second rotation.

The longitudes show that the solar activity has been restricted to a quite definite region.

In north latitude all the groups, save one, were between longitudes 180° and 360°. All southern groups, save two, were between longitudes 130° and 360°.

When compared with the observations made during nearly the same period, one year earlier, all the phenomena indicate less solar activity.

There was no positive testimony collected as to the appearance presented by spots when upon the limb.

ELEMENTS OF COMET *b* 1900 (BORRELLY-BROOKS),

BY C. D. PERRINE.

From Mr. CRAWFORD's observation of July 25th, and my own of July 30th, and August 4th, I have computed the following elements:

$$T = 1900 \text{ August } 3^d.20726 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \omega &= 12^\circ\ 26'\ 13''.2 \\ \Omega &= 328\ \ \ 0\ 30.1 \\ i &= 62\ 30\ 46.3 \end{aligned}\right\} 1900.0$$

$$\log q = 0.006390$$

Residuals O – C: $\Delta\lambda' \cos\beta' = +1''.1$, $\Delta\beta' = -0''.9$

Mt. Hamilton, California, 1900 *August* 13.

CONSTANTS FOR THE EQUATOR OF 1900.0.

$$\begin{aligned} x &= r[9.945799] \sin(\ 86^\circ\ 21'\ 14''.2 + v) \\ y &= r[9.686698] \sin(283\ \ \ 9\ \ \ 4.5 + v) \\ z &= r[9.996636] \sin(\ \ \ 0\ 10\ \ \ 7.0 + v) \end{aligned}$$

The comet has faded rapidly since discovery. On August 4 it was barely visible without telescopic aid, the nucleus being only of about the 10th magnitude. The rapid loss of light still continues.

On July 28 the comet was examined with the 36-inch refractor. The nucleus was then of 8.5 magnitude, and distinctly elongated, although no separation could be detected.

OBSERVATIONS OF COMET *b* 1900 (BROOKS),

MADE WITH THE 12-INCH TELESCOPE OF THE LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,

BY RUSSELL TRACY CRAWFORD.

1900 Mt. Hamilton M.T.	*	No. Comp.	☌ – * $\Delta\alpha$	☌ – * $\Delta\delta$	☌'s apparent α	☌'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			m s	′ ″	h m s	° ′ ″		
July 25 14 48 18	1	8 , 8	+0 39.35	+2 20.8	2 45 48.91	+18 39 14.6	*n*9.613	0.579
Aug. 9 13 47 22	3	10 , 8	–0 43.10	–3 45.9	3 13 9.70	+62 16 1.9	*n*9.954	*n*9.544
10 15 49 2	4	12 , 8	+1 16.85	–6 21.1	3 17 2.92	+64 47 29.2	*n*9.850	*n*0.415
11 14 45 13	5	10 , 10	–1 37.29	–4 49.4	3 21 6.87	+67 0 4.8	*n*9.958	*n*0.517

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	2 45 6.69	+2.87	+18 36 44.5	+9.3	Connected with *2
2	2 45 2.85	+2.87	+18 44 57.0	+9.2	Auwers, Berlin A, A.G.C. 766
3	3 13 47.96	+4.84	+62 19 52.2	–4.4	Krueger, Helsingfors-Gotha, A.G.C. 2926
4	3 15 40.94	+5.13	+64 53 55.4	–5.1	Krueger, Helsingfors-Gotha, A.G.C. 2951
5	3 22 38.78	+5.38	+67 5 0.2	–6.0	Geelmuyden, Christiania, A.G.C. 596

In all cases $\Delta\alpha$ was obtained by transits.

An incomplete observation was made on the 24th of July, at which time the nucleus was estimated to be 6½ magnitude. On Aug. 11 the nucleus was estimated to be of the 11th magnitude, and the whole head to be of the 9½ magnitude.

Lick Observatory, University of California, 1900 *Aug.* 21.

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PTOLEMY'S PROBLEM.

By G. W. HILL.

In the case of the inferior planets PTOLEMY discovered the position of the line of apsides and the eccentricity of the eccentric circle by observations of greatest elongations. When two were found of the same magnitude, but situated on opposite sides of the mean Sun, the position of the line of apsides was discovered by bisecting the angle between the mean positions of the Sun. And the eccentricity was found by comparing the magnitude of the greatest elongations when the mean Sun was at either end of the line of apsides.

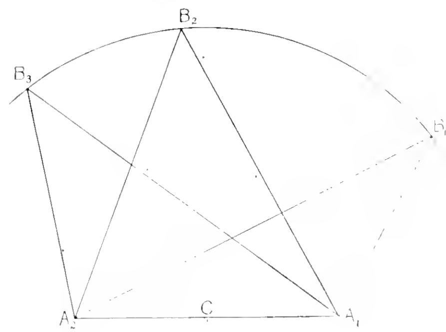
These devices are impracticable in the case of the superior planets, and PTOLEMY had recourse to observations made when the true longitude of the planet differed by 180° from the mean longitude of the Sun. Observations fulfilling this condition cannot generally be obtained, but, by interpolation between several observations in close proximity, the desired data may be got. At these special times the Earth, planet and center of the epicycle are in a right line: thus, as we do not have to deal with distance, but only with orientation, we need not pay any attention to the epicycle. It is understood that the period of the planet is known, there are then three unknowns to be determined, viz.: the position of the line of apsides, the eccentricity of the eccentric circle, and the epoch of the mean longitude; thus three observations are necessary.

What I have ventured to call PTOLEMY'S Problem may be stated in a geometrical form as follows:

On a given common base to construct three triangles such that, while their vertices are equidistant from the middle point of the base, the differences of the angles at each end of the base may be equivalent to given angles.

In the figure, let A_1A_2 be the common base, C its middle point, and B_1, B_2, B_3 the vertices of the three triangles. The angles $B_1A_1B_2$, $B_1A_1B_3$, $B_1A_2B_2$, $B_1A_2B_3$ are known. Adopt A_1C as the linear unit, and let A_2 be the point from which the observations are made, while A_1 is the point about which the center of the epicycle rotates uniformly. Take C for the origin of a system of rectangular co-ordinates, CA_1 being the positive direction of the axis of x. Let the general equations of the radii A_1B and A_2B be severally

$$y = \tan\mu.(x-1) \quad , \quad y = \tan\nu.(x+1)$$



where μ and ν denote severally the mean and true anomaly measured, as with PTOLEMY, from the aphelion. In order to have the rectangular coördinates of the point B, we solve these equations, regarding x and y as the unknowns, and get

$$x = \frac{\tan\mu + \tan\nu}{\tan\mu - \tan\nu} = \frac{\sin(\mu+\nu)}{\sin(\mu-\nu)}$$

$$y = \frac{2\tan\mu\tan\nu}{\tan\mu - \tan\nu} = \frac{\cos(\mu-\nu) - \cos(\mu+\nu)}{\sin(\mu-\nu)}$$

It will be more convenient to employ as the variables $\chi = \mu + \nu$ and $\psi = \mu - \nu$. Then, as a constant quantity,

$$BC^2 = x^2 + y^2 = \frac{\sin^2\chi + (\cos\chi - \cos\psi)^2}{\sin^2\psi}$$

But, if $2u^{-1}$ is put for $x^2 + y^2 + 1$, this equation can be given the form

$$u - u\cos\chi\cos\psi = \sin^2\psi$$

This equation constitutes the relation between the true and mean anomalies in PTOLEMY'S theory of eccentric circles. It holds for each of the three observations, and

we thus have the data requisite for the determination of the three unknowns u, χ and ψ. If the latter symbols are used for the first observation we should add to them certain known arcs for the second and third observations. But it is conducive to symmetry to suppose that the known arcs to be added to χ and ψ in the several observations in their order are $\alpha_1, \alpha_2, \alpha_3$, and again $\beta_1, \beta_2, \beta_3$. It will shorten the writing of some of the formulas if we impose upon the β the condition $\beta_1+\beta_2+\beta_3 = 0$. As to the α, the computation is shortened if we suppose one of them vanishes. To abbreviate we write ψ for $\psi+\beta_i$, then the equations for solution are

$$\begin{aligned} u - u\cos(\chi+\alpha_1)\cos\psi_1 &= \sin^2\psi_1, \\ u - u\cos(\chi+\alpha_2)\cos\psi_2 &= \sin^2\psi_2, \\ u - u\cos(\chi+\alpha_3)\cos\psi_3 &= \sin^2\psi_3. \end{aligned}$$

By developing $\cos(\chi+\alpha)$ we have the modified form:

$$\begin{aligned} u - \cos\alpha_1\cos\psi_1 \,.\, u\cos\chi + \sin\alpha_1\cos\psi_1 \,.\, u\sin\chi &= \sin^2\psi_1 \\ u - \cos\alpha_2\cos\psi_2 \,.\, u\cos\chi + \sin\alpha_2\cos\psi_2 \,.\, u\sin\chi &= \sin^2\psi_2 \\ u - \cos\alpha_3\cos\psi_3 \,.\, u\cos\chi + \sin\alpha_3\cos\psi_3 \,.\, u\sin\chi &= \sin^2\psi_3 \end{aligned}$$

Let these equations be regarded as linear, and as determining the unknowns u, $u\cos\chi$, and $u\sin\chi$. Employ Δ to denote the determinant formed from the coefficients of the equations, and S to denote summation with respect to the cyclical permutation of the subscripts 1, 2, 3; thus, of three terms only one will be written, the remaining two being derived from this by the mentioned cyclical permutation. Then the expression for Δ is

$$\Delta = S.\sin(\alpha_1-\alpha_2)\cos\psi_1\cos\psi_2$$

and the values of the unknowns u, $u\cos\chi$, $u\sin\chi$ are given by the expressions

$$\begin{aligned} \Delta u &= S.\sin(\alpha_2-\alpha_3)\cos\psi_2\cos\psi_3\sin^2\psi_1 \\ \Delta u\cos\chi &= S.[\sin\alpha_2\cos\psi_2-\sin\alpha_3\cos\psi_3]\sin^2\psi_1 \\ \Delta u\sin\chi &= S.[\cos\alpha_2\cos\psi_2-\cos\alpha_3\cos\psi_3]\sin^2\psi_1 \end{aligned}$$

The substitution of these values in the identity

$$(\Delta u\cos\chi)^2 + (\Delta u\sin\chi)^2 \equiv (\Delta u)^2$$

gives an equation involving only the unknown ψ, and which serves to determine this quantity.

With advantage these expressions may be transformed: thus, in case of the second,

$$\Delta u\cos\chi = S.\sin\alpha_1\cos\psi_1[\sin^2\psi_3-\sin^2\psi_2]$$

and by applying the formula

$$\sin^2 x - \sin^2 y = \sin(x-y)\sin(x+y)$$

this becomes, mindful of the relation $\beta_1+\beta_2+\beta_3 = 0$,

$$\Delta u\cos\chi = \tfrac{1}{2}S.\sin\alpha_1\sin(\beta_3-\beta_2)[\sin 3\psi+\sin(\psi-2\beta_1)]$$

In like manner

$$\begin{aligned} \Delta u\sin\chi &= \tfrac{1}{2}S.\cos\alpha_1\sin(\beta_3-\beta_2)[\sin 3\psi+\sin(\psi-2\beta_1)] \\ \Delta u &= \tfrac{1}{4}S.\sin(\alpha_2-\alpha_3)[2\sin(\beta_1-\beta_2)\sin(\beta_1-\beta_3) \\ &\quad +\cos(\beta_2-\beta_3)+2\sin(\beta_1-\beta_2)\sin(2\psi-\beta_3) \\ &\quad +2\sin(\beta_1-\beta_3)\sin(2\psi-\beta_2)-\cos(4\psi+\beta_1)] \end{aligned}$$

If we put

$$\begin{aligned} A_1 {}^{\cos}_{\sin}\alpha_1 &= \pm\tfrac{1}{2}S.\sin\alpha_1\sin(\beta_3-\beta_2){}^{\cos}_{\sin}2\beta_1 \\ A_2 &= \tfrac{1}{2}S.\sin\alpha_1\sin(\beta_3-\beta_2) \\ A_3 {}^{\cos}_{\sin}\alpha_2 &= \pm\tfrac{1}{2}S.\cos\alpha_1\sin(\beta_3-\beta_2){}^{\cos}_{\sin}2\beta_1 \\ A_4 &= \tfrac{1}{2}S.\cos\alpha_1\sin(\beta_3-\beta_2) \\ A_5 &= \tfrac{1}{4}S.\sin(\alpha_2-\alpha_3)[2\cos(\beta_2-\beta_3)-\cos 3\beta_1] \\ A_6 {}^{\cos}_{\sin}\alpha_3 &= -\tfrac{1}{4}S.\sin(\alpha_2-\alpha_3)[\cos(\beta_2-\beta_3){}^{\cos}_{\sin}2\beta_1 \pm {}^{\cos}_{\sin}\beta_1] \\ A_7 {}^{\cos}_{\sin}\alpha_4 &= -\tfrac{1}{4}S.\sin(\alpha_2-\alpha_3){}^{\cos}_{\sin}\beta_1 \end{aligned}$$

we shall have

$$\begin{aligned} \Delta u\cos\chi &= A_1\sin(\psi+\alpha_1)+A_2\sin 3\psi \\ \Delta u\sin\chi &= A_3\sin(\psi+\alpha_2)+A_4\sin 3\psi \\ \Delta u &= A_5+A_6\cos(2\psi+\alpha_3)+A_7\cos(4\psi+\alpha_4) \end{aligned}$$

From these expressions can be derived the algebraic equation on which the solution of the problem depends. Let us adopt $\tan\psi = x$ as the unknown. Then

$$\sin\psi = x(1+x^2)^{-\frac{1}{2}}, \quad \cos\psi = (1+x^2)^{-\frac{1}{2}},$$
$$\sin 2\psi = 2x(1+x^2)^{-1}, \quad \cos 2\psi = (1-x^2)(1+x^2)^{-1}$$
$$\sin 3\psi = 3x(1+x^2)^{-\frac{1}{2}} - 4x^3(1+x^2)^{-\frac{3}{2}},$$
$$\sin 4\psi = 4x(1-x^2)(1+x^2)^{-2},$$
$$\cos 4\psi = (1-4x^2+2x^4)(1+x^2)^{-2}$$

For brevity put

$$\Delta u(1+x^2)^{\frac{3}{2}} = \rho$$

then

$$\begin{aligned} \rho\cos\chi &= B_1(1+x^2)+B_2x+B_3x^3 \\ \rho\sin\chi &= B_4(1+x^2)+B_5x+B_6x^3 \\ \rho x(1+x^2) &= B_7+B_8x+B_9x^2+B_{10}x^3+B_{11}x^4 \end{aligned}$$

where the coefficients B have the following values:

$$\begin{aligned} B_1 &= -S.\sin\alpha_1\sin(\beta_3-\beta_2)\sin\beta_1\cos\beta_1 \\ B_2 &= S.\sin\alpha_1\sin(\beta_3-\beta_2)(1+\cos^2\beta_1) \\ B_3 &= -S.\sin\alpha_1\sin(\beta_3-\beta_2)\sin^2\beta_1 \\ B_4 &= -S.\cos\alpha_1\sin(\beta_3-\beta_2)\sin\beta_1\cos\beta_1 \\ B_5 &= S.\cos\alpha_1\sin(\beta_3-\beta_2)(1+\cos^2\beta_1) \\ B_6 &= -S.\cos\alpha_1\sin(\beta_3-\beta_2)\sin^2\beta_1 \\ B_7 &= \tfrac{1}{2}S.\sin(\alpha_2-\alpha_3)[\cos(\beta_2-\beta_3)+\cos\beta_1]\sin^2\beta_1 \\ B_8 &= S.\sin(\alpha_2-\alpha_3)[\cos(\beta_2-\beta_3)\cos\beta_1+1]\sin\beta_1 \\ B_9 &= \tfrac{1}{4}S.\sin(\alpha_2-\alpha_3)[2\cos(\beta_2-\beta_3)+2\cos\beta_1-\cos 3\beta_1] \\ B_{10} &= \tfrac{1}{2}S.\sin(\alpha_2-\alpha_3)[2\cos(\beta_2-\beta_3)\cos\beta_1-1]\sin\beta_1 \\ B_{11} &= \tfrac{1}{4}S.\sin(\alpha_2-\alpha_3) \\ &\qquad [4\cos(\beta_2-\beta_3)\cos\beta_1-1-4\cos^2\beta_1]\cos\beta_1 \end{aligned}$$

The equation in x is then

$$\left\{\begin{array}{l} \;\;[B_1(1+x^2)+B_2x+B_3x^3]^2 \\ +[B_4(1+x^2)+B_5x+B_6x^3]^2 \end{array}\right\}(1+x^2) = [B_7+B_8x+B_9x^2+B_{10}x^3+B_{11}x^4]^2$$

and thus is of the eighth degree. We do not elaborate it further, as it is less laborious to employ the expressions for Δu, $\Delta u\cos\chi$ and $\Delta u\sin\chi$.

As an illustration take the example given by Ptolemy in the case of *Mars*. Here, at the three selected oppositions of *Mars* with the mean Sun, the observed longitudes of the planet in their order were 81°, 148° 50′ and 242° 34′.

From the known period it is gathered that, exclusive of whole circumferences, motion in the mean longitude between the first and second observations was 81° 14′, and between the second and third 95° 28′. Making $\alpha = 0°$ and adopting the condition $\beta_1 + \beta_2 + \beta_3$, these data make

$$\alpha_1 = 0° \qquad \alpha_2 = 149°34' \qquad \alpha_3 = 338°16'$$
$$\beta_1 = -9°50'40'' \qquad \beta_2 = +\ 4°\ 3'20'' \qquad \beta_3 = +\ 5\ 17'20''$$

The substitution of these values in the formulas gives (logarithms are in [ ]).

$$Ju\cos\chi = [9.0480661]\sin(\psi + 170°32'22''.35) - [9.0182551]\sin 3\psi$$
$$Ju\sin\chi = [9.3825734]\sin(\psi - 8°\ 2'37''.59) + [9.3860693]\sin 3\psi$$
$$Ju = -0.08286966 + [8.8855859]\cos(2\psi + 95°49'15''.62) + [8.9513935]\cos(4\psi + 8°21'48''.29)$$

By moving ψ from 0° to 180° we discover the following six real solutions of the problem:

	1	2	3
	° ′ ″	° ′ ″	° ′ ″
ψ	1 37 22.78	2 47 43.58	71 28 20.82
χ	132 11 58.15	283 57 57.55	294 21 58.28
log u	8.0893041	8.2967311	9.9836184
	4	5	6
ψ	85 9 41.39	85 30 8.91	96 52 54.68
χ	77 23 26.50	284 47 47.15	117 14 48.82
log u	9.9958862	0.0008790	9.9886814

The two remaining roots of the equation of the 8th degree are imaginary.

Although all the six solutions satisfy the equations, the second is the only one which fulfills all the conditions of the problem. Those not involved in the equations are the following: μ and ν must together lie between 0° and 180° or between 180° and 360°, and, in the first case, μ must exceed ν, and, in the second case, ν must exceed μ. Every right line drawn through a point has two orientations differing 180°; in the equations this duplicity is left undecided, but, in the problem, that orientation must be chosen which is directed towards the point of intersection on the circumference.

The values of the mean and true anomalies, at the times of the several observations, given by the second solution are:

μ	ν
° ′ ″	° ′ ″
318 27 30.57	325 30 26.98
40 11 30.57	33 20 26.98
135 39 30.57	127 1 26.98

Whence it follows that the longitude of the apogee is 115° 29′ 33″.01; and, from u we deduce that the eccentricity of the orbit $= \dfrac{1}{BC} = 0.1000026$. PTOLEMY's values of these quantities are 115° 30′ and 0.1. His procedure in treating the problem virtually consists in the assumption that the eccentricity is so small that, for a first approximation, we may put unity for $\cos\psi$ in the equation defining the connection between the two anomalies. This makes the values of the unknowns depend on equations of the first degree. The linearity of the equations is maintained in the following approximations by computing the length of certain lines in the geometrical figure from the elements of the preceding approximation. PTOLEMY's method is ingenious but tedious, at least in the narration of it.

OBSERVATIONS OF ONE HUNDRED NEW DOUBLE STARS.

SECOND CATALOGUE.

By WILLIAM J. HUSSEY.

In *A.J.* 480 I published observations of one hundred double stars which I had then discovered. The present paper, which may be regarded as a continuation of that one, contains measures of another hundred new pairs of the same class. With respect to the distance between the components the stars of the present list have the following classification:

0″.25 or less,	9 pairs
0.26 to 0.50,	16 "
0″.51 to 1″.00,	22 pairs
1.01 to 2.00,	26 "
2.01 to 5.00,	27 "

These stars have been discovered with the 12- and 36-inch telescopes. Most of the measures have been made with the latter instrument. The power generally used was 1000, except for the very close pairs for which 1500 and 1900 were employed.

The positions of the stars are for the epoch 1900.

101. DM. +51°746.
$\alpha = 3^h 25^m 41^s$; $\delta = +51°38'.8$.

	°	″	M	M
1900.198	245.3	0.72	9.5	9.5
.204	249.3	0.75	9.3	9.5
1900.20	247.3	0.74	9.4	9.5

102. DM. +18°959.
$\alpha = 3^h 29^m 33^s$; $\delta = +48°20'.4$.

	°	″	M	M
1900.198	60.5	2.98	9.0	10.0
.204	62.1	3.13	9.2	11.0
1900.20	61.3	3.06	9.1	10.5

103. DM. +49°1014.
$\alpha = 3^h 38^m 5^s$; $\delta = +49°32'.8$.

	°	″	M	M
1900.198	209.7	0.78	8.0	8.2
.204	205.7	0.87	8.2	8.5
1900.20	207.7	0.84	8.1	8.4

104. DM. −12°982.

$\alpha = 4^h\ 41^m\ 11^s\ :\ \delta = -12°\ 7'.9.$

	°	″	M	M
1900.067	265.7	0.90	7.7	11.5
.128	262.6	1.02	. .	. .
1900.10	264.2	0.96	7.7	11.5

There is also a companion of the 9th magnitude in the direction 289° at a distance of 10″.7. With the principal star it forms Σ 596.

105. DM. +21°915.

$\alpha = 5^h\ 37^m\ 12^s\ :\ \delta = +21°\ 21'.6$

1900.204	190.2	1.47	9.0	10.5
.297	192.3	1.45	9.0	11.0
1900.25	191.3	1.46	9.0	10.8

106. DM. −11°1396.

$\alpha = 6^h\ 4^m\ 39^s\ :\ \delta = -11°\ 39'.8.$

1900.128	333.0	0.90	9.0	9.2
.207	333.0	0.80	9.0	9.5
.243	333.1	0.88	9.0	9.2
1900.19	333.0	0.86	9.0	9.3

107. DM. −10°1143.

$\alpha = 6^h\ 11^m\ 4^s\ ;\ \delta = -10°\ 48'.1.$

1900.128	324.5	0.36	8.5	8.5
.207	325.2	0.34	8.5	8.7
.243	326.0	0.31	8.8	9.0
1900.19	325.2	0.35	8.6	8.7

108. DM. −10°1452.

$\alpha = 6^h\ 12^m\ 38^s\ :\ \delta = -10°\ 41'.4.$

1900.128	331.2	3.39	9.0	12.5
.207	330.6	3.22	9.0	11.5
.243	331.5	3.42	9.0	12.0
1900.19	331.1	3.31	9.0	12.0

109. DM. −10°1516.

$\alpha = 6^h\ 20^m\ 41^s\ :\ \delta = -10°\ 34'.8.$

1900.128	71.1	0.36	9.2	9.2
.207	64.5	0.45	9.2	9.2
.242	70.1	0.32	9.5	10.0
1900.19	68.7	0.38	9.3	9.5

110. DM. −10°1521

$\alpha = 6^h\ 21^m\ 16^s\ :\ \delta = -10°\ 5'.6.$

1900.207	131.0	2.27	9.2	9.5
.242	132.1	2.24	9.5	9.8
1900.22	131.7	2.25	9.4	9.6

111. DM. −11°1728.

$\alpha = 6^h\ 55^m\ 24^s\ :\ \delta = -11°\ 52'.0.$

1900.217	17.2	2.88	8.7	8.8
.243	19.0	3.14	8.5	8.5
.246	18.0	2.98	8.8	8.8
1900.21	18.1	3.00	8.7	8.7

112. DM. −11°1747.

$\alpha = 6^h\ 57^m\ 8^s\ :\ \delta = -11°\ 9'.5.$

	°	″	M	M
1900.246	188.0	0.53	7.5	8.0
.249	191.1	0.58	7.5	8.5
.262	191.0	0.55	. .	. .
1900.25	191.0	0.55	7.5	8.2

113. DM. −13°1919.

$\alpha = 7^h\ 12^m\ 31^s\ :\ \delta = -13°\ 48'.6.$

1900.051	53.1	1.78	8.5	12.0
.128	55.3	1.64	8.5	13.0
.220	53.2	1.82	7.5	13.0
1900.13	53.9	1.75	8.2	12.7

114. DM. −13°2182.

$\alpha = 7^h\ 38^m\ 0^s\ :\ \delta = -14°\ 3'.8.$

1900.220	218.3	1.61	8.8	12.5
.243	220.7	1.56	8.5	13.5
1900.23	219.5	1.58	8.6	13.0

115. DM. −13°2439.

$\alpha = 8^h\ 8^m\ 31^s\ :\ \delta = -13°\ 36'.2.$

1900.292	129.6	0.88	9.0	10.0
.297	129.1	1.04	9.0	10.0
.311	125.6	1.14	9.0	10.0
1900.30	128.1	1.02	9.0	10.0

116. DM. −10°2495.

$\alpha = 8^h\ 17^m\ 48^s\ ;\ \delta = -10°\ 22'.0.$

1900.226	171.5	1.74	9.0	9.5
.292	167.6	1.85	9.0	9.5
.311	172.4	1.86	9.0	9.2
1900.28	170.5	1.82	9.0	9.4

This is the principal component of *h* 784. The companion noted by HERSCHEL is about the 10th magnitude, in the direction 7°.0, at a distance 16″.92.

117. DM. −11°2388.

$\alpha = 8^h\ 29^m\ 10^s\ ;\ \delta = -11°\ 52'.1.$

1900.207	1.3	1.43	8.5	13.0
.220	3.0	1.57	8.5	13.0
1900.21	2.2	1.50	8.5	13.0

118. DM. −14°2612.

$\alpha = 8^h\ 35^m\ 50^s\ ;\ \delta = -14°\ 17'.9.$

1900.207	324.4	1.90	9.0	10.5
.243	323.8	1.70	9.0	10.0
.311	325.0	. .	9.0	11.0
1900.26	324.4	1.80	9.0	10.5

119. DM. −13°2668.

$\alpha = 8^h\ 40^m\ 42^s\ ;\ \delta = -13°\ 44'.7.$

1900.229	356.7	2.74	8.5	9.5
.243	356.9	2.95	8.3	8.8
.246	356.0	2.72	8.5	9.5
1900.24	356.5	2.80	8.4	9.3

120. DM. −13°2670.

$\alpha = 8^h\ 41^m\ 26^s\ :\ \delta = -13°\ 59'.8.$

	°	″	M	M
1900.229	63.3	0.47	8.5	8.8
.246	58.6	0.42	8.5	8.8
1900.24	61.0	0.45	8.5	8.8

121. DM. −10°2642.

$\alpha = 8^h\ 42^m\ 19^s\ :\ \delta = -10°\ 31'.2.$

1900.229	99.5	3.88	8.8	11.5
.243	99.9	3.88	. .	. .
.246	95.8	4.03	8.8	12.0
1900.24	98.4	3.93	8.8	11.8

122. DM. −12°2719.

$\alpha = 8^h\ 53^m\ 4^s\ :\ \delta = -12°\ 42'.5.$

1900.240	258.9	0.91	9.0	11.0
.311	258.7	0.90	8.8	10.5
1900.28	258.8	0.90	8.9	10.8

123. DM. +63°820.

$\alpha = 8^h\ 58^m\ 9^s\ :\ \delta = +63°\ 25'.9.$

1900.424	228.2	0.45	8.8	9.0
.442	228.3	0.60	9.0	9.2
1900.43	228.3	0.52	8.9	9.1

124. DM. +61°1102.

$\alpha = 9^h\ 1^m\ 23^s\ :\ \delta = +60°\ 56'.8.$

1900.442	131.8	1.96	8.5	13.0
.450	129.1	2.03	8.5	11.0
1900.45	130.4	2.00	8.5	12.0

125. DM. −12°2839.

$\alpha = 9^h\ 10^m\ 23^s\ :\ \delta = -12°\ 27'.0.$

1900.204	104.3	3.11	8.5	11.5
.220	105.5	3.05	8.5	12.5
.377	102.6	3.31	8.5	12.5
1900.27	104.1	3.16	8.5	12.2

126. DM. −11°2604.

$\alpha = 9^h\ 13^m\ 17^s\ ;\ \delta = -11°\ 54'.0.$

1900.204	87.5	2.75	8.5	10.0
.220	87.5	2.91	8.5	11.0
.246	87.9	2.89	8.6	11.0
1900.22	87.6	2.85	8.5	10.7

127. DM. −10°2854.

$\alpha = 9^h\ 26^m\ 52^s\ ;\ \delta = -10°\ 58'.4.$

1900.298	87.3	0.65	9.5	10.0
.377	92.4	0.61	9.2	9.5
1900.34	89.9	0.63	9.4	9.8

128. DM. −11°2993.

$\alpha = 10^h\ 55^m\ 18^s\ :\ \delta = -11°\ 12'.9.$

1900.240	46.8	1.18	8.5	11.0
.297	42.9	1.09	8.5	11.0
.377	48.5	1.05	8.5	11.5
1900.30	46.1	1.11	8.5	11.2

129. DM. $-12°3393$.

$\alpha = 11^h\ 13^m\ 15^s$; $\delta = -12°\ 50'.5$.

	°	″	M	M
1900.240	350.0	0.59	9.0	11.5
.377	350.4	0.72	9.0	10.0
1900.31	350.2	0.66	9.0	10.8

130. DM. $-10°3239$.

$\alpha = 11^h\ 13^m\ 52^s$; $\delta = -11°\ 13'.2$.

1900.240	134.8	1.20	8.0	8.2
.243	134.5	1.20	8.5	8.6
.262	134.0	1.16	8.0	8.3
1900.25	134.4	1.19	8.2	8.4

131. DM. $-13°3409$.

$\alpha = 11^h\ 31^m\ 26^s$; $\delta = -13°\ 21'.0$

1900.243	156.6	3.12	9.0	10.5
.273	160.4	3.24	9.0	10.0
.374	157.2	3.29	9.0	10.0
1900.30	158.1	3.22	9.0	10.2

132. DM. $-11°3161$.

$\alpha = 11^h\ 55^m\ 42^s$; $\delta = -11°\ 35'.9$.

1900.128	60.9	1.45	8.0	9.0
.243	61.7	1.41	8.0	9.0
.374	62.7	1.44	8.0	9.0
1900.25	61.8	1.44	8.0	9.0

133. DM. $-21°3491$.

$\alpha = 12^h\ 7^m\ 4^s$; $\delta = -21°\ 57'.8$.

1900.298	329.6	1.52	8.5	8.5
.300	328.2	1.56	8.5	8.5
.412	330.7	1.54	9.0	10.0
1900.34	329.5	1.54	8.7	9.0

134. DM. $-11°3337$.

$\alpha = 12^h\ 31^m\ 39^s$; $\delta = -11°\ 49'.8$.

1900.374	54.5	2.67	8.5	10.0
.377	57.1	2.55	8.5	10.5
.415	55.6	2.48	8.5	11.0
1900.39	55.7	2.57	8.5	10.5

135. DM. $-12°3700$.

$\alpha = 12^h\ 43^m\ 49^s$; $\delta = -13°\ 4'.9$.

1900.243	353.6	3.16	8.6	9.8
.298	352.8	3.31	8.5	9.0
.374	353.1	3.58	9.0	9.5
.377	352.7	3.48	8.5	9.0
1900.32	353.0	3.38	8.7	9.3

136. DM. $-17°3745$.

$\alpha = 12^h\ 45^m\ 13^s$; $\delta = -18°\ 2'.5$.

1900.292	133.4	0.78	9.0	9.2
.412	130.1	0.73	9.0	9.5
1900.35	131.7	0.75	9.0	9.4

137. DM. $-11°3421$.

$\alpha = 12^h\ 57^m\ 50^s$; $\delta = -11°\ 37'.1$.

	°	″	M	M
1900.415	120.9	3.62	9.0	9.2
.494	120.0	3.57	9.2	9.5
.513	121.1	3.54	9.0	9.0
1900.47	120.7	3.58	9.1	9.2

138. DM. $-6°3957$.

$\alpha = 14^h\ 10^m\ 44^s$; $\delta = -6°\ 35'.8$.

1900.128	58.6	0.44	8.8	8.8
.377	58.9	0.47	8.8	9.0
.423	58.3	0.56	8.5	8.7
1900.31	58.6	0.49	8.7	8.8

139. DM. $-10°3865$.

$\alpha = 14^h\ 11^m\ 50^s$; $\delta = -11°\ 11'.5$.

1900.240	118.2	0.90	9.5	9.5
.377	121.3	0.98	9.0	9.5
.418	117.9	0.82	9.0	9.2
1900.34	119.1	0.90	9.2	9.4

140. DM. $-12°4079$.

$\alpha = 14^h\ 27^m\ 8^s$; $\delta = -12°\ 33'.8$.

1900.377	179.7	1.01	8.8	9.0
.418	183.3	1.16	8.5	9.0
.423	184.1	1.24	8.5	8.8
.470	182.4	1.25	8.2	8.8
1900.42	182.4	1.16	8.5	8.9

141. DM. $-10°3967$.

$\alpha = 14^h\ 43^m\ 47^s$; $\delta = -10°\ 24'.7$.

1900.371	323.9	0.42	7.5	8.5
.418	324.0	0.33	7.5	8.5
.470	322.3	0.37	7.5	9.0
1900.42	323.4	0.37	7.5	8.7

142. DM. $-12°4165$.

$\alpha = 14^h\ 50^m\ 19^s$; $\delta = -12°\ 48'.1$.

1900.470	10.1	2.45	8.5	12.5
.491	13.7	2.51	8.5	12.0
.494	10.0	2.50	8.5	12.5
1900.48	11.3	2.49	8.5	12.3

143. DM. $+55°1733$.

$\alpha = 15^h\ 4^m\ 42^s$; $\delta = +55°\ 38'.8$.

1900.576	125.9	0.69	9.0	9.5
.579	126.8	0.75	9.2	9.5
.620	128.6	0.79	9.0	9.2
1900.59	127.1	0.74	9.1	9.4

144. DM. $+20°3075$.

$\alpha = 15^h\ 6^m\ 5^s$; $\delta = +20°\ 13'.7$.

1900.576	242.1	0.81	8.5	11.5
.579	238.5	0.68	9.0	10.5
.588	247.8	0.59	8.8	10.5
.620	241.1	0.57	8.8	11.5
1900.59	242.4	0.66	8.8	11.0

145. DM. $+53°1772$.

$\alpha = 15^h\ 11^m\ 11^s$; $\delta = +52°\ 57'.9$.

	°	″	M	M
1900.576	129.3	1.81	9.0	12.5
.579	128.2	. .	9.0	12.5
.620	131.0	2.07	9.0	12.5
1900.59	129.5	1.94	9.0	12.5

146. DM. $+21°2759$.

$\alpha = 15^h\ 16^m\ 32^s$; $\delta = +21°\ 25'.5$.

1900.5888	172.4	0.27	8.8	9.0
.620	171.4	0.26	8.5	9.0
.623	171.7	0.23	8.8	9.0
1900.61	171.8	0.25	8.7	9.0

147. DM. $+53°1774$.

$\alpha = 15^h\ 17^m\ 33^s$; $\delta = +53°\ 30'.3$.

1900.415	293.7	0.50	9.0	9.4
.423	293.5	0.53	9.2	9.8
.620	295.3	0.54	9.5	9.7
1900.49	294.2	0.52	9.2	9.6

148. DM. $+55°1748$.

$\alpha = 15^h\ 19^m\ 51^s$; $\delta = +55°\ 37'.7$.

1898.400	200.5	1.46	. .	. .
1900.412	200.6	1.42	9.0	10.0
.415	201.0	1.56	9.0	9.5
1899.71	200.7	1.48	9.0	9.8

149. DM. $+54°1715$.

$\alpha = 15^h\ 21^m\ 52^s$; $\delta = +54°\ 34'.0$.

1900.415	294.0	0.22	7.0	7.0
.423	293.3	0.21	7.0	7.2
.620	299.9	0.18	7.5	7.6
.623	295.2	0.21	7.0	7.0
1900.52	295.6	0.21	7.1	7.2

150. DM. $+21°2774$.

$\alpha = 15^h\ 24^m\ 18^s$; $\delta = +20°\ 58'.5$.

1900.576	26.3	4.40	9.0	9.5
.579	27.7	4.56	9.0	9.5
.588	25.9	4.12	9.0	9.2
1900.58	26.6	4.16	9.0	9.4

151. DM. $-13°4200$.

$\alpha = 15^h\ 29^m\ 32^s$; $\delta = -13°\ 20'.3$.

1900.374	310.9	1.00	8.8	13.0
.415	313.0	1.28	8.5	12.5
.418	308.9	1.09	8.0	13.0
1900.40	310.9	1.12	8.4	12.8

152. DM. $+52°1905$.

$\alpha = 15^h\ 44^m\ 14^s$; $\delta = +52°\ 17'.1$.

1900.415	246.9	3.35	7.5	11.5
.423	248.0	3.70	8	11
.579	245.6	3.55	7.8	12.0
1900.47	246.8	3.53	7.8	11.5

153. DM. −12°4353.

$\alpha = 15^h\ 46^m\ 25^s$; $\delta = -12°\ 14'.2$.

	°	″	M	M
1900.415	78.9	0.30	8.0	8.5
.418	78.8	0.33	7.5	7.8
.470	81.5	0.36	7.8	7.8
1900.43	79.7	0.33	7.8	8.0

154. DM. +54°1787.

$\alpha = 15^h\ 57^m\ 57^s$; $\delta = +54°\ 14'.8$.

1900.415	269.6	1.47	7.5	12.0
.423	268.5	1.37	8.5	11.5
.579	272.9	1.53	7.5	12.0
1900.47	270.3	1.46	7.8	11.8

155. DM. −12°4431.

$\alpha = 16^h\ 3^m\ 4^s$; $\delta = -12°\ 28'.8$.

1900.418	62.5	0.86	9.0	9.0
.494	62.1	0.88	9.0	9.2
.510	61.6	0.79	9.0	9.0
1900.47	62.2	0.84	9.0	9.1

156. DM. −11°4086.

$\alpha = 16^h\ 5^m\ 22^s$; $\delta = -11°\ 48'.3$.

1900.374	86.3	3.12	8.8	11.0
.415	84.6	2.98	8.8	13.0
.418	83.9	2.96	8.8	12.5
1900.40	84.9	3.02	8.8	12.2

157. DM. −12°4487.

$\alpha = 16^h\ 15^m\ 54^s$; $\delta = -12°\ 7'.4$.

1900.494	262.7	1.26	9.0	9.2
.494	264.7	1.35	9.0	9.2
.502	262.2	1.20	9.0	9.5
.510	263.7	1.21	8.8	9.0
1900.50	263.3	1.25	9.0	9.2

158. DM. −11°4140.

$\alpha = 16^h\ 21^m\ 33^s$; $\delta = -11°\ 51'.9$.

1900.494	134.3	0.45	8.8	9.0
.494	134.7	0.46	9.0	9.2
.510	134.5	0.46	8.5	8.8
1900.50	134.5	0.46	8.8	9.0

159. DM. −11°4233.

$\alpha = 16^h\ 49^m\ 15^s$; $\delta = -11°\ 23'.3$.

1900.494	151.6	1.36	8.5	9.0
.494	151.7	1.25	8.5	9.0
.502	151.8	1.33	8.5	9.2
1900.50	151.7	1.31	8.5	9.1

160. DM. +10°3099.

$\alpha = 16^h\ 51^m\ 49^s$; $\delta = +10°\ 23'.9$.

1900.527	202.8	0.59	9.0	9.0
.549	203.8	0.67	9.0	9.5
.588	203.5	0.56	8.8	9.0
1900.55	203.4	0.61	8.9	9.2

161. DM. −14°4508.

$\alpha = 16^h\ 52^m\ 14^s$; $\delta = -14°\ 36'.8$.

	°	″	M	M
1900.470	16.5	2.94	8.5	12.5
.513	16.4	3.09	8.5	12.0
.530	16.3	2.85	9.0	12.0
1900.50	16.4	2.96	8.7	12.2

162. DM. −16°4386.

$\alpha = 16^h\ 53^m\ 32^s$; $\delta = -16°\ 44'.9$.

1900.470	237.9	0.42	8.0	8.5
.513	235.4	0.36	8.5	8.5
.530	235.5	0.38	8.0	8.5
1900.50	236.3	0.39	8.2	8.5

163. DM. −12°4641.

$\alpha = 16^h\ 55^m\ 39^s$; $\delta = -12°\ 4'.2$.

1900.513	338.1	0.25	9.0	9.5
.530	334.5	0.29	8.8	9.0
.538	333.5	0.33	9.0	9.2
1900.53	335.4	0.29	8.9	9.2

164. DM. −12°4655.

$\alpha = 16^h\ 58^m\ 54^s$; $\delta = -12°\ 32'.0$.

1900.510	341.3	1.77	6.5	11.5
.513	341.0	1.78	6.5	12.5
.530	342.0	1.80	6.5	12.5
1900.52	341.4	1.78	6.5	12.2

165. DM. −14°4540.

$\alpha = 17^h\ 0^m\ 7^s$; $\delta = -14°\ 13'.6$.

1900.510	41.5	0.52	9.5	11.5
.513	43.5	0.66	9.0	11.5
.530	40.0	0.67	8.8	11.0
1900.52	41.7	0.62	9.1	11.3

166. DM. −12°4664.

$\alpha = 17^h\ 1^m\ 29^s$; $\delta = -12°\ 54'.5$.

1900.510	297.3	1.28	9.0	12.0
.513	299.7	1.17	9.0	12.5
.530	301.5	1.18	9.0	11.5
1900.52	299.5	1.21	9.0	12.0

167. DM. +10°3147.

$\alpha = 17^h\ 3^m\ 51^s$; $\delta = +9°\ 58'.2$.

1900.549	58.9	0.51	9.5	10.0
.588	61.3	0.65	9.5	9.5
.591	58.3	0.58	9.5	9.8
1900.58	59.5	0.58	9.5	9.8

168. DM. −17°4731.

$\alpha = 17^h\ 4^m\ 11^s$; $\delta = -17°\ 53'.2$.

1900.513	111.1	0.37	. .	. .
.530	108.9	0.35	. .	. .
.533	107.2	0.34	8.5	8.5
1900.52	109.1	0.35	8.5	8.5

169. DM. −16°4436.

$\alpha = 17^h\ 5^m\ 44^s$; $\delta = -16°\ 22'.0$.

	°	″	M	M
1900.513	220.6	0.12	8.0	8.0
.530	223.5	0.14	8.0	8.0
.533	225.2	0.13	8.0	8.2
1900.52	223.1	0.13	8.0	8.1

170. DM. +9°3339.

$\alpha = 17^h\ 7^m\ 30^s$; $\delta = +9°\ 52'.9$.

1900.541	273.3	1.58	8.5	11.0
.549	274.5	1.74	8.5	10.5
.588	273.4	1.80	8.5	11.0
1900.56	273.7	1.71	8.5	10.8

171. DM. −17°4806.

$\alpha = 17^h\ 10^m\ 43^s$; $\delta = -17°\ 30'.1$.

1900.533	189.5	1.80	9.5	10.5
.538	190.5	1.82	9.0	11.5
.549	191.3	1.59	9.0	10.5
1900.54	190.4	1.74	9.2	10.8

There is also a 10th magnitude companion in the position, 203°.0 ; 6″.40.

172. DM. +11°3153.

$\alpha = 17^h\ 12^m\ 40^s$; $\delta = +11°\ 19'.6$.

1900.541	347.9	0.82	9.1	12.0
.549	346.8	0.60	9.5	11.5
.588	347.7	0.65	9.0	11.5
1900.56	347.5	0.69	9.2	11.7

173. DM. −10°4479.

$\alpha = 17^h\ 16^m\ 9^s$; $\delta = -10°\ 57'.5$.

1900.442	359.4	0.70	8.5	8.8
.470	358.7	0.71	8.5	8.8
.494	359.0	0.76	8.5	9.0
1900.47	359.0	0.72	8.5	8.9

174. DM. −16°4541.

$\alpha = 17^h\ 18^m\ 54^s$; $\delta = -16°\ 0'.0$.

1900.533	43.0	1.87	8.8	12.5
.538	44.4	2.00	8.8	13.0
.549	42.3	2.15	8.5	13.0
1900.54	43.2	2.01	8.7	12.8

175. DM. −12°4754.

$\alpha = 17^h\ 21^m\ 35^s$; $\delta = -12°\ 3'.7$.

1900.418	68.3	4.59	8.5	12.0
.442	68.2	4.68	8.8	13.0
.470	67.7	4.66	8.5	12.0
1900.44	68.1	4.64	8.6	12.3

176. DM. +8°3425.

$\alpha = 17^h\ 24^m\ 31^s$; $\delta = +8°\ 15'.9$.

1900.549	345.4	0.28	9.5	9.8
.588	346.2	0.25	9.0	9.2
.591	342.5	0.25	9.5	9.5
.597	343.7	0.23	9.0	9.2
1900.58	344.5	0.25	9.2	9.4

177. DM. −14°4665.
$\alpha = 17^h 21^m 50^s$; $\delta = -14° 41'.7$.

	°	″	M	M
1900.533	87.5	0.36	8.0	9.5
.538	84.1	0.36	8.5	9.5
.549	84.1	0.40	8.8	9.5
1900.54	85.2	0.37	8.4	9.5

178. DM. −13°4639.
$\alpha = 17^h 26^m 12^s$; $\delta = -13° 31'.4$.

1900.418	176.5	2.60	9.0	9.0
.442	178.5	2.57	8.8	9.0
.470	177.2	2.56	9.0	9.2
1900.44	177.4	2.58	8.9	9.1

179. DM. +11°3194.
$\alpha = 17^h 27^m 0^s$; $\delta = +11° 16'.9$.

1900.541	51.1	2.03	8.8	9.0
.549	51.8	2.24	8.8	9.0
.588	52.9	2.24	8.8	8.8
1900.56	51.9	2.17	8.8	8.9

180. DM. −13°4661.
$\alpha = 17^h 29^m 48^s$; $\delta = -13° 55'.7$.

1900.442	221.7	0.47	9.0	9.2
.470	222.7	0.45	8.5	8.5
.491	224.0	0.50	. .	. .
1900.47	222.8	0.47	8.7	8.8

181. DM. −15°4635.
$\alpha = 17^h 33^m 40^s$; $\delta = -15° 42'.1$.

1900.538	97.1	0.18	9.2	9.8
.549	93.2	0.20	9.0	9.0
.568	94.3	0.21	9.5	10.0
1900.55	94.9	0.20	9.2	9.6

182. DM. −13°4704.
$\alpha = 17^h 34^m 43^s$; $\delta = -13° 15'.5$.

1900.491	10.1	1.46	9.0	10.0
.494	10.1	1.39	9.0	9.0
.513	11.4	1.41	9.0	9.0
1900.50	10.5	1.42	9.0	9.3

183. DM. −14°4726.
$\alpha = 17^h 35^m 47^s$; $\delta = -14° 27'.0$.

1900.538	293.8	1.07	8.5	10.5
.549	295.5	1.09	8.8	9.2
.568	295.1	1.26	9.0	9.2
1900.55	294.8	1.14	8.8	9.6

184. DM. −15°4651.
$\alpha = 17^h 36^m 29^s$; $\delta = -15° 40'.5$.

1900.538	273.4	4.45	8.5	10.0
.549	273.9	4.50	8.5	9.5
.568	274.8	4.50	8.5	9.0
1900.55	274.0	4.48	8.5	9.5

185. DM. −16°4599.
$\alpha = 17^h 37^m 45^s$; $\delta = -16° 46'.0$.

	°	″	M	M
1900.533	296.8	4.90	8.0	12.5
.538	299.5	4.73	8.5	12.0
.568	298.5	4.68	8.5	12.0
1900.55	298.3	4.77	8.3	12.2

186. DM. −18°4645.
$\alpha = 17^h 41^m 36^s$; $\delta = -18° 4'.2$.

1900.530	341.3	1.01	7.0	11.0
.533	336.9	1.02	7.5	11.0
.538	341.8	0.96	7.0	12.0
1900.53	340.0	1.00	7.2	11.3

187. DM. −16°4622.
$\alpha = 17^h 43^m 15^s$; $\delta = -16° 12'.9$.

1900.533	86.9	4.53	8.3	11.5
.538	86.0	4.55	8.5	12.5
.562	87.6	4.51	8.5	13.0
1900.54	86.8	4.53	8.4	12.3

188. DM. −13°4770.
$\alpha = 17^h 46^m 6^s$; $\delta = -13° 35'.1$.

1900.491	48.7	0.46	9.0	10.5
.494	48.0	0.51	9.0	10.2
.513	49.7	0.46	9.0	11.5
1900.50	48.8	0.48	9.0	10.7

189. DM. −13°4779.
$\alpha = 17^h 47^m 25^s$; $\delta = -13° 37'.6$.

1900.480	231.3	1.19	7.5	8.5
.491	230.2	1.18	7.5	8.0
.494	233.7	1.27	7.5	9.5
1900.49	231.7	1.21	7.5	8.7

190. DM. −13°4807.
$\alpha = 17^h 53^m 4^s$; $\delta = -13° 3'.6$.

1900.415	216.1	0.46	9.0	10.5
.494	217.3	0.49	9.5	10.5
.513	221.0	0.49	9.0	10.5
1900.47	218.1	0.48	9.2	10.5

191. DM. −13°4812.
$\alpha = 17^h 54^m 5^s$; $\delta = -13° 44'.6$.

1900.415	120.3	4.30	9.0	9.0
.491	121.3	4.18	9.5	9.5
.513	121.2	4.42	8.8	9.0
1900.47	120.9	4.30	9.1	9.2

192. DM. −14°4841.
$\alpha = 17^h 54^m 18^s$; $\delta = -14° 29'.7$.

1900.538	136.5	2.56	9.0	12.5
.568	135.0	2.52	9.0	13.0
.620	137.0	2.51	9.0	13.0
1900.58	136.2	2.53	9.0	12.8

193. DM. −14°4870.
$\alpha = 17^h 58^m 22^s$; $\delta = -14° 15'.0$.

	°	″	M	M
1900.538	123.5	0.56	9.5	9.8
.568	123.6	0.62	9.5	9.5
.620	119.7	0.72	9.5	9.5
1900.58	122.3	0.63	9.5	9.6

194. DM. −17°5007.
$\alpha = 17^h 58^m 49^s$; $\delta = -17° 2'.1$.

1900.562	309.7	0.49	8.5	11.0
.568	307.6	0.41	8.8	9.5
.620	301.1	0.44	8.5	10.0
.626	303.3	0.46	8.5	9.5
1900.59	305.4	0.45	8.6	10.0

195. DM. −17°5052.
$\alpha = 18^h 4^m 57^s$; $\delta = -17° 9'.7$.

1900.560	70.4	1.05	8.5	13.0
.563	70.7	0.96	8.5	13.0
.568	76.1	1.05	8.5	13.0
.620	72.8	1.22	8.5	12.5
1900.58	72.5	1.07	8.5	12.9

196. DM. +8°3621.
$\alpha = 18^h 9^m 7^s$; $\delta = +8° 56'.9$.

1900.588	346.7	0.23	9.2	9.5
.591	345.4	0.28	8.8	9.2
.597	343.4	0.24	9.0	9.0
1900.59	345.1	0.25	9.0	9.2

197. DM. +10°3473.
$\alpha = 18^h 14^m 58^s$; $\delta = +10° 14'.4$.

1900.571	28.0	0.40	8.5	9.5
.588	29.1	0.32	8.0	9.0
.591	27.0	0.36	8.0	9.5
1900.58	28.0	0.36	8.2	9.3

198. DM. +8°3780.
$\alpha = 18^h 33^m 34^s$; $\delta = +8° 44'.8$.

1900.415	192.5	0.20	8.0	8.0
.418	191.5	0.21	8.5	8.8
.470	197.2	0.26	. .	. .
.571	199.8	0.20	9.0	9.0
1900.47	195.2	0.22	8.5	8.6

199. DM. +11°3642.
$\alpha = 18^h 47^m 27^s$; $\delta = +11° 40'.7$.

1900.591	5.4	0.21	8.5	9.0
.597	3.5	0.25	8.8	9.2
.623	1.5	0.23	8.8	9.0
1900.60	3.5	0.23	8.7	9.1

200. τ *Capricorni*.
$\alpha = 20^h 33^m 41^s$; $\delta = -15° 18'.3$.

1900.626	270.5	0.17	5.5	7.0
.647	269.0	0.16	5.5	6.5
1900.64	269.8	0.17	5.5	6.8

Lick Observatory, Mt. Hamilton, Cal., 1900 *August* 29.

ON THE STAR LALANDE 14755,

By R. T. A. INNES.

[Communicated by DAVID GILL, C.B., etc., H.M. Astronomer at the Cape of Good Hope.]

This star was looked for by ARGELANDER and not found. He writes in the 7th volume of the Bonn Observations, p. 205: "Zu verschiedenen Zeiten vergebens gesucht. Vergleicht man die Stelle mit meiner Zone 281, so finden sich alle übrigen an beiden Orten gemeinschaftlich, nur der fragliche Stern fehlt bei mir, während ich unter No. 163 einen Stern 8ᵐ habe, der bei Lalande fehlt. Beide würden übereinkommen wenn man bei Lalande läse F2 23 31 st 23 21 und Z D 69 15 50 st 69 0 50. Bei der Eile, mit der beobachtet wurde, scheint mir diese Correctur nicht ganz unwahrscheinlich."

These assumptions were adopted at the Paris Observatory, and we find in the Paris Catalogue, No. 9309, Lal. 14755,

		Paris–Lalande	
$\alpha = 7^h\ 27^m\ 21^s.43$	$\delta = -20°\ 38'\ 38''.6$	$+0^s.18$	$-8''.4$

In spite of the excellence of this agreement I venture to point out that it is incorrect.

We have for 1875:

	Mag.	α	δ	
Lalande 14755		$7^h\ 27^m\ 11^s.8$	$-20\ 23.1$	From B.A.C. edition.
S.DM. $-20°2007$	10	7 27 13.4	$-20\ 23.9$	
Kap.C.P.D. $-20°2569$	9.9	7 27 12.6	$-20\ 23.3$	

SCHÖNFELD gives no reference, but KAPTEYN identifies the C.P.D. star both with Lalande and DM. $-20°2007$. Whilst I was making a map for the variable star 2690 *X Puppis*, I was struck by the faintness of the C.P.D. star identified as LALANDE'S, and I at once assumed it must be variable. At the telescope I found that the small scale of my map and the large number of faint stars thereabouts made identification a little difficult. I therefore made a larger map from the sky on the 19th June, 1899. Professor KAPTEYN also considered the Lalande star as a probable variable, as it is contained in a valuable series of "Suspected Variables" which he transmitted to Dr. GILL in 1899, and which reached the Cape on the 30th of August.

My observations now enable me to say that the star of the DM. and C.P.D. is not the star observed by LALANDE. LALANDE'S star is about 15″ or 20″ S. pr. the C.P.D. star, and this is not inconsistent with the positions above-quoted.

Sometime in Feb. Mar. 1899 I noted a $10^M.5$ star S. pr. the C.P.D. star. This star was invisible in June when I made the new map, but on Sept. 30 I saw it again; in fine, the observations stand thus, taking the C.P.D. star as $= 9^M.9$.

1899 (Feb.–Mar.)	$(10^M.4)$	
June–Aug.	inv.	3 obs.
Sept. 30	11.5	
Oct. 27	seen	
Nov. 11	9.85	
12	9.85	
18	9.7	
21	9.5	

It is therefore beyond doubt that this is Lalande 14755, and that it is variable. It is very remarkable that this is one of the few stars for which LALANDE gives no magnitude. The period of *X Puppis* has until now been given as 415 days or half thereof, but it is really about 26 days, which is more in accordance with the small variation of light which the star undergoes.

It is well in this connection to recall the circumstances attending the star Ö.A. 21384. Professor WEISS writes, "An dieser Stelle in Wien wiederholt vergeblich gesucht. Wohl blos der folgende Stern mit Doppelfehler: in A.R. um 1ᵐ zu klein und Mikr. zu lesen 9° 14′ statt 8° 51′. Die Position wäre dann für 1850.0: $\alpha = 21^h\ 17^m\ 50^s.22$, $\delta = -29°\ 39'\ 15''.3$" (*Weiss's Argelander*, pp. 500–1).

These corrections make the star identical with Ö.A. 21385, $\alpha = 21^h\ 17^m\ 50^s.65$, $\delta = -29°\ 39'\ 14''.6$.

Nevertheless Ö.A. 21384 really exists in ARGELANDER'S uncorrected position. It is the variable star *S Microscopii*.

Great caution is thus evidently required in assuming errors.

Royal Observatory, Cape of Good Hope, 1899 Nov. 22.

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DOUBLE-STAR MEASURES,

By W. A. COGSHALL.

The following measures were made with the 24-inch refractor of the Lowell Observatory. The stars are all taken from the "First Catalogue of Double and Multiple Stars discovered at the Lowell Observatory," between 18^h and 24^h right-ascension (epoch 1900.0). When weather and seeing permitted I have endeavored to obtain three nights' observation of each star, but during the fall and winter this has been impossible in some cases, especially with very difficult stars and those far south.

Each observation is composed of four settings of the micrometer for position angle, and three double distances. The magnitudes are means of estimates made at each observation. Those measures made by Mr. BOOTHROYD are followed by "B."

λ_1 348.

$\alpha = 18^h\ 8^m\ 51^s.2 \quad : \quad \delta = -24°\ 32'\ 2''.6$

9.3 : 12.5

t	θ	ρ
	°	"
1899.641	299.2	1.11
.647	301.1	0.96
.673	305.9	0.78
1899.654	302.0	0.96

This star is erroneously identified in the λ_1 catalogue as Cord. G.C. 24847. It is Cord. G.C. 24836.

λ_1 349.

$\alpha = 18^h\ 9^m\ 39^s.0 \quad ; \quad \delta = -18°\ 40'\ 37''.5$

9.0 : 13.7

1899.641	129.8	10.01
.675	130.5	10.18
1899.658	130.1	10.09

λ_1 352.

$\alpha = 18^h\ 19^m\ 12^s.2 \quad ; \quad \delta = -30°\ 16'\ 9''.5$

7.5 : 14

1899.575	75.6	3.01
.602	69.2	3.02
.608	71.1	. . *
1899.595	72.0	3.03

*Measure stopped by clouds.

λ_1 353.

$\alpha = 18^h\ 24^m\ 31^s.2 \quad : \quad \delta = -33°\ 3'\ 17''.3$

AB

1899.689

Seeing very good, but I cannot make out any duplicity in this star. Star small and steady.

$\frac{AB}{2}\,C$

6 ; 11.5

t	θ	ρ
	°	"
1899.630	201.1	3.33
.649	201.6	3.38
.673	199.5	3.11
1899.651	200.7	3.27

$\frac{AB}{2}\,D \qquad D = 13$

1899.630	338.1	28.54
.649	338.7	28.57
.673	338.8	28.74
1899.651	338.6	28.62

λ_1 354.

$\alpha = 18^h\ 25^m\ 34^s.8 \quad ; \quad \delta = -18°\ 28'\ 15''.5$

6 ; 13.7

1899.647	182.8	25.01
.675	183.9	24.80
.689	182.2	21.52
1899.670	183.0	21.78

λ_1 355.

$\alpha = 18^h\ 30^m\ 2^s.8 \quad : \quad \delta = -19°\ 17'.6$

6.3 : 12.8

1899.610	241.1	11.72
.616	239.8	11.80
.692	239.2	11.75
1899.639	240.1	11.76

This star is not Cord. G.C. 25376 as originally published, but is S.DM. —19 5097.

λ_1 356.

$\alpha = 18^h\ 36^m\ 19^s.0 \quad : \quad \delta = -29°\ 34'\ 23''.6$

8.3 ; 8.5

t	θ	ρ
	°	"
1898.698	131.7	3.31
1899.427	131.1	3.35
.471	134.0	3.52
1899.199	134.3	3.39

λ_1 357.

$\alpha = 18^h\ 38^m\ 0^s.1 \quad : \quad \delta = -29°\ 31'\ 39''.5$

1899.725

Have looked for this star on four different nights, but have been unable to find anything like it.

λ_1 358.

$\alpha = 18^h\ 38^m\ 33^s.4 \quad : \quad \delta = -25°\ 53'\ 40''.2$

7.5 : 8.5

1899.471	30.7	1.60
.573	29.5	1.66
1899.522	30.1	1.63

λ_1 360.

$\alpha = 18^h\ 40^m\ 18^s.8 \quad ; \quad \delta = -22°\ 29'\ 47''.9$

5.0 : 14.3

1899.689	209.1	12.06
.709	210.8	11.91
.725	207.9	. .
1899.708	209.3	12.00

λ_1 361.

$\alpha = 18^h\ 43^m\ 6^s.7$; $\delta = -35°\ 12'\ 28''.6$

7.8 : 7.8

t	θ	ρ
1899.638	230.8	1.40
.668	230.9	1.05
.675	229.5	1.18
1899.660	230.1	1.21

λ_1 362.

$\alpha = 18^h\ 43^m\ 44^s.1$; $\delta = -20°\ 26'\ 18''.2$

6.3 : 13.7

1899.589	1.8	17.67
.602	1.3	17.66
.605	1.6	18.08
1899.599	1.6	17.80

λ_1 363.

$\alpha = 18^h\ 48^m\ 1^s.2$; $\delta = -32°\ 33'\ 21''.1$

9.0 ; 14

1899.610	95.3	5.10
.717	95.1	5.15
1899.663	95.2	5.13

λ_1 364.

$\alpha = 18^h\ 49^m\ 41^s.6$; $\delta = -28°\ 15'\ 25''.1$

8.2 : 8.5

1899.485	95.0	0.41	B
1899.689	95.8	0.66	
.742	97.8	0.37	
1899.716	96.8	0.51	C

λ_1 367.

$\alpha = 18^h\ 57^m\ 29^s.1$; $\delta = -34°\ 31'\ 2''.6$

8.2 : 9.0

1899.185	7.0	1.16
.592	9.1	1.30
.673	8.1	0.75*
1899.583	8.1	1.07

*Seeing bad.

λ_1 368.

$\alpha = 18^h\ 58^m\ 20^s.4$; $\delta = -31°\ 5'\ 46''.1$

8 : 11.5

1899.127	305.3	16.63	B
1899.625	307.1	16.29	
.627	305.1	16.51	
1899.626	306.1	16.40	C

λ_1 369.

$\alpha = 18^h\ 58^m\ 41^s.2$; $\delta = -21°\ 58'\ 14''.9$

4.0 : 12.7

1899.675	239.8	33.47
.687	234.0	33.69
.689	240.6	33.68
1899.684	238.1	33.61

λ_1 370.

$\alpha = 19^h\ 2^m\ 4^s.3$; $\delta = -35°\ 34'\ 48''.7$

A B

7.5 ; 11.5

t	θ	ρ
1899.617	93.4	2.27
.649	92.7	2.42
.671	92.9	2.41
1899.656	93.0	2.37

A C. $C = 10.3$

1899.617	69.3	7.11
.649	70.3	7.26
.671	69.6	7.23
1899.656	69.7	7.20

A D. $D = 12.5$

1899.617	153.5	17.86
.649	156.9	18.51
.671	155.1	18.16
1899.656	155.2	18.18

D E. $E = 13.3$

1899.617	223.6	5.52
.649	225.3	5.91
.671	225.4	5.45
1899.656	224.8	5.63

λ_1 371.

$\alpha = 19^h\ 6^m\ 7^s.1$; $\delta = -22°\ 5'\ 27''.2$

8.0 ; 12.8

1899.427	330.1	7.33
.471	330.8	7.47
.573	. .	7.06
.675	329.6	7.13
1899.536	330.2	7.25

λ_1 372.

$\alpha = 19^h\ 12^m\ 28^s.6$; $\delta = -33°\ 27'\ 14''.8$

8 ; 8

1899.641	119.0	0.56
.706	116.7	0.52
.722	110.6	0.48
1899.690	115.4	0.52

λ_1 375.

$\alpha = 19^h\ 20^m\ 32^s.6$; $\delta = -26°\ 30'\ 59''.1$

7.7 : 11.8

1899.592	167.6	12.25
.600	167.7	12.31
.610	167.0	12.22
1899.601	167.4	12.27

λ_1 376.

$\alpha = 19^h\ 21^m\ 9^s.6$; $\delta = -31°\ 34'\ 52''.0$

8.7 : 8.7

1899.471	112.6	0.86
.675	116.8	0.45
.706	111.2	0.79
1899.617	111.5	0.70

λ_1 377.

$\alpha = 19^h\ 18^m\ 41^s.1$; $\delta = -32°\ 25'.9$

8.2 ; 9.7

t	θ	ρ
1899.185	298.1	1.77
.586	296.1	1.58
.613	297.7	1.76
1899.561	297.5	1.70

This star is not Cord. Z.C. 19h895 but is Cord. DM. −32°15154.

λ_1 378.

$\alpha = 19^h\ 22^m\ 16^s.4$; $\delta = -32°\ 11'\ 35''.6$

8.2 ; 9.0

1899.427	2.6	10.13	
.485	3.0	10.37	
1899.456	2.8	10.25	B

λ_1 379.

$\alpha = 19^h\ 22^m\ 27^s.4$; $\delta = -36°\ 11'\ 17''.8$

10.2 : 12.7

1899.717	270.7	17.34
.722	271.1	17.09
.733	270.8	17.00
1899.724	270.9	17.14

λ_1 380.

$\alpha = 19^h\ 28^m\ 3^s.1$; $\delta = -38°\ 13'.7$

8.7 ; 9.5

1899.706	30.8	6.31
.714	32.0	6.51
.742	32.1	6.19
1899.721	31.6	6.34

This star is not Cord. DM. −38°13542 as identified by Dr. SEE, but is Cord. DM. −38°13557.

λ_1 381.

$\alpha = 19^h\ 26^m\ 0^s.4$; $\delta = -27°\ 57'\ 37''.5$

9.0 ; 10.2

1899.171	13.9	1.60
.485	13.0	1.72
.512	13.5	1.45
1899.489	13.5	1.59

λ_1 383.

$\alpha = 19^h\ 29^m\pm$; $\delta = -19°\ 25'\pm$

Have not been able to find this star.

λ_1 384.

$\alpha = 19^h\ 29^m\ 38^s.0$; $\delta = -23°\ 31'\ 41''.4$

7.7 ; 11.2

1898.741	167.5	5.91
.573	167.7	5.81
.575	167.1	5.71
1899.574	167.5	5.79

λ_1 385.

$\alpha = 19^h\ 31^m\ 7^s.7$; $\delta = -21°\ 51'\ 19''.8$

A B

8.0 : 14.5

1898.703	6.7	1.60
1899.586	3.2	4.34
.600	0.5	3.78
1899.593	1.9	4.06

A C. C = 11.7

t	θ	ρ
	°	″
1898.703	300.3	27.90
1899.586	301.2	27.27
.600	300.1	27.57
1899.593	300.6	27.42

λ_1 388.

$\alpha = 19^h\ 32^m\ 52^s.3$; $\delta = -39°\ 58'\ 14''.7$

7.5 ; 10.7

1899.714	198.7	2.59
.717	200.5	2.69
.742	196.7	2.42
1899.724	198.6	2.57

λ_1 389.

$\alpha = 19^h\ 33^m\ 49^s.1$; $\delta = -23°\ 39'\ 18''.6$

1899.750

Seeing best for a long time. Found star 5m east of meridian. Star is certainly double, but very difficult. Elongated under 1000 diameters. Angle settings 118°, 115°, 113°. Distance cannot be over 0″.15.

λ_1 390.

$\alpha = 19^h\ 33^m\ 58^s.0$; $\delta = -21°\ 13'\ 58''.2$

7.5 ; 13.5

1899.725	86.8	14.34

This star is O. A. 19835 and not O. A. 19840, as originally published.

λ_1 391.

$\alpha = 19^h\ 35^m\ 43^s.1$; $\delta = -30°\ 33'\ 56''.2$

8.5 ; 9.5

1899.673	35.9	6.08
.689	41.7	6.04
.706	36.1	6.07
1899.689	37.9	6.06

λ_1 392.

$\alpha = 19^h\ 36^m\ 58^s.1$; $\delta = -39°\ 8'\ 30''.1$

8.5 ; 12.5

1899.638	123.0	9.03
.647	126.9	9.76
.706	124.6	9.25
1899.664	124.8	9.35

λ_1 393.

$\alpha = 19^h\ 40^m\ 11^s.0$; $\delta = -27°\ 52'\ 44''.5$

1899.750

Cannot see this star double. Seeing best for a long time. Eye-pieces up to 1000 diameters.

λ_1 394.

$\alpha = 19^h\ 40^m\ 21^s.7$; $\delta = -25°\ 7'\ 16''.3$

7.7 ; 9.2

1899.641	288.2	0.51
.695	289.3	0.46
.709	285.5	0.52
1899.682	287.7	0.50

λ_1 395.

$\alpha = 19^h\ 42^m\ 12^s.1$; $\delta = -26°\ 54'\ 19''.5$

9 ; 9.7

t	θ	ρ
	°	″
1898.714	108.7	1.58
1899.512	106.0	1.86
.573	107.2	1.78
1899.543	106.6	1.82

λ_1 396.

$\alpha = 19^h\ 43^m\ 55^s.9$; $\delta = -23°\ 58'\ 7''.8$

8 ; 12

1899.575	269.6	6.35
.586	271.0	6.18
.592	270.5	6.51
1899.584	270.4	6.45

λ_1 397.

$\alpha = 19^h\ 45^m\ 18^s.4$; $\delta = -36°\ 47'\ 12''.5$

8.7 ; 9.5

1899.706	188.0	7.66
.711	186.4	7.91
.733	188.3	7.94
1899.717	187.6	7.84

λ_1 398.

$\alpha = 19^h\ 49^m\ 29^s.5$; $\delta = -38°\ 35'\ 44''.9$

7 ; 12.7

1899.647	309.1	10.43
.722	310.2	9.82
.733	309.4	10.08
1899.701	309.7	10.11

λ_1 399.

$\alpha = 19^h\ 50^m\ 17^s.6$; $\delta = -36°\ 38'\ 33''.5$

8.8 ; 13.2

1899.714	31.8	4.86
.717	31.6	4.02
.722	31.9	4.27
1899.718	31.8	4.38

λ_1 400.

$\alpha = 19^h\ 54^m\ 20^s.6$; $\delta = -24°\ 13'\ 41''.5$

8.5 ; 10

1899.471	30.6	1.60
.485	27.7	1.58
.573	29.2	1.40
1899.510	29.2	1.53

λ_1 401.

$\alpha = 19^h\ 55^m\ 27^s.4$; $\delta = -23°\ 0'\ 43''.5$

1899.668 Have followed this star 10m without seeing any companion. Have strong doubts of its reality. Sky very dark, and definition fair. No trouble to see λ_1 408, 10° farther south, and 15m.

.671 Same result.

.725 Followed 5m with screen in eye-piece. Nothing visible.

λ_1 402.

$\alpha = 19^h\ 56^m\ 58^s.0$; $\delta = -36°\ 52'\ 28''.0$

7 ; 12.2

t	θ	ρ
	°	″
1899.630	310.1	21.38
.706	309.5	21.26
.711	309.7	20.96
1899.682	309.8	21.20

λ_1 404.

$\alpha = 19^h\ 59^m\ 9^s.3$; $\delta = -33°\ 16'\ 58''.9$

1899.742

Star found, but measure prevented by bad seeing.

λ_1 405.

$\alpha = 20^h\ 0^m\ 29^s.6$; $\delta = -28°\ 39'\ 24''.6$

8.3 ; 8.7

1899.586	234.0	0.55
.641	230.7	0.50
.728	231.7	0.47
1899.652	232.1	0.51

λ_1 406.

$\alpha = 20^h\ 1^m\ 32^s.7$; $\delta = -19°\ 51'\ 33''.4$

8.3 ; 11.3

1898.703	5.1	2.53	
.760	4.0	2.68	
1899.485	4.3	2.51	
1898.983	4.5	2.58	B

λ_1 407.

$\alpha = 20^h\ 2^m\ 21^s.5$; $\delta = -39°\ 0'\ 49''.0$

8.2 ; 8.5

1899.647	259.7	3.15
.717	260.6	3.03
.799	259.9	2.83
1899.721	260.1	3.00

λ_1 408.

$\alpha = 20^h\ 5^m\ 19^s.0$; $\delta = -32°\ 8'\ 34''.9$

8 ; 14.7

1899.668	193.6	16.26
.675	194.0	16.61
.695	194.8	15.53
1899.679	194.1	16.13

λ_1 409.

$\alpha = 20^h\ 6^m\ 40^s \pm$; $\delta = -20°\ 33' \pm$

I have not been able to find this star.

λ_1 410.

$\alpha = 20^h\ 7^m\ 17^s.2$; $\delta = -34°\ 24'\ 56''.8$

6 ; 9

1899.793	9.7	2.25
.807	8.8	2.43
1899.800	9.3	2.34

This star is Cord. G.C. 27676. The next catalogue number was accidentally used in the λ_1 catalogue.

λ_1 411.

$\alpha = 20^h\ 8^m\ 22^s.4$: $\delta = -20°\ 32'\ 25''.9$
8 : 14

t	θ	ρ
	°	″
1899.578	5.2	1.77

λ_1 414.

$\alpha = 20^h\ 13^m\ 22^s.0$: $\delta = -27°\ 29'\ 42''.8$
8.2 : 9.5

1899.471	51.4	2.20
.619	52.8	2.14
.638	50.9	2.20
1899.576	51.6	2.18

λ_1 416.

$\alpha = 20^h\ 16^m\ 19^s.0$; $\delta = -28°\ 2'\ 40''.8$
8.7 : 8.7

A B

1899.641	244.5	0.98
.663	250.0	1.24
.706	248.2	1.21
1899.670	247.6	1.14

A C. $C = 12.8$

1899.641	251.9	27.60
.663	251.6	27.55
.706	251.5	26.15
1899.670	251.7	27.10

λ_1 417.

$\alpha = 20^h\ 17^m\ 21^s.5$: $\delta = -35°\ 42'\ 24''.8$
8.5 : 11.5

1899.714	137.8	3.11
.722	136.6	2.95
1899.718	137.2	3.03

λ_1 418.

$\alpha = 20^h\ 18^m\ 11^s.7$; $\delta = -25°\ 17'\ 53''.1$
8.5 : 11.5

1898.703	56.1	[1.72]
.760	52.3	2.91
1899.471	52.6	2.65
1898.978	53.7	2.78

λ_1 419.

$\alpha = 20^h\ 22^m\ 34^s.0$: $\delta = -32°\ 15'\ 33''.8$
9 : 13.5

1898.760	68.6	7.07	B
1899.675	68.1	6.15	C

λ_1 420.

$\alpha = 20^h\ 26^m\ 11^s.9$: $\delta = -22°\ 1'\ 48''.0$
8.5 : 13.5

1899.600	82.3	1.72
.728	88.0	1.50
1899.664	85.1	1.61

λ_1 423.

$\alpha = 20^h\ 34^m\ 4^s.0$: $\delta = -29°\ 13'\ 26''.0$
8.0 ; 9.0

t	θ	ρ
	°	″
1899.575	21.6	0.58
.586	23.0	0.69
.617	22.3	0.60
1899.603	22.3	0.62

λ_1 424.

$\alpha = 20^h\ 34^m\ 50^s.7$; $\delta = -33°\ 47'\ 47''.1$
9.0 : 9.8

1899.714	335.6	11.25
.733	334.9	11.27
.788	335.5	11.23
1899.745	335.3	11.25

λ_1 425.

$\alpha = 20^h\ 36^m\ 58^s.4$; $\delta = -29°\ 7'\ 39''.1$
8.0 : 11.2

1898.671	224.4	8.09
1899.512	224.1	7.94
1899.091	224.2	8.01

λ_1 426.

$\alpha = 20^h\ 38^m\ 30^s.2$: $\delta = -24°\ 0'\ 49''.6$
8.5 ; 10.5

1899.594	87.3	. . *
.689	87.5	7.40
.807	88.4	6.98
1899.697	87.7	7.19

*Clouds stopped measure.

λ_1 427.

$\alpha = 20^h\ 38^m\ 47^s.5$; $\delta = -23°\ 33'\ 3''.0$

1899.750

Have looked for this star several times without finding it. After finding by chart I could see nothing of any companion. Sky very dark. Seeing good.

λ_1 428.

$\alpha = 20^h\ 38^m\ 52^s.3$: $\delta = -35°\ 31'\ 38''.0$
7 ; 14

1899.807	182.9	4.37

λ_1 430.

$\alpha = 20^h\ 45^m\ 50^s.8$: $\delta = -34°\ 15'\ 18''.1$
7.5 ; 12.8

1899.471	347.6	13.21
.644	348.4	12.91
.706	347.2	12.84
1899.606	347.7	12.99

λ_1 431.

$\alpha = 20^h\ 46^m\ 14^s.3$; $\delta = -19°\ 48'.0$
8.4 : 13.2

1899.600	345.6	2.44
.687	343.8	1.97
.689	341.4	1.96
.695	341.2	1.94
1899.668	342.9	2.08

λ_1 432.

This star is identical with the last, and both original identifications are wrong. The star is S.DM. −19 5940.

λ_1 433.

$\alpha = 20^h\ 51^m\ 24^s.9$; $\delta = -24°\ 40'\ 20''.2$
8.2 ; 8.4

t	θ	ρ
	°	″
1898.671	42.3	2.42
.703	. .	2.59
.791	41.7	2.66
.802	42.0	2.61
1898.742	42.0	2.57

λ_1 434.

$\alpha = 20^h\ 51^m\ 31^s.3$; $\delta = -22°\ 1'\ 2''.2$
8.7 : 10.5

1898.671	153.2	4.30	
.684	150.3	4.09	
1898.677	151.7	4.19	B
1898.760	150.5	4.04	C

λ_1 435.

$\alpha = 20^h\ 57^m\ 12^s.4$; $\delta = -28°\ 7'\ 28''.3$
7.5 ; 8.5

1899.660	299.1	0.32

λ_1 438.

$\alpha = 21^h\ 0^m\ 11^s.3$: $\delta = -35°\ 1'\ 39''.4$

1899.739

Could not find this star.

λ_1 439.

$\alpha = 21^h\ 1^m\ 16^s.8$; $\delta = -25°\ 24'\ 19''.0$
4.7 ; 11.2

1899.675	186.3	26.10
.687	186.8	25.94
.695	186.0	25.74
1899.686	186.4	25.93

λ_1 440.

$\alpha = 21^h\ 4^m\ 9^s.1$; $\delta = -26°\ 27'\ 24''.5$
8.3 ; 12.7

1898.684	72.9	10.04	
.711	72.2	10.31	
.760	69.0	10.40	
1898.718	71.4	10.25	B

λ_1 441.

$\alpha = 21^h\ 15^m\ 12^s.0$; $\delta = -25°\ 7'.7$
8 ; 8.2

1898.742	16.5	1.92
.760	16.0	1.99
.791	14.8	1.90
1898.764	15.8	1.94

This star is not Cord. DM. 25°15323, but is Cord. DM. 25 15377.

λ_1 442.

$\alpha = 21^h\ 11^m\ 22^s.4 \quad : \quad \delta = -39°\ 36'\ 7''.5$

10.2 : 11

t	θ	ρ
	°	″
1899.714	31.2	5.57
.720	32.6	5.84
.750	31.0	5.70
1899.728	31.6	5.70

λ_1 443.

$\alpha = 21^h\ 12^m\ 55^s.7 \quad : \quad \delta = -27°\ 2'\ 2''.7$

8.2 ; 10.7

t	θ	ρ
1898.684	282.9	1.79
.703	281.8	1.74
.722	279.5	1.72
1898.703	281.4	1.75

λ_1 444 C.

$\alpha = 21^h\ 13^m\ 45^s.6 \quad : \quad \delta = -24°\ 11'\ 30''.1$

8.2 : 13.8

t	θ	ρ
1898.758	231.2	12.62
1899.586	232.7	12.06
.675	233.9	11.92
1899.631	233.3	11.99

λ_1 445.

$\alpha = 21^h\ 19^m\ 25^s.6 \quad : \quad \delta = -39°\ 38'\ 36''.7$

7.7 ; 11.2

t	θ	ρ
1899.714	222.7	2.20
.720	220.8	2.21
.722	221.6	2.53
1899.719	221.7	2.31

λ_1 446.

$\alpha = 21^h\ 20^m\ 57^s.5 \quad ; \quad \delta = -22°\ 50'\ 41''.0$

4.2 : 12.3

t	θ	ρ	
1898.758	12.8	21.38	
.760	13.8	21.76	
1899.512	14.7	21.40	
1899.010	13.8	21.51	B

λ_1 447.

$\alpha = 21^h\ 23^m\ 4^s.4 \quad ; \quad \delta = -39°\ 15'\ 51''.8$

8 ; 10.8

t	θ	ρ
1899.619	25.3	7.05
.647	26.6	6.85
.660	27.5	6.72
1899.642	26.5	6.87

λ_1 448.

$\alpha = 21^h\ 24^m\ 38^s.3 \quad : \quad \delta = -24°\ 51'\ 54''.2$

8 : 12.2

t	θ	ρ
1899.471	252.1	1.46
.512	250.4	1.42
.586	248.2	1.38
1899.523	250.3	1.42

λ_1 449.

$\alpha = 21^h\ 25^m\ 48^s.9 \quad ; \quad \delta = -19°\ 40'\ 36''.6$

6.3 : 12.7

t	θ	ρ
	°	″
1899.641	200.6	1.99
.695	200.6	1.87
.742	201.1	1.75
1899.693	200.8	1.87

λ_1 450.

$\alpha = 21^h\ 30^m\ 54^s.6 \quad : \quad \delta = -33°\ 57'\ 12''.6$

8.7 : 13.1

t	θ	ρ
1899.619	251.4	11.23
.675	253.7	11.43
.687	253.4	10.92
1899.660	252.8	11.19

λ_1 451.

$\alpha = 21^h\ 33^m\ 26^s.3 \quad : \quad \delta = -30°\ 45'\ 19''.3$

7.2 : 12.3

t	θ	ρ
1899.647	257.4	11.82
.687	256.6	11.42
.689	258.4	12.23
1899.674	257.5	11.82

λ_1 452.

$\alpha = 21^h\ 35^m\ 15^s.6 \quad : \quad \delta = -26°\ 17'\ 56''.3$

7.8 : 13.2

t	θ	ρ
1898.760	99.3	12.12
.843	101.3	11.56
1899.619	101.8	11.56
1899.074	100.8	11.75

λ_1 453.

$\alpha = 21^h\ 36^m\ 16^s.2 \quad : \quad \delta = -25°\ 6'\ 30''.1$

7.3 : 12.2

t	θ	ρ	
1898.714	326.3	11.65	B
1899.614	327.6	11.51	
.666	326.8	11.56	
.695	326.5	11.70	
1899.668	327.0	11.59	

λ_1 454.

$\alpha = 21^h\ 36^m\ 19^s.0 \quad : \quad \delta = -23°\ 42'\ 53''.4$

6.0 : 13.2

t	θ	ρ
1898.711	201.1	5.35
.725	198.9	5.52
1898.720	200.0	5.44

λ_1 455.

$\alpha = 21^h\ 38^m\ 37^s.2 \quad : \quad \delta = -36°\ 4'\ 18''.1$

7.5 ; 13.3

t	θ	ρ
1899.687	89.2	3.22
.728	88.7	3.72
.807	90.3	3.65
1899.740	89.4	3.53

λ_1 456.

$\alpha = 21^h\ 39^m\ 29^s.0 \quad ; \quad \delta = -20°\ 34'\ 13''.7$

8 : 11.5

t	θ	ρ
	°	″
1899.586	59.4	2.98
.644	56.2	3.19
1899.615	57.8	3.09

λ_1 458.

$\alpha = 21^h\ 40^m\ 59^s.7 \quad ; \quad \delta = -27°\ 3'\ 4''.4$

8.7 : 9.7

t	θ	ρ
1899.471	90.5	0.88
.641	90.9	0.97
.647	86.4	0.74
1899.586	89.3	0.86

λ_1 459.

$\alpha = 21^h\ 42^m\ 59^s.0 \quad : \quad \delta = -32°\ 51'\ 53''.1$

1899.750

Can see no companion to this star. Seeing very much better than usual.

λ_1 460.

$\alpha = 21^h\ 45^m\ 21^s.5 \quad : \quad \delta = -20°\ 45'\ 18''.3$

7.7 : 7.8

t	θ	ρ
1899.660	116.3	0.59
.666	120.6	0.64
.714	118.0	0.48
1899.680	118.3	0.57

λ_1 461.

$\alpha = 21^h\ 50^m\ 35^s.3 \quad : \quad \delta = -27°\ 45'\ 48''.1$

8.7 : 11.5

t	θ	ρ	
1898.701	59.6	3.91	
1899.512	62.0	3.91	
1899.107	60.8	3.91	B
1899.722	60.6	3.60	C

λ_1 462.

$\alpha = 21^h\ 52^m\ 39^s.0 \quad : \quad \delta = -35°\ 39'\ 29''.2$

7.8 ; 11

t	θ	ρ
1899.647	195.5	10.88
.692	195.2	10.77
.711	196.0	11.09
1899.683	195.6	10.91

λ_1 463.

$\alpha = 21^h\ 52^m\ 55^s.3 \quad : \quad \delta = -19°\ 7'.6$

8.3 : 13.2

t	θ	ρ	
1898.725	116.8	15.68	
.760	117.2	14.42	
.782	118.5	15.30	
1898.756	117.5	15.13	B

This star is S.DM. 19°6197, and not O.A. 21786.

λ_1 164.

$\alpha = 21^h\ 57^m\ 18^s.0 \quad : \quad \delta = -16^\circ\ 45'\ 0''.5$

I have not been able to find this star.

λ_1 165.

$\alpha = 21^h\ 58^m\ 12^s.5 \quad : \quad \delta = -25^\circ\ 19'\ 47''.0$

7.2 : 13.5

t	θ	ρ
1899.687	191.0	2.27
.695	193.7	2.19
.722	191.5	2.50
1899.701	192.1	2.42

λ_1 167.

$\alpha = 21^h\ 59^m\ 33^s.5 \quad : \quad \delta = -27^\circ\ 19'\ 59''.5$

8.7 : 14

1898.856	120.2	8.17
1899.610	117.5	8.20
1899.233	118.8	8.33

λ_1 168.

$\alpha = 22^h\ 3^m\ 5^s.8 \quad : \quad \delta = -36^\circ\ 32'\ 40''.7$

7.5 : 13.2

1899.647	112.1	25.58
.666	109.8	25.41
.692	111.3	25.80
1899.668	111.1	25.61

λ_1 169.

$\alpha = 22^h\ 3^m\ 6^s.9 \quad : \quad \delta = -26^\circ\ 15'\ 30''.2$

8 : 8

1899.586	320.6	0.35

Elongated at times. Not certain. Distance of no value. Duplicity doubtful.

1899.660

Certainly not double now. Seeing best for months. No trace of duplicity.

λ_1 170.

$\alpha = 22^h\ 4^m\ 56^s.2 \quad : \quad \delta = -24^\circ\ 1'\ 37''.9$

8.2 : 9.2

1898.684	39.2	1.55
.760	35.2	2.04
.876	33.1	1.57
1898.773	35.9	1.72

λ_1 171.

$\alpha = 22^h\ 16^m\ 1^s.5 \quad : \quad \delta = -28^\circ\ 58'\ 48''.9$

10.2 : 12

1898.722	35.2	1.78
1899.512	33.9	5.02
.750	34.0	1.60
1899.631	34.0	1.81

λ_1 172.

$\alpha = 22^h\ 15^m\ 57^s.3 \quad : \quad \delta = -25^\circ\ 51'\ 15''.7$

11 : 11.1

t	θ	ρ	
1898.760	58.0	5.74	
1899.512	56.9	5.44	
1899.136	57.1	5.59	B

λ_1 173.

$\alpha = 22^h\ 22^m\ 47^s.5 \quad : \quad \delta = -39^\circ\ 38'\ 11''.7$

5.7 : 12.7

1899.692	116.0	27.35
.744	116.5	27.35
1899.718	116.2	27.35

λ_1 174.

$\alpha = 22^h\ 23^m\ 49^s.3 \quad : \quad \delta = -29^\circ\ 10'\ 13''.6$

1899.711 Can see no companion. Seeing not very good.

.744 If double, not wider than 0″.15. Certainly no 0″.6 companion visible now. Slight elongation in about 300°.

.750 Probably double in about 300°, but certainly under 0″.25.

λ_1 176.

$\alpha = 22^h\ 39^m\ 42^s.4 \quad : \quad \delta = -23^\circ\ 37'\ 39''.6$

8.2 : 13.8

1898.856	38.4	3.5 ±	
1899.512	42.6	5.66*	
1899.184	40.5	4.58 ±	B

*Measure stopped by dawn.

λ_1 179 C.

$\alpha = 23^h\ 7^m\ 42^s.9 \quad : \quad \delta = -24^\circ\ 38'\ 57''.8$

8 : 13.5

1898.922	51.2	11.00
1899.586	53.8	11.47
.610	57.6	11.52
1899.373	54.2	11.33

λ_1 180.

$\alpha = 23^h\ 9^m\ 51^s.7 \quad : \quad \delta = -36^\circ\ 15'\ 22''.5$

8.2 : 9.2

1899.619	298.6	0.66
.647	297.1	0.60
.660	300.0	0.68
1899.642	298.9	0.65

λ_1 181.

$\alpha = 23^h\ 11^m\ 1^s.4 \quad : \quad \delta = -26^\circ\ 53'\ 39''.6$

8.7 : 8.7

1898.671	112.3	2.87
.714	112.3	2.91
.722	111.0	2.97
1898.702	111.9	2.92

λ_1 182.

$\alpha = 23^h\ 14^m\ 23^s.2 \quad : \quad \delta = -23^\circ\ 46'\ 18''.6$

7 : 13.3

t	θ	ρ	
1898.755	95.9	14.71	
.856	95.6	13.88	
.862	93.0	14.45	
1898.824	94.8	14.35	B

λ_1 183.

$\alpha = 23^h\ 20^m\ 34^s.8 \quad : \quad \delta = -36^\circ\ 21'\ 43''.8$

9 : 9.7

1899.660	294.8	1.95
.668	293.1	1.95
1899.664	294.0	1.95

λ_1 185.

$\alpha = 23^h\ 21^m\ 19^s.1 \quad : \quad \delta = -22^\circ\ 17'\ 26''.8$

6.7 : 12.7

1899.610	136.3	..
.647	134.8	5.19
.666	134.7	5.12
.675	132.5	5.10
1899.650	134.6	5.14

λ_1 186.

$\alpha = 23^h\ 20^m\ 43^s.6 \quad : \quad \delta = -23^\circ\ 4'\ 8''.2$

7.7 : 10.2

AB

1899.600	56.3	1.40
.675	57.9	1.24
.689	52.2	1.13
1899.655	55.5	1.26

This star and λ_1 484 are identical. The catalogue number given to λ_1 484 is correct with place as above.

AC

1899.600	144.4	22.54
.675	144.1	22.65
.689	145.1	22.85
1899.655	144.5	22.68

λ_1 187.

$\alpha = 23^h\ 25^m\ 46^s.2 \quad : \quad \delta = -33^\circ\ 50'\ 25''.5$

8.2 : 10

1898.671	298.1	10.22	B
1899.692	299.5	9.78	
.744	298.0	9.90	
1899.718	298.7	9.84	C

λ_1 189.

$\alpha = 23^h\ 28^m\ 24^s.8 \quad : \quad \delta = -36^\circ\ 49'\ 1''.1$

7 : 11.7

1899.660	116.4	19.89
.692	116.2	19.87
.796	115.7	19.86
1899.716	116.1	19.87

λ_1 490.

$\alpha = 23^h\,28^m\,32^s.6$; $\delta = -35^\circ\,3'\,50''.8$

8 ; 12.7

AB

t	θ	ρ
	°	"
1898.862	225.4	8.35
1899.914	228.4	8.47

AC. $C = 12.5$

1898.862	336.0	21.81
1899.914	335.9	21.29

λ_1 491.

$\alpha = 23^h\,28^m\,35^s.7$: $\delta = -34^\circ\,12'\,51''.8$

7.5 ; 13

1898.925	72.4	17.53
1899.647	72.2	17.76
.666	72.4	17.66
1899.413	72.3	17.65

λ_1 492.

$\alpha = 23^h\,30^m\,23^s.5$: $\delta = -28^\circ\,2'\,17''.5$

6 ; 7.5

1899.586	270.8	0.45
.796	267.7	0.39
1899.691	269.2	0.42

λ_1 493.

$\alpha = 23^h\,32^m\,54^s.6$: $\delta = -25^\circ\,46'\,25''.3$

7.5 ; 12.5 ; 13

$A, \frac{BC}{2}$

t	θ	ρ
	°	"
1899.605	206.4	37.53
.619	208.1	36.64
.744	206.9	37.35
1899.656	207.1	37.16

BC

1899.605	267.9	1.66
.619	272.8	0.88
.744	271.6	1.00
1899.656	270.8	1.18

λ_1 495.

$\alpha = 23^h\,45^m\,14^s.9$: $\delta = -34^\circ\,7'\,3''.1$

8 : 13

1899.647	15.1	2.54
.666	14.4	2.66
.668	17.4	2.62
1899.660	15.5	2.61

λ_1 496.

$\alpha = 23^h\,46^m\,16^s.3$; $\delta = -32^\circ\,59'\,11''.9$

7.3 : 12.5

1899.675	191.0	16.43
.692	192.8	16.75
.744	192.5	16.74
1899.704	192.1	16.63

λ_1 497.

$\alpha = 23^h\,46^m\,43^s.2$: $\delta = -28^\circ\,52'\,58''.5$

8.2 : 12.3

t	θ	ρ
	°	"
1898.925	68.9	1.62
1899.586	69.1	1.86
.602	69.6	1.87
1899.371	69.3	1.78

λ_1 498.

$\alpha = 23^h\,47^m\,15^s.8$; $\delta = -28^\circ\,55'\,46''.7$

8.7 ; 12

1898.925	174.7	1.34
1899.586	175.3	1.17
.602	175.2	1.20
1899.371	175.1	1.21

λ_1 500.

$\alpha = 23^h\,59^m\,45^s.3$: $\delta = -33^\circ\,2'\,49''.0$

1899.692

Have examined this star a number of times, but without being able to recognize it without the original chart. To-night, while the seeing is rather poor, I can see no trace of any duplicity.

Lowell Observatory, Flagstaff, Arizona.

ON THE PROBABLE MOTION IN THE STELLAR SYSTEM "KRUEGER 60."

BY ERIC DOOLITTLE.

Among the stars observed by BURNHAM at the Lick Observatory during the years 1888 to 1892 were 67 stars noted as double in KRUEGER's catalogue of the *Astronomische Gesellschaft*. Of these, number 60 (which is number 13170 in the A.G. Catalogue, Zone $+55^\circ$ to $+65^\circ$), was measured as follows:

	°	"				
1890.788	178.8	2.32	9.0	12.0	*AB*	$1n\beta$
	56.3	26.82		9.3	*AC*	

In 1898, measures secured on five good nights showed that the relative positions of the stars had greatly changed, and this was confirmed by measures made during the past month, the change being so rapid that observations separated by two years, only, are sufficient to clearly indicate its character. These observations are,

	°	"				
1898.146	140.66	3.19	9.1	10.5	*AB*	$5n$
	58.71	34.39		9.4	*AC*	
1900.737	134.02	3.18	9.1	11.1	*AB*	$4n$
	59.19	36.18		9.4	*AC*	

There are no other measures on these stars.

In order to ascertain which two, of the three stars, are in motion, the difference in right-ascension and declination between *A* and the three neighboring stars A.G. Catal. 13149, 13177 and 13199 was measured. These latter appear to be fixed, or at least the change, if any, in their relative positions since 1873 is less than 1″. The mean of the three resulting positions of *A* compare with KRUEGER's values as follows:

	α	δ	
1900.82	$22^h\,23^m\,29^s.83$	$+57^\circ\,4'\,7''.90$	Ep.1875.0.
1873.7	32.35	16.3	

From the micrometric measures of *AC*, given above, assuming the change due wholly to the motion of *A*, we find that this is *0″.93 in the direction 247°.9*. The residuals from the A.G. Catalogue position for the date 1873.7 under this hypothesis are $\Delta\alpha = -0^s.33$, $\Delta\delta = -1''.0$.*

This motion is hence approximately the true one, and the whole change in *AC* is due to the above proper motion of *A*.†

* These, of course, include also the errors of my own measures of $\Delta\alpha$ and $\Delta\delta$.

† KRUEGER's description of the system is, "Dupl. 12″, prec. com. 9.3m." The motion here derived would give for the position of the companion in 1873.7: 41°.0, 11″.76. As KRUEGER's estimates of distance are seldom 2″ in error, and as in only one instance is the error as great as 4″, BURNHAM's first measure alone (26″.82) serves to suggest a large change.

The star, *B*, was not seen by KRUEGER.

This proper motion of *A* does not account for the large change in *AB*. Were *B* fixed, its position in 1890.79 should have been: 228°.2, 8″.68. Though the measures are so few, they indicate that there is here a faint star with a large proper motion, attended by a minute companion, which either has a large proper motion of its own, or else is in rapid revolution about the primary.

The Flower Observatory, 1900 October 30.

THE SHORT-PERIOD VARIABLE STAR, S.DM. −21°1019,

BY R. T. A. INNES.

[Communicated by Dr. DAVID GILL, C.B., etc., H.M. Astronomer at the Cape of Good Hope.]

The position of this star is (1875) 1h 50m 56s −21° 21′.9, and its variation is referred to in the *Astr. Nach.*, No. 2987 and the *A.J.*, No. 168.

The observations made here, at which the time was also recorded, are as follows:

1899	Cape M.T. h	m	Mag.
Feb. 27	7	20	10.0
Mar. 1	8	26	10.0
7	8	21	9.15
13	10	6	9.25
17	7	17	9.9
	8	10	9.3
		51	9.3
19	6	53	9.25
	7	11	9.3
		31	9.4
		51	9.4
21	8	33	9.63
	10	31	9.75
22	8	59	9.6
Apr. 27	6	22	9.5
28	6	17	9.0 good
29	7	32	9.85 v.g.
		38	9.7
		41	9.7
May 2	6	22	9.7
	7	20	9.3
		41	9.1 v.g.
1	6	25	9.7
		30	9.6
		46	9.45
10	6	17	9.8
	7	0	9.75
Nov. 5	8	8	9.85
7	8	21	9.2 ±
		28	9.3
		33	9.33
		55	9.35
	9	0	9.35
		26	9.37
10	8	13	9.1
		57	9.4

1899	Cape M.T. h	m	Mag.
Nov. 10	9	9	9.4
		26	9.4
		32	9.4
		41	9.4
11	9	7	9.1
		11	9.15
		25	9.25
		38	9.25
	10	7	9.35
		27	9.37
		48	9.35
	11	27	9.37
12	9	57	9.6
	10	17	9.7
		29	9.6
		31	9.6
		49	9.6
	11	2	9.7
13	8	17	9.75
	9	5	9.7
17	7	55	9.45
18	10	6	9.3
		33	9.3
21	8	52	9.45
	9	4	9.45
		29	9.4
		42	9.45
22	9	10	9.45
23	8	47	9.6 cl'dy
26	9	32	9.75
		38	9.75
27	7	11	9.75
28	9	12	9.45
		19	9.45
29	8	9	9.8
Dec. 1	7	50	9.45

Royal Observatory, Cape of Good Hope, 1900 February 2.

1899	Cape M.T. h	m	Mag.
Dec. 1	8	23	9.5
		55	9.5
	9	33	9.5
2	9	26	9.3
		39	9.3
4	8	35	9.7
	9	4	9.5
		16	9.5
6	8	7	9.6
7	10	24	9.7
8	7	53	9.6
9	7	53	9.2
	8	3	9.27
		32	9.2
13	9	30	9.3
15	8	51	9.55
16	13	56	9.9
	14	46	9.92
19	8	1	9.45
		27	9.43
	9	6	9.45
20	8	49	9.25
	9	9	9.2
22	8	11	9.7
		26	9.7
24	8	15	9.85
		47	9.6
	10	32	9.2
		47	9.3

1899	Cape M.T. h	m	Mag.
Dec. 27	8	11	9.3
1900			
Jan. 2	8	24	9.15
3	8	13	9.3
		23	9.3 yel. 4
4	8	14	9.65
		27	9.8
		49	9.85
5	8	7	9.55 clds.
		9	9.5 "
		37	9.6
6	8	8	9.15
		49	9.45
	11	8	9.7
		23	9.7
		46	9.7
	12	29	9.7
		52	9.75
	13	2	9.75
7	9	9	9.2
8	10	5	9.75
9	9	41	9.65
10	8	18	9.3
12	9	2	9.65
13	8	28	9.47
14	7	56	9.25
	8	14	9.3
		40	9.3
15	9	47	9.65

It will be noticed that the phases are repeated about every four and every seven days. The observations 1899 Nov. 5 to 1900 Jan. 15 are represented very well by a period of 13h 58m.9, but this period will not represent the observations 1899 Feb. 27 to May 10; a period of 13h 48m gives a somewhat poor representation of the earlier observations.

At some future period it is purposed to resume observations of this variable.

P.S. — I recently announced the variability of Lal. 14755, but I now find that this was previously discovered by Mr. PERRY (see *A.J.* 398) = *Z Puppis*. A maximum occurred on 18 Dec. 1899 = 8M.0.

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THE

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No. 487.

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NOTES ON VARIABLE STARS. — No. 33,

BY HENRY M. PARKHURST.

4315 *R Comae*. Although the erection of a tall building adjoining my observatory on the north has compelled most of my observations to be made in the bright twilight, there seems to be no material inaccuracy from that source. The shortening of the period shown by the correction suggests a new sine formula to replace that given in *A.J.* 384:

$$2389899.2 + 361.5\,E + 20 \sin (10^\circ E + 120^\circ).$$

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date		E	Corr.	W	Mag.	Factors			Remarks
			Julian	Calendar								
				1899							d	
7944	*T Pegasi*	Max.	4871	Aug. 4	31	+38	9	8.97	1.05	1.18	28	1899
"	"	Max.	5249	Aug. 17	35	+43	9	9.55	1.85	1.97	34	1900
7999	*X Aquarii*	Max.	4946	Oct. 18	5	+27	6	7.6	–	–	–	
8230	*S Aquarii*	Max.	4933	Oct. 5	52	– 6	7	–			–	
8290	*R Pegasi*	Max.	4867	July 31	17	–88	1	–	–		–	Very uncertain
8369	*W Pegasi*	Max.	4854	July 18	2	–	5	7.5	0.90	0.90	21	Assuming period 338 days
8373	*S Pegasi*	Max.	4915	Sept. 17	40	+ 5	9	8.23	1.91	1.38	25	
"	"	Max.	5227	July 26	41	–	E	–	–	–	–	Approximately accurate
8512	*R Aquarii*	Max.	4991	Dec. 2	83	+19	5	6.29	2.10	1.60	–	Subtangent approximation
8622	*W Ceti*	Max B	5041	Jan. 21			6		–		–	Another max? Period 358 days?
103	*T Andromedae*	Min.	5005	Dec. 16	11	+25	2		–		–	Interruption 102 days
906	*R Trianguli*	Min.	4979	Nov. 20	13	–17	1	–			–	Interruption 124 days
"	"	Max.	5112	Apr. 2	13	– 3	6				–	Probably later
1113	*U Arietis*	Max.	4960	Nov. 1	–	–	1					Estimated interval 348 days
1222	*R Persei*	Min.	5026	Jan. 6	67	+21	2	14.1	1.3	1.3	156	Interruption 142 days
"	"	Max.	5119	Apr. 9	67	+18	5	–	–	–	–	
1717	*V Tauri*	Max.	5085	Mar. 6	59	–	1E	–				Perry's observations
2689	*Z Puppis*	Max.	5070	Feb. 19	–	–	4r				–	Period appears irregular
3264	*W Cancri*	Max.	5146	May 6	9	+23	9	9.65	0.15	0.82	9	
3493	*R Leonis*	Max.	5112	Apr. 2	167	–32	5	–	–	–	–	Possibly earlier
3994	*S Leonis*	Max.	5144	May 4	76	–67	8	10.26	0.98	1.55	13	Corr. slowly increasing
4315	*R Comae*	Max.	5225.2	July 21	70	–9.1	9	9.29	0.58	0.57	9	El. *A.J.* 384. See note above
4377	*T Virginis*	Max.	5158	May 18	42	+ 8	9	9.64	1.25	1.77	29	
4596	*U Virginis*	Max.	5186	June 15	60	–12	9	8.35	3.31	2.76	28	
4665	*RT Virginis*	Max.	5146	May 6	1	–	9	8.64	1.76	2.69	41	Last interval 394 days
4847	*S Virginis*	Max.	5229	July 28	47	+25	5	–				
5070	*Z Virginis*	Max.	5237	Aug. 5	24	+20	2	–				From the light-curve
5194	*V Bootis*	Min.	5215	July 14	23	+ 5	9	10.50	2.00	1.35	48	
5237	*R Bootis*	Min.	5156	May 16	69	– 4	4	–			–	
5249	*V Librae*	Max.	5220	July 19	26	– 2	8	8.90	1.13	1.12	28	Elements. *A.J.* 444
5338	*U Bootis*	Max.	5261	Aug. 29	42	+54	9	10.09	2.12	1.84	37	Obs. entire period
5405	*RT Librae*	Min.	5171:	June –		–	E	–	–	–	–	Period assumed 250 days
5430	*T Librae*	Max.	5196	June 25	34	– 1	3	–	–	–	–	
5494	*S Librae*	Min.	5214	July 13	50	+13	2	–	–	–	–	Possibly a little later

Individual Observations.

Including Observations by Arthur C. Perry.

7944 *T Pegasi.*

(Continued from 431.)

Julian	Calendar	Mag.
	1899	
4822.7	June 16	10.4
4835.7	29	10.26_2
4837.6	July 1	10.86_2
4839.6	3	9.88_2
4846.6	10	9.80_2
4854.6	18	9.44_2
4866.7	30	9.17
4867.7	31	8.71_2
4875.6	Aug. 8	8.95_2
4883.6	16	9.17_2
4892.6	25	9.69_2
4895.5	28	9.71_2
	1900	
5220.6	July 19	10.73_2
5230.6	29	9.83_2
5232.6	31	9.69_2
5248.6	Aug. 16	9.38_2
5257.5	25	9.57_2
5263.5	31	9.66_2
5268.5	Sept. 5	9.89_2
5275.5	12	9.88_2

7999 *X Aquarii.*

(Continued from 431.)

Julian	Calendar	Mag.
	1899	
4882.6	Aug. 15	11.2
4903.6	Sept. 5	9.4
4911.5	13	8.89_2
4914.5	16	8.45_2
4918.5	20	8.43_2
4921.5	23	8.00_2
4925.5	27	8.19_2
4928.5	30	8.22_2
4938.5	Oct. 10	8.14_2
4947.5	19	7.56_2
4953.5	25	8.21_2

8230 *S Aquarii.*

(Continued from 400.)

Julian	Calendar	Mag.
	1899	
4911.6	Sept. 13	8.5
4914.6	16	8.36_2
4918.5	20	7.83_2
4926.5	28	8.61_2
4929.5	Oct. 1	7.11_2
4932.5	4	7.78_2
4940.5	12	7.85_2
4948.5	20	8.22_2

8290 *R Pegasi.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4867.7	July 31	7.70_2
4875.6	Aug. 8	8.35_2
4875.6	8	8.7P
4883.6	16	8.93_2
4895.5	28	9.24
4897.6	30	9.1P
4905.6	Sept. 7	9.6P
4910.5	12	9.71_2
4918.5	20	9.54_2
4928.5	30	10.77_2
4928.6	30	10.1P

8369 *W Pegasi.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4845.6	July 9	7.6
4846.6	10	7.78_2
4854.6	18	7.50_2
4867.7	31	8.11_2
4875.6	Aug. 8	8.12_2
4875.6	8	8.1P
4876.6	9	8.3
4883.6	16	8.75_2
4897.6	30	9.0P

8373 *S Pegasi.*

(Cont. from 464. Comp. Stars 464)

Julian	Calendar	Mag.
	1899	
4866.7	July 30	9.03_2:
4867.7	31	8.66_2
4875.6	Aug. 8	8.8P
4875.7	8	8.98_2
4883.6	16	8.90_2
4892.6	25	8.40_2
4897.6	30	8.6P
4905.6	Sept. 7	8.8P
4910.5	12	8.35_2
4913.5	15	8.22_2
4918.5	20	8.09_2
4921.5	23	8.39_2
4926.5	28	8.57_2
4928.6	30	8.5P
4940.5	Oct. 12	8.33_2
4965.5	Nov. 6	9.3P
4984.5	25	9.8P
	1900	
5229.6	July 28	8.3
5235.6	Aug. 3	8.19_2
5248.6	16	8.94_2

8512 *R Aquarii.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4914.6	Sept. 16	8.71_2
4918.5	20	8.99_2
4926.5	28	8.81_2
4929.6	Oct. 1	8.16_2
4932.6	4	7.85_2
4940.5	12	7.60_2
4950.5	22	7.03
	1900	
5026.5	Jan. 6	7.15_2
5036.5	16	7.76_2
5041.5	21	8.26_2

8622 *W Ceti.*

(Continued from 464.)

Julian	Calendar	Mag.
	1899	
4927.6	Sept. 29	10.2
4929.6	Oct. 1	9.75_2
4932.5	4	9.15_2
4938.5	10	9.7
4948.5	20	10.23_2
4949.5	21	9.95_2
	1900	
5026.5	Jan. 6	7.85_2
5041.5	21	7.27_2
5049.5	29	7.65_2
5053.5	Feb. 2	7.51_2

103 *T Andromedae.*

(Cont. from 468. Comp. Stars 346)

Julian	Calendar	Mag.
	1899	
4913.6	Sept. 15	8.6
4926.5	28	10.01_2
4932.5	Oct. 4	9.80_2
4940.6	12	10.00_2
	1900	
5042.5	Jan. 22	9.0]
5054.5	Feb. 3	10.5]
5082.5	Mar. 3	9.61_2

906 *R Trianguli.*

(Continued from 468.)

Julian	Calendar	Mag.
	1899	
4911.6	Sept. 13	9.5
4915.6	17	9.65_2
4926.5	28	10.36_2
4932.5	Oct. 4	10.09_2
	1900	
5056.5	Feb. 5	8.53_2
5082.5	Mar. 3	7.72
5095.5	16	8.26_2
5096.5	17	8.30_2
5103.5	24	6.42_2
5111.5	Apr. 1	6.44_2
5115.5	5	6.83_2
5118.5	8	6.31_2

1113 *U Arietis.*

(Cont. from 468. Comp. Stars 314)

Julian	Calendar	Mag.
	1899	
4938.5	Oct. 10	9.7]
4940.6	12	9.7]
	1900	
5028.5	Jan. 8	8.6
5042.5	22	9.0
5054.5	Feb. 3	9.5
5065.5	14	10.0

1222 *R Persei.*

(Continued from 403.)

Julian	Calendar	Mag.
	1899	
4940.6	Oct. 12	8.88_2
	1900	
5082.5	Mar. 3	11.56_2
5095.5	16	10.24_2
5096.5	17	10.50_2
5103.5	24	9.80_2
5110.5	31	9.19_2
5115.5	Apr. 5	9.76_2
5118.5	8	8.50_2
5123.5	13	8.84_2

1717 *V Tauri.*

(Continued from 468.)

Julian	Calendar	Mag.
	1900	
5056.6	Feb. 5	10.5P
5081.6	Mar. 2	9.3P

2689 *Z Puppis.*

(Cont. from 468. Comp. Stars 403)

Julian	Calendar	Mag.
	1899	
4965	Nov. 6	10.3P
	1900	
5056	Feb. 5	8.0P
5065.5	14	8.35_2
5070.5	19	8.59_2

2689 *Z Puppis.* — Cont.

Julian	Calendar	Mag.
	1900	
5078.5	Feb. 27	9.14_2
5081	Mar. 2	8.7P
5089.5	10	8.83_2
5101.5	22	8.94_2
5110.5	31	8.86_2
5117.5	Apr. 7	9.09
5129.5	19	9.82_2

3264 *W Cancri.*

(Cont. from 441. Comp. Stars 384)

Julian	Calendar	Mag.
	1900	
5103.5	Mar. 24	11.5
5107.5	28	12.5]
5110.5	31	12.5
5129.5	Apr. 19	11.02_2
5136.5	26	10.19_2
5141.5	May 1	10.35_2
5142.5	2	9.70_2
5145.5	5	9.38_2
5150.5	10	9.79_4
5152.5	12	9.71_4

3493 *R Leonis.*

(Continued from 441.)

Julian	Calendar	Mag.
	1899	
4731.5	Mar. 17	9.58
	1900	
5107.5	Mar. 28	5.2
5113.5	Apr. 3	4.78_2
5116.5	6	5.20_2
5119.5	9	5.43_2
5123.5	13	5.84_2
5136.5	26	5.98_2
5145.5	May 5	5.73_2

3994 *S Leonis.*

(Continued from 441.)

Julian	Calendar	Mag.
	1900	
5107.5	Mar. 28	12.7
5110.6	31	12.18_2
5129.5	Apr. 19	11.16_2
5137.6	27	10.08_2
5156.6	May 16	10.66_2
5162.6	22	10.62_2
5169.6	29	10.99_2
5176.6	June 5	11.08_2

4315 *R Comae.*

(Continued from 470.)

Julian	Calendar	Mag.
	1900	
5186.6	June 15	11.90_2
5193.6	22	11.22_2
5200.6	29	10.93_4
5208.6	July 7	9.40_2
5212.6	11	10.14_4:
5214.6	13	9.39_2
5217.6	16	9.84_3
5220.6	19	9.55_4
5223.6	22	9.27_4
5225.6	24	9.28_5
5228.6	27	9.40_{10}
5229.6	28	9.56_{10}
5230.6	29	9.58_{10}
5231.6	30	9.74_{10}
5232.6	31	9.94_8

4377 *T Virginis.*

(Continued from 470.)

Julian	Calendar	Mag.
	1900	
5103.5	Mar. 24	12.5]
5113.5	Apr. 3	11.3:
5129.6	19	10.2
5135.5	25	10.07_2
5137.6	27	9.6
5139.5	29	10.10_4
5147.6	May 7	10.23_2
5156.6	16	9.40_2
5162.6	22	9.89_2
5169.6	29	10.32_2
5176.6	June 5	9.60_2
5186.6	15	9.87_2

4596 *U Virginis.*

(Continued from 470.)

Julian	Calendar	Mag.
	1900	
5107.5	Mar. 28	11.5
5117.5	Apr. 7	11.5:
5129.6	19	10.5
5135.5	25	10.42_2
5160.6	May 20	9.4
5160.6	20	9.34_2
5167.6	27	8.70_2
5176.6	June 5	8.05_2
5181.6	10	8.42_2
5186.6	15	8.37_2
5193.6	22	8.53_2
5202.6	July 1	8.07_2
5214.6	13	9.04_2

4665 *RT Virginis.*

(Continued from 470.)

Julian	Calendar	Mag.
	1900	
5107.5	Mar. 28	9.1
5113.5	Apr. 3	8.83_2
5116.5	6	9.11_2
5119.5	9	8.98_2
5125.6	15	8.99_2
5136.6	26	9.17_2
5147.6	May 7	8.67_2
5154.6	14	9.09_2
5167.6	27	9.04_2
5177.6	June 6	8.81_2
5188.6	17	9.08_2

4847 *S Virginis.*

(Cont. from 470. Comp. Stars 384)

Julian	Calendar	Mag.
	1900	
5107.6	Mar. 28	12.6
5116.6	Apr. 6	11.8]
5129.6	19	12.4]
5136.6	26	13.72_2
5137.6	27	12.52_2
5160.6	May 20	11.25
5172.6	June 1	11.74_2
5186.6	15	11.14_2
5196.6	25	10.92_2
5208.6	July 7	9.52_2
5218.6	17	8.20_2
5229.6	28	8.25_2
5232.6	31	8.38_2

5070 *Z Virginis.*

(Continued from 470.)

Julian	Calendar 1900	Mag.
5129.6	Apr. 19 to	
5196.6	June 25	13.0]
	6 dates	
5217.6	July 16	11.2

5194 *V Bootis.*

(Cont. from 470 Comp. Stars 333)

Julian	Calendar 1900	Mag.
5118.6	Apr. 8	8.39_2
5125.6	15	7.56_2
5135.6	25	8.54_2
5139.6	29	8.60_2
5147.5	May 7	8.85_2
5151.5	14	8.62_2
5188.7	June 17	10.36_2
5203.6	July 2	10.09_2
5214.6	13	11.31_2
5225.6	24	9.98_2

5194 *V Bootis.*—Cont.

Julian	Calendar 1900	Mag.
5231.6	July 30	10.18_2
5236.5	Aug. 4	10.47_2
5211.5	9	9.70_2

5237 *R Bootis.*

(Cont. from 470 Comp. Stars 333)

Julian	Calendar 1900	Mag.
5113.6	Apr. 13	12.0 :
5129.6	19	12.0 :
5137.6	27	11.94_2
5167.6	May 27	11.3 :
5169.6	29	12.39_2

5249 *V Librae.*

(Cont. from 470. Comp. Stars 388)

Julian	Calendar 1900	Mag.
5162.6	May 22	13]
5176.6	June 5	11.5
5177.6	6	11.45_2
5186.6	15	10.51_2

5249 *V Librae.*—Cont.

Julian	Calendar 1900	Mag.
5193.6	June 22	9.98_2
5201.6	30	9.47_2
5215.6	July 11	8.79_2
5218.6	17	8.81_2
5225.6	24	8.92_2
5231.6	30	9.21_2
5236.5	Aug. 4	9.33_2

5338 *U Bootis.*

(Continued from 476.)

Julian	Calendar 1900	Mag.
5113.5	Apr. 3	10.1
5116.6	6	10.71_2
5125.6	15	10.57_2
5136.6	26	11.20_2
5137.6	27	10.92_2
5167.6	May 27	10.89_2
5177.6	June 6	10.31
5188.6	17	10.44_2

5338 *U Bootis.*—Cont.

Julian	Calendar 1900	Mag.
5202.6	July 1	10.87_2
5214.6	13	10.30_2
5228.6	27	9.99_2
5235.6	Aug. 3	10.89_2
5216.5	14	10.39_2
5250.5	18	10.09_2
5260.5	28	9.99_2
5265.5	Sept. 2	10.15_2
5266.5	3	10.16_2
5267.5	4	10.13_2
5272.5	9	10.15_2
5279.5	16	10.32_2
5287.5	24	10.90_2
5301.5	Oct. 11	11.04_2

5405 *RT Librae.*

(Continued from 476.)

Julian	Calendar 1900	Mag.
5139.6	Apr. 29 to	
5236.5	Aug. 4	12]
	9 dates	

5430 *T Librae.*

(Continued from 414.)

Julian	Calendar 1900	Mag.
5139.6	Apr. 29	11.5]
5165.6	May 25	11.5]
5176.6	June 5	11.5]
5188.6	17	11.35_2
5196.6	25	11.37_2
5202.6	July 1	11.31_2
5217.6	16	11.8

5491 *S Librae.*

(Continued from 476.)

Julian	Calendar 1900	Mag.
5165.6	May 25	9]
5176.6	June 5	10.6
5188.6	17	10.87_2
5193.6	22	11.16_2
5201.6	30	11.26_2
5217.6	July 16	11.30_2

Comparison Stars, 1893–1900.

7999 *X Aquarii.*

Star	DM.	Mag.	n
G	−21°6197	7.53	11
N	−20°6427	8.33	1
P	−21°6193	8.67	30
1 *Q*	−20°6414	8.52	1
R	−21°6191	8.90	26
T	−21°6198	9.08	10
1 *T*	−20°6428	9.09	11
U	−21°6190	9.40	4
W	−21°6192	9.88	3
a	5*n*2*p* *P*	9.88	5
b	4*s*2*p* *R*	10.07	5

8290 *R Pegasi.*

Star	DM.	Mag.	n
D	+10°4887	7.71	7
G	+10°4891	8.07	2
I	+9°5156	7.52	15
N	+9°5154	8.70	14
1 *N*	+9°5164	8.65	10
T	+10°4877	9.36	4
1 *T*	+9°5161	9.39	13
U	+9°5155	9.68	3
W	+9°5163	10.17	11
Z	+9°5159	10.09	21
d	1*n*2*f* *V*	10.88	3

8512 *R Aquarii.*

Star	DM.	Mag.	n
C	−16°6345	5.52	5
D	−16°6317	7.03	3
H	−15°6491	7.40	13
J	−16°6341	7.87	22
1 *J*	−15°6487	7.67	8
L	−15°6494	8.02	17
Q	−16°6353	8.94	27
1 *T*	−16°6356	9.72	14
2 *T*	−15°6485	9.57	4
1 *W*	−15°6479	10.49	8
X	−16°6354	10.83	9

8622 *W Ceti.*

Star	DM.	Mag.	n
D	−14°6588	7.03	6
E	−15°6539	7.32	8
L	−15°6532	8.72	35
N	−15°6528	8.78	21
1 *T*	−15°6527	9.92	6
2 *T*	−15°6534	9.55	9
3 *T*	−15°6512	9.15	10
U	−15°6529	9.72	29
1 *U*	−15°6536	9.70	13
1 *X*	−15°6530	10.21	11
2 *X*	−15°6533	10.10	7

NORMAL POSITIONS OF *CERES*.

By G. W. HILL.

Having been at some pains to collect the observations of *Ceres* for the century it has now been known, and having formed normals from that part of the material which seemed suitable for the purpose, I have concluded to publish the results apart from any comparison with a definite theory. The elaboration of the latter involves so much work, that, although something has been done, I can hardly hope to finish it; but the labor of forming the normals need not be lost.

Ceres has been observed at every opposition since its discovery; but on two occasions, which will be noticed in the list to be given, the material is so scanty and discordant that no normals were formed. Some of the observations in the decade following the discovery of the planet were unreduced, especially those made at Palermo and Milan; these have been reduced as well as the data permit. But, for the remaining material, the published reduction has been accepted without the application of any corrections. No accurate ephemeris of the planet was published for the interval 1801–1830; accordingly, approximate tables were constructed giving the heliocentric position. The theory employed was that in *A.J.*, Vol. XVI, pp. 57–62. But only the ten largest equations were tabulated, as that seemed sufficient for the purpose of normal-forming.

The following is a description of the material used in each opposition. As it was concluded to form but one normal for each of these occasions, the aim has been to limit the range of observation to 10 days. Where the observations are found in out of the way places, the place of publication is given.

1801. — For this normal we have only the observations of PIAZZI at Palermo, between Jan. 1–Feb. 11. They have been reduced anew. The observation at the transit instrument on Jan. 18 is in error in some incorrigible way and is rejected.

1802. — 29 observations at Palermo, Mar. 2–Apr. 19; 9 at Vienna, Mar. 3–Mar. 20; 6 at Greenwich, Mar. 6–Apr. 21; 13 at Paris, Apr. 7–Apr. 30; 27 at Seeberg, Mar. 3–Apr. 19.

1803. — 7 observations at Greenwich, June 23–July 18; 17 at Paris, June 25–July 27; 13 at Seeberg, July 1–July 23; 13 at Milan, June 27–July 26; 28 at Palermo, June 17–July 23. The Milan observations are in the *Effemeridi Milano.*

1804. — 5 observations at Paris, Sept. 13–Oct. 6; 19 at Seeberg, Sept. 13–Oct. 21; 12 at Milan, Sept. 19–Oct. 17; 15 at Palermo, Oct. 2–Oct. 25.

1806. — 6 observations at Palermo, Jan. 8–Feb. 8; 1 at Milan, Jan. 4–Jan. 12; 2 at Ofen, Jan. 22–Jan. 29.

1807. — 17 observations at Palermo, May 2–May 24; 15 at Paris, Apr. 21–May 24; 9 at Göttingen, Apr. 26–May 7; 5 at Padua, Apr. 10–Apr. 24; 15 at Milan, Apr. 19–May 13.

1808. — 21 observations at Palermo, Aug. 2–Aug. 25; 1 at Gottingen, July 25; 9 at Milan, July 28–Aug. 14.

1809. — 4 observations at Palermo, Nov. 11–Nov. 18; 4 at Milan, Nov. 1–Nov. 6; 8 at Paris, Oct. 23–Nov. 9.

1811. — 4 observations at Greenwich, Mar. 9–Mar. 22; 8 at Seeberg, Feb. 17–Feb. 25; 8 at Milan, Feb. 13–Feb. 25; 3 at Paris, Feb. 18–Feb. 27.

1812. — 6 observations at Palermo, June 8–June 19; 7 at Milan, June 6–June 13; 2 at Greenwich, June 11–June 15.

1813. — 3 observations at Vienna, Sept. 18–Sept. 20; 7 at Wilna, Sept. 2–Sept. 10; 4 at Copenhagen, Sept. 8–Sept. 18. These observations are in the *Berl. Jahrbuch*, 1817, p. 146.

1814. — There has been found but one complete observation at Greenwich on Dec. 16, and a R.A. at Königsberg on Dec. 3. As the two R.A.s disagree, no normal has been formed for this opposition.

1816. — 4 observations at Paris, Apr. 19–May 1; 12 at Königsberg, Mar. 26–Apr. 18.

1817. — As but one observation was found, which was made at Greenwich July 19, it was thought not worth while to give a normal for this opposition.

1818. — 5 observations at Greenwich, Oct. 7–Oct. 16; 8 at Königsberg, Oct. 3–Oct. 20.

1820. — 2 observations at Mannheim, Jan. 31–Feb. 2; 3 at Berlin, Feb. 8–Feb. 14; 3 at Munich, Feb. 5–Feb. 8. The first and second sets are in the *Berl. Jahrbuch* for 1823 and 1824.

1821. — 5 observations at Greenwich May 16–May 26; 5 at Paris, May 21–June 1; 9 at Königsberg, May 8–June 3.

1822. — 3 observations at Paris, Aug. 17–Aug. 29; 6 at Königsberg, Aug. 18–Sept. 3.

1823. — 3 observations at Greenwich, Dec. 1–Dec. 9; 5 at Paris, Nov. 10–Dec. 7.

1825. — 2 observations at Greenwich, Mar. 11–Mar. 18; 7 at Paris, Mar. 17–Mar. 29; 6 at Königsberg, Mar. 8–Mar. 19; 7 at Göttingen, Mar. 9–Apr. 7.

1826. — 6 observations at Greenwich, June 23–July 1; 4 at Königsberg, June 27–July 2.

1827. — 8 observations at Königsberg, Sept. 20–Sept. 30.

1829. — 3 observations at Göttingen, Jan. 22–Feb. 11.

1830. — 8 observations at Greenwich, Apr. 19–May 3; 7 at Königsberg, Apr. 19–May 5; 9 at Göttingen, Apr. 24–May 5; 8 at Åbo, Apr. 17–May 12; 10 at Vienna, Apr. 17–May 12.

1831. — 6 observations at Greenwich, July 22–Aug. 7; 3 at Cambridge, July 22–July 31; 5 at Vienna, July 20–Aug. 6.

1832. — 4 observations at Königsberg, Oct. 29–Nov. 7; 6 at Altona, Oct. 21–Nov. 16; 6 at Kremsmünster, Oct. 16–Nov. 10.

1834. — 4 observations at Greenwich, Feb. 7–Feb. 21; 6 at Königsberg, Feb. 6–Mar. 3; 7 at Mannheim, Jan. 31–Feb. 23; 6 at Cracow, Feb. 13–Mar. 1; 8 at Munich, Feb. 9–Feb. 20; 11 at Vienna, Feb. 9–Mar. 2.

1835. — 3 observations at Greenwich, May 28–June 15; 10 at Königsberg, June 2–June 17; 10 at Kremsmünster, June 2–June 22; 8 at Vienna, June 3–June 27.

1836. — 6 observations at Greenwich, Aug. 17–Sept. 30; 5 at Vienna, Sept. 9–Sept. 29; 1 at Helsingfors, Sept. 7; 3 at Cracow, Sept. 5–Sept. 16; 2 at Kremsmünster, Aug. 31–Sept. 1.

1837. — 4 observations at Greenwich, Nov. 17–Jan. 4; 4 at Paris, Dec. 3–Dec. 26; 1 at Königsberg, Dec. 1; 3 at Vienna, Dec. 10–Dec. 30; 1 at Kremsmünster, Dec. 15.

1839. — 7 observations at Greenwich, Mar. 27–May 2; 8 at Paris, Mar. 26–Apr. 26; 9 at Königsberg, Mar. 26–Apr. 20; 6 at Vienna, Apr. 5–Apr. 29; 1 at Kremsmünster, Mar. 24–Apr. 23.

1840. — 3 observations at Greenwich, July 27–Aug. 11; 11 at Paris, July 11–Aug. 11; 3 at Vienna, July 21–Aug. 6.

1841. — 6 observations at Greenwich, Oct. 12–Nov. 12; 6 at Paris, Oct. 11–Oct. 28; 1 at Königsberg, Oct. 5; 8 at Vienna, Oct. 18–Nov. 11; 7 at Kremsmünster, Oct. 11–Nov. 5.

1843. — 2 observations at Greenwich, Jan. 30–Feb. 6; 2 at Paris, Jan. 19–Feb. 13; 2 at Königsberg, Jan. 13–Feb. 3; 5 at Kremsmünster, Feb. 1–Feb. 15.

1844. — 7 observations at Greenwich, Apr. 28–June 1; 11 at Paris, May 1–June 1; 9 at Königsberg, May 15–June 1; 9 at Kremsmünster, May 3–June 1; 11 at Hamburg, May 7–May 30.

1845. — 8 observations at Greenwich Aug. 15–Sept. 9; 5 at Paris, Aug. 22–Sept. 6; 9 at Königsberg, Aug. 11–Sept. 5; 8 at Kremsmünster, Aug. 9–Sept. 8.

1846.—4 observations at Greenwich, Nov. 3–Dec. 4; 2 at Paris, Nov. 20–Nov. 26; 5 at Bonn, Nov. 10–Dec. 1.

1848. — 7 observations at Greenwich, Mar. 7–Apr. 14; 8 at Paris, Mar. 12–Apr. 14; 2 at Königsberg, Mar. 13–Mar. 20; 9 at Hamburg, Mar. 22–Apr. 4.

1849. — 4 observations at Greenwich, July 13–July 26; 13 at Paris, June 18–July 13; 5 at Königsberg, July 7–July 13; 3 at Leipzig, July 11–July 15.

1850. — 9 observations at Greenwich, Sept. 6–Oct. 5; 1 at Paris, Sept. 6.

1852. — 6 observations at Greenwich, Dec. 10–Jan. 23; 7 at Kremsmünster, Jan. 1–Jan. 21.

1853. — 10 observations at Greenwich, Apr. 7–May 25; 5 at Paris, Apr. 17–May 10; 4 at Königsberg, Apr. 22–May 17; 6 at Kremsmünster, Apr. 25–May 24; 1 at Bonn, Apr. 13.

1854. — 4 observations at Greenwich, July 20–Aug. 29; 11 at Paris, Aug. 2–Aug. 30; 7 at Kremsmünster, July 24–Aug. 14; 3 at Bonn, July 25–Aug. 13.

1855. — 3 observations at Greenwich, Oct. 15–Nov. 10; 5 at Paris, Oct. 19–Nov. 11; 4 at Kremsmünster, Oct. 28–Nov. 13; 2 at Bonn, Oct. 22–Nov. 2.

1857. — 8 observations at Greenwich, Feb. 16–Mar. 16; 3 at Paris, Feb. 24–Mar. 11; 10 at Königsberg, Feb. 3–Mar. 1; 10 at Berlin, Jan. 31–Feb. 27; 13 at Kremsmünster, Feb. 14–Mar. 18; 11 at Bonn, Feb. 5–Feb. 25.

1858. — 11 observations at Greenwich, May 18–June 18; 7 at Paris, May 19–June 11; 12 at Königsberg, May 29–June 18.

1859. — 10 observations at Greenwich, Aug. 19–Sept. 19; 10 at Paris, Aug. 19–Sept. 18; 7 at Königsberg, Aug. 22–Sept. 19.

1860. — 7 observations at Greenwich, Nov. 15–Dec. 19; 3 at Paris, Nov. 18–Dec. 20; 3 at Königsberg, Nov. 30–Dec. 4; 3 at Berlin, Nov. 22–Dec. 6; 2 at Kremsmünster, Dec. 6–Dec. 21.

1862. — 13 observations at Greenwich, Mar. 25–May 5; 11 at Paris, Mar. 30–May 5; 3 at Berlin, Apr. 2–Apr. 9; 1 at Vienna, May 2; 3 at Copenhagen, Apr. 15–Apr. 21; 1 at Königsberg, Apr. 9; 8 at Kremsmünster, Apr. 2–May 3.

1863. — 7 observations at Greenwich, July 2–Aug. 7; 13 at Paris, July 5–Aug. 3; 1 at Leiden, July 12; 2 at Kremsmünster, July 19–July 28; 1 at Berlin, July 3.

1864. — 11 observations at Greenwich and Paris, Oct. 3–Nov. 3; 20 at Königsberg, Oct. 3–Oct. 11; 8 at Leiden, Oct. 2–Oct. 20; 5 at Cracow, Oct. 16–Oct. 23; 1 at Washington, Oct. 25–28.

1866. — 5 observations at Greenwich and Paris, Jan. 22–Feb. 23; 10 at Leiden, Jan. 15–Feb. 15; 4 at Washington, Jan. 31–Feb. 26.

1867 — 11 observations at Greenwich and Paris, May to June 4; 6 at Konigsberg, May 8–May 31; 5 at Bonn, May 18–May 30; 2 at Leipzig, June 1–June 2; 2 at Leiden, May 17–June 1; 2 at Kremsmünster, June 5–June 6.

1868. — 7 observations at Greenwich and Paris, Aug. 25–Sept. 8; 5 at Kremsmünster, Aug. 26–Sept. 4; 4 at Leiden, Aug. 8–Aug. 25; 11 at Warsaw, Aug. 10–Sept. 8; 9 (in Dec.) at Padua, Aug. 15–Sept. 5; 3 at Washington, Aug. 13–Aug. 29.

1869. — 6 observations at Greenwich and Paris, Nov. 8–Dec. 4; 2 at Berlin, Nov. 12–Dec. 1; 3 at Leipzig, Nov. 12–Nov. 29; 4 at Warsaw, Nov. 23–Dec. 10.

1871. — 4 observations at Greenwich, Mar. 21–Apr. 4; 5 at Kremsmünster, Mar. 13–Mar. 24; 5 at Berlin, Mar. 1–Mar. 24; 3 at Leiden, Feb. 28–Mar. 13.

1872.—11 observations at Greenwich and Paris, June 14–July 12; 3 at Königsberg, July 6–July 12; 2 at Kremsmünster, July 7–July 10; 2 at Berlin, June 20–June 29; 2 at Leipzig, June 23–July 8; 3 at Neufchâtel, June 14–June 22.

1873.—13 observations at Greenwich and Paris, Sept. 19–Oct. 17; 2 at Königsberg, Sept. 28–Oct. 13; 2 at Kremsmünster, Sept. 11–Oct. 3; 2 at Berlin, Sept. 18–Sept. 21; 1 at Vienna, Oct. 10; 3 at Leiden, Sept. 20–Sept. 27; 4 at Madrid, Sept. 21–Sept. 27.

1875. — 9 observations at Greenwich and Paris, Jan. 5–Jan. 28; 5 at Washington, Dec. 11–Dec. 23.

1876. — 16 observations at Greenwich and Paris, Apr. 21–May 19; 1 at Vienna, May 11; 6 at Washington, Apr. 22–May 13.

1877. — 7 observations at Greenwich and Paris, July 24–Aug. 27.

1878. — 7 observations at Greenwich and Paris, Oct. 25–Nov. 19; 4 at Hamburg, Nov. 6–Nov. 20.

1880. — 5 observations at Greenwich, Feb. 11–Mar. 15; 4 at Königsberg, Mar. 6–Mar. 14; 5 at Pulkowa, Feb. 16–Feb. 23; 1 at Washington, Nov. 5.

1881. — 11 observations at Greenwich, May 30–June 30; 1 at Hamburg, June 29; 3 at Washington, June 12–June 24.

1882. — 8 observations at Greenwich, Sept. 2–Oct. 21.

1883. — 10 observations at Greenwich, Nov. 20–Dec. 15; 10 at Washington, Nov. 17–Dec. 17.

1885. — 14 observations at Greenwich, Mar. 27–Apr. 25; 6 at Hamburg, Mar. 28–Apr. 28; 3 at Berlin, Apr. 24–Apr. 28.

1886. — 2 observations at Greenwich, July 5–Aug. 7; 6 at Paris, July 22–Aug. 10.

1887. — 6 observations at Greenwich, Sept. 29–Oct. 31; 9 at Paris, Oct. 14–Nov. 3.

1889. — 8 observations at Greenwich, Jan. 8–Feb. 15.

1890. — 9 observations at Greenwich, May 11–June 9; 8 at Paris, May 21–June 9.

1891. — 11 observations at Paris, Aug. 24–Sept. 16.

1892. — 6 observations at Greenwich, Nov. 1–Nov. 30.

1894. — 10 observations at Greenwich, Feb. 20–Mar. 27; 10 at Paris, Mar. 10–May 24.

1895. — 6 observations at Greenwich, June 5–July 9; 9 at Toulouse, June 15–July 6.

1896. — 4 observations at Greenwich, Sept. 9–Oct. 23.

1897. — 9 observations at Greenwich, Nov. 30–Dec. 31.

The dates of the normals are for Greenwich mean noon, and the given values of the coördinates are *true* not *apparent*. In the column headed No. Obs. where there are two numbers, the first belongs to the R.A. and the second to the Decl.; where but one number is given this is common to both. In the preceding list the number of observations given is that of R.A. except in one case where there was none.

Normal Positions of *Ceres*.

Date	True R.A.	True Decl.	No. Obs.
	h m s	° ′ ″	
1801 Jan. 21	3 26 31.25	+16 57 21.6	22–21
1802 Mar. 31	12 2 57.76	+17 59 18.4	84–80
1803 July 8	18 37 7.29	−28 11 23.2	78–67
1804 Oct. 1	0 37 31.42	+ 12 53 56.6	51–49
1806 Jan. 21	6 41 58.91	+30 31 28.2	12–11
1807 May 6	14 50 31.06	− 5 18 26.0	61
1808 Aug. 10	21 15 12.22	−29 25 56.6	31–29
1809 Nov. 3	2 15 57.21	+ 1 54 55.8	16
1811 Feb. 24	10 29 53.65	+27 0 11.2	23
1812 June 12	17 18 55.70	−23 7 54.0	15
1813 Sept. 11	23 36 0.01	−20 0 20.1	11
1816 Apr. 12	13 26 22.53	+ 7 10 11.5	16–15
1818 Oct. 13	1 41 7.26	1 33 1.8	13
1820 Feb. 7	8 28 16.06	+31 56 18.1	8–7
1821 May 25	15 58 28.51	−11 53 8.3	19
1822 Aug. 24	22 26 18.54	−25 50 22.0	9–8
1823 Nov. 24	3 19 13.11	+13 18 15.5	8–6
1825 Mar. 19	12 2 38.57	+18 26 28.2	22
1826 June 28	18 37 13.12	−27 55 32.5	10
1827 Sept. 26	0 36 36.66	−13 10 13.1	8
1829 Jan. 30	6 21 16.94	+30 40 39.0	3
1830 May 1	14 13 30.55	− 1 9 0.1	13
1831 July 29	21 18 10.16	−28 36 19.4	11–10
1832 Oct. 31	2 40 56.18	+ 1 1 6.1	16–10
1834 Feb. 17	10 21 31.16	+27 17 16.5	12–11
1835 June 10	17 14 5.88	−22 35 32.3	31
1836 Sept. 10	23 30 5.77	−19 38 21.2	17–16
1837 Dec. 15	5 1 35.07	+22 16 8.7	13
1839 Apr. 13	13 18 4.42	+ 8 12 23.7	34
1840 Aug. 2	19 38 43.35	−31 15 29.4	15–16
1841 Oct. 26	1 23 32.81	− 5 50 54.2	28–26
1843 Feb. 5	8 18 38.23	+32 2 25.3	11
1844 May 17	15 59 1.89	−14 10 35.2	50–48
1845 Aug. 29	22 16 34.94	−26 38 14.5	30
1846 Nov. 19	3 17 36.12	+13 10 1.4	11
1848 Mar. 28	11 44 36.17	+19 59 17.3	26–22
1849 July 5	18 22 26.38	−28 11 15.6	25–23
1850 Sept. 18	0 37 56.11	−13 4 21.3	10
1852 Jan. 8	6 32 47.89	+29 9 31.1	13
1853 May 5	14 30 14.15	− 3 1 41.6	26–25
1854 Aug. 13	20 57 56.49	−30 15 21.5	25–28
1855 Nov. 4	2 30 18.51	+ 3 8 17.2	14–16
1857 Feb. 22	10 7 26.90	+28 27 6.2	55
1858 June 6	17 11 32.29	−22 2 35.3	30
1859 Sept. 1	23 31 59.71	−20 13 37.5	27
1860 Dec. 1	5 4 30.93	+21 15 0.0	18–17
1862 Apr. 21	13 5 28.14	+ 9 5 26.2	40–37
1863 July 16	19 18 49.23	−30 4 5.7	24
1864 Oct. 16	1 26 23.54	− 6 8 12.6	27–28
1866 Feb. 4	8 10 14.16	+32 5 59.4	19–21
1867 May 26	15 45 39.55	−13 43 22.6	28
1868 Aug. 27	22 42 40.98	−26 50 32.8	30–39
1869 Nov. 23	3 37 5.90	+12 35 19.5	15–16
1871 Mar. 19	11 43 0.40	+20 30 36.0	17
1872 June 30	18 20 14.97	−27 44 34.7	23–22
1873 Sept. 30	0 23 7.31	−14 34 29.6	27
1875 Jan. 5	6 25 8.51	+28 40 38.6	14
1876 May 6	14 21 47.04	− 2 2 16.2	23
1877 Aug. 17	20 48 16.32	−30 14 10.7	7
1878 Nov. 7	2 22 8.73	+ 2 25 48.7	11–10
1880 Feb. 28	9 53 17.79	+29 23 14.7	15
1881 June 16	16 51 4.83	−21 55 13.7	15–16
1882 Sept. 23	23 7 52.33	−22 28 41.0	8–7
1883 Dec. 4	1 58 28.24	+20 45 39.7	20–13
1885 Apr. 1	13 3 26.43	+ 9 53 57.6	23
1886 July 30	19 27 9.79	−31 2 52.7	8
1887 Oct. 22	1 16 15.76	− 7 4 20.9	15–14
1889 Feb. 4	8 0 10.55	+32 13 15.4	8
1890 May 27	15 35 47.70	−12 51 32.2	17–18
1891 Sept. 6	21 57 14.06	−27 52 12.0	11
1892 Nov. 18	3 33 18.75	+11 15 45.3	6
1894 Mar. 15	11 33 59.87	+21 29 10.2	20–21
1895 June 22	18 20 58.40	−27 0 17.7	15–14
1896 Sept. 26	0 20 25.27	−11 58 1.5	4–5
1897 Dec. 19	6 29 58.43	+27 2 21.4	9

ON PISTOR AND MARTINS'S PRISMATIC REFLECTING CIRCLE.

By T. H. SAFFORD.

Chauvenet's excellent treatise on Spherical and Practical Astronomy seems to contain an oversight which probably accidentally arises from the method employed in the construction of the work, and should not be used as a cause for unfavorable criticism.

On page 92 of Volume II, he says, Section 78: "The sextant of all astronomical instruments is the most especially adapted to the purposes of the investigator and the scientific explorer."

This statement, for the better students who elect astronomy in their college course, seems to need modification as the "Handbuch der Nautischen Instrumente," published by the Hydrographical Bureau of the German Admiralty, as well as a similarly authorized "Handbuch der Navigation," contains a description of the Pistor and Martins Prismatic Circle which shows the superiority of the later invented instruments both theoretically and practically, as I have tested by the use of the prismatic circle which Wil-

liams College Observatory possesses, procured of the firm of BUFF & BURGER, formerly of 9 Province Court, Boston.

The circle alluded to was made by WAGENER of Berlin, and is excellently well made so far as I can judge. It has the smaller dimensions described in the German books. The sextants which the College possesses include an older one by a good firm, SPENCER, BROWNING & RUST, of London, which was purchased of an old ship captain, who found it so difficult to read off that he supposed his eyesight was in fault. Consequently it was redivided by JOHN BLISS & Co., 128 Front Street, New York City, a well-known firm of dealers in nautical instruments. At a future time I hope its errors will be investigated by my pupils. The College possesses all the necessary apparatus for the purpose. The chief advantage in the prismatic circle of PISTOR and MARTINS, or those like them, is that as a circle it enables the observer to eliminate at once the eccentricity of the alhidade by reading the two opposite verniers.

For the other advantages of this instrument see page 130 of Volume II of CHAUVENET'S "Manual," a book which is preferred by our students, from easily understood causes, to BRÜNNOW'S English translation of his own spherical astronomy, and, of course, to DOOLITTLE'S "Practical Astronomy," and other similar smaller works. The smaller dimensions of the prismatic circle require more delicate handling than the larger. Those who wish to see how the circle of larger dimensions endures the tests made under very unfavorable circumstances of transportation, &c., may be referred to Prof. BACKLAND'S paper, "*Astronomische Ortsbestimmungen von Nördlichen Russland*," in Vol. 7 of the *Mélanges Mathématiques et Astronomiques*.

Williams College Observatory.

SOUTHERN VARIABLES.

BY R. T. A. INNES.

[Communicated by Dr. DAVID GILL, C.B., etc., H.M. Astronomer at the Cape of Good Hope.]

On page (94) of Vol. I of the *Cape Photographic Durchmusterung*, Professor KAPTEYN remarks that the variability of the star at $17^h\,49^m\,32^s$, $-49°\,24'.9$ (1875) is all but proved by the Cape "*Carte du ciel*" plates. This star is C.P.D. $-49°10361$. Observations were commenced in May, 1898, and soon showed a range of magnitude from $9^M.0$ to $9^M.8$, but it was not until the night of Oct. 3, 1899, that its period was even roughly ascertained. It was then found to have a period of under $7^h\,30^m$. The shortness of this period put many of the observations out of count, as the date only, without the hour and minute, had been recorded.

The remaining observations are annexed. Assuming a period of $0^d.3115$ (or about $7^h\,28^m\,36^s$) and reducing all the observations to the period Oct. 3, 1899, $7^h\,30^m$ to 15^h, they have been plotted. I have drawn two curves through the observations, and the deviations from one or the other are well within the errors of observation. The observations have been corrected for the light equation, before being plotted, by the formula

$$-7^m.5\cos(\odot-88°)$$

The range of magnitude is from $8^M.9$ to $9^M.75$, and the form of either curve much resembles that of ordinary long-period variables. Excluding cluster-variables, this is the shortest period variable known. As to the two curves even and odd maxima will not account for them. All that has been derived with any certainty is the average period and amplitude of the curve. Observations extending over 8 or 9 hours or more, on several successive evenings, will throw further light on the variations of the curve. These will be undertaken in due course. Meanwhile, it may be pointed out that the curve seems to be subject to irregularities analogous to that of *Mira Ceti*.

GILLIS'S Polar Zones 9192. $9^M.0$.

$13^h\,9^m\,32^s$, $-83°\,34'.1$.

This star is included in one of Professor KAPTEYN'S lists of stars not found on the C.P.D. plates. It is variable:

		M
1899 Aug.	4	10.2
	9	10 ±
	17	10.3 (red)
Sept.	1	9.9
. . .	. . .	. . .
Oct.	6	8.7
	10	8.2 red 6.5
	14	8.2
	18	7.7
	22	7.7 red

1850. *S Pictoris.* (*A.J.* 468.)

A maximum occurred in August, 1899:

	M
1899 May 31	invisible
Aug. 27	8.3 red 6
Sept. 30	9.1 red 7.5
Oct. 3	9.25 red 8
10	9.4
22	9.7

Between the different observed maxima (or thereabouts) I find the following intervals and periods with their estimated extreme errors:

5076 ± 35	123 ± 3^d	12 periods
820 ± 20	110 ± 10	2
1728 ± 12	132 ± 3	1
1758 ± 4	139 ± 1	4

or an average period of 128^d ± 5.

C.P.D. 41 1681. (A.J. 468.)

Dr. ROBERTS of Lovedale, has kindly informed me that the period of this *Algol*-variable is about $6^d.11$, and not $12^d.906$ as I had supposed. My observations are not inconsistent with the shorter period, and a re-reduction of them also gives $6^d.11$. To use the C.P.D. *minimum* of 1890 May 10, we must suppose 515 intervals and a period of $6^d.1123$.

Ö.A. 13411. (A.J. 468.)

Observations of a recent maximum give a new interpretation to the old observations. The rise to a maximum is very sudden, with a long stay there — fading not more than $0^M.4$ in 50 days.

Hence the Cape observations may be taken thus:

		Max.	O—C
	1899 Aug. 10	$8^M.4$	0^d
	1898 Aug. 27	8.8	− 3
	1897 Sept. 24	8.8	+ 5
	1896 Oct. 1	9.0	− 9
And Cordoba	1879 June 29	8.75	−82
Argelander	1851 May 20	8.0	0

The column O—C assumes a period of 345^d, but the discordance of the Cordoba maximum is too large to allow this result much weight.

OBSERVATIONS OF C.P.D. 49 10361.

1899	C.M.T.	Mag.	1899	C.M.T.	Mag.	1899	C.M.T.	Mag.	1899	C.M.T.	Mag.
	h m			h m			h m			h m	
July 18	8 50	9.5	Sept 24	8 18	9.65	Oct. 3	10 26	8.9	Oct. 13	10 4	9.25
19	7 28	9.65	25	6 56	9.0		10 33	8.9		10 20	9.35
22	7 21	9.1		7 27	9.0		10 45	8.95	14	6 56	9.0
26	6 20	9.1		9 57	9.1		10 54	9.1		8 4	9.25
27	6 12	9.1		10 17	9.5		11 7	8.9		8 14	9.25
28	5 12	9.75	26	6 50	9.1		11 21	9.1		8 26	9.4
Aug. 1	8 13	9.0		7 18	9.1		11 23	9.2	18	6 57	9.65 twi.
11	6 28 ±	9.0		8 50	9.1		11 30	9.25		7 11	9.6
16	9 11	9.6	27	6 12	9.55		11 36	9.3 low		7 34	9.4
17	6 30	9.75		7 24	9.5		11 42	9.3 low		7 39	9.3
31	11 13	9.3		8 13	9.65		11 47	9.3 low		7 42	9.3
Sept. 1	7 22	9.1	28	6 53	9.65	6	6 50	9.5 twi.		7 47	9.25
2	6 57	9.2	30	8 38	9.35		7 12	9.6		7 58	9.15
	8 29	9.3	Oct. 1	7 56	9.5		7 29	9.65		8 5	9.05
12	7 9	9.15		8 29	9.5		7 57	9.65		8 19	9.0
	8 37	9.6	3	8 6	9.75	10	7 2	9.3	22	7 8	9.67
16	7 26	9.1		9 11	9.7		7 11	9.4		7 16	9.7
17	10 18	9.65		9 17	9.55		7 23	9.35		7 38	9.65 Decid'ly brighter
18	9 53	9.55		9 50	9.3		7 45	9.45		7 55	9.7
19	6 31	9.6		9 52	9.2		8 11	9.45		8 17	9.7
	7 11	9.65		9 54	9.1		8 33	9.5	24	7 2	9.35
	8 13	9.6		9 58	9.0		8 49	9.6		7 16	9.4
20	9 6	9.1		10 1	9.05		9 7	9.6 good		7 55	9.4
23	7 31	9.65		10 12	9.0		9 17	9.55		8 16	9.5
24	7 32	9.7		10 20	8.9	13	9 48	9.2			

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THE CASE ALMUCANTAR.

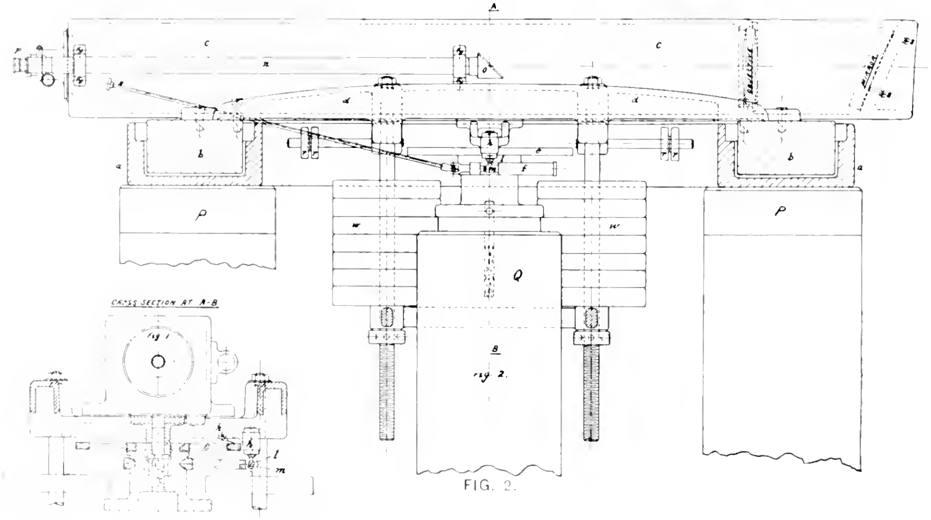

FIG. 2.

FIG. 1.

THE CASE ALMUCANTAR.

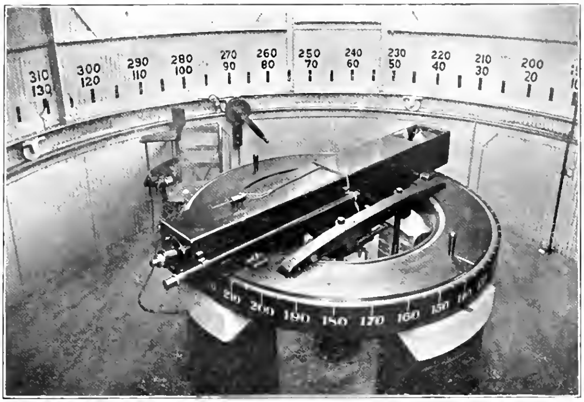

FIG. 3

FIG. 4.

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THE CASE ALMUCANTAR.

By CHARLES S. HOWE.

In 1879 Dr. S. C. CHANDLER invented an astronomical instrument which he called an almucantar. He used the instrument for determinations of time, latitude, right-ascensions and declinations of stars — in fact for all the work of a transit circle. The almucantar consisted of a telescope fastened to a rectangular support which floated in mercury. The box containing the mercury could be turned in azimuth. The telescope was adjusted at a fixed vertical angle, and hence by turning the instrument in azimuth, the central horizontal wire traced out in the sky a circle parallel to the horizon or an almucantar. The time when any star passed this almucantar either in ascending or descending could be determined. In short, for the fixed meridian of the transit circle was substituted a fixed circle parallel to the horizon. The advantages claimed for this new method of observing were. 1st — that it eliminated all flexure. 2d — that it eliminated all errors of refraction which depend upon zenith-distance; the only change of refraction which could affect the results being those due to change of pressure and temperature at a fixed zenith-distance. 3d — that the deviation of the central wire from the fundamental plane of reference was much less than the deviation of the transit circle from the meridian. 4th — stars could be observed all around the circle of reference instead of over about one-half of it as in the transit circle.

The first almucantar constructed had a telescope $1\frac{1}{2}''$ in diameter, and $25''$ focal length. The results obtained with this instrument were presented to the American Association for the Advancement of Science at the Boston meeting in 1880. These results were so satisfactory that a larger instrument having a telescope of $4''$ aperture and $44''$ focal length was built. A full description of this instrument and the work accomplished with it is given in Vol. 17, Annals of the Harvard College Observatory. The telescope was mounted on a light wooden float about $31'' \times 6''$, with ends $10'' \times 8''.5$ projecting at right angles. The float and telescope complete weighed 31 lbs. The clock corrections obtained agreed very closely with those obtained with the large transit circle of the Harvard Observatory. A series of latitude observations extending throughout a year showed the variation of latitude, although its true character was not then recognized. Determinations of right-ascension and declination showed a close agreement with those obtained at Greenwich.

A similar instrument was built at the same time for Mr. C. H. ROCKWELL of Tarrytown, N.Y., and has been used ever since in his private observatory.

The work accomplished with this instrument interested me deeply, and it seemed to me desirable that a large almucantar with optical power equal to that of the best transit circles should be built and used for research work. About a year ago the opportunity came to me to build such an instrument. The rough sketches were submitted to WARNER & SWASEY, who completed the drawings and constructed the instrument. The optical parts were made by BRASHEAR. It was decided to have an object glass of $6''$ aperture. Dr. CHANDLER had suggested that an instrument might be constructed with a horizontal telescope, the light being reflected into it by means of a plane mirror. I adopted this suggestion because I thought the wind would have much less effect upon the instrument. Dr. CHANDLER's almucantar had a very light float. I decided upon a heavy float of cast iron. The reason for this change was that a heavy iron float would be less liable to change of form on account of changes of temperature, etc., and the wind would have less effect upon it. In order to test the stability of a heavy float, one was constructed of cast iron, rectangular in form, and weighing 150 lbs. There was no arrangement for altering the center of gravity with respect to the center of floatation, and the method of preventing the float from moving up against the sides of the box introduced some friction which might alter the level of the float. A delicate astronomical level indicated that, after disturbance, the float always returned to very nearly the same position. The changes were so slight that it was a question whether they were in the float or in the level. In order to test the question in another way, Dr. E. W. MORLEY of Western Reserve University, arranged an interferometer in such a way that changes of level would be indicated. The distance between two wave lengths of light was equal to a

change of 0″.8 in level, and we could easily estimate tenths of this, or 0″.08. Long series of measurements with this instrument indicated that the float almost invariably returned to the same position after disturbance, and the variations noted could be easily accounted for by friction of the pins which held it in place, and the action of the meniscus of the surface of mercury. The form of the float was next to be decided. If the float was rectangular as in Dr. CHANDLER'S instrument, it would weigh at least 1000 lbs., the mercury and the box to contain it would weigh at least 500 lbs. more. This weight would have to be turned in azimuth for every star. Some years ago Professors MORLEY and MICHELSON while working with the interferometer had constructed a float in the form of a ring. Dr. MORLEY assured me that this worked admirably, the vibrations subsiding almost instantly after disturbance. This form was therefore decided upon for the float. The result of these changes has been an instrument embodying the floating principle of the original almucantar but totally different in appearance.

Fig. 2 shows a vertical section of the instrument through the center.

Figs. 3 and 4 show the instrument in the observatory.

Four stone piers, two of which are shown at P, P, in Fig. 2, support an iron ring-shaped box, a, a. This box contains the mercury, and in it floats the ring-shaped float, b, b, which has an outer diameter of fifty-seven inches, a width of eight inches, and a depth of four and a half inches. The distance between the float and the box is one-quarter of an inch, except at the top, where it is one inch. The float supports the horizontal telescope tube, c, c, and the frame, d, d, from which hang four rods carrying the weights w, w. The telescope and the frame can be screwed down to the float. The float, telescope tube and frame weigh about eighteen hundred pounds. The addition of weights w changes the position of the center of gravity with respect to the center of floatation. A central pier, Q, supports a twelve-inch setting circle, e, which is read from the eye-end of the instrument by means of the telescope n. The central pier also supports a collar, f, which has a clamp, not shown in the figure, and a slow-motion rod, s. The only connection between the float and the clamp collar is the pin, l, which plays in a slot, m. When unclamped the pin, l, carries the collar, e, around as it is turned in azimuth. The clamp fastens the collar to the central pier, and the instrument cannot turn in azimuth, but it is as free to move in any other direction as when unclamped.

Fig. 1, which is a vertical section through the center at right angles to Fig. 2, shows the method by which the float is kept from touching the sides of the box, a, a. x is a block of iron resting on the central pier, Q, and terminating in a cylinder open at the top. t is an iron ball at the end of a rod coming down from the bottom of the telescope tube, and fitting into the open end of the cylinder, which is filled with watch oil. Although the instrument weighs twenty-three hundred pounds, the slightest pressure of the finger turns it around this ball.

In the telescope tube are the six-inch objective of sixty inches focal length and the plane mirror seven inches in diameter which reflects light into the telescope. The mirror rests against the screws, s, s, and is fixed at such an angle as to reflect stars which are at an angle above the horizon equal to the latitude. The screws, s, s, permit a slight adjustment of the mirror.

The eye-piece has a reticle of seventeen horizontal wires arranged in three tallies. No micrometer is needed as all determinations of time, latitude, right-ascension and declination, are made from the times at which stars pass the central wire of the reticle.

This instrument differs materially from the transit-circle, as it has neither pivots, level, fine circle, reading microscopes or micrometer. Instead of the three corrections of the transit circle, the almucantar has one — that of zenith-distance.

Some of the advantages of the circular float are: — First — that it can be easily turned in azimuth. Second — that the motion of the float produces a motion in the mercury, and keeps it at the same temperature throughout. If the mercury is not agitated, one part of it will tend to become colder than the rest, and this will change the angle of the float.

The instrument was ready for use in March of this year. Since then I have been engaged in learning how to use it to best advantage, in adjusting the weights and in determining its changes of zenith-distance, after disturbance, by means of a delicate level. It is too early yet to speak of the character of its work. I might say, however, that it appears remarkably steady. It can be turned in azimuth 360 degrees without changing a delicate level placed on the telescope tube, more than seven seconds of arc. All oscillation seems to have ceased in one minute after setting. Stars can be observed at intervals of three minutes. Most of this time is taken up in setting the instrument and dome and making notes. So far as oscillations go, observations could be made once in two minutes. Under rather unfavorable circumstances readings with a level indicated that the probable error of the setting was 0″.1. This is made up of three parts, viz.: — the sluggishness of the level, the error of reading and the error of the instrument. After deducting the first two from the 0″.1, there would seem to be very little left for the almucantar. Other observations would seem to indicate that it is more accurate than any level which cannot be reversed. In using it there are no tiresome determinations of level or collimation. Time determinations can be made much faster, and I believe reduced in a shorter time than with the transit circle. A subsequent paper will give some results which have been obtained.

Case Observatory, Cleveland, Ohio, 1900 Nov. 5.

SYSTEMATIC OBSERVATIONS OF OCCULTATIONS OF STARS BY THE MOON,

MADE AT THE DEARBORN OBSERVATORY OF NORTHWESTERN UNIVERSITY.

By G. W. HOUGH, Director.

The computation for immersion has usually been made for the earliest hour in the evening at which it was possible to see a 9th magnitude star. The occultation could then be observed with the least interference with other regular work.

By means of expanded tables, in connection with a graphical method, the computation for all stars which may be occulted during the hour chosen, may be made in about fifteen minutes.*

If, however, an emersion is to be observed it is desirable to have the prediction as exact as possible, which will require about ten minutes of additional labor.

The occultations have been observed with the 18½-inch refractor, and a power of 190.

From New Moon to First Quarter, there is no difficulty in observing any DM. star, but near the time of Full Moon, a star smaller than the 8th magnitude is difficult to see. But few emersions have been observed for the reason that my work is usually confined to the early part of the night.

The time has been recorded, either with the printing or recording chronograph, and the error of the sidereal clock determined in the same night. Unless otherwise stated the phenomenon was instantaneous, and was recorded without error.

The observation of an emersion, near the time of Full Moon, when the dark limb is not visible, is difficult, and unless the eye is directed to the exact spot where the star should appear, the observation will be recorded too late by an unknown quantity.

With a single exception, which is indicated, all the observations have been made at the dark limb of the Moon.

The Standard Time is that of the 90th meridian, or 6^h slow by Greenwich M.T.

In the table of observations —

D = disappearance — Immersion.
R = reappearance — Emersion.

*Vide *Popular Astronomy*, January, 1898.

OCCULTATION OF STARS BY THE MOON.

No.	Date		Name	Mag.	Phase	Sidereal Time	Standard Time	Remarks
	1897					h m s	h m s	
1	Aug.	9	χ *Sagittarii*	5.4	D	17 36 23.9	8 44 18.9	0s.2 late
2		9	"		R	18 12 52.9	9 18 7.4	2s late probably
3	Aug.	10	S.DM. −20°5272	7	D	18 48 23.5	9 19 40.8	
4		11	S.DM. −17°6193	7	D	17 3 21.1	7 31 2.7	
5		12	S.DM. −11°5756	6.8	D	21 11 31.5	11 34 33.6	
6		14	λ *Piscium*	4.5	D	21 13 1.6	11 28 11.6	
7	Sept.	2	A.W.E. 12146 N	7	D	18 50 9.7	7 51 0.8	
8		2	A.W.E. 12146 S	7	D	18 51 8.0	7 51 59.0	
9		2	B.A.C. 5214	7	D	19 21 12.8	8 22 28.7	
10		7	S.DM −18°5805	6	D	20 56 54.9	9 37 15.7	
11		8	S.DM. −13°6008	6.5	D	20 2 5.4	8 39 9.3	
12		8	S.DM. −13°6012	7.5	D	20 45 43.0	9 22 39.8	
13		17	DM. +26°797	8.3	R	3 33 40.2	15 31 6.9	
14		17	DM. +26°796	7	R	3 46 46.1	15 47 10.7	0s.5 late
15	Oct.	3	53 *Sagittarii*	7	D	19 41 54.3	6 40 43.8	
16		3	" "	7	R	21 1 16.0	7 59 52.5	2s late
17		3	B.A.C. 6727	6	D	19 51 49.0	6 50 36.9	
18		3	" "		R	21 9 18.0	8 7 53.2	1s late
19		5	S.DM. −14°6039	6.5	D	21 44 36.5	8 2 19.5	1s late
20		5	S.DM. −14°6047	6.5	D	21 34 26.3	8 22 6.1	
21		6	B.A.C. 7774	6	D	19 26 41.0	6 13 45.3	
22		30	A.W.E. 15265	7	D	20 10 16.7	5 22 52.0	
23		30	χ_1 *Sagittarii*	5.6	D	22 54 3.7	8 6 12.2	
24	Nov.	2	S.DM. −10°5830	8.8	D	22 10 38.6	7 11 6.5	
25		6	DM. +14°152	8.3	D	22 10 33.4	6 55 17.7	
26	Dec.	4	Schj. 9694	7.7	D	3 32 33.6	10 38 7.3	
27		5	26 *Arietis*	6	D	0 9 19.4	7 0 12.7	0s.2 late
28		25	B.A.C. 7053	9.5	D	0 25 50.4	5 57 32.8	
29		25	o *Capricorni*	6.2	D	0 26 42.3	5 58 24.6	

No.	Date	Name	Mag.	Phase	Sidereal Time	Standard Time	Remarks
	1897				h m s	h m s	
30	Dec. 26	Schj. 8679	7.3	D	0 30 49.4	5 58 35.1	
31	26	S.DM. −14°6029	9	D	0 41 38.1	6 9 22.0	
32	27	*θ Aquarii*	4.4	D	23 40 31.3	5 4 29.3	
33	27	"		R	0 19 52.1	5 43 43.7	1s late probably
34	27	W.B. XXII, 223	8.6	D	0 30 32.0	5 51 21.8	
35	27	B.A.C. 7804	7	D	3 13 18.7	8 36 41.9	
36	29	DM. +3°4905	8.8	D	1 13 38.0	6 29 29.0	
37	29	"	10	D	1 18 56.3	6 34 46.4	
38	29	DM. +3°4906	9	D	1 22 1.5	6 37 51.1	
39	29	DM. +3°4913	8.8	D	3 15 37.1	9 1 3.1	
40	29	DM. +3°4918	8.7	D	4 7 35.6	9 22 58.0	
41	Jan. 4	W.B. IV, 868	8.4	D	0 46 50.0	5 39 9.9	
42	5	125 *Tauri*	6	D	0 8 1.1	4 56 31.4	
43	26	DM. +6°42	9	D	4 2 57.9	7 28 15.7	
44	26	45 *Piscium*	7	D	4 10 26.1	7 35 12.8	
45	26	DM. + 6°44	9.2	D	4 11 29.9	7 36 16.2	
46	29	DM. +20°462	8.2	D	2 12 17.7	5 56 30.8	
47	Feb. 2	DM. +25°1215	7.8	D	4 27 7.2	7 24 19.5	Moisture in eye-piece. Star faint. Record possibly late
48	3	W.B. VII, 172	7.8	D	5 32 43.5	8 26 19.2	
49	3	W.B. VII, 179	8.3	D	5 48 20.4	8 41 53.5	
50	5	*a_2 Cancri*	4	D	6 45 2.5	9 30 34.5	
51	23	Σ 67	8.2	D	4 50 34.5	6 25 38.9	Double star. $p = 7°.0$, $s = 1''.86$. Mag. 8.2–9. Companion about 1s later
52	24	DM. +15°250	8.8	D	5 51 19.8	7 22 18.4	
53	24	DM. +15°251	7.8	D	5 54 53.8	7 25 51.8	
54	25	26 *Arietis*	6.5	D	5 7 8.0	6 31 17.9	
55	25	DM. +19°372	9.1	D	7 20 33.0	8 47 21.0	
56	26	DM. +22°475	8	D	5 5 46.0	6 29 0.2	
57	26	DM. +22°479	8.1	D	6 3 14.2	7 26 48.9	
58	Mar. 2	DM. +24°1457	7.5	D	7 16 18.8	8 23 28.0	
59	3	DM. +21°1679	7.5	D	6 29 28.5	7 32 19.4	Star suddenly diminished to one-half brightness before disappearance. This is a very close double = Ho 247. 1887.22 $p = 101°.3$ $s = 0''.36$ ($7^M.5$–8^M).
60	3	DM. +21°1680	8	D	7 43 40.4	8 46 19.2	
61*	28	DM. +25°918	8.4	D	7 37 0.2	7 1 52.1	
62	28	DM. +25°919	9.3	D	7 39 3.6	7 3 55.4	
63	28	Σ 776	8.1	D	8 35 25.3	8 0 7.9	1869.01 $p = 102°.5$, $s = 2''.20$ ($8^M.1$–9^M) J4a
64	28	"	9	D	8 35 31.2	8 0 13.9	
65	28	DM. +25°940	6.8	D	8 51 52.3	8 16 32.2	
66	28	DM. +25°941	8	D	8 57 10.4	8 21 49.5	
67	29	DM. +24°1310	9.3	D	7 46 35.6	7 7 30.3	
68	29		9.6	D	7 50 46.9	7 11 10.9	
69	29	DM. +24°1311	9.4	D	7 58 33.5	7 19 26.3	
70	29	DM. +24°1315	9.1	D	8 5 39.5	7 26 31.1	
71	29	DM. +24°1316	9	D	8 8 52.0	7 29 43.1	
72	29		9.5	D	8 14 18.1	7 35 8.6	
73	30	DM. +22°1678	9.2	D	7 43 19.4	7 0 18.7	
74	30	DM. +22°1679	9.3	D	8 5 30.3	7 22 26.0	
75	30	DM. +22°1680	9.3	D	8 11 35.0	7 28 29.7	
76	30	DM. +22°1683	9.3	D	8 14 45.4	7 31 39.6	
77	30	DM. +22°1687	7	D	9 2 50.7	8 19 37.0	Moved telescope in R.A. Rec. prob. 0s.5 late
78	31	DM. +19°1963	7.6	D	7 43 13.7	6 56 17.2	Double star = Ho 524. $p = 343°.7$, $s = 3''.88$ (8^M–11^M)
79	Apr. 1	DM. +14°2033	7.7	D	10 8 52.4	9 17 36.1	
80	2	10 *Sextantis*	6	D	10 28 5.4	9 32 50.0	
81	3	W.B. X, 665	7.5	D	10 50 22.4	9 51 7.5	
82	7	S.DM. −18°3766	7.8	R	10 53 16.8	9 38 17.8	0s.5 late
83	25	DM. +24°1188	9	D	10 3 58.8	7 38 21.5	
84	25		9.5	D	10 16 12.8	7 51 3.4	
85	25	DM. +24°1190	9.5	D	10 24 6.7	7 55 26.6	
86	25	DM. +24°1191	9.5	D	10 32 20.3	8 6 38.3	
87	25		10	D	10 35 18.3	8 9 35.9	
88	25		9.5	D	10 36 32.8	8 10 50.2	
89	25	DM. +24°1194	9.5	D	10 37 22.8	8 11 40.0	

* Disappeared slowly.

No.	Date	Name	Mag.	Phase	Sidereal Time	Standard Time	Remarks
	1898				h m s	h m s	
90	Apr. 25	DM. +24°1197	9.3	D	10 39 19.9	8 13 36.8	Gradual disappearance. Possibly double
91	26	DM. +22°1597	9.5	D	10 34 54.5	8 5 16.2	
92	26	DM. +22°1596	8	D	10 42 36.5	8 12 57.0	
93	26	DM. +22°1599	9.5	D	11 2 16.4	8 33 3.6	
94	26	Σ 1035	7.8	D	11 52 1.7	9 22 10.8	Double star. $p = 41°.4$, $s = 8''.41$ ($7^M.8$ $7^M.8$) 1866.68 $\Delta 3^a$
95	26	"	7.8	D	11 52 6.0	9 22 15.1	
96	29	DM. +11°2061	7.7	D	11 23 53.6	8 42 19.6	
97	30	DM. + 6°2301	6.7	D	11 11 14.9	8 25 17.0	
98	May 23	DM. +23°1533	9	D	12 8 25.8	7 52 22.6	
99	23	DM. +22°1512	9	D	12 12 7.6	7 56 3.8	
100	23	DM. +23°1535	9.4	D	12 17 43.0	8 1 38.3	
101	24	DM. +20°1889	7.5	D	12 10 52.2	7 50 52.7	
102	24	79 *Geminorum*	6.3	D	12 42 11.8	8 22 7.2	
103	24	DM. +20°1895	9.5	D	12 46 5.6	8 26 0.3	
104	24	DM. +20°1897	9.5	D	13 2 8.2	8 42 0.3	
105	25	DM. +16°1752	9.4	D	12 25 49.1	8 1 51.6	
106	25	DM. +17°1867	9.3	D	12 45 1.1	8 21 3.1	
107	26	DM. +12°2021	8.5	D	13 28 23.1	9 0 19.4	
108	26	DM. +12°2022	9	D	13 34 39.6	9 6 31.6	
109	30	S.DM. − 8°3355	9.5	D	13 12 34.1	8 28 19.0	
110	30	S.DM. − 8°3356	9.1	D	13 27 6.8	8 43 19.4	
111	June 1	S.DM. −19°3846	7.5	D	13 10 8.2	8 18 31.7	
112	23	DM. + 9°2256	9.2	D	14 36 36.3	8 18 15.6	
113	23	DM. + 9°2257	9.2	D	14 39 11.1	8 21 20.2	
114	25	DM. − 1°2521	7.2	D	15 16 31.1	8 50 12.0	
115	25	DM. Companion	10	D	15 14 48.7	8 48 29.9	
116	27	S.DM. −12°3735	9.3	D	15 5 56.3	8 31 47.4	
117	27	S.DM. −12°3736	9.1	D	15 12 18.1	8 38 7.9	
118	27	S.DM. −12°3738	8.9	D	15 29 26.0	8 55 13.0	
119	28	S.DM. −17°3958	8.5	D	15 10 7.6	8 32 1.8	
120	28	S.DM. −17°3961	8	D	15 56 37.0	9 18 23.6	
121	30	*α Scorpii*	1	D	14 32 22.0	7 16 30.6	1898.49 $p = 271°.9$, $s = 3''.13$ (1^M 8^M) Ho 4
122	30	Companion	8	D	14 32 17.2	7 16 25.8	
123	30	3 *Scorpii*	6.7	D	14 55 2.2	8 9 7.1	
124	July 23	S.DM. − 5°3395	9.3	D	16 35 28.8	8 18 51.2	
125	23	S.DM. − 5°3397	9.1	D	16 50 3.7	8 33 23.7	Faint $10^m.5$
126	25	S.DM. −15°3704	9.3	D	16 21 16.7	7 59 19.2	
127	25	S.DM. −15°3705	7.8	D	16 38 49.9	8 14 20.0	
128	26	S.DM. −20°4043	7.3	D	16 14 21.0	7 45 59.2	$0^s.2$ late
129	26	S.DM. −20°4047	7.9	D	16 50 17.6	8 22 19.8	
130	27	A.W.E. 11920	7	D	16 11 6.9	7 38 49.7	
131	30	B.A.C. 6369	6	D	16 28 30.1	7 44 22.3	
132	Sept. 20	A.W.E. 12292	7	D	19 37 16.5	7 28 10.6	$0^s.2$ late
133	Nov. 26	D.M. +21°147	6.8	D	0 18 42.8	8 15 19.9	
134	Dec. 15	S.DM. −19°5721	7	D	23 47 14.7	5 59 19.6	
135	17	S.DM. − 9°5884	8.3	D	23 3 26.04	5 7 46.3	
136	17	S.DM. − 9°5900	9.5	D	0 31 23.5	6 35 29.3	
137	18	S.DM. − 3°5521	7.8	D	23 3 10.3	5 3 31.6	Clock error uncertain $\pm 0^s.5$. Clouded
138	22	DM. +17°339	7.3	D	2 35 33.0	8 19 38.9	
	1899						
139	Jan. 15	*κ Piscium*	–	D	2 6 39.7	6 16 28.5	
140	15	North 3′	9.5	D	2 0 34.4	6 10 21.1	
141	15	DM. + 0°1997	8.9	D	2 21 8.7	6 30 55.1	
142	16	DM. + 6°21	8.7	D	2 38 58.6	6 44 16.2	
143	18	DM. +16°235	9.2	D	4 11 50.7	8 12 30.7	
144	19	DM. +19°132	6.8	D	3 20 43.9	7 14 36.9	
145	21	DM. +21°1566	8.5	D	3 33 1.7	7 7 13.2	
146	24	DM. +21°1574	8.6	D	5 2 2.2	8 35 59.1	
147	27	*o Leonis*	3.8	D	4 52 26.7	8 11 37.1	Bright limb
148	Feb. 12	DM. +3°4908	8.8	D	3 18 25.2	6 7 51.9	
149	12	DM. +3°4909	7.2	D	4 3 50.6	6 23 14.7	Star disappeared gradually. Possibly double.

No.	Date	Name	Mag.	Phase	Sidereal Time	Standard Time	Remarks
	1898				h m s	h m s	
150	Feb. 15	DM. +19 394	7.5	D	6 19 4.7	8 26 18.9	
151	18	DM. +21 816	8.6	D	6 8 17.0	8 3 45.2	
152	20	DM. +22 1558	7	D	1 51 6.6	6 11 55.2	0s.2 late
153	Mar. 13	DM. +12 161	9	D	6 17 10.9	7 12 6.9	
154	19	DM. +22 1473	7.2	D	7 14 30.9	7 15 12.0	
155	22	DM. +12 1991	8.7	D	8 5 30.1	7 54 50.1	
156	23	DM. + 8 2296	8.7	D	8 57 8.7	8 12 21.1	
157	Apr. 11	DM. +23 961	8.9	D	10 30 56.5	8 19 26.8	
158	11	North 3′	10	D	10 31 59.2	8 50 29.1	
159	15	DM. +22 1352	7.5	D	10 13 21.9	8 57 57.3	
160	16	DM. +20 1769	9	D	9 50 59.7	8 1 14.8	
161	16	DM. +20°1770	8.2	D	10 17 51.2	8 28 31.8	
162	17	DM. +17 1771	9	D	9 30 31.6	7 37 27.1	
163	17	DM. +17 1776	8.5	D	10 22 18.0	8 29 2.0	
164	17	DM. +17 1778	7	D	10 37 40.1	8 44 21.9	
165	19	DM. + 9°2226	7.5	D	9 39 8.3	. . . .	
166	June 12	DM. +12°1779	9.2	D	13 38 14.3	8 1 15.2	
167	13	DM. + 7°2206	8.3	D	13 32 36.0	8 54 41.9	
168	16	S.DM. − 6°3518	6.1	D	15 16 13.9	9 26 15.2	
169	July 17	S.DM. −21 1065	6	D	17 22 29.3	9 30 16.5	
170	19	A.W.E. 13222	9	D	17 7 16.6	9 7 14.5	
171	Aug. 15	A.W.E. 12850	7	D	17 21 33.1	7 35 19.5	
172	15	South 3′	9	D	17 11 57.1	7 28 41.2	
173	15	A.W.E. 12875	7.5	D	18 19 30.6	8 33 7.2	
174	17	*ν¹ Sagittarii*	5	D	17 18 13.5	7 24 38.2	
175	19	S.DM. −15°5818	6	D	18 21 31.7	8 19 21.3	
176	Sept. 13	A.W.E. 111131	7.8	D	18 40 34.2	7 0 6.0	
177	13	P = 95	9	D	18 22 41.3	6 42 16.0	
178	13	A.W.E. 11112	8	D	19 12 39.2	7 32 5.8	
179	15	S.DM. −17 6011	7	D	20 5 31.5	8 16 57.6	Clock error uncertain ±0s.5. Clouded
180	Oct. 8	A.W.E. 12525	9	D	19 30 58.5	6 12 1.4	
181	8		10	D	19 25 51.9	6 6 58.6	
182	12	S.DM. −18 5615	8.8	D	20 36 42.2	7 1 53.6	
183	21	*τ Tauri*	4.1	R	. . . . .	10 16 0.0	Chronograph failed to record. Reappeared first as 9M. Close double. p=n.p., s=0″.15 to 0″.4 (4M.4–9M) Vide *A.J.* 474
184	23	A.G.Berlin 2441	8.6	D	0 19 18.6	10 0 38.6	
185	Nov. 6	A.W.E. 13771	8	D	20 50 8.2	5 36 59.8	
186	19	B.A.C. 1970	6.5	R	1 7 13.6	9 2 16.1	
187	Dec. 7	S.DM. −11°5618	9.1	D	23 15 10.5	5 59 45.0	
188	7	S.DM. −11 5619	8.7	D	23 29 7.8	6 13 10.1	

Occultations Observed During the Total Eclipse of the Moon, Dec. 27, 1898.

Name	Mag.	Phase	Sidereal Time	Standard Time	Remarks
			h m s	h m s	
579 Pulkowa	9.1	D	0 4 7.5	5 28 58.7	
DM. +22°1402	9.1	D	0 5 11.6	5 30 2.6	
DM. +23 1125	7.1	D	0 10 36.1	5 35 26.5	Disappeared slowly
North prec. +22 1410	9.4	D	0 20 32.7	5 45 21.2	
DM. +22 1410	7.7	D	0 21 9.6	5 45 58.0	

Observations were discontinued to make photographs of the eclipse.

The actinism or photographic power of the eclipsed moon was found to be $\frac{1}{1500}$ that of the uneclipsed moon.

The plate used was, Seed 27.

As the light of the eclipsed moon is always colored, it is obvious that its actinism or photographic power will depend on the kind of plate employed; and possibly on its manipulation previous to development, owing to the effect of preliminary or supplementary exposure.

FILAR-MICROMETER OBSERVATIONS OF ASTEROIDS.

MADE WITH THE 12-INCH EQUATORIAL OF THE DETROIT OBSERVATORY, ANN ARBOR, MICH.,

By SIDNEY D. TOWNLEY.

◯	1897 Ann Arbor M.T.	*	No. Comp.	Planet — * $\Delta\alpha$	Planet — * $\Delta\delta$	Planet's Apparent α	Planet's Apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
				m s	′ ″	h m s	° ′ ″		
137	July 21 10 52 37	1	12 , 8	−0 30.89	−2 1.5	19 57 56.96	+ 2 10 37.4	n9.085	0.754
137	22 9 53 25	1	10 , 8	−1 12.42	−4 12.5	19 57 15.14	+ 2 7 56.5	n9.338	0.756
137	23 11 1 22	2	12 , 8	−0 39.90	+1 27.7	19 56 30.13	+ 2 1 46.1	n8.945	0.755
200	29 11 11 23	1	10 , 8	+0 46.18	−0 7.1	20 41 2.51	−21 53 11.9	n9.067	0.895
200	30 12 2 41	1	12 , 10	−0 14.34	−1 23.0	20 40 2.03	−21 54 27.9	n7.283	0.900
200	31 11 8 32	4	8 , 7	−1 10.51	−2 30.7	20 39 5.86	−21 55 35.6	n9.011	0.897
200	Aug. 1 12 9 28	6	10 , 10	−0 35.72	+1 38.3	20 38 3.53	−21 56 43.2	8.509	0.899
79	July 30 13 0 38	5	10 , 8	+0 53.81	−1 24.1	20 24 32.32	−11 17 34.3	9.143	0.847
79	Aug. 1 13 6 35	7	10 , 10	−0 26.74	+0 11.4	20 22 35.55	−11 25 23.5	9.225	0.845
79	6 11 30 13	8	10 , 10	−1 11.34	+1 9.9	20 17 53.87	−11 45 31.2	8.500	0.853
194	27 11 15 53	9	12 , 10	+0 5.93	+1 16.8	22 27 18.97	− 6 58 13.0	n8.931	0.823
194	30 11 27 25	10	10 , 8	+0 57.32	−0 27.1	22 25 40.00	− 7 55 12.3	n8.578	0.830
194	Sept. 14 10 29 35	11	8 , 6	−1 30.21	+0 14.0	22 18 36.48	−12 25 16.1	n8.350	0.857
115	Oct. 2 13 16 31	12	10 , 8	−0 12.79	+1 7.3	0 25 31.21	+25 37 26.0	9.311	0.442
115	4 11 22 34	13	10 , 8	−0 14.53	−3 17.4	0 23 1.62	+25 30 47.4	n7.920	0.401
1	Dec. 27 10 15 9	14	8 , 6	−1 12.47	+1 35.8	6 20 57.74	+27 43 34.0	n9.313	0.392
1	28 9 17 10	15	10 , 8	−0 29.93	+2 56.4	6 19 55.68	+27 47 53.0	n9.396	0.414
1	32 9 10 14	16	10 , 10	−0 10.15	+2 34.3	6 15 14.89	+28 4 14.1	n9.448	0.428

Mean Places for 1897.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	19 58 23.96	+3.89 +3.90	+ 2 12 31.1	+ 7.8 + 7.9	$\frac{1}{15}$(7 A.G.C. Alb. 6962 + Weisse B. 1416 [+ 4 Munich$_1$ 22791 + 3 Schj. 7773)
2	19 57 6.42	+3.91	+ 2 3 10.3	+ 8.1	DM. +1°4182. Micr. determination from 3
3	19 59 43.09	+3.92	+ 2 9 2.0	+ 8.4	$\frac{1}{4}$(3 A.G.C. Alb. 6972 + Weisse Bessel 1451)
4	20 40 11.79	+ 1.57 + 1.58 + 1.58	−21 53 47.3	+12.5 +12.4 +12.4	$\frac{1}{17}$(3 Greenwich 1850, 1338 + Munich$_1$ 25873 [+ 3 C.G.C. 28162 + 2 Cape 1850, 4123 [+ 7 Rade. 5587 + Yarn.$_2$ 9268)
5	20 23 34.23	+ 1.28	−11 16 20.8	+10.9	$\frac{1}{2}$(Weisse Bessel 525 + Munich$_1$ 24616)
6	20 38 34.66	+ 1.59	−21 58 33.7	+12.2	$\frac{1}{2}$(Munich$_1$ 25752 + Oelt. Arg. 20793)
7	20 22 58.00	+ 1.29	−11 25 45.8	+10.9	$\frac{1}{4}$(Weisse Bessel 510 + 3 Santini$_1$ 2272)
8	20 19 0.88	+ 1.33	−11 49 52.0	+10.9	$\frac{1}{2}$(Munich$_1$ 24280 + Santini$_1$ 2260)
9	22 27 8.73	+4.31	− 6 59 52.6	+22.8	$\frac{1}{3}$(Mun.$_1$ 31136 + Rade. 6014 + Cord. G.C. 30738)
10	22 24 38.34	+4.34	− 7 55 37.9	+22.7	Munich$_1$ 31057 [+ 3 Santini 2520)
11	22 20 2.22	+4.47	−12 25 52.1	+22.3	$\frac{1}{5}$(Weisse B. 372 + 3 Mun. 30902 + Rüm. 10191
12	0 25 39.54	+4.46	+25 35 49.7	+29.0	Lick Obs. Mer. Circle observations
13	0 23 11.68	+4.47	+25 33 35.3	+29.5	Lick Obs. Mer. Circle observations
14	6 22 3.75	+6.46	+27 41 57.8	+ 0.4	$\frac{1}{6}$(5 A.G.C. Camb. Eng. 3255 + Weisse B. 543)
15	6 20 19.14	+6.47	+27 44 55.7	+ 0.9	A.G.C. Camb. Eng. 3227
16	6 16 18.54	+6.50	+28 2 8.2	+ 1.6	$\frac{1}{5}$(4 A.G.C. Camb. Eng. 3176 + Weisse B. 375)

NOTES.

July 31. Observations interrupted by clouds.

Oct. 2. The record for the position of the asteroid is, "115 north preceding," but a comparison of the reading of the micrometer and the micrometer circle with those preceding and following dates, make it almost certain that the record should have been "115 south preceding."

All reductions to mean place have been made with the Struve-Peters constants.

Positions of the stars 12 and 13 were obtained through the courtesy of Director KEELER and Professor TUCKER of the Lick Observatory.

OBSERVATIONS OF THE STARS *KRUEGER* 60 AND β 1291,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

BY E. E. BARNARD.

In the *Astronomical Journal*, No. 486, Mr. ERIC DOOLITTLE calls attention to a remarkable case of motion in a small double star. This object is the preceding component of a wide double noted by KRUEGER in his meridian observations in 1875. In measuring this list of KRUEGER stars in 1890, Mr. BURNHAM found that the preceding component of No. 60 was an unequal double.

Mr. DOOLITTLE finds from measurements in 1898 and 1900 that a decided change has taken place in the relative position of these stars since Mr. BURNHAM'S measures ten years ago. This star is likely to be of considerable interest, and I have therefore made a series of measures with the 40-inch, introducing two other smaller stars to more thoroughly explain hereafter the character of the motion.

A and *B*. (β 1291).

	°	″	
1900.937	132.06	3.30	9.2 , 10.5
.940	133.19	3.25	9.1 , 10.5
.943	133.59	3.24	
.948	133.71	3.23	
1900.942	133.39	3.25	9.1 , 10.5

A and *C*. (KRUEGER 60).

	°	″	
1900.937	59.27	36.87	9.5
.940	59.52	36.65	9.4
.943	59.25	36.67	
.948	59.16	36.64	
1900.942	59.30	36.71	9.4

A and *D*.

	°	″	
1900.943	21.25	21.25	15.5
.948	20.86	21.33	15.5
1900.945	21.05	21.29	15.5

Yerkes Observatory, Williams Bay, Wis., 1900 *Dec.* 15.

A and *E*.

	°	″	
1900.937	98.85	68.00	12.5
.940	99.06	67.76	13.0
.943	99.00	67.73	13.5
1900.940	98.97	67.83	13.0

There is a fainter star, $16\frac{1}{2}^m$ by estimation from *A*, 245° : 17″.

The following are all the measures of the principal stars:

A and *B*.

		″			
1890.79	178.8	2.32	9.0 , 12.0	1*n*	β
1898.45	140.7	3.19	9.1 , 10.5	5*n*	Doolittle
1900.74	134.0	3.18	9.1 , 11.1	4*n*	Doolittle
1900.94	133.4	3.25	9.1 , 10.5	4*n*	Barnard

A and *C*.

		″			
1890.79	56.3	26.82	9.3	1*n*	β
1898.45	58.7	34.39	9.4	5*n*	Doolittle
1900.74	59.2	36.18	9.4	4*n*	Doolittle
1900.94	59.3	36.71	9.4	4*n*	Barnard

Mr. DOOLITTLE has shown that *A* has a proper motion of 0″.93 in the direction 247°.9.

Considering the rarity of proper motion in small stars like these, it is improbable that *A* and *B*, so close together, are moving independent of each other through space. It would seem more probable they are a physical pair. A few years, however, will settle this point.

For some reason *AB*, which is really a BURNHAM star, is omitted from the General Catalogue of his double stars (*Pub. Yerkes Observatory*, Vol. I, 1900), but it should certainly be assigned a number and be incorporated in any catalogue of his discoveries, and I would suggest β 1291 as a designation for *AB*.

CONTENTS.

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CHANGES IN THE ANNUAL ELLIPTICAL COMPONENT OF THE POLAR MOTION.

By S. C. CHANDLER.

In *A.J.* 446 it was shown that the observations of 1890–97 make manifest progressive changes in the direction of the major axis of the very eccentric ellipse in which the pole of the earth's figure revolves annually about its mean position, and in the date when the pole reaches its maximum distance from this mean position. That is, by the elements there given, the ellipse appears to have turned in this interval of seven years through an angle of about twenty-five degrees, the element ω having diminished from about 45° east to about 20° east of the Greenwich meridian; and the date of maximum separation appears to have been retarded by nearly a month, the element L having increased from 7° to 34°. It was also surmised in that article that the singular variations in this component of the polar motion betrayed by the observations between 1830 and 1890, discussed in *A.J.* 422, were attributable to the same phenomenon, although it was not then practicable to trace the exact relation between them.

The subject is important, and it is extremely desirable that all available evidence should be brought to light and properly weighed. The present article is directed to this end, and gives the discussion of all the series of observations that existing astronomical records prior to 1890 provide for the solution of the problem. The result goes to corroborate the reality of the changes above spoken of, and indicates that the line of apsides is revolving from east to west, or in a direction contrary to that of the pole in its orbit, in a long period of say seventy-five years, that is, at a rate of about five degrees annually; and that the length of the annual period oscillates about its mean value, the fluctuations having a long periodical character, with a cycle of about sixty years.

The selection of the data for the investigation of this problem is limited by the condition requiring simultaneous series of observations of sufficient accuracy made in widely different longitudes. I have accordingly taken, for the epoch 1865, the Pulkowa vertical-circle observations of *Polaris* 1862–70, the Leyden meridian-circle observations of *Polaris* 1864–68, and the Washington prime-vertical observations of α *Lyrae* 1862–67; and, for the epoch 1883, the Pulkowa prime-vertical observations 1875–82, the Berlin universal transit observations 1884–85, the Cambridge almucantar observations 1884–85, the Washington prime-vertical observations 1882–84, and the Madison meridian-circle observations 1883–90. This list, I think, exhausts the suitable matter at hand. The process to be followed is to eliminate from each series the effect of the 427-day term of the latitude-variation, after correction to a uniform value of the aberration-constant, and for stellar parallax where possible; from the residuals to find the constants of the annual term of the latitude-variation; and finally, by combination of these constants for all the series at each epoch to compute the elements of the ellipse for 1865 and 1883. First the treatment of the several series will be described.

Pulkowa, Vertical Circle, Polaris, 1862–70. In my determination of the aberration, parallax and latitude-variation from this series I have given, *A.J.* 293, p. 35, in Table III the values n, which, being a combination of simultaneous upper and lower culminations which eliminates the aberration and parallax, represents the latitude-variation, $\varphi - \varphi_0$. From these values were subtracted the 427-day term by eq. (33), *A.J.* 106, using the constants

$$T_1 = 2402327\,, \quad \theta = 0^\circ.840\,, \quad r_1 = 0''.140$$

The residuals were then arranged by calendar dates, and grouped by weights, as follows:

	n	p		n	p
	″				
Mar. 15	−0.09	15	Sept. 18	−0.02	20
27	−0.10	28	Oct. 7	−0.02	15
Apr. 22	−0.02	30	23	−0.08	20
May 16	−0.04	20	Nov. 27	+0.02	18
July 2	+0.17	22			

Putting u and $\pi = 0$, in eq. (28), *A.J.* 287, p. 177, we have by eq. (35)

$$X + \eta \sin\odot + \zeta \cos\odot = n$$

Then we find by least-squares from the above series, with equal weights,

$$X = -0''.026\,, \quad \eta = +0''.085\,, \quad \zeta = -0''.060$$

whence, since $\eta = -r_2 \sin G'$, $\zeta = -r_2 \cos G'$, we have

$$r_2 = 0''.101\,, \quad G' = 305^\circ.2$$

Leyden, Meridian Circle, Polaris, 1864–68. The meridian altitudes of *Polaris* will be found on pp. 227–231, Vol. II of the Leyden Annals. These were reduced to 1866.0, and the appropriate corrections applied for division-errors and flexure, in the different positions of the instrument, for both culminations, and for each circle, and reduction of the reflected observations to the center of the instrument. The latitudes thus deduced were reduced to the aberration 20''.500 and parallax 0''.033, the combined correction being $\mp 0''.060 \sin\odot \mp 0''.017 \cos\odot$ (upper sign for upper culminations, lower for lower); and finally the 427-day term was subtracted, using the same constants as for foregoing series. Then grouping and taking means by weights we have

	''	p		''	p
Mar. 11	−0.02	9	Sept. 15	−0.01	8
Apr. 15	+0.06	8	Oct. 15	−0.09	11
May 15	+0.02	14	Nov. 15	+0.01	5
June 20	+0.07	6	Dec. 15	−0.08	2

Giving the last group half-weight, we find

$$X = -0''.007\,, \quad \eta = +0''.017\,, \quad \zeta = +0''.030$$

whence $r_2 = 0''.056 \quad G' = 237^\circ.8$

Washington, Prime Vertical, α Lyrae, 1862–67. The values which I gave on p. 69, *A.J.* 219, were reduced to the aberration 20''.500 and parallax 0''.09, and for the 427-day term as above. Grouping, we get

	''	p		''	p
Jan. 20	−0.21	50	Aug. 11	+0.15	50
Mar. 7	+0.03	50	Sept. 28	+0.17	50
Apr. 28	+0.06	50	Nov. 1	−0.05	50
June 9	+0.17	50	Dec. 8	−0.18	50
July 9	+0.13	50			

giving equal weights we find,

$$X = +0''.022\,, \quad \eta = +0''.161\,, \quad \zeta = -0''.052$$

hence $r_2 = 0''.172\,, \quad G' = 287^\circ.5$

Pulkowa, Prime Vertical, 1875–82. These observations were discussed in *A.J.* 297, p. 69, where I have given the normal equations, by means of which we can readily and rigorously apply the necessary corrections. The zero there assumed for the epoch of the 427-day term was $T = 2407091$. For this investigation I propose to adopt, for this and the following series at the same epoch, the constants for the 427-day term,

$$T_1 = 2407082 + 428.6\,E, \quad r_1 = 0''.173$$

For the Pulkowa meridian this epoch is 2407118. By attending to the explanations on p. 177, *A.J.* 287, it will be easily seen that, in this case, $\Delta T = +27$; consequently, we have $y = -0''.0667$, $z = -0''.1596$. Now the normal equations give us for the several stars,

δ *Cassiop.*	$Y =$	+0.0968	$-0.0482\,y$	$+0.3268\,z$
υ *Urs. Maj.*		− .1480	+ .0470	+ .0638
ι *Draconis*		+ .2416	+ .0525	+ .6892
ο *Draconis*		+0.0108	−0.2085	−0.1168
δ *Cassiop.*	$Z =$	−0.2420	$-0.2965\,y$	$+0.5301\,z$
υ *Urs. Maj.*		+ .0004	+ .0290	+ .4163
ι *Draconis*		+ .0674	+ .0497	+ .2447
ο *Draconis*		−0.1315	+0.0240	+0.0905

Substituting the above values of y and z we have

	Y ''	w_1	Z ''	w_2
δ *Cassiop.*	+0.048	49.6	−0.307	20.3
υ *Urs. Maj.*	− .161	27.2	− .068	9.1
ι *Draconis*	+ .128	12.6	+ .025	47.8
ο *Draconis*	+0.043	14.4	−0.148	49.3

We can reduce these to the aberration 20''.500, by putting $u = +0''.055$ in the equations, $\eta = Y - qu$, $\zeta = Z - pu$, the values of p and q being

	p	q
δ *Cassiop.*	−0.063	+0.813
υ *Urs. Maj.*	− .245	− .712
ι *Draconis*	+ .812	− .547
ο *Draconis*	+0.974	+0.184

Making these substitutions above we find for the various stars

	η ''	w_1	ζ ''	w_2
δ *Cassiop.*	+0.003	49.6	−0.304	20.3
υ *Urs. Maj.*	− .122	27.2	− .055	9.1
ι *Draconis*	+ .098	12.6	− .020	47.8
ο *Draconis*	+0.033	14.4	−0.202	49.3

So that, finally, for this series, we have

$$\eta = -0''.015\,, \quad \zeta = -0''.146\,, \quad r_2 = 0''.147\,, \quad G' = 5^\circ.9$$

This process is much more expeditious, and gives rigorously the same result that we should get by correcting the individual observations for the 427-day term and the aberration-reduction.

Berlin, Universal Transit, 1884–85. The values of the latitude given by KÜSTNER, A.N. 2993, s. 275, were reduced to the aberration 20''.500, and corrected for the 427-day term by the constants given in the foregoing series. Grouped by calendar dates, using weights, we have

	''	p		''	p
1884 Apr. 13	+0.06	4	1884 Oct. 20	+0.01	10
May 10	+ .11	7	1885 Apr. 7	− .08	5
Aug. 15	+0.14	13	May 11	−0.02	3

The solution (equal weights) gives

$$r_2 = 0''.125\,, \quad G' = 289^\circ.7$$

Cambridge, Almucantar, 1884–85. Since the results given in *A.J.* 248 were printed I have subjected them to thorough rediscussion. The original values were dependent upon the system of declinations employed (*B.J.*). But this is itself affected by a term depending on the latitude-variation, as I have shown in *A.J.* 296 and 402, and this fact has since been confirmed at Berlin. I have therefore applied the corrections necessary to eliminate this term from the declinations, given for the 336 *Hauptsterne* in *A.J.* 402, as well as Auwers's corrections to the places, given in *A.N.* 3508–9, and have then recomputed the latitudes. I have also tried to improve the results before Nov. 8, 1884, which were defective. The corrected values, to be substituted for those in *A.J.* 248, in the table at top of col. 2 of p. 60, are given below in the column $\Delta\varphi$ (C—O).

	$\Delta\varphi$ (C—O)	n		$\Delta\varphi$ (C—O)	n
1884	″	″	1885	″	″
June 19	+0.02	−0.19	Feb. 17	−0.16	+0.32
Sept. 7	+ .24	− .32	Mar. 13	+ .07	+ .06
Nov. 14	+ .22	− .14	Apr. 13	− .27	+ .35
Dec. 14	+ .12	+ .02	May 21	− .47	+ .45
1885 Jan. 11	+0.10	+0.07	June 17	−0.04	−0.04

The changes from the original values are small, except for the first two, which at best are relatively uncertain, and must be given half weight in the following solution. Changing the signs, to represent $\Delta\varphi$ (O—C) $= \varphi - \varphi_0$, and subtracting the 427-day term by the same formula as for the two foregoing series, we get the column n. The solution gives $r_2 = 0''.278$, $G' = 184^\circ.7$. These results are still referred to Struve's aberration, but it is to be noted that, from the system of observation employed, the effect of any difference in the aberration is largely eliminated from the latitude results.

Washington, Prime Vertical, 1882–84. The details of this series were published by Prof. S. J. Brown in *A.J.* 263–4. I have made an elaborate discussion of the sixteen stars of the list most numerously observed, by the same methods as for the various Pulkowa series in Vols. XI and XII of the *Journal*. From this unprinted discussion I take the final normal equations, after eliminating the 427-day term and reducing the absolute terms to the aberration $20''.500$.

$$\begin{aligned} 17.936\,\eta \quad + \ 15.360\,\zeta &= \ +0''.638 \\ 15.360\,\eta \quad + 103.265\,\zeta &= \ +9.231 \end{aligned}$$

whence $\eta = -0''.0161$, $\zeta = +0''.0918$; which give $r_2 = 0''.093$, $G' = 170^\circ$.

Madison, Meridian Circle, 1883–90. These results for latitude of the Washburn Observatory are given by Comstock in Vol. IV, part 4, pp. 108–119, of the Annals. I have first corrected the individual results for the systematic error of the *B.J.* declinations used depending on the latitude-variation. The data were then combined in 32 groups, and normals formed for the five unknowns of the method in *A.J.* 287. To eliminate the difference due to difference of declination-zero, spoken of by Comstock on pp. 120–121 of the Annals cited above, the constant was eliminated from each of the portions 1883–85 and 1886–90, and the elimination-equations were then summed. The absolute terms of the equations in η and ζ were then reduced to the values used for the 427-day term in this investigation for this epoch, thus eliminating y and z. Thus we have

$$\begin{aligned} 16.926\,\eta \quad + \ 2.709\,\zeta &= \ +0.624 \\ 2.709\,\eta \quad + 14.445\,\zeta &= \ +4.094 \end{aligned}$$

which give

$$\eta = -0''.009\,, \quad \zeta = +0''.285\,, \quad r_2 = 0''.285\,, \quad G' = 178^\circ$$

These values are dependent on Struve's aberration. The labor of reducing to the constant $20''.500$ would be very great, hence I leave the results as they stand. The difference of $0''.055$ consequently remains, being distributed between η and ζ in some unknown manner.

For convenience I now assemble all the above results in the first part of the accompanying table, headed "Observed Constants of the Annual Term." The process of determining from these data the elements of the annual ellipse for the two epochs 1865 and 1883 will be evident enough from the formulas given in the geometrical synopsis of the theory in *A.J.* 406; but for convenience I rewrite here the equations to be employed, retaining the original numbering, but in the order in which they are to be applied.

$$\left.\begin{aligned} r_2 \sin G' &= -\mu' \sin\lambda + \nu' \cos\lambda \\ r_2 \cos G' &= +\mu \ \sin\lambda - \nu \ \cos\lambda \end{aligned}\right\} \quad (32)$$

$$\left.\begin{aligned} \mu \ &= m \sin M \qquad & \nu \ &= n \sin N \\ \mu' &= m \cos M \qquad & \nu' &= n \cos N \end{aligned}\right\} \quad (31)$$

$$\tan 2L = \frac{m^2 \sin 2M + n^2 \sin 2N}{m^2 \cos 2M + n^2 \cos 2N} \quad (16)$$

$$\tan\omega = \frac{m \sin (L - M)}{n \sin (L - N)} \quad (9)$$

$$a = 2\frac{m \sin (L - M)}{\sin\omega} = 2\frac{n \sin (L - N)}{\cos\omega} \quad (10)$$

$$b = 2\frac{m \cos (L - M)}{\cos\omega} = -2\frac{n \cos (L - N)}{\sin\omega} \quad (12)$$

We can get better than in any other way, I think, an idea of the trustworthiness of the conclusions to be drawn from this investigation, by means of the degree of consistency between the elliptical elements deduced by combining the series at each epoch, two and two. In the second portion of the table, therefore, are given the coordinates found by eq. (32) and (31) for such combinations; and in the third portion of the table the elements found by eq. (16), (9), (10) and (12). The values denoted "Combination of all" are not the mean values of the columns above them, but are derived from a least-square solution of the auxiliary constants μ', μ, ν' and ν for each epoch.

OBSERVED CONSTANTS OF ANNUAL TERM.

		G'	$r_0 \sin G'$	$r_0 \cos G'$	λ	$\sin\lambda$	$\cos\lambda$
			EPOCH 1865				
Pulkowa, 1865	0.104	305.2	−0.085	+0.060	−30.4	−0.503	+0.864
Leyden, 1866	0.056	237.8	−0.047	−0.030	− 4.5	−0.078	+0.997
Washington, 1865	0.172	287.5	−0.164	+0.052	+77.0	+0.974	+0.225
			EPOCH 1883				
Pulkowa, 1879	0.147	5.9	+0.015	+0.146	−30.4	−0.503	+0.864
Berlin, 1884	0.125	289.7	−0.117	+0.042	−13.4	−0.233	+0.972
Cambridge, 1884	0.278	184.7	−0.023	−0.277	+71.1	+0.946	+0.325
Washington, 1883	0.093	170.0	+0.016	−0.092	+77.0	+0.974	+0.225
Madison, 1887	0.285	178.2	+0.009	−0.285	+89.4	+1.000	+0.010

RECTANGULAR AND POLAR COORDINATES OF THE MAJOR APSIDES.

	μ	ν	r'	r	m	n	M	N
			EPOCH 1865					
Pulkowa–Washington	+0.128	+0.033	−0.173	−0.088	0.133	0.194	14	207
Leyden–Washington	+0.155	+0.059	−0.059	+0.025	0.165	0.064	21	157
Combination of all	+0.120	+0.023	−0.107	−0.016	0.122	0.108	11	188
			EPOCH 1883					
Pulkowa–Cambridge	+0.025	−0.293	+0.002	+0.001	0.295	0.002	275	27
Berlin–Washington	−0.042	−0.099	−0.110	−0.020	0.108	0.112	247	190
Pulkowa–Washington	−0.011	−0.118	+0.024	−0.101	0.118	0.104	265	283
Berlin–Cambridge	−0.016	−0.284	−0.117	+0.025	0.285	0.120	267	168
Pulkowa–Madison	−0.009	−0.285	+0.023	0.000	0.285	0.023	268	0
Berlin–Madison	−0.009	−0.285	−0.118	+0.024	0.285	0.120	268	168
Combination of all	−0.005	−0.228	−0.056	−0.013	0.228	0.058	269	193

ELEMENTS OF ELLIPSE.

	T	L	ω	a	b
	EPOCH 1865				
Pulkowa–Washington	July 14	113	146	0.17	0.05
Leyden–Washington	July 8	107	107	0.34	0.08
Combination of all	July 1	100	132	0.33	0.01
	EPOCH 1883				
Pulkowa–Cambridge	Apr. 1	10	90	0.59	0.00
Berlin–Washington	Jan. 27	307	43	0.27	0.14
Pulkowa–Washington	Mar. 25	3	18	0.31	0.05
Berlin–Cambridge	Mar. 19	358	94	0.57	0.23
Pulkowa–Madison	Mar. 19	358	90	0.57	0.05
Berlin–Madison	Mar. 22	0	95	0.57	0.23
Combination of all	Mar. 20	358	86	0.45	0.11

The minuteness of the observed quantities on which these elements depend would lead us to expect and tolerate much larger discordances than the results actually show, as naturally attributable to unavoidable errors of observation. The harmony between the results of the various pairs is quite surprising when the delicacy of the problem is considered. It seems too great to be ascribed to mere coincidence, and I am inclined to regard the mean elements for each of these epochs 1865 and 1883 as real though rough approximations to the truth.

The results of this investigation up to this point being so much more definite than I had dared to anticipate, I have been tempted to pursue the matter further perhaps than it can be legitimately followed, and to inquire whether the most precise series of observations prior to 1860 can give us any notion as to the position of the ellipse during the first half of the century. Upon examination this endeavor is not so hopeless as it may appear on its face. At any rate the importance of the subject requires that the attempt should be made and go for what it is worth. No apology is therefore made for presenting the following speculation.

There are, of course, no existing American series of observations before 1850 of sufficient accuracy to furnish — by comparison with such series in Europe as STRUVE's prime-vertical, PETERS's vertical-circle and POND's mural-circle work — a complete solution of the problem, that is, the determination of all four elements of the ellipse. But it is to be noted that observations in one longitude provide two equations definitely, so that, if two of the elements are assumed as known, the other two can be found on this hypothesis. Now, it is to be observed that the evidence so far seems to indicate that the dimensions of the ellipse are not subject to alteration, as will be seen in the table given further on. Whether this invariability is true or not, however, it is proposed, on the hypothesis that a and b are constant, to find the elements L and ω for the epochs 1830 and 1842 from the observed data hereafter discussed.

The proper equations may be found by putting $\lambda = 0$ in eq. (32), then by (31) above and eq. (6), *A.J.* 406, we have, for this meridian,

$$r_2 \sin G' = \frac{a}{2} \cos\omega \sin L - \frac{b}{2} \sin\omega \cos L$$

$$r_2 \cos G' = \frac{a}{2} \cos\omega \cos L + \frac{b}{2} \sin\omega \sin L$$

whence
$$\tan(L - G') = \frac{b}{a} \tan\omega$$

$$r_2 = \tfrac{1}{2} \sqrt{a^2 \cos^2\omega + b^2 \sin^2\omega}$$

$$\sin\omega = \frac{1}{e}\left[1 - 4\left(\frac{r_2}{a}\right)^2\right]^{\frac{1}{2}}$$

where e is the eccentricity of the ellipse. Therefore, if we find by observation the quantities r_2 and G' for the meridian of reference, the elements L and ω can be computed, subject to the ambiguity of the different roots of the equation which determines ω. The selection of the proper root is governed by comparison with extraneous data.

To apply this method let us first examine the separate series competent to furnish r_2 and G'.

Pulkowa, Vertical Circle, Polaris, 1842–44. This series is investigated in *A.J.* 287, p. 179. To eliminate the 427-day term from the normal equations there given in the first column, we adopt the constants

$$T_1' = 2394174\,, \quad r_1 = 0''.09$$

These give $y = -0''.059$, $z = -0''.067$, which being substituted give the normals in X, η and ζ, as follows:

$$\begin{aligned}
+231.00\,X + 63.98\,\eta + 6.06\,\zeta &= +\overset{''}{2}.33\\
+63.98\,X + 78.66\,\eta + 18.93\,\zeta &= -5.46\\
+6.06\,X + 18.93\,\eta + 152.34\,\zeta &= -10.55
\end{aligned}$$

whence

$$\eta = -0''.083\,, \quad \zeta = -0''.060\,, \quad r_2 = 0''.102\,, \quad G' = 54^\circ$$

These values are independent of the aberration.

Pulkowa, Prime Vertical 1840–55. The discussion will be found in *A.J.* 296, the normals on p. 59. Taking the same constants as above for the 427-day term, we find

$$y = -0''.017\,, \quad z = -0''.088$$

To reduce to the aberration-constant 20″.500 put

$$u = +0''.055$$

Introducing these quantities into the normals for η and ζ, we have, for the seven stars observed by STRUVE during 1840–42,

$$\begin{aligned}
+69.190\,\eta + 3.882\,\zeta &= -\overset{''}{1}.455\\
+3.882\,\eta + 56.160\,\zeta &= -7.816
\end{aligned}$$

whence

$$\eta = -0''.013\,, \quad \zeta = -0''.139\,, \quad r_2 = 0''.139\,, \quad G' = 6^\circ$$

Similarly for all the observations from 1840–55 we get

$$\begin{aligned}
+77.628\,\eta + 3.090\,\zeta &= -\overset{''}{1}.014\\
+3.090\,\eta + 65.702\,\zeta &= -7.879
\end{aligned}$$

whence

$$\eta = -0''.008\,, \quad \zeta = -0''.120\,, \quad r_2 = 0''.120\,, \quad G' = 4^\circ$$

Comparing now the values just derived from STRUVE's prime-vertical and PETERS's vertical-circle observations we have

	η	ζ	r_2	G'
	″	″	″	°
Vert. Circle, 1842–44	−0.083	−0.060	0.102	54
Pr. Vert., 7 stars, 1840–42	−0.013	−0.139	0.139	6
Pr. Vert., all stars, 1840–55	−0.008	−0.120	0.120	4
Adopted	−0.045	−0.095	0.105	25

Introducing the adopted values, $r_2 = 0''.105$, $G' = 25^\circ$, in the foregoing formulas we find with the mean dimensions of the ellipse found in *A.J.* 446, $a = 0''.275$, $b = 0''.085$,

$$\sin\omega = \pm 0.68$$

The four roots corresponding to the double sign of the radical and to the two ends of the ellipse are, after referring them to Greenwich by applying the longitude of Pulkowa,

73°, 167°, 253°, 347°

By referring to the table of elements hereafter given, and to the diagram, it appears that, if the line of apsides is in continuous motion as all the later observations indicate, the first and last roots may be excluded. The other two lie near the third quadrant, and selection between them is at present indeterminate. A similar computation, using different values for the 427-day constants ($r_1 = 0''.06$, $T_1 = 2391161$), gives the corresponding roots 158° and 262°. I think we may legitimately infer from this determination that for the epoch 1842 ω lies somewhere in the third quadrant, and in the table of elements I have placed the value in the middle to signify this, but it is subject to large uncertainty either way. The corresponding value of L is 42°, with a range of uncertainty between 25° and 115°.

Pond, Greenwich Mural Circles, 1825–36. These results of the reduction of 7176 declinations from double-altitudes of 36 stars are given in *A.J.* 313, 315. I regard the constants of the latitude-variation deduced from this series, from its quality, continuity and extent, as by all odds more trustworthy than those furnished by any other that we possess. The constants of the annual term which it gave are

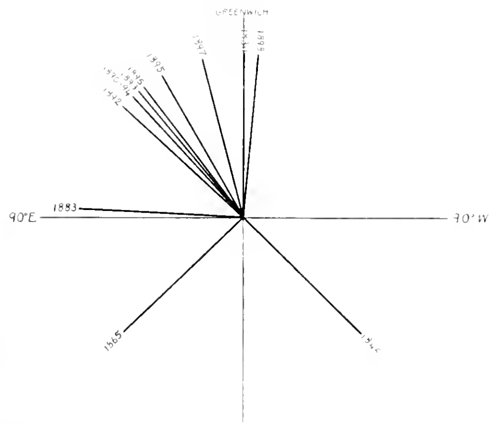
	r_a ″	G °
1825.3–1830.6	0.170	7.6
1830.7–1836.0	0.155	17.0

These require no correction for aberration, since I employed the constant 20″.500 in the reduction of the observations, nor are they dependent on the 427-day term, since the series covers two harmonic seven-year cycles, which permitted a perfect elimination in this respect.

The value of r_a is slightly in excess of the assumed semi-axis major of the ellipse. We may therefore adopt at once, for 1830, $\omega = 0°$ or 180°. Also consequently $L = G' = 15°$ or 195°. Consistently with the data for the other epochs we select the first values as the only ones admissible on the hypothesis of continuous motion of the apsides which we are investigating. If this hypothesis should not prove to be true the values $\omega = 180°$ and $L = 195°$ are the alternatives. The ambiguity cannot be removed independently of such considerations, by the nature of the case, in default of corresponding observations at this epoch in other longitudes.

All the foregoing results for the elements of the annual term are now incorporated with the table in *A.J.* 446, giving the annual results from 1890 to 1897, and are given below, where I have also added the results for 1898 of a provisional reduction of the data given in the *Centralbureau Report* of 1900.

To facilitate a study of the motion of the line of apsides the diagram has been prepared. The center represents the mean position of the pole of the earth's figure. The line vertically upward is directed to Greenwich; that at right angles to this corresponds on the left to 90° east, and on the right to 90° west longitude. For each date a line is drawn from the center representing the position of the major axis at that date, corresponding to the end of the



ellipse for which ω is given in the table. In interpreting the diagram the conclusions drawn must take into account the ambiguity spoken of above for the epochs 1830 and 1842, as to the proper end of the ellipse to be used. From 1865 to 1898 there is no such ambiguity, and the only uncertainty is that due to error of observation.

	T	L °	ω	a ″	b ″
1830	April 6 ::	15 ::	0 ::	—	—
1842	May 3 ::	42 ::	225 ::	—	—
1865	July 1	100	132	0.33	0.01
1883	Mar. 20	358	86	.45	.11
1890	Mar. 29	7	13	.27	.13
1892	April 3	13	18	.32	.08
1893	Mar. 31	9	10	.26	.09
1894	April 10	19	18	.26	.08
1895	Mar. 29	7	30	.30	.09
1896	April 13	23	37	.27	.07
1897	April 25	34	15	.33	.11
1898	May 10	50	355	0.25	0.05

The diagram has a very fruitful significance in whatever way it is interpreted. To arrive at the correct interpretation, consider first the points between 1890 and 1898. These show, aside from the accidental deviations of individual years, a turning towards the east in that interval of, say, 40° to 50°. The line for 1883 accords with such a motion within its greater degree of uncertainty. That for 1865 is again in sufficient accord with a general continuous motion in the same direction. The rate between 1865 and 1898 is, roughly indicated, between 4° and 5° annually.

The same rate continued backward to 1842 would bring the position to the middle of the third quadrant, where the observations place it. This seems to justify our exclusion of the roots of the equation in the first quadrant. A continuation of the motion brings us back to 1830, where POND's observations place it. From a consideration of the whole data, if we take a period of 75 years, which gives an average annual rate of 4°.8, and take for 1894 a value of $\omega = 33°$ we find a sufficient consistency with all the observed positions,* as follows, where the values for 1891, '94 and '97 are the means of 1890–92, 1893–95 and 1896–98, respectively.

VALUE OF ω.

	Observed	Hypothesis
	°	°
1830	360 ::	340
1842	253 ::	282
1860	180 ::	196
1865	132	172
1867	155 ::	163
1873	132 ::	134
1883	86	86
1891	16	47
1894	39	33
1897	16	19

The only wide discordance here is for 1865. In this case the agreement of the hypothetical value (172°) is better with that given by the combination Pulkowa–Washington (146°) than with that for Leyden–Washington (107°). The Pulkowa determination of the constants is undoubtedly more certain than that for Leyden, on account of the greater number and better distribution of the observations, and the larger interval which they cover. Perhaps it would have been better to have assigned weights, indeed, in the combination of the various determinations of the constants. But this would have brought in an element of arbitrary judgement from which I desired to be free in a demonstration of this kind.

The hypothesis, then, of a revolution of the line of apsides in a period of about 75 years seems to harmonize with all the data given by observation, and unless these data can be shown to be illusory this hypothesis seems to be the most natural way of accounting for the phenomena. Nevertheless it is to be observed that there is another hypothesis which numerically will answer, namely, the one suggested on p. 108, *A.J.* 446, of an oscillatory motion of the apsides through a large angle. The reason for this alternative possibility is that the equations which determine L by means of the tangent, permit two values differing 180°, corresponding to the two ends of the ellipse. The observations cannot distinguish between the two. The diagram may be changed to represent this alternative hypothesis by prolonging the lines through the center and taking the other ends. But here the inherent improbability of this hypothesis appears, for it seems to be a violent supposition that the apsis which moves so regularly towards the west from 1890 to 1898 could have occupied the position 266° only seven years earlier. Therefore this alternative hypothesis appears to be unnatural and improbable.

A third possible hypothesis, namely, that the position of the ellipse is subject to no regular law whatever, seems to be contradicted by the continuous series between 1865, 1883, 1890 and 1898.

Coming now to the question of the variations of the period from an exact year, a consideration of the values of L (or T) makes manifest the fact that the annual period is not constant. From 1890 to 1895 it was about a year, the dates of the return of the pole to the end of the major axis under observation being very near the beginning of April. After 1895, however, this date is retarded through April to the beginning of May in 1898. Between 1865 and 1890 on the contrary the returns were notably accelerated. Roughly speaking, the period between 1865 and 1890 was about 361 days; between 1890 and 1895, nearly 365 days; and between 1895 and 1898 it has certainly lengthened. A corresponding lengthening is indicated between 1842 and 1865, in which interval it was 368 days. If the values of L be plotted it will be seen that they rudely indicate a period of fluctuation of about 60 years.* It may be recalled that a period somewhat longer than this was deduced in *A.J.* 422 from a consideration of the values of the epoch G of minimum latitude for the annual term for the meridian of Greenwich. It appears to me that the fact of some such long periodical oscillation of four or five days on either side of a mean period of a year is not open to much doubt.

It should be said in conclusion that for some of the shorter series included in this investigation the quantities derived from the observations, r_2 and G', can be modified, but not essentially so, by the particular values employed to represent the 427-day term; but that the more extended series, like Pulkowa 1863–70, Washington 1862–67, Pulkowa 1875–82 and Madison 1883–90, are quite independent of any variation due to this source.

* In this table I have substituted for 1842, as can now legitimately be done, the third root of p. 69; and have added the following values, found by that method since the above paper was written, from NYRÉN's vertical-circle observations, 1871–75, and by the Greenwich Reflex Zenith-Tube, 1857–63 and 1864–70.

	r_2	G'	L	ω
	"	°	°	°
1860	0.166	323	143	180
1867	0.134	313	53	155
1873	0.058	297	61	132

* All the values of L on p. 70, and the additional ones in the foregoing note, are consistent with the expression,

$$L = 51° - 45° \cos (t - 1887)\, 6°$$

the differential of which gives the corresponding expression for the variation of the period from a year.

$$+4^{d}.7 \sin (t - 1887)\, 6°$$

NEW COMET — *c* 1900 (GIACOBINI).

(SCIENCE OBSERVER, *Special Circular, No. 128.*)

A message received during the night of December 23 announces the discovery of a comet by GIACOBINI, at Nice, on December 20. A later position from Europe and others which have been kindly communicated by Professor CAMPBELL of Lick Observatory are here given:

POSITIONS.

Greenw. M.T.	R.A.	Decl.	Observers
1900	h m s	° ′ ″	
Dec. 20.313	22 32 0.0	−22 0 0	Giacobini
21.271	22 57 0.0	−22 45 0	Giacobini
24.6025	22 59 10.2	−22 41 40	Aitken
26.6280	23 11 23.6	−22 57 59	Aitken
28.6197	23 23 17.1	−23 7 30	Aitken

From the three Lick positions Mr. AITKEN has computed the following orbit:

ELEMENTS.

T = 1900 December 1.41 Gr. M.T.

$$\left.\begin{aligned}\omega &= 175^\circ\ 54'\\ \Omega &= 192\ \ 39\\ i &= 31\ \ \ 1\end{aligned}\right\}\text{Mean Eq. 1900.0}$$

q = 0.9769

EPHEMERIS.

Greenw. Midnight	R.A.	Decl.	Light
1901	h m s	° ′	
January 3	23 57 41	−23 14	.074
7	0 20 8	−23 2	
11	0 41 36	−22 37	
15	1 2 8	−22 2	.055

Light at discovery = 1.

OBSERVATIONS OF COMET *c* 1900 (GIACOBINI),

BY R. G. AITKEN.

1900 Mt. Hamilton M.T.	*	No. Comp.	☌′ — * $\Delta\alpha$	☌′ — * $\Delta\delta$	☌′s apparent α	☌′s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			s	′ ″	h m s	° ′ ″		
Dec. 24 6 20 35	1	10 , 8	+16.62	+1 22.2	22 59 10.17	−22 44 41.1	9.301	0.870
26 6 57 42	4	10 , 10	+30.96	−3 3.2	23 11 23.71	−22 58 2.1	9.428	0.861
28 6 45 58	7	10 , 10	+10.20	−4 21.0	23 23 18.54	−23 7 27.1	9.377	0.867

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	22 58 50.01	+3.54	−22 46 21.6	+18.3	Gould, Cordoba Gen. Catal. 31348
2	23 9 3.68	+3.55	−22 55 12.5	+18.1	Gould, Cordoba Zones 23ʰ202
3	23 10 23.16	+3.55	−22 48 26.2	+18.1	Eastman, 2d Washington Catal. 4933
4	23 10 49.20	+3.55	−22 55 17.0	+18.1	Micrometer comparison with (2) and (3)
5	23 19 41.46	+3.56	−23 3 4.8	+17.9	Gould, Cordoba Gen. Catal. 31716
6	23 20 13.56	+3.56	−23 4 8.2	+17.8	Gould, Cordoba Gen. Catal. 31733
7	23 23 4.77	+3.57	−23 3 23.9	+17.8	Micrometer comparison with (5) and (6)

The first two observations were made with the 12-inch, the last, with the 36-inch refractor. In all cases $\Delta\alpha$ was measured directly with the micrometer.

The second position differs slightly from the one telegraphed, because the comparison-star was later connected with star (3), as it was found that star (2) had but one observation in GOULD's Zones. The mean of the two places for star (4) was adopted.

The comet is faint and small, irregular in outline, with a condensation slightly preceding and south of the center. The 36-inch shows a nucleus estimated at the 15th magnitude, and a short, fan-shaped extension in the north following quadrant. The comet is comparable to an 11th magnitude star in brightness.

Lick Observatory, University of California, 1900 *Dec.* 31.

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RESULTS OF OBSERVATION WITH THE ZENITH TELESCOPE, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.*

By C. L. DOOLITTLE.

A statement of the provisional results of observation with the Zenith Telescope at this place, from 1898 Oct. 8, to 1899 Sept. 28, may be found in the *Astronomical Journal* for January, 1900.

The values of the latitude which follow may be considered definitive up to January 10, 1900. The observations during the following February and March were by ERIC DOOLITTLE, my own activity having suffered a temporary suspension as the result of an accident. The values after 1900 January 10 are to be regarded as provisional. It is intended to make a re-adjustment of this part of the series in connection with the work of the present year.

The correction to STRUVE'S Constant of Aberration has been derived from the observations from 1898 Oct. 8 to 1899 Nov. 27.

The result being	+ 0″.0961
Giving for this constant	**20.542**

This value has been employed in the final latitude-reduction.

In this connection it may not be out of place to say that a thorough revision of the latitude work at Bethlehem has been for some time in progress, with a view to its publication in full. The series extending from 1891 January 19 to 1895 August 19, is now in the printer's hands. It is to be published by the Philosophical Society at Philadelphia. This series gives for the value of the constant of aberration,

20″.537

Four values have now been derived from my zenith-telescope observations, viz:

1892–1893	20″.552	at Bethlehem
1894–1895	20.537	"
1896–1898	20.580	at Philadelphia
1898–1900	20.542	"

The details of the latitude determinations given below presumably call for no explanation.

LATITUDE = 39° 58′ +.

	Group IV	No.	I	No.
1898	″		″	
Sept. 6	2.28	3	..	..
8	2.27	9	..	..
10	2.62	7	..	..
11	2.32	9	..	..
12	2.10	8	..	..
13	2.12	9	..	..
16	2.46	6	..	..
17	2.24	9	..	..
18	2.32	7	..	..
20	2.20	8	..	..
21	2.30	9	..	..
26	2.29	6	..	..
27	2.40	9	..	..
Oct. 8	2.10	7	2.19	9
9	2.31	8	2.18	8
12	2.22	8	2.13	10
15	2.25	8	..	..
16	2.31	9	2.15	9
17	2.30	9	..	..
19	2.25	9	2.19	10
20	2.15	9	2.31	10
22	2.08	9	2.09	10
23	2.21	9	2.30	8
24	2.22	9	2.17	10
27	2.36	9	2.33	10
28	2.10	8	..	..
Nov. 1	1.98	9	2.05	10
2	2.18	9	2.21	10
3	2.36	9	2.13	10
4	2.17	9	..	..
6	2.27	9	2.18	10

	Group IV	No.	I	No.
1898	″		″	
Nov. 7	2.27	9	2.19	10
11	..	..	2.24	10
12	2.31	6	..	..
14	..	..	2.25	9
15	2.02	9	2.19	8
20	2.33	9	2.08	10
21	2.07	9	..	..
25	2.10	9	..	..
Dec. 16	..	..	1.91	10
18	..	..	2.18	10
23	..	..	2.04	10
24	..	..	2.13	10
25	..	..	2.19	10
26	..	..	1.93	10
29	..	..	1.98	10
1899				
Jan. 1	..	..	1.91	10
2	..	..	2.16	4
7	..	..	2.11	10
11	..	..	1.95	10
14	..	..	2.18	10
15	..	..	1.94	10
18	..	..	2.25	10
19	..	..	2.09	10
20	..	..	2.06	7
21	..	..	1.89	10
22	..	..	2.20	10
23	..	..	1.95	9
25	..	..	1.77	10
Feb. 1	..	..	1.82	8
5	..	..	2.00	8

	Group I	No.	II	No.
1899	″		″	
Feb. 8	1.91	10	1.94	9
9	1.98	10	1.84	10
10	2.05	10	2.01	8
11	1.91	10	1.93	9
15	1.95	10	1.92	10
19	1.75	10	..	..
20	2.12	10	1.98	8
21	2.00	9	1.80	7
24	1.99	10	1.88	10
25	1.84	9	..	..
27	1.94	9	1.96	10
Mar. 1	1.98	10	..	..
5	1.88	10	1.93	10
6	1.63	10	..	..

	Group I	No.	II	No.
1899	″		″	
Mar. 7	..	..	2.01	10
10	1.72	4	..	..
12	..	..	2.08	8
16	..	..	1.91	10
20	..	..	2.03	10
23	..	..	2.07	9
24	..	..	1.82	10
Apr. 1	..	..	2.12	10
3	..	..	2.08	10
4	..	..	2.10	10
5	..	..	2.00	10
9	..	..	1.63	10
10	..	..	1.86	10

Group II		No.	III	No.
1899 May 9	2.29	10	2.13	10
10	1.93	9	1.91	8
11	2.08	10	1.93	10
15	1.96	10	2.01	10
20	2.02	9	2.10	10
21	2.16	10	2.13	10
24	1.96	10	2.07	9
25	2.02	10	2.15	10
26	1.93	9	1.91	10
28	2.22	9	2.21	2
30	2.09	10	2.06	6
June 2	2.19	10	2.22	8
3	1.87	10	2.03	10
4	1.95	10	2.15	10
7	..	..	2.11	10
8	2.11	10	2.10	10
10	..	..	2.11	1
15	..	..	2.08	7
16	..	..	2.19	10
18	..	..	2.06	10
19	..	..	2.03	6
21	..	..	2.23	10
22	..	..	2.04	10
27	..	..	2.01	10
29	..	..	2.19	10
30	..	..	2.27	10
July 1	..	..	2.10	10

Group III		No.	IV	No.
1899 July 2	2.08	10	2.15	9
3	2.12	10	2.05	9
4	1.97	10	2.23	7
9	2.11	10	2.15	9
10	2.19	10	2.20	8
11	2.08	10	2.26	6
12	..	..	2.25	9
14	2.31	10	2.05	9
18	2.11	10	2.11	9
26	2.21	9	..	..
30	2.20	10	2.26	9
31	2.16	10	2.18	9
Aug. 1	2.06	10	2.01	9
6	2.38	10	2.15	9
7	2.42	10	2.27	9
8	2.22	10	..	..
13	2.27	5	2.11	9
15	2.14	10	2.28	9
16	2.33	10	2.15	9
17	2.29	2	2.33	8
Sep. 13	..	..	2.33	9
14	..	..	2.31	9
15	..	..	2.17	9
16	..	..	2.16	9
17	..	..	2.29	9
21	..	..	2.33	9
22	..	..	2.22	9
26	..	..	2.35	9
27	..	..	2.11	9
28	..	..	2.16	9

Group IV		No.	I	No.
1899 Oct. 1	2.09	8	2.25	7
6	2.17	9	2.62	9
9	2.18	9	2.26	9
12	2.65	9	2.55	9
16	2.02	9	..	..
17	2.33	8	..	..
18	..	..	2.20	10
19	2.38	9	..	..
20	2.17	9	2.55	10
21	2.51	9	2.31	10
24	2.41	8	2.15	9
Nov. 1	..	..	2.12	10
Nov. 2	2.42	6	..	..
4	2.26	9	2.21	9
5	..	..	2.23	10
6	2.20	9	2.31	10
7	2.25	8	..	..
8	..	..	2.33	10
9	2.24	9	2.11	10
10	2.21	8	..	..
12	2.45	9	2.26	10
13	2.43	9	2.35	10
16	2.44	9	2.50	9
20	2.29	9	2.18	10
21	2.33	9	..	..
23	..	..	2.32	1
24	2.37	9	2.29	10
25	2.34	9	2.21	10
26	2.29	9	2.23	9
27	2.18	9	2.35	10
Dec. 13	..	..	2.41	8
15	..	..	2.29	10
16	..	..	2.19	10
19	..	..	2.48	10
21	..	..	2.23	10
22	..	..	2.31	10
24	..	..	2.21	10
25	..	..	2.21	10
26	..	..	2.26	10
28	..	..	2.23	10
30	..	..	2.25	9
1900 Jan. 1	..	..	2.06	10
2	..	..	2.20	10
3	..	..	2.12	8
6	..	..	2.12	10
8	..	..	2.41	10
9	..	..	2.21	7
10	..	..	2.14	10

Weighted Mean Date	Latitude	No.
1898 Sept. 11	39° 58′ 2″.398	51
21	2.287	48
Oct. 14	2.237	125
28	2.187	139
Nov. 13	2.187	126
Dec. 28	2.038	104
1899 Jan. 22	2.018	102
Feb. 13	1.945	124
Mar. 1	1.913	126
30	1.962	99
May 17	2.055	155
June 2	2.070	131
23	2.131	97
July 10	2.146	164
Aug. 7	39 58 2.285	167
1899 Sept. 20	39° 58′ 2″.244	90
Oct. 13	2.355	143
Nov. 4	2.269	125
21	2.340	163
Dec. 20	2.325	88
1900 Jan. 4	2.229	84
Feb. 2	2.168	111
28	2.244	135
May 14	2.007	157
June 5	1.965	133
24	2.039	64
July 11	2.037	163
29	2.082	153
Aug. 13	2.087	158
Sept. 13	39 58 2.115	99
Whole number		3627

The values from 1898 to 1900 Jan. 4 are definitive: those subsequent are provisional.

NOTE ON A SUSPECTED NEW VARIABLE.

By ALBERT S. FLINT.

I desire to call the attention of observers of variables to the star DM. +25°3803, 7.5 mag., whose place for 1900.0 is R.A. $19^h\,17^m\,32^s$, Decl. +25° 23′. This is one of the comparison-stars in my present series of parallax observations with the 12.2 cm. meridian circle; and I have been comparing its magnitude, in the course of my regular observations, with that of DM. +25°3802, 7.0 magnitude, preceding 11^s, north 1′.8. I have now thirty-five of these comparisons, and in three of them the deviation from the mean difference of the estimated magnitudes is so marked, that I feel assured it is due to variability of DM. +25°3803, which in each instance appeared abnormally faint. I had previously made fifteen estimates, on as many different dates, of the magnitudes of this star and of Br. 2459, 6.1 mag., following $3^m\,46^s$, and DM. +24°3758, 7.5 mag., following $6^m\,15^s$. On one of these dates the magnitude of DM. +25°3803 appeared so faint that I made a check-mark against the record, to signify that the estimate occasioned surprise, but was confirmed. I have plotted the entire series of my estimates on all four stars, extending from Sept. 20, 1898, to Oct. 8, 1900, but see no marked deviation, except on the four dates already referred to.

The variations from the mean difference of the estimated magnitudes in each case are presented in the following table, where the stars are designated 1, 2, 3, 4, in the order of right-ascension.

	1 – 2	3 – 2	4 – 2
1898 Oct. 5.3	–	+1.0	+0.5
1899 Sept. 26.3	+0.5	+0.8	+0.4
1900 May 2.7	+0.7	+0.2	–0.2
1900 Sept. 12.3	+0.8	+0.6	+0.6

On May 2, 1900, stars 3 and 4 appeared much fainter than normal, owing to increasing daylight, so that their differences from star 2 on this date are not comparable with those for the other dates. No clouds were noticed on the first three nights of these four, and on the last a special note was made that the sky appeared perfectly clear.

I had thought to obtain some more comparisons with an equatorial telescope the present season, after my regular observations are over for the evening, but the star is very low now at that time. I hope, however, that some observers may yet follow it with early evening observations.

Washburn Observatory, Madison, Wis., 1900 Nov. 26.

NOTES ON VARIABLE STARS. — No. 34.

By HENRY M. PARKHURST.

6849 *R Aquilae*. Rejecting my late minimum observations, obtaining from the elements of the Second and Third Catalogues normal maxima for eleven earlier dates, and including my maxima in 1893, 1894 and 1900, I derive the following approximate elements, representing the maxima within an average of four days:

$$2399167 + 336.7\ E + 150 \sin (4^\circ E + 0^\circ)$$

Results of Observations.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors			Remarks
				1900							d	
5501	*S Serpentis*	Max.	5241	Aug. 9	72	+58	9	8.22	1.20	3.43	19	
5511	*RS Librae*	Max.	5190	June 19	18	+22	2					
5566	*RU Librae*	Max.	5197	June 26	14	–83	7					Period 314 days?
5593	*W Librae*	Max.	5166	May 26	39		E					Probably earlier
5675	*V Coronae*	Min.	5268	Sept. 5	9	39	9	11.66	1.01	1.37	15	Perhaps later
5677	*R Serpentis*	Max.	5247	Aug. 15	75	–20	9	6.14	0.71	1.80	19	
5770	*R Herculis*	Max.	5148	May 8	40	–16	9	9.09	1.55	1.89	38	Correction diminishing
5798	*RU Herculis*	Max.	5350:	Nov.			1		–			Interruption, new building
5856	*W Ophiuchi*	Max.	5201	June 30	21	–28	6					Possibly earlier
5887	*V Ophiuchi*	Min.	5193	June 22	32	+19	9	9.16	1.62	2.24	11	Curve irregular
"	"	Max.	5247	Aug. 15	32	–103	9	7.64	1.48	6.22	16	Probably later
5903	*Y Scorpii*	Max.	5201	June 30	24	+ 6	5		–			Period assumed 381 days
6044	*S Herculis*	Max.	5240	Aug. 8	52	+17	9	7.07	1.31	2.24	21	+2, transition elem. *A.J.* 388
6132	*R Ophiuchi*	Max.	5218	July 17	52	–29	9	7.46	2.17	2.19	33	
6160	*RT Herculis*	Max.	5295	Oct. 2	5	–52	5	9.5				Period suggested *A.J.* 421
6225	*RS Herculis*	Min.	5229	July 28	7	– 1		•				*A.J.* 456, 476
"	"	Max A	5310	Oct. 17	7	+ 2	5	9.6				
"	"	Max B	5336	Nov. 12	7	0	3	8.1			–	Mean interval max. 28 days
[6141]	— *Herculis*	Max.	5317	Oct. 24			6	9.4				
6624	*T Serpentis*	Max.	5248	Aug. 16	12	–35	2				–	Probably earlier
6682	*X Ophiuchi*	Min.	5291	Sept. 28	16	– 5	9	8.31	3.64	2.99	50	Assuming $M-m$ 125, *A.J.* 311
6849	*R Aquilae*	Max.	5298	Oct. 5	18	0	9	6.55	1.06	1.46	38	From elements above
6894	*S Lyrae*	Max.	5271	Sept. 8	6	+31	8	9.67	0.93	1.25	22	Probably secondary
6921	*S Sagittarii*	Max.	5307	Oct. 14	54	15	4		–		–	
[7040]	— *Aquilae*	Max.	5180	June 9	–		E				–	Probably not later
7118	*X Aquilae*	Min.	5300	Oct. 7	8		E					
7120	*χ Cygni*	Max.	5184	June 13	123		E					Obsns. tend to confirm elements
7155	*RR Aquilae*	Max.	5303	Oct. 10	5	88	9	9.06	1.61	2.16	18	Last 3 intervals have been 398d
7162	*RS Aquilae*	Max.	5400:	Jan. (1901)			1		–	–		Period apparently about 400d
7234	*R Capricorni*	Max.	5201	June 30	13	– 24	1			–	–	Probably earlier
[7241]	— *Aquilae*	Max.	5193.1	June 22	77	– 0.6	7				–	Adopting period 7.87 days
"	"	Max.	5218.0	July 17	80	+ 0.8	9					
"	"	Max.	5226.0	July 25	81	+ 0.9	7				–	
"	"	Max.	5273.0	Sept. 10	87	+ 0.	8					

INDIVIDUAL OBSERVATIONS.

Including Observations by ARTHUR C. PERRY.

5501 *S Serpentis.*

(Cont. from 476. Comp. Stars 388.)

Julian	Calendar	Mag.
	1900	
5118.6	Apr. 8 to	
5136.6	26	13]
	4 dates	
5137.6	Apr. 27	12.33_2
5169.6	May 29	11.76_2
5189.6	June 18	11.3
5195.6	24	10.89_2
5201.6	30	10.76_2
5209.6	July 8	10.58_2
5220.6	19	9.24_2
5230.6	29	8.22_2
5236.6	Aug. 4	8.32_2
5242.6	10	7.89_2
5243.5	11	8.50_2
5246.5	14	8.31_2
5255.5	23	7.86_2
5261.6	29	8.69_2
5266.5	Sept. 3	8.65_2

5511 *RS Librae.*

(Cont. from 476. Comp. Stars 388.)

Julian	Calendar	Mag.
	1900	
5176.6	June 5	9.4
5188.6	17	9.45_2
5193.6	22	9.46_2
5201.6	30	10.35_2

5566 *RU Librae.*

(Continued from 476.)

Julian	Calendar	Mag.
	1900	
5172.6	June 1	11.2
5176.6	5	11.33_2
5186.6	15	10.52_2
5190.6	19	10.34_2
5196.6	25	9.72_2
5202.6	July 1	9.71_2
5215.6	14	12.43_2

5593 *W Librae.*

(Continued from 421.)

Julian	Calendar	Mag.
	1900	
5172.6	June 1 to	
5196.6	25	13]
	4 dates	

5675 *V Coronae.*

(Continued from 421.)

Julian	Calendar	Mag.
	1900	
5118.6	Apr. 8	9.85_2
5125.6	15	9.95_2
5135.6	25	9.99_2
5139.6	29	10.25_2
5147.5	May 7	10.15_2
5151.5	11	9.96_2
5165.6	25	10.59
5203.6	July 2	10.00_2
5214.6	13	10.03_2
5225.6	24	10.21_2
5231.6	30	10.87_2
5236.5	Aug. 4	10.29_2
5241.5	9	11.27_2

5675 *V Coronae.* Cont.

Julian	Calendar	Mag.
	1900	
5250.5	Aug. 18	11.35_2
5260.5	28	11.74_2
5265.5	Sept. 2	11.28_2
5266.5	3	11.93_2
5267.5	4	11.63_2
5272.5	9	11.61_2
5279.5	16	11.31_2
5291.5	Oct. 1	11.83_2

5677 *R Serpentis.*

(Cont. from 476. Comp. Stars 476.)

Julian	Calendar	Mag.
	1900	
5189.6	June 18	10.0
5195.6	24	10.20_2
5201.6	30	10.07_2
5209.6	July 8	9.46_2
5220.6	19	8.82_2
5230.6	29	7.20_2
5236.6	Aug. 4	6.82_2
5242.5	10	6.28_2
5243.5	11	5.89_2
5246.5	14	6.23_2
5255.5	23	6.27_2
5261.5	29	6.14_2
5266.5	Sept. 3	6.64_2

5770 *R Herculis.*

(Cont. from 476. Comp. Stars 476.)

Julian	Calendar	Mag.
	1900	
5118.6	Apr. 8	10.03_2
5135.6	25	9.35_2
5139.6	29	9.45_2
5147.6	May 7	9.24_2
5156.6	16	9.42_2
5167.6	27	9.55_2
5188.6	June 17	9.84_2

5798 *RU Herculis.*

(Continued from 476.)

Julian	Calendar	Mag.
	1900	
5189.6	June 18	12]
5202.6	July 1	13]
5217.6	16	13]
5229.6	28	12.6
5246.6	Aug. 14	12.6
5255.6	23	12.6
5280.5	Sept. 17	11.3
5287.5	24	11.3
5309.5	Oct. 16	10.0
5310.5	17	9.83_2

5856 *W Ophiuchi.*

Julian	Calendar	Mag.
	1900	
5172.6	June 1	9.8
5176.6	5	9.19_2
5186.6	15	10.33_2
5190.6	19	10.00_2
5196.6	25	9.61_2
5202.6	July 1	9.77_2
5215.6	14	9.80_2
5228.6	27	10.72_2

5887 *V Ophiuchi.*

(Cont. from 456. Comp. Stars 456.)

Julian	Calendar	Mag.
	1900	
5162.6	May 22	9.27_2
5167.6	27	9.21_2
5169.6	29	9.06_2
5180.6	June 9	9.23_2
5189.6	18	9.00_2
5196.6	25	9.52_2
5202.6	July 8	8.81_4
5215.6	14	9.18_2
5228.6	27	7.97_2
5235.6	Aug. 3	8.15_2
5241.6	9	7.11_2
5246.6	14	7.91_2
5257.5	25	7.64_2
5262.5	30	7.49_2
5266.6	Sept. 3	7.95_2
5267.5	4	7.80_2
5272.5	9	7.50_2
5279.5	16	7.78_2
5281.5	18	7.80_2
5304.5	Oct. 11	7.67_2

5903 *Y Scorpii.*

(Continued from 476.)

Julian	Calendar	Mag.
	1900	
5189.6	June 18	10.9
5190.6	19	10.96
5196.6	25	11.06
5502.6	July 1	10.79_2
5217.6	16	11.42_2

6044 *S Herculis.*

(Cont. from 456. Comp. Stars 388.)

Julian	Calendar	Mag.
	1899	
1928.6	Sept. 30	7.5r
	1900	
5139.6	Apr. 29	12.5]
5162.6	May 22	10.4
5165.6	25	10.0
5172.6	June 1	9.51_2
5188.6	17	9.39_2
5195.6	24	8.36_2
5201.6	30	8.96_2
5208.6	July 7	7.91_2
5215.6	14	7.72_2
5228.6	27	7.38_2
5235.5	Aug. 3	6.98_2
5242.5	10	7.20_2
5243.6	11	6.63_2
5246.6	14	7.88_2
5257.5	25	6.70_2
5262.5	30	6.88_2

6132 *R Ophiuchi.*

(Cont. from 476. Comp. Stars 476.)

Julian	Calendar	Mag.
	1900	
5180.6	June 9	8.7
5181.6	10	9.13_2
5191.6	20	7.94_2
5200.6	29	7.70_2
5208.6	July 7	7.52_2
5218.6	17	7.55_2
5229.6	28	7.41_2

6132 *R Ophiuchi.* Cont.

Julian	Calendar	Mag.
	1900	
5235.6	Aug. 3	7.61_2
5241.6	9	7.90_2
5243.5	11	7.70_2
5255.5	23	9.00_2

6160 *RT Herculis.*

(Continued from 476.)

Julian	Calendar	Mag.
	1900	
5135.6	Apr. 25 to	
5255.6	Aug. 23	13]
	6 dates	
5280.5	Sept. 17	10.4
5281.5	18	9.61_2
5294.5	Oct. 1	9.47_2
5308.5	15	9.48_2
5313.5	20	9.91_2
5318.5	25	9.89_2
5336.5	Nov. 12	10.56_2

6225 *RS Herculis.*

(Continued from 476.)

Julian	Calendar	Mag.
	1899	
1928.6	Sept. 30	8.8r
	1900	
5189.6	June 18	10.9
5202.6	July 1	11.5
5217.6	16	12]
5229.6	28	12
5246.6	Aug. 14	12
5255.6	23	11.5
5280.5	Sept. 17	10.8
5281.5	18	11.23_2
5294.5	Oct. 1	10.47_2
5308.5	15	9.67_2
5313.5	20	9.59_2
5318.5	25	10.02_2
5336.5	Nov. 12	8.06_2

[6441] — *Herculis.*

Julian	Calendar	Mag.
	1900	
5242.6	Aug. 10	11]
5262.6	30	11.1
5280.5	Sept. 17	9.9
5281.5	18	9.91_2
5294.5	Oct. 1	9.44_2
5308.5	15	9.31_2
5313.5	20	9.26_2
5318.5	25	9.12_2
5336.5	Nov. 12	9.52_2

6624 *T Serpentis.*

(Continued from 476.)

Julian	Calendar	Mag.
	1899	
1928.6	Sept. 30	9.6r
	1900	
5246.6	Aug. 14	9.1
5248.6	16	9.32_2
5255.5	23	9.66_2
5261.5	29	9.79_2
5265.5	Sept. 2	9.99_2
5272.6	9	10.06_2
5287.5	24	11.38_2

6682 *X Ophiuchi.*

(Continued from 421.)

Julian	Calendar	Mag.
	1897	
4224.5	Oct. 26	8.8r
	1900	
5218.6	July 17	7.9
5229.6	28	8.8
5230.6	29	7.81_2
5232.6	31	7.87_2
5236.6	Aug. 1	8.33_2
5242.6	10	8.23_2
5249.5	17	8.12_2
5260.5	28	8.17_2
5265.5	Sept. 2	8.15_2
5267.5	4	7.81_2
5272.5	9	8.43_2
5276.5	13	8.21_2
5280.5	17	8.11_2
5313.5	Oct. 20	8.69_2
5318.5	25	7.97_2
5331.5	Nov. 7	7.70_2
5340.5	16	7.59_2
5346.5	22	7.78_2

6849 *R Aquilae.*

(Continued from 476.)

Julian	Calendar	Mag.
	1900	
5218.6	July 17	10.2
5229.6	28	10.0
5230.6	29	9.70_2
5255.6	Aug. 23	9.91_2
5258.6	26	9.38_2
5262.5	30	9.38_2
5263.5	31	8.75_2
5267.6	Sept. 4	8.59_2
5273.5	10	8.15_2
5275.5	12	7.64_2
5280.5	17	7.08_2
5287.5	24	6.48_2
5304.5	Oct. 11	5.97_2
5313.5	20	7.35_2
5318.5	25	6.60_2
5330.5	Nov. 6	7.18_2
5340.5	16	7.29_2
5347.5	23	7.25_2

6894 *S Lyrae.*

(Cont. from 476. Comp. Stars 339.)

Julian	Calendar	Mag.
	1900	
5228.6	July 27	14.0
5246.6	Aug. 14	10.9
5255.6	23	10.52_2
5261.6	29	9.89_2
5266.6	Sept. 3	9.60_2
5272.6	9	9.69_2
5279.5	16	9.58_2
5282.5	19	10.12_2
5290.5	27	10.01_2

6921 *S Sagittarii.*

(Continued from 482.)

Julian	Calendar	Mag.
	1900	
5228.6	July 27	13]
5246.6	Aug. 14	13]
5257.6	25	13]

6921 *S Sagittarii*.–Cont.

Julian	Calendar 1900	Mag.
5280.5	Sept. 17	12.2
5282.5	19	11.98_2
5312.5	Oct. 19	9.87_2
5318.5	25	10.55_2

[7040] — *Aquilae*.
(Continued from 482.)

Julian	Calendar 1900	Mag.
5205.6	July 4	9.4
5211.6	10	9.36_2
5220.6	19	9.63_2

7118 *X Aquilae*.
(Cont. from 482. Comp. Stars 339.)

Julian	Calendar 1900	Mag.
5229.6	July 28 to	
5368.5	Dec. 14	12]
	9 dates	
5337.5	Nov. 13	12.35_2

7120 *χ Cygni*.

Julian	Calendar 1900	Mag.
5190.6	June 19	6.4
5191.6	20	6.25_2
5193.6	22	6.49_2
5203.6	July 2	6.54_2
5211.6	10	6.45_2
5214.6	13	6.52_2
5230.6	29	7.32_2

7155 *RR Aquilae*.
(Cont. from 482, Comp. Stars 393.)

Julian	Calendar 1900	Mag.
5229.6	July 28	12]
5249.6	Aug. 17	12]
5260.6	28	11.3
5276.5	Sept. 13	9.30_2
5282.5	19	9.35_2

7155 *RR Aquilae*. Cont.

Julian	Calendar 1900	Mag.
5308.5	Oct. 15	9.06_2
5313.5	20	8.93_2
5330.5	Nov. 6	9.81_2
5331.5	7	9.46_2
5334.5	10	9.53_2

7162 *RS Aquilae*.
(Cont. from 482. Comp. Stars 464.)

Julian	Calendar 1900	Mag.
5249.6	Aug. 17 to	
5314.5	Oct. 21	12]
	4 dates	
5337.5	Nov. 13	12.2
5347.5	23	12.0
5368.5	Dec. 14	10 : :

7234 *R Capricorni*.
(Cont. from 482. Comp. Stars 464.)

Julian	Calendar 1900	Mag.
5205.6	July 4	10.6
5217.6	16	10.8

[7244] — *Aquilae*.
(Continued from 482.)

Julian	Calendar 1900	Mag.
5189.6	June 18	8.61_2
5190.6	19	8.71_2
5191.6	20	8.33_2
5193.6	22	8.25_4
5195.6	24	8.78_2
5214.6	July 13	8.49_2
5215.6	14	8.89_2

[7244] — *Aquilae*.–Cont.

Julian	Calendar 1900	Mag.
5217.6	16	8.10_4
5218.6	17	8.26_4
5220.6	19	9.46_2
5223.6	July 22	9.14_2
5225.6	24	8.54_2
5228.6	27	9.24_2
5268.5	Sept. 5	9.28_2
5270.6	7	9.80_2
5272.6	9	8.65_2
5273.5	10	8.67_2

Comparison-Stars, 1893–1900.

5566 *RU Librae*.

Star	DM.	Mag.	n
I	−15°4144	8.01	4
1*P*	−15°4136	8.76	7
1*Q*	−14°4227	8.79	7
U	−14°4234	10.11	2
X	−14°4231	9.88	4
a	9*s*7*f* 1*Q*	10.81	10
c	9*n*1*p* 1*Q*	11.11	2
d	5*p* *a*	11.19	9
f	2*p* *d*	11.56	3

5675 *V Coronae*.

Star	DM.	Mag.	n
G	+40°2932	7.76	10
P	+40°2931	9.31	26
2*P*	+40°2934	9.11	22
S	+39°2923	9.50	13
Y	+39°2920	9.89	34
Z	+40°2930	10.13	10
c	5*s*15*p* *V*	10.61	14
e	8*n*6*f* *U*	11.19	10
f	*np* *c*	11.85	2

5798 *RU Herculis*.

Star	DM.	Mag.	n
D	+25°3039	8.03	5
G	+26°2790	8.40	3
L	+25°3031	8.81	6
M	+25°3036	9.22	2
N	+25°3042	9.09	11
R	+25°3044	9.35	4
S	+24°2984	8.79	3
T	+25°3046	9.41	6
c	15*p* *T*	10.90	3

6225 *RS Herculis*.

Star	DM.	Mag.	n
G	+23°3091	7.49	3
L	+23°3092	8.08	5
1*U*	+22°3126	9.00	6
2*U*	+23°3094	9.23	21
X	+22°3128	9.82	19
W	+23°3095	9.42	7
2*Z*	+23°3096	9.69	4
3*Z*	+23°3097	9.77	7
a	1*n*5*p* *V*	10.60	1

NOTE ON THE REDUCTION OF THE *EROS* OBSERVATIONS,

BY GEORGE C. COMSTOCK.

In the reduction of micrometric observations, such as those of *Eros*, it is very convenient to assume that the mean of a number of micrometer settings represents the relative positions of planet and comparison-star at the mean of the observed times, and within rather narrow limits this common practice is unobjectionable. It is not, however, immediately apparent just how narrow those limits must be assumed, and the purpose of the present paper is to determine for the special case presented by micrometric observations of the planet *Eros*, made in 1900–1901, in accordance with the recommendations of the Paris Conference, the maximum interval during which the relation between the measured quantities and the time may be assumed linear, and within which, therefore, the mean of a set of observations may be reduced as if it were a single observation.

For this purpose it is necessary to consider separately three independent sources of changes in the observed coordinates, viz: (*a*), the geocentric motion of the planet as represented by an ephemeris. (*b*), the change of parallax due to the observer's share in the diurnal motion of the earth. (*c*), the varying effect of differential refraction. For each of these cases we shall use a system of rectangular coordinates whose fundamental plane is tangent to the celestial sphere at a point, α_0, δ_0, near the planet, *e.g.*, the center of the field of view of the telescope. This point of tangency will be taken as the origin of coordinates, the positive axis of x being directed toward the north pole, and the positive axis of y in the direction of increasing right-ascensions.

Geocentric Motion. If the right-ascension and declination of the planet be represented by α and δ, these coordinates will be connected with the x and y above defined by the relations

$$\left.\begin{aligned} \tan(\delta_0+x) &= \tan\delta \sec(\alpha-\alpha_0) \\ \sin y &= \cos\delta \sin(\alpha-\alpha_0) \end{aligned}\right\} \qquad (A$$

In these equations x and y are functions of the time, t, through α and δ; and for either coordinate, *e.g.*, x, we may write

$$x = x_0 + \frac{dx}{dt}(t-t_0) + \tfrac{1}{2}\cdot\frac{d^2x}{dt^2}(t-t_0)^2 + \quad \text{etc.}$$

and obtain as the condition for the required limit, $t-t_0$, within which the variation of x may be treated as linear.

$$\frac{1}{2}\frac{d^2 \cdot}{dt^2}(t-t_0)^2 = \epsilon$$

where ϵ represents the maximum negligible error in the reduction.

Differentiating equations A twice with respect to t, and dropping, as insignificant, all terms containing either x or y as a factor, we obtain

$$\text{B)}\quad \begin{cases} \dfrac{d^2x}{dt^2} = \dfrac{d^2\delta}{dt^2} + \sin\delta\cos\delta\left(\dfrac{d\alpha}{dt}\right)^2 \\[2ex] \dfrac{d^2y}{dt^2} = \cos\delta\,.\,\dfrac{d^2\alpha}{dt^2} - 2\sin\delta\,.\,\dfrac{d\alpha}{dt}\,.\,\dfrac{d\delta}{dt}\end{cases}$$

If we adopt the sidereal minute as the unit of time, and represent by $\alpha', \alpha'', \delta', \delta''$, the first and second differences of the tabular α and δ furnished by a daily ephemeris, we shall have, to a sufficient degree of approximation,

$$\frac{d\alpha}{dt} = \frac{\alpha'}{1441} \qquad \frac{d^2\alpha}{dt^2} = \frac{\alpha''}{(1441)^2}$$

$$\frac{d\delta}{dt} = \frac{\delta'}{1441} \qquad \frac{d^2\delta}{dt^2} = \frac{\delta''}{(1441)^2}$$

where the several differences, α', α'', etc., are supposed to be expressed in seconds of arc. Introducing these values into equations B together with the factor arc $1''$ required for homogeneity we obtain

$$\text{C)}\quad \begin{cases} (1441)^2\dfrac{d^2x}{dt^2} = \delta'' + \dfrac{\sin\delta\cos\delta(\alpha')^2}{206265} \\[2ex] (1441)^2\dfrac{d^2y}{dt^2} = \cos\delta\,.\,\alpha'' - \dfrac{2\sin\delta\,.\,\alpha'\,.\,\delta'}{206265}\end{cases}$$

from which numerical values of the required differential coefficients may be readily obtained.

If we assume as the limit of permissible error in the reductions $\epsilon = 0''.01$ we shall find, in minutes of time,

$$\text{D)}\quad \begin{cases} (t-t_0)_x = [9.151] \div \sqrt{\dfrac{d^2x}{dt^2}} \\[2ex] (t-t_0)_y = [9.151] \div \sqrt{\dfrac{d^2y}{dt^2}}\end{cases}$$

where $t-t_0$ represents *half* of the interval within which observations may be combined. By means of these equations I have computed the following typical values of this limit:

1900, 1901	Oct. 1	Nov. 30	Jan. 29
$(t-t_0)_x$	84^{m}	31^{m}	40^{m}
$(t-t_0)_y$	37	25	31

We may generalize these results in the statement that if a given set of observations does not extend over a period greater than one hour the individual settings may be combined as if the planet's motion were strictly uniform, and the resulting error will never much exceed $0''.01$, and will seldom reach this limit.

Parallax. By applying to the expressions for the effect of parallax upon the x and y-coordinates of the planet, an analysis essentially similar to the preceding, I obtain the following expressions for the limiting $t-t_0$ at which the effect of second order terms in the change of parallax becomes equal to $0''.01$:

$$\text{(E)}\quad \begin{cases} (t-t_0)_x = \dfrac{C}{\sqrt{\rho\pi\cos\varphi'\sin\delta\cos t}} \\[2ex] (t-t_0)_y = \dfrac{C}{\sqrt{\rho\pi\cos\varphi'\sin t}}\end{cases}$$

where the notation is that of CHAUVENET, Vol. I, p. 123 and $\log C = 1.512$ when $t-t_0$ is to be expressed in minutes of time.

It is evident from equations E that the limits represented by these equations will vary widely at different hour-angles, but a few trials with assumed values of π, δ and t, shows that in general these limits are fairly represented by the statement that the double interval $2(t-t_0)$ can not much exceed 20^{m} without there being introduced into one of the coordinates an error greater than $0''.01$.

Differential Refraction. The effect of refraction upon differences of rectangular coordinates is represented by equations of the form

$$\Delta x = \mu x + \lambda y \quad , \quad \Delta y = \nu y + \lambda x$$

and for the required second derivatives of these quantities with respect to the time we have

$$\frac{d^2}{dt^2}\Delta x = \frac{d^2\mu}{dt^2}\,.\,x + \frac{d^2\lambda}{dt^2}\,.\,y$$

$$\frac{d^2}{dt^2}\Delta y = \frac{d^2\nu}{dt^2}\,.\,y + \frac{d^2\lambda}{dt^2}\,.\,x$$

The analytical expressions for these differential coefficients are too cumbrous for convenient use, and I have, therefore, computed for an assumed latitude of 43°, and an assumed declination of 50°, an ephemeris of μ, ν, λ, at intervals of 30^{m} of hour-angle, and from the tabular second differences of this ephemeris I have derived numerical values, μ'', ν'', λ'', which in the given case suffice to determine the numerical values of

$$\frac{d^2}{dt^2}\Delta x \quad \text{and} \quad \frac{d^2}{dt^2}\Delta y$$

Strictly, a similar procedure would be required in every case which may arise; but, since only approximate values of these coefficients are needed, it will usually be sufficient, in the case of observations made at northern observatories during the latter part of the year 1900, to interpolate values of μ'', ν'', λ'', from the following table, with the actual zenith-distance of the planet as argument, instead of employing as argument the hour-angle with which they were actually compared for the special one defined above. The numerical factor 10^{8} corresponds to the adoption of the second of arc and minute of time as units in terms of which the several quantities are expressed.

$$10^8 \frac{d^2x}{dt^2} = \mu'' x + \lambda'' y \quad . \quad 10^8 \frac{d^2y}{dt^2} = \nu'' y + \lambda'' x$$

t	z	μ''	ν''	λ''
0^h	7°	+0	+1	±1
4	41	2	2	2
5	50	3	2	2
6	58	6	3	5
7	66	19	3	10
8	73	+58	+4	±24

λ'' has the same sign as $\sin t$

As an example of the application of the table let us assume that at a zenith-distance of 60°, and east of the meridian, the planet was compared with a star which furnished as the differences of rectangular coordinates

$$x = +300'' \quad . \quad y = -300''$$

With the argument $z = 60^\circ$ we find from the table

$$10^8 \frac{d^2x}{dt^2} = (+9)(+300) + (-6)(-300) = +1500$$

$$10^8 \frac{d^2y}{dt^2} = (+3)(-300) + (-6)(+300) = -2700$$

The limit $t - t_0$ within which second-order terms do not exceed 0″.01 is then given by the equations

$$(t-t_0)_x = 10^4 \sqrt{\frac{0.02}{1500}} = 21^m$$

$$(t-t_0)_y = 10^4 \sqrt{\frac{0.02}{2700}} = 27^m$$

It is to be observed that here, as in all preceding cases, $(t-t_0)$ represent a time interval measured from the middle of a set of observations, and that the total interval, over which the observations may extend without neglecting in the case of any single result a second-order term greater than 0″.01, is $2(t-t_0)$.

Washburn Observatory, 1900 Dec. 24.

ON A NEW COMPONENT OF THE POLAR MOTION.

By S. C. CHANDLER.

The object of this note is to state briefly the principal result of an investigation that has been some time in hand, and is now complete. Space does not permit the present communication of the details. These will be printed later in sufficient fullness to enable astronomers to form an opinion on the validity of the main conclusion, which is as follows.

In addition to the 428-day and the annual components of the polar motion, already made known, there is a third component with a period of 436 days, and a radius of 0″.09, thus considerably smaller than that of the other terms. The evidence is very extensive, and extremely clear, and I think the reality of this third motion is beyond reasonable doubt. For the present it is assumed to be circular, existing observations being inadequate to determine its form.

The reason why this term has been hitherto overlooked is manifest. The period is so near that of the 428-day term that the two have been confounded in the methods of discussion used. In fact, what we have been studying heretofore has been the resultant. I will here briefly describe the relations that subsist between these twin motions, treating the smaller as parasitic on the larger.

The periods are $P' = 428.5$ and $P'' = 436.0$, corresponding to the daily angular velocities $\theta' = 0^\circ.8401$ and $\theta'' = 0^\circ.8257$; and the radii are $r' = 0''.140$ and $r'' = 0''.090$. Taking $T = 2405331 + 428.5\,\mathrm{E}$, and $\tau = (P'' - P')\,\mathrm{E} = 7^d.5\,\mathrm{E}$, we have the coordinates of the pole for any date t by

$$\left.\begin{aligned} x_1 &= r' \sin(t-T)\,\theta' + r'' \sin(t-T-\tau)\,\theta'' \\ y_1 &= r' \cos(t-T)\,\theta' + r'' \cos(t-T-\tau)\,\theta'' \end{aligned}\right\} \quad (1)$$

the positive y-axis being directed to Greenwich and the positive x-axis to 90° east. The resultant is a compound harmonic motion in the nature of an alternately expanding and contracting spiral-like curve, that for any given instant may be treated as a circle with the radius

$$r_1 = r''(k + \cos\sigma)\sec U\theta' \qquad (2)$$

and the epoch (minimum latitude at Greenwich)

$$T_1 = T + U \qquad (3)$$

the anomaly U being determined by the expression

$$\tan U\theta' = \frac{\sin\sigma}{k + \cos\sigma} \qquad (4)$$

where $\sigma = \tau\theta'' + (t-T)(\theta' - \theta'')$. $k = \dfrac{r'}{r''}$ (5)

Then we have

$$\left.\begin{aligned} x_1 &= r_1 \sin(t-T_1)\,\theta' \\ y_1 &= r_1 \cos(t-T_1)\,\theta' \end{aligned}\right\} \quad (6)$$

as the coordinates of the pole in virtue of this compound motion that we have hitherto treated as a simple one under the name of the 428-day term.

The length of the period, obtained through the differentials of (3), (4) and (5), is

$$P = P' + (P'' - P')\,\frac{k\cos\sigma + 1}{(k + \cos\sigma)^2}\cos^2 U\theta' \qquad (7)$$

or

$$P = 428^d.5 + 7^d.5\,\frac{1.555\cos\sigma + 1}{(1.555 + \cos\sigma)^2}\cos^2 U\theta'$$

the limits of which are $P = 428.5 + \dfrac{7^d.5}{1 \pm 1.555}$ indicating a variation of the period between 431.4 and 415.0 days.

the mean length being 428.5 days. These fluctuations are embraced in a cycle of about 57 periods, or 67 years. But the changes of period are of a remarkable character. It remains during five-sixths of the cycle between its mean value and the upper limit, or between 428.5 and 431.4 days; then suddenly shortens to minimum, 415 days, and immediately rapidly lengthens. Similarly the variations of the radius of motion are singularly asymmetrical. It is at present about 0″.07, and approaching its minimum dimension of 0″.05. The decrease from 0″.17 to 0″.11 between 1890 and 1897, pointed out in *A.J.* 446, is an illustration of the action of the above law. We shall soon have a similar test of the law in its operation on the period, which between 1850 and 1890 persisted in the neighborhood of 430 days, and is now 428 days, but ought to shorten to the minimum value, 415 days, within the next few years. Of course an accurate prediction cannot be made as to when this interesting phase will become perceptible, because the length of the harmonic cycle, which depends on the difference of the two component periods, is imperfectly defined by existing observations. For this reason the above description should be regarded merely as outlining the general nature of the fluctuations rather than as specifying sharply their numerical limits.

OBSERVATIONS OF COMET *b* 1900 (*BORELLY BROOKS*),

BY R. G. AITKEN.

1900	Mt. Hamilton M.T.	*	No. Comp.	☌ − * Δα	☌ − * Δδ	☌'s apparent α	☌'s apparent δ	log pΔ for α	log pΔ for δ
	h m s			m s	′ ″	h m s	° ′ ″		
Sept. 3	9 23 36	1	18	−3 38.79	+6 35.9	13 5 11.82	+80 39 47.5	0.441	0.470
8	12 33 5	2	16	−2 21.43	+4 15.2	13 37 13.10	+77 25 5.6	0.007	0.859
14	10 13 42	3	16	+1 20.79	−6 5.9	13 57 15.57	+74 20 22.4	0.180	0.660
16	7 11 6	4	10 , 8	− 26.84	+4 28.5	14 2 0.90	+73 29 48.0	0.199	*n*9.672
20	7 28 6	5	10 , 6	− 57.64	− 59.7	14 10 19.32	+71 52 47.8	0.171	9.544
27	7 1 17	7	10 , 6	− 36.32	−5 27.5	14 23 32.27	+69 36 55.4	0.117	9.415
29	7 16 37	8	10 , 6	+ 46.74	−1 23.3	14 26 50.80	+69 4 38.1	0.114	9.934
Oct. 10	7 5 0	10	8 , 6	−1 31.77	+1 14.5	14 43 34.19	+66 54 4.3	0.077	0.167
12	7 10 8	12	18	− 3.21	−2 11.5	14 46 30.39	+66 37 42.8	0.073	0.253
21	7 13 54	14	8 , 6	+ 48.28	+ 30.3	14 59 35.07	+65 50 0.8	0.059	0.415

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	13 8 57.37	−6.76	+80 33 17.5	−5.9	Hamburg Zone (+80° +81°) No. 173
2	13 39 42.20	−4.67	+77 20 51.0	−3.6	Kasan A.G. Catal. 2418
3	13 55 58.41	−3.63	+74 26 31.4	−3.1	Dorpat A.G. Zones, +74°562
4	14 2 31.06	3.32	+73 25 22.3	−2.8	Dorpat A.G. Zones, +73°616
5	14 11 19.92	−2.96	+71 53 50.3	−2.8	Micrometer comparison with (6)
6	14 15 51.05	−3.00	+71 58 19.1	−2.3	Dorpat A.G. Zones, +72°636
7	14 24 11.17	2.58	+69 42 25.8	−2.9	Micrometer comparison with (9)
8	14 26 6.55	−2.49	+69 6 1.7	−3.3	Christiania A.G. Catal. 2151
9	14 26 21.52	2.58	+69 41 56.3	−2.7	Christiania A.G. Catal. 2152
10	14 45 8.26	2.30	+66 52 53.8	−4.0	Micrometer comparison with (11)
11	14 46 29.58	2.34	+67 0 57.6	−3.8	Christiania A.G. Catal. 2205
12	14 46 35.92	−2.32	+66 40 1.8	−4.5	Christiania A.G. Catal. 2206
13	14 58 23.78	−2.36	+65 52 18.1	−5.1	Christiania A.G. Catal. 2235
14	14 58 19.45	2.36	+65 49 35.6	−5.1	Micrometer comparison with (13)

The last two observations were made with the 36-inch equatorial, the others with the 12-inch. On the nights of Sept. 3, 8, 14 and Oct. 12, the position-angle and distance of the comet with reference to the comparison-star were measured. On the other nights Δα was measured directly with the micrometer.

Work on *Eros*, and later, unfavorable weather prevented me from looking for the comet again until Dec. 22. On that date I found it very close to its predicted place; but before a measure could be made it was blotted out by rising fog. It appeared to be about as bright as a star of the 15th magnitude. There has been no opportunity for subsequent examination.

Lick Observatory, University of California, 1901 *Jan.* 7.

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SOUTHERN VARIABLE STARS,

By ALEXANDER W. ROBERTS.

As the published elements of many of the Southern Variables are out of accord with more recent observations, it has been thought desirable to give without delay such elements as have been determined with some degree of certainty from observations taken at Lovedale during the years 1891-99.

A full discussion of these observations is delayed until they have been supplemented by a series of observations made with the new equatorial, now being constructed by COOKE. It is confidently expected that the new instrument will yield measures of greater precision than can be secured with an ordinary telescope; thus the light-curves, especially of the short-period variables, so determined, will form a necessary complement to the results based on the older measures.

No refinements of observation, however, will materially alter the general characteristics of variation—epoch, period, type of variation and similar elements, which we obtain from a consideration of observations extending over many years. And as an exhibition of the general features of variation, rather than the minuter details, is the purpose of this investigation, no good reason exists for further delaying its publication.

The work at Lovedale is strictly confined to the observation of variable stars south of $-30°$, and as far as possible every known variable in this part of the sky, visible at any portion of its period in a $3\frac{1}{4}$-inch telescope, is under regular observation.

The present list of stars therefore includes nearly all the Southern Variables in CHANDLER's Third Catalogue, together with some of the more important variables recently discovered at the Cape, Cordova and Arequipa.

The identification of each star has been made a matter of some care, and the positions for 1900 of the catalogue stars have been determined from all available data. As the places of many of the uncatalogued stars are not known with the accuracy necessary for a ready identification of the variable when near a minimum, the places of the following stars have been determined with the ring micrometer on the $3\frac{1}{4}$-inch telescope:—

(1023) *R Horologii*	(1662) *R Caeli*
(1635) *R Reticuli*	(3663) *Z Carinae*
(4056) *RS Centauri*	(7483) *U Pavonis*
(4188) *U Centauri*	(7813) *R Gruis*
(5320) *S Lupi*	(8040) *S Gruis*
(6170) *RW Scorpii*	(8593) *R Tucanae*
(6331) *RU Scorpii*	

The position of the following catalogue stars was also similarly determined, as there seemed some doubt as to their identity or position:—

(21) *V Sculptoris*	(7077) *T Pavonis*
(6062) *RR Scorpii*	(7266) *RT Sagittarii*

No great precision can be claimed for the positions secured (although their accuracy can be certified for within the limits required in the catalogue), still they are sufficiently accurate to indicate that in the case of one or two stars, notably that of (1023) *R Horologii*, the positions given in CHANDLER's Third Catalogue require material correction. The accurate determination of the position of a variable is of some moment when the star comes first under examination near its minimum limit. A faulty position under these conditions may mean the loss of many nights' observations.

As already indicated, the elements of most of the stars contained in this catalogue are determined solely from Lovedale observations. When other measures were available I did not hesitate to make use of them either for correcting or supplementing my own results.

I am accordingly indebted to Mr. INNES, Professor WEST, Mr. WILLIAMS and Colonel MARKWICK, not only for their published observations, but for valuable manuscript notes which they have been good enough to place at my disposal.

The work that these observers have done both along the narrower lines of discovery, and the higher plane of systematic observation, should be encouragement to many other southern observers to take up this branch of astronomical research.

I have already in previous papers on Southern Variable Stars spoken of my obligation to Professor PICKERING. His efficient, courteous help, has been an aid to me in no small degree.

Special reference in this same connection must be made to five stars on the list:—

(2935) *Z Puppis*	(5618) *T Normae*
(3669) *W Velorum*	(6370) *RZ Scorpii*
(4216) *Z Hydrae*	

The elements given are found by relating my own observations to those of INNES. Without these additional data the Lovedale observations alone would not be sufficient to yield reliable elements of any one of the five stars.

It is not expedient, as a rule, to depart from a well established and generally accepted mode of astronomical reckoning, but the change introduced in this catalogue of considering the first maximum passage of 1900 (or in the case of an *Algol*-variable, the first minimum passage) as the epoch of the variable has, I hope, something to commend it. That it leads to uniformity and definiteness will be admitted. Perhaps southern observers in dealing either with their own observations, or with any observations, will relate them to this date.

The limits of variation have been thus determined. For magnitudes brighter than $6^M.0$ the U.A. scale has been adopted. Fainter magnitudes have been determined by the selection of the following empirical reference points: (1), $6^M.8$, the faintest magnitude visible to the naked eye on a clear night; (2), $9^M.2$, the lowest magnitude seen in my one-inch telescope; (3), $11^M.4$, the faintest magnitude seen in my $3\frac{1}{4}$-inch telescope.

The minima of many of the stars are fainter than $11^M.4$. When there was strong evidence, from a consideration of the observations above $11^M.0$, that the variation of the star is of a regular nature, the light-curve was computed for the full range. Minima so determined are marked by a note of interrogation.

For minima between $11^M.4$ and $11^M.8$ this indirect mode of determination is not markedly inferior to one directly obtained. For minima, however, lower than $12^M.0$ the area of uncertainty increases so rapidly that little reliance can be placed in the inferred results. As the relation of increasing to decreasing rate of variation is bound to be of extreme importance in the working out of a theory of long-period variation, it is to be hoped that those who possess large instruments in the Southern Hemisphere will make the necessary observations at minima of the fainter stars. It would be matter of regret if our knowledge of some of the most remarkable variables in the southern sky were confined to that portion of their variation immediately preceding or following their maxima.

The last two columns of the catalogue indicate the extreme dates during which the variable has been under examination at Lovedale, and the number of observations made during that time. It may be mentioned that no observations were made in 1897. Thus, 1891–1899 will be considered as meaning 1891–1896 and 1898–99.

The total number of observations dealt with is considerably over twenty-three thousand.

The notes at the end of the catalogue deal with the general features of variation rather than with the history of any of the stars, this being the main purpose of the paper.

Frequently in the notes there is reference to the typical curve of short-period variation. Stars are described as according with, or departing from this normal type. As the variation of all the southern short-period stars differs but slightly from this mean curve, it is pertinent to the present investigation to consider what are the chief typical features of short-period variation, and indirectly, by contrast, to indicate other types of variation.

The most outstanding feature of short-period variation is the rapidity of the increasing rate of variation, as compared with the decreasing rate.

The mean value of this important ratio as obtained from a preliminary examination of the light-curves of fourteen short-period variables, is 3 : 1. The majority of the stars dealt with range very near this value; one or two have a much higher ratio. Three stars have a slower relative ascending rate, but no star has a ratio of unity or under it.

That the ascending rate of variation is much more rapid than the descending rate may be considered therefore as a fundamental law of short-period variation.

It is frequently stated that the reverse of this law holds good in the case of *Algol* stars. There are in the southern sky six stars of undoubted *Algol*-type, and about none of these can it be said with certainty, that its ascending rate of variation is distinctly slower than the descending. In every instance the ratio of increasing to decreasing rate of variation is approximately one of equality.

Then, again, with regard to long-period variables, though the law of rapid increasing variation holds good in many instances, it is by no means general. There are variable stars of this class, as the notes to the catalogue indicate, whose ascending rate of variation is slower than the descending rate.

The number of exceptions is few, but still they are sufficient to indicate that the law has not the same significance with regard to long-period variation as it has with regard to short-period variation. Further, the mean ratio over the whole class of long-period variables is much lower than is the case with the short-period variables.

A second feature of short-period variation is its regularity.

If a mean light-curve for any short-period variable be computed, either graphically or analytically, and applied to the observed magnitudes, then the (O–C) discordances will be found to be only slightly greater than the errors of observation.

The natural interpretation of this is that whatever causes underlie short-period variation, they operate with extreme regularity.

In the case of long-period variation such uniformity is the exception. Not only is the period in which the star completes its variation usually irregular, but frequently successive light-curves differ so radically as almost to suggest different types of variation. There are, of course, long-period variables whose amplitude, period and type of variation is quite regular, as regular as the variation of any

short-period variable, but such regularity is, as already stated, exceptional. A third characteristic of short-period variation is its narrow limits. If we take the twenty short-period variables whose elements are given in this catalogue, we will find that the average range of variation of the group is only one magnitude, and that in no single case is there a range of variation amounting to two magnitudes. The lowest value is that of (3777) *Y Carinae*, $0^{M}.5$; and the highest the recently discovered variable (5608) *U Normae*, $1^{M}.6$.

If now we consider long-period variation we find that the average range of variation for thirty stars, whose limits are known with some degree of accuracy, is $3^{M}.7$, a value nearly four times greater than that yielded by the short-period variables.

Had it been possible to determine the lower limits of many of the stars in this catalogue, this value would be considerably increased. Let us regard $4^{M}.0$ as the mean range in magnitudes of long-period variables.

A range of one magnitude for short-period variation and four magnitudes for long-period variation does not, at first sight, represent a fundamental difference in the character of the two types of variation. If, however, we indicate the respective ranges in light-grades we will at once appreciate the reality of the difference.

A range of one magnitude, the average range of the southern short-period variables, means that at a maximum an average short-period variable is two and a half times brighter than it is at a minimum. A range of four magnitudes, the assumed average range of the southern long-period variables (an average value it may be premised under the actual value), indicates that the typical long-period variable is at a maximum forty times brighter than it is at a minimum.

The great increase in brightness of some of the long-period stars is indeed remarkable. For instance the star (5096) *R Centauri* is at least three hundred times brighter at a maximum than it is at a minimum. It is difficult to conceive how such a stupendous increase in light and heat should not end in the complete disintegration of the star.

It has sometimes been stated that all long-period variables are red in color, and short-period variables white. On this matter I am not able to speak with certainty or with confidence, as my own appreciation of color is poor, arising either from actual inability to readily distinguish different shades of color, or from want of experience.

It is evident, of course, from a consideration of the material supplied by many observers, that a majority of long-period variables are red, and that a majority of short-period variables are not red; but that all short-period stars are white I am inclined to doubt.

Several of the southern short-period variables have an orange tinge, and two at least (2935) *Z Puppis*, and (5823) *S Normae*, have a more decided trace of red.

But if it be considered that only a small percentage of stars are red or reddish, the unsoundness of the conclusion that because more are apparently not red stars in the list of short-period variables, therefore all variable stars of this class are white, will be at once apparent. The simple fact that one star is slightly red disposes of the deduction as untenable, at least so far as southern variables are concerned.

An attempt was made in 1892 and 1893 to bring the question of color variation into the scheme of work at Lovedale, but for reasons already indicated, the investigation was not continued.

That stars do change in color I think certain, and it may be that much of the photographic evidence of stellar variation which has not been confirmed by eye observations is due probably to changes of color rather than of intensity.

The foregoing conclusions have arisen from a consideration of some of the general questions which the summarizing of the work of the past nine years has brought prominently before my own mind. It will be noticed that they deal simply with the facts of variation.

The consideration of theories advanced to explain the facts lies beyond the province of this paper.

I do not purpose either dealing, at present, with the place or mode of observing as carried on at Lovedale. A more fitting place for this will be the detailed consideration of all the observations.

I may be permitted, however, to refer to one matter, as there is some possibility of misconception arising from reticence regarding it. Col. MARKWICK, in dealing recently with the distribution of variable stars, remarked on the large number of short-period variables in *Carina* and the neighborhood of this constellation. He suggested as an explanation that special attention may have been directed to this part of the sky. As far as this observatory is concerned, his explanation is correct. A search for short-period variables was begun in *Crux* in 1891, the intention being to work round the whole sky. The number of new variables discovered at Harvard and the Cape made it impossible to continue this search, and at the same time carry on the work of observing all southern variable stars. The former was accordingly given up, as being the less important, after a few constellations had been carefully examined.

The growth of this branch of astronomical research since 1891 can be well exemplified by two sets of figures, one representing the number of known variables south of $-30°$ at January 1891, and the other the number as at January 1900.

	Jan. 1891	Jan. 1900
Algol-variables	0	6
Short-period variables	4	23
Long-period variables	10	74
Irregular variables	1	4
Total	15	107

I cannot conclude the prefatory portion of my paper without making some acknowledgement of my great indebtedness to Dr. DAVID GILL. His sympathetic advice and help was always mine when most needed.

Ch. No.	Catalogue	Star	1900 R.A.	1900 Decl.	Annual Variation R.A.	Annual Variation Decl.	Magnitude Max.	Magnitude Min.
			h m s	° ′	s	′		
21	C.DM. −39°16	V Sculptoris	0 3 34	−39 17.1	+3.05	+0.33	8.8	<12.0 ?
62	A.G.C. 457	S Sculptoris	0 10 19	32 36.1	3.04	.33	5.8–6.5	11.8 ?
110	C.P.D. −62°28	S Tucanae	0 18 23	62 13.7	2.87	.33	9.0	11.8 ?
146	A.G.C. 404	T Sculptoris	0 24 17	38 27.7	2.96	.33	8.6	11.4
401	C.DM. −30°375	U Sculptoris	1 6 50	30 38.8	2.84	.32	9.0	<13.0 ?
494	A.G.C. 1377	R Sculptoris	1 22 22	33 3.7	2.77	.31	6.2–6.5	8.5– 8.8
1023	. . . .	R Horologii	2 50 33	50 17.9	1.98	.25	5.6–6.6	10.8–11.6
1066	Z.C. $3^{h}1547$	T Horologii	2 57 40	51 2.2	1.92	.24	8.5	11.6 ?
1635	. . . .	R Reticuli	4 32 30	63 14.2	0.61	.12	7.0–8.5	12.0 ?
1654	L. 1567	R Doradus	4 35 36	62 16.4	0.70	.12	4.8–5.5	6.8
1662	. . . .	R Caeli	4 37 2	38 25.8	2.08	.12	7.2–8.0	<12.9 ?
1701	A.G.C. 5428	R Pictoris	4 43 29	49 25.6	1.60	.11	7.2	8.6
1850	Z.C. $5^{h}283$	S Pictoris	5 8 18	48 37.7	1.59	.07	8.2	<12.0 ?
1894	A.G.C. 6135	T Columbae	5 15 38	33 48.7	2.19	.06	7.0	11.5 ?
2059	C.DM. −31°2732	S Columbae	5 43 10	31 43.7	+ 2.25	.02	9.0	<12.0 ?
2141	. . . .	R Octantis	5 56 18	86 26.0	−18.37	+0.00	6.8	<12.0 ?
2583	L. 2691	L_2 Puppis	7 10 29	44 28.7	+ 1.82	−0.10	3.4–4.6	5.8– 6.2
2776	Z.C. $7^{h}3056$	W Puppis	7 42 39	41 57.1	1.99	.14	8.0	11.2
2781	C.P.D. −41°1681	RR Puppis	7 43 31	41 7.6	2.02	.14	10.0	11.0
2852	L. 3105	V Puppis	7 55 22	48 58.4	1.73	.16	4.1	4.65–4.85
2933	C.DM. −34°4482	Y Puppis	8 8 51	34 50.3	2.29	.18	8.8	9.2
2935	A.G.C. 10946	Z Puppis	8 9 14	34 16.6	2.30	.18	7.0	8.5
3040	L. 3393	V Carinae	8 26 41	59 47.3	1.23	.20	7.4	8.1
3055	Z.C. $8^{h}2388$	X Carinae	8 29 7	58 53.2	1.31	.20	7.9	8.60–8.68
3087	A.G.C. 11668	T Velorum	8 34 26	47 0.7	1.95	.21	7.65	8.5
3355	A.G.C. 12793	V Velorum	9 19 15	55 32.0	1.82	.26	7.5	8.2
3416	A.G.C. 13052	S Velorum	9 29 27	44 45.9	2.26	.26	7.8	9.3
3417	A.G.C. 13053	U Velorum	9 29 28	45 4.3	2.25	.26	8.2	8.6
3418	L. 3932	R Carinae	9 29 44	62 20.8	1.52	.26	4.5–5.5	9.2–10.0
3495	L. 4033	*l* Carinae	9 42 30	62 2.8	1.65	.28	3.6	5.0
3569	A.G.C. 13624	RR Carinae	9 54 51	58 23.0	1.94	.29	7.8	8.6
3637	L. 4189	S Carinae	10 6 11	61 3.6	1.92	.29	5.8–6.6	9.0
3663	. . . .	Z Carinae	10 10 27	58 21.5	2.07	.30	10.0	<12.0 ?
3669	C.P.D. −53°3515	W Velorum	10 11 31	53 58.9	2.23	.30	8.8	<11.4
3777	Z.C. $10^{h}2067$	Y Carinae	10 29 25	57 59.0	2.25	.31	8.1	8.6
3847	A.G.C. 14720	η Carinae	10 41 11	59 9.5	2.32	.31	. .	. .
3922	L. 4542	U Carinae	10 53 44	59 11.8	2.43	.32	6.8	8.0
4056	. . . .	RS Centauri	11 16 6	61 19.5	2.61	.33	8.4	12.0 ?
4216	C.DM. −32°8314	Z Hydrae	11 41 37	32 42.8	3.01	.33	9.2	10.0
4225	C.DM. −41°6787	X Centauri	11 44 12	41 12.0	2.99	.33	7.6	11.8 ?
4260	Z.C. $11^{h}3351$	W Centauri	11 50 2	58 41.8	2.98	.33	8.2	12.0 ?
4364	L. 5060	S Muscae	12 7 24	69 35.7	3.19	.33	6.4	7.3
4415	L. 5108	T Crucis	12 15 51	61 43.6	3.24	.33	6.85	7.6
4429	A.G.C. 16882	R Crucis	12 18 8	61 1.5	3.26	.33	6.8	7.9
4488	. . . .	U Centauri	12 27 59	54 6.5	3.30	.33	8.1–8.6	11.8 ?
4536	L. 5236	R Muscae	12 35 59	68 51.5	3.61	.33	6.5	7.6
4611	L. 5311	S Crucis	12 48 27	57 53.3	3.52	.33	6.5	7.6
4896	L. 5615	T Centauri	13 36 2	33 5.5	3.43	.31	5.3–6.0	9.0
4935	Z.C. $13^{h}2483$	RT Centauri	13 42 30	36 21.7	3.50	.30	8.8	11.3
5096	A.G.C. 19295	R Centauri	14 9 22	59 26.9	4.28	.28	5.6–6.2	9.0–11.8?
5099	L. 5861	RR Centauri	14 9 55	−57 23.3	+4.20	−0.28	7.4	7.85

Ch. No.	$M-m$	Elements of Maximum, Greenwich Mean Time	Dates	Obsns.
21	–	J 241 5188 + 295.5 E	1896–99	64
62	161 ?	241 5350 + 366.5 E	1894–99	124
110	75 ?	241 5193 + 238.5 E	1896–99	71
146	85	241 5169 + 201.5 E (See notes)	1896–99	89
401	–	241 5065 + 328.0 E	1898–99	46
494	171	241 5341 + 376.4 E (Sec. max., see notes)	1894–99	251
1023	154	241 5229 + 405.0 E	1893–99	152
1066	100 ?	241 5193 + 218.2 E	1899	20
1635	–	241 5021 + 273.4 E	1891–99	155
1654	100	241 5135 + 315.0 E (Irregular, see notes)	1891–99	382
1662	–	241 5258 + 398.0 E	1893–99	119
1701	–	241 5080 + 160.0 E (Irregular, see notes)	1896–99	67
1850	–	241 5305 + 428.5 E	1896–99	56
1894	105	241 5097 + 225.0 E	1898–99	57
2059	–	241 5149 + 325.5 E	1898–99	47
2141	–	241 5170 + 330.0 E	1895–99	44
2583	59	241 5108 + 140.15 E (Sec. max., see notes)	1891–99	594
2776	54	241 5078 + 120.8 E	1896–99	122
2781	–	Min. 1900 Jan. 1^d 20^h 34^m + 6^d 10^h 19^m $25^s.0$ E *Algol*-type	1899	73
2852	–	Min. 1900 Jan. 1^d 5^h 5^m + 1^d 10^h 54^m $26^s.7$ E *Algol*-type	1894–99	479
2933	–	. . . . (Irregular, see notes)	1898–99	41
2935	11.8	241 5050.3 + 41.26 E	1899	30
3040	2.16	241 5026.78 + 6.6951 E	1892–99	710
3055	–	Min. 1900 Jan. 1^d 2^h 14^m + 12^h 59^m $29^s.9$ E *Algol*-type	1893–99	1084
3087	1.40	241 5022.78 + 1.6392 E	1892–99	469
3355	0.97	241 5021.64 + 4.3709 E	1892–99	575
3416	–	Min. 1900 Jan. 1^d 3^h 44^m + 5^d 22^h 24^m $21^s.1$ E *Algol*-type	1894–99	648
3417	30	241 5078 + 62 E Irregular	1894–99	109
3418	126	241 5180 + 309.7 E + cos. (10° E – 180°)	1891–99	557
3495	13	241 5044.5 + 35.523 E	1891–99	780
3569	120	241 5051 + 365.0 E (See notes)	1895–99	86
3637	78	241 5043 + 149.1 E	1891–99	653
3663	–	241 5276 + 391.0 E	1895–99	53
3669	60 ?	241 5057 + 185.8 E	1899	10
3777	1.07	241 5021.40 + 3.6401 E	1893–99	521
3847	–	. . . . (Irregular)	1894–99	150
3922	5.5	241 5034.0 + 38.7397 E	1891–99	729
4056	80 ?	241 5032 + 168.0 E	1899	41
4216	18	241 5062 + 52.5 E	1899	20
4225	121 ?	241 5234 + 313.9 E	1896–99	54
4260	90 ?	241 5202 + 204.3 E	1896–99	84
4364	3.45	241 5029.18 + 9.657 E	1891–99	736
4415	2.07	241 5028.32 + 6.7322 E	1893–99	158
4429	1.40	241 5027.39 + 5.82485 E	1891–99	739
4488	106 ?	241 5046 + 216.8 E	1895–99	96
4536	0.26	241 5021.23 + 0.882495 E (Sec. var., see notes)	1891–99	759
4611	1.49	241 5026.92 + 4.68989 E	1891–99	696
4896	46	241 5078 + 90.4 E (See notes)	1891–99	222
4935	120	241 5048 + 249.2 E	1899	10
5096	167 ?	241 5131 + 569.0 E (Sec. max.)	1891–99	112
5099	0.1514	Max. 1900 Jan. 1^d 4^h 57^m + 0^d 7^h 16^m $5^s.5$ E	1894–99	739

Ch. No	Catalogue	Star	1900 R.A.	1900 Decl.	Annual Variation R.A.	Annual Variation Decl.	Magnitude Max.	Magnitude Min.
			h m s	° ′	s	′		
5192	L. 5954	V Centauri	14 25 23	−56 26.6	+4.27	−0.27	6.4–6.6	7.8
5320	. . . .	S Lupi	14 46 44	46 12.2	4.00	.25	8.6–9.2	<12.0 ?
5396	. . . .	S Apodis	14 59 21	71 40.4	5.92	.24	10.0	<11.4
5465	L. 6264	R Triang. Austr.	15 10 49	66 7.7	5.31	.22	6.7	7.4
5608	Z.C. 15h2254	U Normae	15 34 37	54 59.3	4.61	.20	8.8	10.4
5618	S. 8527	T Normae	15 36 21	54 40.0	4.60	.20	7.0–7.5	<11.4
5682	A.G.C. 21504	R Lupi	15 46 59	35 59.9	3.88	.18	9.0–9.6	<12.0 ?
5713	L. 6578	S Triang. Austr.	15 52 12	63 29.5	5.35	.18	6.4	7.4
5751	A.G.C. 21762	U Triang. Austr.	15 58 25	62 38.3	5.30	.17	7.75	8.4
5823	L. 6713	S Normae	16 10 35	57 39.2	4.95	.15	6.6	7.4–7.55
5949	L. 6887	R Arae	16 31 26	56 47.6	4.96	.13	6.8	7.9
6050	A.G.C. 22855	RS Scorpii	16 48 22	44 56.3	4.34	.10	6.0–7.6	12.0 ?
6062	C.DM. −30 13626	RR Scorpii	16 50 16	30 25.2	3.82	.10	6.2–7.0	12.2 ?
6071	A.G.C. 22956	RV Scorpii	16 51 48	33 27.2	3.92	.10	6.9	8.0
6170	. . . .	RW Scorpii	17 8 19	33 19.0	3.93	.07	10.0	<12.0 ?
6275	. . . .	S Octantis	17 25 54	86 46.0	26.41	.04	8.2	11.8 ?
6331	. . . .	RU Scorpii	17 35 12	43 42.0	4.34	.04	8.2–8.6	<12.0
6370	C.P.D. −35°7270	RZ Scorpii	17 41 36	35 39.9	4.03	.03	9.0	<11.4
6386	L. 7459	RY Scorpii	17 44 16	33 40.5	3.96	.02	7.5	9.0
6429	C.P.D. −49°10361	S Arae	17 51 27	49 25.2	4.63	−0.01	9.6	10.8
6500	A.G.C. 24674	R Pavonis	18 3 17	63 38.1	5.77	+0.00	8.0	11.7 ?
6546	L. 7646	RS Sagittarii	18 10 59	34 8.5	3.98	.02	6.6	6.9–7.6
6608	C.DM. −33°13234	RV Sagittarii	18 21 21	33 22.9	3.95	.03	7.8–8.4	<12.0 ?
6686	Z.C. 18h1903	U Cor. Austr.	18 34 17	37 55.6	4.10	.05	8.4	11.0
6760	L. 7856	κ Pavonis	18 46 38	67 21.5	6.21	.07	3.8	5.2
6841	C.DM. −37°13027	R Cor. Austr.	18 55 10	37 5.5	4.05	.08	10.2	<11.0
6900	L. 8051	RY Sagittarii	19 10 1	33 41.8	3.92	.10	6.5	<11.0
7077	L. 8174	T Pavonis	19 39 30	72 1.1	6.81	.14	8.0	<12.0 ?
7121	A.G.C. 27193	S Pavonis	19 46 47	59 27.2	5.10	.15	7.2–7.6	9.0–9.6
7151	L. 8276	RU Sagittarii	19 51 50	42 6.9	4.14	.16	7.0	11.4
7245	. . . .	R Telescopii	20 7 12	47 18.0	4.30	.18	8.4	<11.4
7266	C.DM. −39 13722	RT Sagittarii	20 11 5	39 25.2	4.00	.18	7.0	<12.0 ?
7483	. . . .	U Pavonis	20 47 12	63 5.2	5.04	.22	8.6	<12.0 ?
7494	Z.C. 20h1539	S Indi	20 48 59	54 42.3	4.47	.22	9.0	<12.0 ?
7544	. . . .	T Octantis	20 47 24	82 30.0	10.33	.23	8.6–9.0	11.8
7685	C.DM. −30°18609	S Microscopii	21 20 48	30 17.0	3.57	.26	8.0–8.5	11.5
7813	. . . .	R Gruis	21 42 5	47 22.6	3.89	.28	8.0	<12.0 ?
7991	Z.C. 22h342	R Piscis Austr.	22 12 19	30 6.2	3.42	.30	8.5	<11.5
8039	C.DM. −38°15044	T Gruis	22 19 51	38 4.4	3.52	.30	7.8–8.6	11.2
8040	. . . .	S Gruis	22 19 56	48 56.4	3.72	.30	8.2	<12.0 ?
8588	Z.C. 23h1353	R Phoenicis	23 51 16	50 20.6	3.13	.33	7.4–8.0	12.0 ?
8593	. . . .	R Tucanae	23 52 12	65 56.4	3.17	.33	10.0	<11.4
8603	L 9672	S Phoenicis	23 53 54	−57 7.9	+3.13	+0.33	7.4	8.2

NOTES ON FOREGOING STARS.

21. Light-curve symmetrical on either side of maximum. Maxima not well defined. In 1898 the rise from $11^{M}.4$ to maximum was slightly slower than the fall to minimum. Slight confusion has arisen as to the identity of this star. It is C.DM. −39°.16. There are some neighboring faint stars which make identification difficult near minimum.

62. The rise to maximum is only slightly more rapid than the fall to minimum. Some maxima are sharply

Ch. No.	$M-m$	Elements of Maximum, Greenwich Mean Time	Dates	Obsns.
5192	1.47	241 5025.52 + 5.49394 E	1894–99	267
5320	–	241 5059 +346.0 E	1896–99	52
5396	–	241 5196 +298.0 E	1898–99	38
5465	1.01	241 5022.00 + 3.3891 E	1891–99	687
5608	4.5	241 5029.0 + 12.71 E	1899	24
5618	–	241 5028 +214.0 E	1899	10
5682	117 ?	241 5024 +234.5 E	1896–99	59
5713	2.10	241 5023.41 + 6.3231 E	1892–99	545
5751	0.63	241 5022.02 + 2.5683 E	1892–99	502
5823	4.4	241 5029.45 + 9.7525 E	1892–99	559
5949	–	MIN. 1900 Jan. 5^d 7^h 35^m + 4^d 10^h 12^m $7^s.9$ E *Algol*-type	1891–99	958
6050	160 ?	241 5339 +332 E	1894–99	128
6062	136 ?	241 5251 +282.7 E	1895–99	103
6071	1.41	241 5026.04 + 6.0622 E	1894–99	230
6170	–	241 5283 +388 E	1896–99	49
6275	107 ?	241 5094 +265 E	1898–99	59
6331	–	241 5372 +373 E	1896–99	76
6370	–	241 5251 +245 E	1899	16
6386	12	241 5029.5 + 39.14 E	1899	15
6429	0.0486	MAX. 1900 Jan. 1^d 7^h 10^m + 10^h 50^m $47^s.4$ E	1899	76
6500	109 ?	241 5106 +229 E	1895–99	119
6546	–	MIN. 1900 Jan. 3^d 2^h 2^m + 2^d 9^h 58^m $36^s.7$ E *Algol*-type	1891–99	512
6608	–	241 5058 +320 E	1898–99	50
6686	58	241 5032 +145 E	1898–99	48
6760	3.8	241 5029.10 + 9.0908 E	1891–99	723
6811	40 ?	241 5050 + 89.2 E	1896–99	45
6900	–	. . . . Irregular, no definite period. (See notes)	1899	44
7077	–	241 5031 +243.9 E	1898–99	60
7121	150	241 5310 +389 E	1895–99	131
7151	94	241 5220 +239 E	1896–99	84
7245	–	241 5331 +372 E	1896–99	49
7266	130 ?	241 5182 +301 E	1896–99	53
7483	–	241 5082 +277 E	1898–99	46
7494	–	241 5100 +405.7 E	1896–99	39
7541	55	241 5021 +205 E	1896–99	57
7685	60	241 5199 +216.1 E	1899	11
7813	–	241 5056 +334.8 E	1895–99	58
7994	–	241 5026 +292.5 E	1896–99	20
8039	64	241 5038 +141 E	1898–99	52
8040	–	241 5330 +410 E	1898–99	54
8588	137 ?	241 5099 +270 E	1895–99	61
8593	–	241 5135 +275 E	1896–99	56
8603	66	241 5058 +151.2 E	1895–99	95

defined, *i.e.*, 1898 = $5^M.8$; others flat, *i.e.*, 1899 = $6^M.4$. As an exemplification of the form of the sharper curves, we may instance the algebraic representation of the 1898 variation near maximum:

Decreasing mag. = $5^M.8 + 0^M.05\,(\tau_0) - 0^M.0001\,(\tau_0)^2$.

Increasing mag. = $5^M.8 + 0^M.065\,(\tau_1)$, τ_1 and τ_0 being the days to and after maximum respectively.

110. Light-curve very flat at a maximum, the star remaining almost constant at $9^M.0$ for about one month.

146. The elements given in the catalogue satisfy the

later Lovedale observations very closely. They fail, however, to meet the earlier observations. The period is most probably subject to inequalities. Innes finds a period of 206.3 days, and West 200 days. A period of 190 days would satisfy the 1896 Lovedale observations.

101. Rise to a maximum very rapid: only 25 days taken to rise from $11^M.0$ to $9^M.0$, while the decreasing period through the same range is 95 days. Ascending and descending portions of the light-curve practically straight lines.

191. Light-curve very irregular. In 1891, 1895 and 1899, a secondary maximum has been observed 60 days before the chief maximum. The two maxima recorded in the U.A. (p. 291), Dec., 1872, and Jan., 1874, are fairly well represented by the elements given in this paper, if we regard the maximum of Dec., 1872, as a secondary maximum. The period given in the U.A., 207 days, fails entirely to meet the observations.

1023. Light-curve regular. Maxima usually sharply defined. A very imperfectly defined secondary minimum has been observed about 70 days after chief minimum. The places of the star for 1900 is R.A. $2^h\ 50^m\ 33^s.2$ Decl. $-50°\ 17'\ 55''$

1066. Variation very regular. Rise to a maximum only slightly more rapid than the descent to a minimum. Innes finds a period of 218.4 days.

1635. Maximum limits irregular, and not well defined. A secondary maximum preceding primary by 20 days, has sometimes been observed. The rise to a maximum is, on the average, twice as rapid as the fall to a minimum. There is considerable irregularity, however, both as to the character and amplitude of the light variations.

1654. Variation irregular: secondary maxima and minima. The full elements are:

Period	315^d	
Chief min.	241 5035	$= 6^M.8 - 7^M.0$
Chief max.	241 5135	$= 4.8 - 5.8$
Secondary min.	241 5175	$= 6.0 - 6.4$
Secondary max.	241 5239	$= 5.0 - 5.8$

The secondary max., as indicated above, has frequently been observed brighter than the primary.

1662. Form of light-curve, as far as observed, regular. Increasing rate of variation equal to decreasing rate. Maxima not sharply defined, the light-curve being very flat: limits of variation irregular.

1701. This star is subject to great irregularities in its variation. Indeed its light-curve is of the same type as Ch. 6900, *RY Sagittarii*, that is frequently a succession of increasing and decreasing phases. The period given in the catalogue is, accordingly, only approximately correct.

1850. Maxima not well defined; increasing rate of variation more rapid than decreasing rate. Innes finds a period of 423 days from a discussion of all recorded observations. The elements given in this catalogue are obtained by relating the Lovedale observations to that recorded in the Zone Catalogue.

1894. Light-curve very regular and symmetrical on either side of maximum. Maxima sharply defined. As the star approaches minimum passage, rate of variation decreases.

2059. Ascent to a maximum rapid; ratio of increasing to decreasing rate of variation = 3 : 1. Star below $11^M.0$ during seven months of its period.

2141. Ascent to a maximum rapid. Only 50 days taken to increase from $11^M.4$ to $6^M.8$. The decreasing period through the same range is 130 days. Light-curve regular. Rate of variation diminishes as the star approaches minimum. Visible to naked eye at maximum.

2583. Variation subject to irregularities both as regards limits and period. The form of the light-curve is also dissimilar for different periods, the star sometimes taking longer to rise to a maximum than to fall to a minimum. The following elements of variation represent the observations from 1891 to 1899.

Period	$140^d.15$
Chief min.	241 5049
Secondary max.	241 5082
Secondary min.	241 5095
Chief max.	241 5098

2776. Light-curve very regular. Minima much more sharply defined than maxima. For thirty days on either side of max. increasing rate of variation equal to decreasing rate. Near minimum, however, the decreasing rate is much slower than the increasing rate.

2781. Variation regular and of the same type as (3416) *S Velorum*. The elements of variation are:

Period	$6^d\ 10^h\ 19^m\ 25^s$
Descending period	$3\ \ 8$
Stationary period at min.	$7\ \ 55$
Ascending period	$3\ \ 2$

The light-changes are accounted for by the revolution in the plane of sight, of two stars, one $3\frac{1}{2}$ times larger than the other, the smaller star being, however, $1\frac{1}{2}$ times brighter than its larger companion. On this supposition a secondary minimum would be practically inappreciable. Innes makes the limits of variation $9^M.7$ and $10^M.7$.

2852. Light-curve of the same type as *U Pegasi*. Variation continuous, and symmetrical on either side of minima. The full elements of variation are:

Period	$1^d\ 10^h\ 54^m\ 26^s.7$
Chief min.	1900 January $1^d\ \ 5^h\ 5^m$
Secondary min.	$1\ \ 23\ \ 0$

These elements are obtained from Lovedale observations alone, but they satisfy the observations made by WILLIAMS in 1886. Spectroscopic observation of this star indicate that it is a close binary. Its light-variation, also, is such as would follow from the revolution of two stars, slightly unequal in brightness, in a plane nearly coincident with the line of sight.

2933. This star varies irregularly between the limits assigned; the observations yield no satisfactory period.

2935. Ascent to a maximum rapid. Light-curve regular, and of short-period type. Maxima well defined.

3040. Light-curve of ordinary short-period type. Maxima and minima sharply defined.

3055. Light-variation very regular. The full elements are:

Period	$12^h\ 59^m\ 29^s.9$
Descending period	3 20
Ascending period	3 20

There is no stationary phase at minimum. There are indications that odd minima are $0^M.08$ fainter than even minima. This would point to a period double that given, or $1^d\ 1^h\ 58^m\ 59^s.8$. This latter supposition would be more in accordance with the theory of the star's variation, viz: two stars of equal magnitude, or nearly so, revolving round one another in a plane coincident with the line of sight.

3087. Light-variation of ordinary short-period type.

3355. Light-variation very regular. Ascent to a maximum rapid. When the announcement of this star's variation was made I assigned it, in error, to the constellation *Carina*. It is on the border of this constellation, but in *Vela*.

3416. Variation very regular, and of well defined type. The full elements of variation are:

Period	$5^d\ 22^h\ 24^m\ 21^s.1$
Descending phase	4 20
Stationary phase at min.	6 18
Ascending phase	4 20

Epoch of middle of min. 1900 Jan. $1^d\ 3^h\ 44^m$

The nearer to min. stationary period the more rapid the variation. The passing into and from the stationary phase is abrupt; the form of the light-curve at these two points being practically a right-angle.

Ch. 2781 has been already referred to as an *Algol*-variable of a similar type. The explanation that naturally suggests itself is that in stars of this type of variation we have two bodies, one bright and small, the other dark and large, revolving round one another; the darker body eclipsing the brighter each revolution.

3417. Variation irregular.

3418. Periodic inequality. Successive light-periods dissimilar both as regards amplitude of variation and form of light-curve. Continuous observation of this star has been made by TEBBUTT for twenty years. These, as well as the Lovedale observations since 1891, indicate two types of light-variation: a sharp, bright, well defined maximum, and a fainter and less distinctly defined maximum.

3493. Light-variation irregular. Minima have been observed, though rarely, as faint as $5^M.5$. The elements given in this paper are determined solely from Lovedale observations. INNES obtains, by a comparison of his own observations with all other available data, the following results:

Epoch of max.	241 5042.3
Epoch of min.	241 5028.9
Period	$35^d.5236$
Limits	$3^M.4$ to $4^M.9$

An irregular and ill-defined secondary max. has frequently been observed.

3569. The period given in this catalogue, one year, is only an approximation. From October, 1898, to April, 1899, the star never varied outside the limits $7^M.8$ and $8^M.0$. A similar constant period was observed in 1896. It is more than probable that the star has no definite regular period of variation.

3637. Both the period and amplitude of variation are subject to irregularities. A secondary maximum preceding the primary by from 20 to 40 days, and $1^M.0$ fainter than it has frequently been observed. One striking peculiarity about this variable is that on the average the rise from min. to max. is longer than the fall from max. to minimum. In some light-periods this reversion of general type is more marked than in others. This is usually the case when the secondary minimum is very pronounced.

3663. Rise to a maximum slow, and maximum passage poorly defined.

3669. Variation regular; ascent to maximum more rapid than descent to a minimum.

3777. A want of correspondence between the computed and observed brightness of this star seems to indicate that its variation is irregular, or that the full interpretation of its light-changes has not yet been obtained.

3847. The Lovedale observations indicate that the light of this star has been practically permanent during the last decade. The observations of FINLAY, SEE, INNES and MARKWICK point to the same conclusion. A careful comparison of all the meridian places of the star was made in order to certify beyond doubt that the same star has been under observation. The mean place for 1900 is,

R.A.	$10^h\ 41^m\ 10^s.79\ \pm 0^s.04$
Decl.	$-59°\ 9'\ 31''.3\ \pm 0''.2$

There is no evidence of proper motion in either coordinate.

3922. Light-variation very regular, and of ordinary short-period type. The rise to maximum is, however, more rapid than is usually the case with this class of variable,

being only one-seventh of the whole period. The average ratio is one-third.

1056. Light-curve regular; ascending period nearly equal to descending. Maxima very imperfectly defined. PICKERING finds a period of 162 days.

1216. Light-variations irregular. The rise to a maximum is rapid, but not continuous. INNES finds minima more distinctly marked than maxima; from observations of minima he also deduces a period of 53 days. In order to connect his observations with those made at Lovedale I have taken as the probable period 52.5 days. A longer period, 62 days, would satisfy the Lovedale observations alone.

1225. Light-curve very regular. Maxima distinctly marked. No secondary deviations noticeable. Ascending and descending variation so continuous that it can be represented by two straight lines.

1260 Variation very regular. Maxima distinctly marked. As the star approaches maximum, increasing rate of variation diminishes until within 20 days of maxima it is almost equal to descending rate.

1364. Light-variation irregular; of short-period type.

4415. Light-variation regular; of short-period type.

4429. Variation very regular; of short-period type. Ascending rate of variation considerably above average rate. Maxima and minima very distinctly marked.

4488. Light-curve irregular. Maxima sometimes flat and poorly defined; at other times sharp and well defined. In this apparent irregularity the star has much in common with many other long-period variables. Between $9^{M}.5$ and minimum, decreasing and increasing rate equal. This, however, does not always hold good.

4536. Variation very interesting. GOULD suspected it to belong to the *Algol*-class of stars. It conforms more, however, to the short-period type of variation. There is evidence of secular change in the period of the star.

4611. Variation very irregular; an almost typical example of short-period variation. Maxima sharply defined. Descent to a minimum as well as ascent to a maximum continuous.

4896. In the determination of the elements of this star, as given in the catalogue, a maximum and minimum observed by MARKWICK in 1894 have not been included. If these be retained, the resulting period is 91.5 days, with the probability of a secular variation. Both the amplitude and rate of variation is subject to slight irregularities. Minima are more distinctly marked than maxima.

4935. Rise to and fall from maximum rapid. Light-curve regular; ascending rate of variation only slightly more rapid than descending rate.

5096. The variation of this star, as far as observed at Lovedale, is very regular. There is a well defined secondary maximum, almost equal in brightness to the chief maximum. The secondary minimum is also well defined. The full elements of variation are:

Period	569^{d}	
Chief max.	2415131 =	$5^{M}.6$
Second. min.	2415244 =	9.0
Second. max.	2415330 =	6.2
Chief min.	2415533 =	11.8

The star is allied in type to L. 6417 according to INNES's investigation of the variation of the latter (*A.J.*, No. 468). In the case of L. 6417, however, chief min. follows chief max.; while in the case of *R Centauri* it immediately precedes it. The range of variation, $6\frac{1}{4}$ magnitudes, is much greater than the average range of long-period variables.

5099. This star may be taken as a good example of a class of variables distinct in character from the ordinary short-period type. The variation is within narrow limits, usually half a magnitude, very rapid, and very regular. In the present case the variation can be represented under the simple form:

$$\text{Mag.} = 7^{M}.62 + 0^{M}.22 \cos(49°.6\, t - M).$$

These stars should more properly be regarded as *Algol*-variables, as they have more in common with this class of variables than with those of the ordinary short-period type.

5192. The period may be double that given in the catalogue, as the 1898, 1899 observations indicate that odd maxima are about $0^{M}.2$ brighter than even maxima. At maximum the rate of variation is rapid.

5320. Light-period regular; maximum not well defined. Ascending period of variation nearly equal to descending.

5396. Maximum not distinctly marked; increasing and decreasing rate of variation slow and apparently equal. Decreasing phase irregular.

5465. Light-curve regular and of short-period type. Lovedale observations do not go to support Dr. GOULD's opinion that the increasing phase of variation is longer than the decreasing: U.A., p. 260. "I find the most probable period to be $3^{d}\,9^{h}.35$, the minima preceding the maxima by about 48 hours." The variation of this star's light is no exception to the general law of short-period variation, that the increasing is of shorter duration than the decreasing phase. With regard to the period Dr. GOULD obtained two sets of observations, the values $3^{d}\,9^{h}\,21^{m}$ and $3^{d}\,9^{h}\,19^{m}.8$. A mean of the two, $3^{d}\,9^{h}\,20^{m}.4$, is almost identical with the period resulting from a discussion of the Lovedale observations alone, viz.: $3^{d}\,9^{h}\,20^{m}.31$. This would indicate that the period of the star is constant.

5608. Rise to and fall from maximum rapid; near minimum, rate of variation less rapid. Maximum not distinctly marked; minimum more definite.

5618. Rise to and fall from maximum equally rapid. Variation very regular; maximum distinctly marked. The

form of the light-curve can be practically represented by two straight lines.

5682. Light-curve symmetrical on either side of maximum. As is the case with many of the southern long-period variables the variation of this star can be represented by two straight lines.

If t = days to or from maximum
M = magnitude at maximum

then the magnitude for any date can be fairly well represented by the expression Mag. $= M + 0^{M}.055\,t$.

5713. Considerable want of correspondence between successive light-curves. The mean light-curve, however, is of the ordinary short-period type.

5751. The light-curve of this variable is apparently regular. It does not, however, conform completely to the ordinary short-period type, as the star is almost stationary at its maximum for about twelve hours. The remainder of the star's variation is quite regular.

5823. In *Ara* according to Stone's Catalogue. Observations seem to indicate that alternate minima are $0^{M}.15$ fainter than the intermediate minimum. This would make the period $19^{d}.805$. The variable departs considerably from the ordinary type of short-period variation, the ascending period being almost of the same duration as the descending period. It conforms more to a type of variation intimately related to the *Algol*-type; that is, the immediate and predominant cause of variation is the obscuration of the light by eclipse. In ordinary short-period variation other causes, of more complex nature, operate in producing the light-changes.

5949. The light-curve on either side of minimum can be represented under the form Mag. $= m + \alpha t + \beta t^2$, where m is the magnitude at minimum, and t the time to or from minimum. The light-changes are completed in $9^{h}\ 12^{m}$. There is no stationary phase at minimum, and only the slightest evidence of a secondary minimum. This would seem to indicate that the occulting star is practically dark.

6050. Variation irregular. At some maxima, star visible to naked eye.

6062. Amplitude of variation irregular. Ascending and descending periods nearly equal. In 1896 the increasing rate from $8^{M}.0$ to max. was only half as rapid as the decreasing rate, through the same position. Both the increasing and decreasing phase are continuous. The determinations of the position of the star are:

C.DM.	(1875)	$16^{h}\ 48^{m}\ 40^{s}.3$	$-30°\ 23'.2$
C.P.D.		16 48 40.2	30 22.6
Hartwig		16 48 40.0	-30 22 43.5
Lovedale		16 48 40.4	30 22 42

6071. Light-curve very regular. Ascent to maximum rapid. Maxima and minima very sharply defined.

6170. Ascending rate of variation only slightly more rapid than descending rate. Star remains stationary at a maximum for about 20 days.

6275. Light-curve flat at maximum. Ascending rate of variation rapid and regular.

6331. Light-curve regular and symmetrical on either side of maximum. The actual maximum passages in 1898 and 1899, however, have not been observed, as they occurred when the star was near the sun. Indeed, it will not be till 1905 or 1906 when good observations for the determination of period can be secured. Although each light-period is, *per se*, very regular, there is considerable difference between successive light-curves. Thus in 1899, the variable took four months, from the beginning of August till the end of November, to rise from $11^{M}.0$ to $8^{M}.6$; while in the preceding period, the same range was completed in a little over two months.

6370. Ascending rate of variation more rapid than descending rate. Identification difficult near minimum.

6386. Maxima and minima limits irregular. Maxima distinctly marked. Period given in the catalogue obtained by relating Lovedale observations to those of West, 1896, and Cordoba, 1899. West's period of 19 days will not meet Lovedale observations. General features of light-curve of short-period type; there are evidences, however, of secondary maxima and minima.

6429. The variation of this star is exceedingly remarkable. Evening observations taken in November, 1899, when the star was near the sun, and an extended series of observations was not possible, seemed to confirm the shorter period of $7^{h}\ 28^{m}$ suggested by Mr. Innes. More continuous morning observations, however, yield the longer period now given. The remarkable feature about this star is its rapid rise to maximum. In $1^{h}\ 10^{m}$ it passes from its minimum to its maximum phase. The rate of increase is $0^{M}.1$ every five minutes. After maximum passage the rate of variation steadily slows down till some three hours before minimum it is almost stationary. The main characteristics of the star's variation are, however, of short-period type.

6500. Variation regular. There seems little difference between the ascending and descending rates of variation. Maxima usually very distinctly marked.

6546. A satisfactory period for this star was obtained by Gould, who also suspected that the variation was peculiar in character. "The result of these comparisons is perplexing in the extreme . . . . . . the problem can only be solved by an extended series of careful observations for which there has not yet been opportunity. The indications are that the star fulfils its period in about 58 hours, the minimum preceding the maximum by not more than a quarter of the period," U.A., p. 288. The full elements of variation are:

Period		2d 9h 58m 36s.7
Chief min.	1900 Jan.	3d 2h 2m
Second. min.		4 6 17
Duration of chief min.		12h 30m
Duration of second. min.		8 0

There is evidence that the period between two minima, primary and secondary, is subject to slow change. As the orbit of the star is eccentric a progressive change in the line of apsides is possible. At its normal brightness the star is easily seen by the naked eye.

6608. Variation irregular as regards maximum limits. For four months of its full period the star is below 11M.0; above this magnitude the increasing rate of variation is slightly more rapid than the decreasing rate. Maxima very distinctly marked.

6686. The variation of this star is very regular. The light-curve can be fairly well represented under the form,

$$\text{Mag.} = 10^{M}.0 + 1^{M}.34 \cos (t\,.\,2^{\circ}.5 - M) + 0^{M}.25 \cos (t\;5^{M}.0 - N)$$

6760. Light-curve regular. The variation does not fully conform to the short-period type. Concerning this star GOULD says (U.A., p. 244) — "The period seems subject to considerable perturbations, and the position of the minimum, while always later than midway between the maxima, appears to be likewise inconstant." The dates given in the U.A. when reduced to a mean period yield the following:

Max.	2405138d.5
Min.	2405134.0
Diff.	4.5

It is this approximate equality of ascending and descending rates of variation that makes *κ Pavonis* an exception to the ordinary type of short-period variation. Although the Lovedale observations place minimum passage nearer the succeeding than the preceding maximum, still the type of light-curve resulting from the observations lacks the essential features of the mean short-period curve. The following determinations may be of interest:

Comparison of Lovedale mean max. with Cordoba mean maximum	9d.0906
Comparison of Lovedale mean min. with Cordoba mean minimum	9d.0914
Period from Lovedale observations only	9d.0908

The accordance between these results is testimony to the constancy of the length of the light-periods.

6811. Variation very irregular. Ascending and descending periods almost equal. The ascending phase is more continuous than the descending. There is some difficulty in securing good estimates of this star's light from the absence of faint stars in its neighborhood.

6900. The peculiar variation of this star has been pointed out by PICKERING (*A.N.* 3362, p. 26), and INNES (*A.J.* 468, p. 95). There is apparently no regular period, though there is evidence of a rough cycle of eighteen months. The character of the star's variation may be exhibited by instancing its light-changes in 1899. It rose steadily to a max. of 8M.8 on May 20, falling to a minimum of 11M.1 on July 10. It was 10M.6 on July 30, and 11M.2 on Aug. 25. Rising to 10M.3 on Sept. 10, it fell again to 10M.8 by Sept. 30. From this date the variable rose steadily to 8M.6 on Oct. 22. A succession of more rapid fluctuations took place till the star was lost in the sun's light. At the commencement of 1900 the star was evidently rising to one of its chief maxima. The extreme limits of variation are at least six magnitudes apart.

7077. Light-curve regular. The ascent to a maximum is rapid and continuous. Maxima distinctly marked. The variable was observed by LACAILLE, and since the star changes rapidly, his observation must have been made near its maximum. Two or three more determinations of maxima will enable a certain relation to be established between present observations and those of LACAILLE.

7121. Variation irregular both as regards form of light-curve period and limits of variation. In 1895 the light-curve was of a very simple form, the increasing and decreasing rates being equal. In 1898 and 1899, however, the increasing rate was much more rapid than the decreasing rate.

7151. Light-curve very regular. Maxima not distinctly marked. The variable is L. 8276, which GOULD identifies with A.G.C. 27305. STONE's identification of L. 8276 with Cape 1880, No. 10723, is in error. When STONE observed this region the variable was near minimum. His star, 10723, is Z.C. 2083. The position of the variable and its neighboring stars are:

			(1875) h m s	° ′
L. 8276	A.G.C. 27305,	C.P.D. 8962	19 50 6	−42 10.8
	A.G.C. 27306,	C.P.D. 8963	50 12	10.0
S. 10723	Z.C. 2083,	C.P.D. 8964	19 50 34	−42 9.9

7245. Light-curve regular. Ascent to a maximum rapid. Maxima sharply defined. The ascending portion of the light-curve is practically a straight line.

7266. Light-curve regular. Ascent to a maximum rapid and continuous. Maxima sharply defined. Close following and north of the variable is a star 10M.4, and some doubt has arisen as to whether this star or the variable is C.DM. −39°13722. With the ring-micrometer both the stars were related to A.G.C. 27749, 27778 and 27783. The following positions were obtained:

		h m s	° ′ ″
Var.	(1875)	20 9 25.0	−39 29 45
∗ 10M.4		20 9 26.7	−39 29 25

The catalogue position of −39°13722 is

$$20^{h}\,9^{m}\,25^{s} \quad -39^{\circ}\,29'\,9''$$

which agrees more closely with the variable than with the $10^M.4$ star.

7483. Rise to a maximum very rapid. Star rises from $11^M.0$ to $9^M.0$ in 16 days. The decreasing period through the same range is 70 days. The light-curve is very regular, and the completed curve, based on measures above $11^M.2$, would indicate a minimum of $13^M.0$, 67 days before maximum. These deductions, however, are not reliable enough to be incorporated in the catalogue. They are given here to indicate the range and character of the star's variation.

7494. Variation above $11^M.0$ regular. Star visible in the 3¼-inch only four months out of the thirteen. Ascending period apparently slightly shorter than descending period.

7544. Ascent to maximum rapid, increasing rate of variation being nearly four times more rapid than decreasing rate. Light-curve, especially ascending portion, very regular. Maxima sharply defined.

7685. Limits of variation irregular. Ascent to a maximum very rapid. INNES finds a period of 217.1 days.

7813. Variation regular and rapid towards maximum. The star increases in brightness from $11^M.0$ to $8^M.0$ in about 30 days. Four months are taken in decreasing through the same range. Maxima sharply defined. The position of the variable as determined with the ring-micrometer is:

$$(1875)\ 21^h\ 40^m\ 27^s.6 \qquad -47°\ 29'\ 28''$$

7994. Light-curve very regular. Ascending period shorter than descending. Star increases from $11^M.5$ to $8^M.5$ in about one month. It is probable, from the form of the light-curve obtained from observations above $11^M.0$, that the ascending period is not of more than 60 days duration.

8039. Light-curve very interesting. It is very regular in form, although the amplitude is variable. The expression:

$$\text{Mag.} = m + a \cos(2°.55\,t - M)$$

will indicate the general character of the variation. As the above formula suggests, there is little appreciable difference between the ascending and descending rates of variation. If anything, the ascending rate is more rapid.

8040. Light-curve regular. Maximum not well defined. The decreasing phase of variation contains a faint secondary maximum. The increasing rate of variation is very rapid; indeed, relatively to the decreasing period, the most rapid of southern long-period variables. The star increases in brightness from $11^M.2$ to $8^M.2$ in one month; six months is taken to decrease through the same three magnitudes.

8588. Light-curve somewhat irregular. Maxima not sharply defined. Ascending rate of variation, on the average, equal to the descending rate. The nearer minimum the star is the slower the rate of variation.

8593. Variation regular so far as observed. Above $11^M.4$ ascending rate of variation slightly more rapid than descending. Maxima not distinctly marked. From a consideration of the form of the curve above $11^M.0$ the minimum cannot be above the fourteenth magnitude.

8603. Variation irregular. The full elements of variation are:

Period	$151^d.2$
Chief max.	2415058
Chief min.	2415143
Second. max.	2415169
Second. min.	2415193

Certain full-light periods have been observed without any indication of a secondary max. or min. taking place. The chief max. and minima are also affected by perturbations. The star was observed by LACAILLE.

I. SUPPLEMENTARY LIST OF STARS.

The following list includes a few stars that have been under examination for some time at Lovedale, but whose variation is either unconfirmed, or not confirmed with sufficient certainty and definiteness as to warrant their incorporation in the catalogue.

Uncatalogued. (854) *S Horologii.*

R.A. = $2^h\ 22^m\ 22^s$; Decl. = $-60°\ 1'.2$.

The variation of this star was announced by PICKERING in *A.N.*, No. 3362, p. 35. The period is stated to be 300 days. It has been looked for 13 times in 1899, but without success. It is probable that its variation is so rapid near maximum that this phase has been missed.

L. 2916. (2742) *R Puppis.*

R.A. = $7^h\ 37^m\ 0^s$; Decl. = $-31°\ 25'.7$.

This supposed variable is the brightest star in the cluster *Her.* 3091. GOULD states that during the time it was under observation at Cordoba it varied between the limits $6^M.5$ and $7^M.4$. YARNALL estimated it in 1869 as $5\frac{1}{2}^M$. The star has been observed over a hundred times during the years 1891–99, but no variation, such as that stated, has been detected. It has, perhaps, varied between $6^M.9$ and $7^M.2$; but certainly not outside these limits.

The neighboring star, L. 2893, which GOULD also suspected of variation, is constant between the limits $7^M.0$ and $7^M.2$.

L. 2999. (2783) *S Puppis.*

R.A. = $7^h\ 43^m\ 50^s$; Decl. = $-47°\ 51'.9$.

LACAILLE estimated this star as of the sixth magnitude. STONE in 1876 made three observations of it, noting it, also, as of the sixth magnitude. It is possible, however, that STONE simply accepted LACAILLE's values. GOULD's observations make it variable between $7^M.2$ and $9^M.0$. The star has been regularly examined at Lovedale from 1891 to 1899, but no departure from $7^M.5$ has been noticed.

L. 3001. (2788) *T Puppis.*

R.A. = $7^h\ 14^m\ 41^s$; Decl. = $-40°\ 24'.1$.

GOULD states that this star is variable between $6^M.5$ and $7^M.2$. During the nine years that the star has been, irregularly, under observation at Lovedale, it has not differed more than $0^M.1$ or $0^M.2$ from the neighboring star L. 3015. Entered in the U.A. as $7^M.0$.

L. 3910. (3109) *N Velorum.*

R.A. = $9^h\ 28^m\ 11^s$; Decl. = $-56°\ 35'$.

This star was suspected of variation by GOULD, who suggested a period of $1^d.25$. I have been unable to detect any certain variation in brightness, although I strongly suspect variation in the color of the star. GOULD was also of opinion that the color changed. Usually it appears a star of a bright yellow color; at times, however, it seems distinctly of a dull shade. Because of this apparent variation, and because of the absence from its neighborhood of good comparison-stars, estimates of its brightness are difficult to make.

L. 4156. (3614) *R Velorum.*

R.A. = $10^h\ 2^m\ 23^s$; Decl. = $-51°\ 42'.1$.

Dr. GOULD says (U.A., p. 276): "This star has been seen by Dr. THOME to vary from the mag. $6\frac{1}{2}$ to $7\frac{1}{2}$, thus putting its variable character beyond question." I am, however, unable to confirm its variation. It has never been observed outside the limits $7^M.4$ and $7^M.6$. Its variation is much more doubtful than that of *N Velorum.*

L. 4168. (3633) *R Antliae.*

R.A. = $10^h\ 5^m\ 27^s$; Decl. = $-37°\ 14'.4$.

Dr. GOULD found this star to fluctuate between $6^M.5$ and $8^M.5$, and gives dates (U.A., p. 295), at which these, or intermediate magnitudes, were observed. In the C.P.D. it is entered as $7^M.6$. In the C.DM. it is marked variable. Professor BAILEY also, in his "Southern Meridian Photometry," regards the star as variable. The star certainly varies in brightness; the extreme limits apparently being $7^M.2$ and $7^M.8$. Although a large number of observations have been secured, I am unable to deduce from them a satisfactory period. In 1900 a systematic effort is being made to deal with this star.

L. 4530. (3908) *T Carinae.*

R.A. = $10^h\ 51^m\ 18^s$; Decl. = $-59°\ 59'.2$.

A series of observations made between 1871 and 1877 (U.A., p. 256), by THOME, seems to justify the conclusion that the star is variable between the limits $6^M.2$ and $6^M.9$. *T Carinae* has, however, been constantly used as a comparison-star for (3922) *U Carinae*, but without any evidence of variation equal to that found by Dr. THOME. The star certainly does not vary outside the limits $6^M.7$ and $7^M.0$, otherwise such variation would be at once manifest in the reduction of the observations of *U Carinae*.

Uncatalogued. (3983) *RS Carinae.*

R.A. = $11^h\ 3^m\ 54^s$; Decl. = $-61°\ 23'.6$.

In *A.N.* 3320 (p. 141), PICKERING states this star to be a *Nova*. Systematic search was made for it in 1898 and 1899. The limit of the $3\frac{1}{4}$-inch, however, is $11^M.4$, and it is probable that the variable was far below this magnitude during these years.

A.G.C. 19416. (5134) *T Lupi.*

R.A. = $14^h\ 15^m\ 43^s$; Decl. $-49°\ 23'.5$.

About this star Dr. GILL states in the introduction to the C.P.D., *Annals of the Cape Observatory*, Vol. IV, p. xi, "Missing in four sweeps on two pairs of plates. Perhaps doubtful traces on M. and C. in the original zones of Cordoba the star is noted red." The star was observed at Lovedale, 5 times in 1896; 5 times in 1898; 32 times in 1899, but no variation greater than $0^M.3$ or $0^M.4$ was detected. Its red color is so manifest that it is doubtful if ever the star will be photographed.

L. 6077. (5319) *R Apodis.*

R.A. = $14^h\ 46^m\ 29^s$; Decl. = $-76°\ 15'.3$.

The note in the U.A. (p. 243), about the variation of this star is: "The variability seems beyond question, but the pressure of other observations has not yet permitted the determination of its period, or the exact limits of its variation. These limits are not nearer than $5^M.5$ and $6^M.2$." My observations, which, however, are few in number, and very irregular, as they extend over eight years, do not prove a variation such as that found by Dr. GOULD. It is possible that the star is an *Algol*-variable. I think the probability that it belongs to any other class is remote.

L. 6193. (6402) *T Triang. Austr.*

R.A. = $15^h\ 0^m\ 24^s$; Decl. = $-68°\ 20'.1$

GOULD considered this star to vary between the limits $7^M.0$ and $7^M.4$, in a period of about one day (U.A., p. 260). Lovedale observations do not confirm this variation. A great many observations have been secured, as it was frequently observed along with *R Triang. Austr.*

It is possible that the apparent variation may be really due to position-error; this would be fulfilled in a period of one day.

Uncatalogued. (6101) *RT Scorpii.*

R.A. = $16^h\ 56^m\ 48^s$; Decl. = $-36°\ 40'$

The limits of this variable are given by PICKERING as $9^M.2$ and $12^M.9$. It has never been seen at Lovedale, although regular search has been made for it. I am convinced, however, of the reality of the star's existence from a photograph of the variable and its immediate neighborhood sent me by Prof. PICKERING. The impression on the photograph is unmistakable, but no star is visible in the telescope, although all the other stars in the photograph are easily seen. The star probably rises to and falls from maximum rapidly.

C.DM. 35°11829. (6324) — *Scorpii.*

R.A. = $17^h 35^m 41^s$; Decl. —35° 11′.8

PICKERING notes this star as varying between the limits $10^M.7$ and $11^M.6$. I am unable to confirm its variation, as during all the time it was under observation in 1899, it never became invisible in the $3\frac{1}{4}$-inch telescope, the limit of which is considered to be $11^M.4$.

Uncatalogued. (6806) *S Cor. Australis.*

R.A. = $18^h 54^m 26^s$; Decl. = —37° 5′.3

The variation of this star is said to be between the limits $<9^M.5$ and $13^M.0$. It has never been seen at Lovedale, although it has been frequently searched for.

Uncatalogued. (6812) *T Cor. Australis.*

R.A. = $18^h 55^m 14^s$; Decl. = —37° 6′.4

The variation of this star is said to be between the limits $<9^M.3$ and $13^M.0$. It has never been seen at Lovedale, although its proximity to (6811) *R Cor. Australis* would make it an easier object to pick up than the preceding star.

A.G.C. 30765. (8093) *R Indi.*

R.A. = $22^h 28^m 53^s$; Decl. = —67° 48′.3

In the Cordoba Zone observations GOULD found this star to vary in magnitude between $9^M.0$ and $11^M.0$. Its variation has not been confirmed at Lovedale, although irregular observations have been made of the immediate neighborhood of the star since 1892.

A.G.C. 31427. (8302) *Y Sculptoris.*

R.A. = $23^h 3^m 40^s$; Decl. = —30° 40′.5

PICKERING considers this star irregularly variable between the limits $8^M.0$ and $8^M.9$, *A.N.*, No. 3362, p. 36.

In *A.J.*, No. 395, p. 88, WEST gives observations confirming the variation between the limits $7^M.7$ and $8^M.4$.

Lovedale observations made in 1898 and 1899 indicate variation between $7^M.7$ and $8^M.8$ in a period, roughly, of $9\frac{1}{2}$ months. The observations are not numerous enough to indicate a full light-curve, and so this star is not included in the catalogue. Its variation, however, is beyond doubt.

These stars are only a few of those observed here. They are also only a small portion of the total number of stars south of —30° either suspected of or proved variable.

If we take the stars in the Zone Catalogue with discordant magnitudes, or the stars whose magnitudes differ considerably in different magnitudes, and also the stars suspected by GOULD, THOME and GILL, then the total number of suspected stars south of —30° will lie between two and three hundred.

There is abundant material here for the best energies of many observers.

OBSERVATIONS OF COMET *b* 1900 (*BROOKS*),

MADE AT THE CHAMBERLIN OBSERVATORY, UNIVERSITY PARK, COLORADO,

By CHARLES J. LING.

The following observations were made with the Bruce Micrometer on the twenty-inch equatorial. The magnifying power was two hundred. The right-ascension observations are chronographic, and the declination bisections were made while the object was drifting through the field. Dr. J. G. PORTER, of Cincinnati Observatory, was kind enough to furnish the places for those stars whose declination exceeded 80°.

1900 Univ. Park M.T.		*	No. Comp.	☄ — * $\Delta\alpha$	☄ — * $\Delta\delta$	☄'s apparent α	☄'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
	h m s			m s	′ ″	h m s	° ′ ″		
July 28	11 52 45	1	20 . 6	— 3 30.76	— 0 25.1	2 48 54.21	+27 8 45.2	n9.702	0.729
	12 7 33	2	19 . 6	— 3 28.77	+11 31.6	2 48 55.27	+27 10 49.7	n9.705	0.714
29	13 15 47	3	20 . 6	— 3 43.73	+11 47.8	2 50 12.10	+30 24 20.8	n9.707	0.602
	13 37 21	4	20 . 6	— 1 14.92	+16 47.2	2 50 13.31	+30 27 10.8	n9.695	0.566
30	15 30 50	5	18 . 8	— 0 14.18	— 2 1.8	2 51 27.99	+33 47 47.2	n9.558	0.260
31	11 49 54	6	20 . 6	— 8 23.74	+11 40.6	2 52 50.30	+36 25 7.3	n9.747	0.643
	12 25 25	7	20 . 6	— 8 30.25	+15 25.6	2 52 51.82	+36 29 46.1	n9.750	0.634
	13 10 33	8	20 . 6	+ 9 30.30	+11 57.8	2 52 54.70	+36 35 36.4	n9.738	0.546
Aug. 2	11 40 52	9	20 . 6	— 3 44.72	+ 0 58.6	2 55 57.79	+42 32 11.8	n9.784	0.673
	12 9 56	10	20 . 8	+ 1 47.78	+ 2 28.8	2 55 59.99	+42 33 51.4	n9.788	0.612
4	11 37 54	11	20 . 8	— 0 30.18	+ 3 29.0	2 59 39.61	+48 28 49.9	n9.831	0.637
	11 52 51	12	20 . 6	— 3 0.72	+16 6.1	2 59 40.99	+48 30 39.5	n9.834	0.602
6	10 3 44	13	20 . 8	— 0 28.92	+ 1 42.2	3 3 56.03	+53 57 36.2	n9.821	0.779
	10 26 16	14	20 . 6	+ 3 15.52	+ 2 27.9	3 3 58.59	+54 0 24.7	n9.846	0.743
15	9 27 13	15	10 . 6	—12 14.18	+ 8 18.0	3 41 55.30	+74 29 53.5	n0.126	0.753
17	9 51 20	16	10 . 6	— 3 19.48	+ 6 37.3	1 1 7.77	+77 59 24.7	n0.243	0.707
20	9 41 1	17	5 . 5	+ 3 59.17	— 9 34.8	4 49 11.13	+82 12 12.4	n0.348	0.761
21	11 21 44	18	5 . 5	+ 9 5.72	—19 27.0	5 21 3.93	+83 26 51.6	n0.520	0.592
22	12 43 39	19	5 . 5	+ 5 9.90	+17 8.6	6 1 9.86	+84 28 37.7	n0.650	0.423
23	12 34 14	20	5 . 5	+ 5 57.41	— 6 2.2	6 13 53.89	+85 13 24.0	n0.671	0.588
25	10 18 33	21	5 . 5	— 5 12.57	— 5 57.9	8 36 6.45	+85 50 54.6	8.884	0.856
Sept. 28	10 23 18	22	20 . 8	— 2 2.36	—10 30.8	14 24 5.39	+69 18 56.2	0.008	0.577

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	2 52 21.88	+ 3.09	+27 9 4.0	+ 6.3	Graham, Cambridge, A.G.C. 1529
2	2 52 20.95	+ 3.09	+26 56 11.7	+ 6.4	" " " 1528
3	2 53 52.65	+ 3.18	+30 12 27.6	+ 5.4	" " " 1537
4	2 54 25.11	+ 3.15	+30 10 18.3	+ 5.3	" " " 1541
5	2 54 38.87	+ 3.30	+33 49 14.6	+ 4.4	Weisse's Bessel II. 1175
6	3 1 10.00	+ 3.34	+36 13 23.6	+ 3.1	" " 1397
7	3 1 18.73	+ 3.34	+36 14 17.5	+ 3.0	Cincinnati (13) 369
8	2 43 20.98	+ 3.42	+36 23 31.5	+ 4.1	Yarnall 1255
9	2 59 38.93	+ 3.58	+42 31 11.9	+ 1.3	Deichmüller, Bonn A.G.C. 2610
10	2 51 8.50	+ 3.62	+42 33 21.0	+ 1.6	" " " 2544
11	3 0 5.95	+ 3.87	+48 25 21.2	— 0.3	" " " 2614
12	3 2 37.86	+ 3.85	+48 11 33.9	— 0.5	" " " 2640
13	3 1 20.76	+ 4.19	+53 55 56.0	— 2.0	Rogers, Cambridge, A.G.C. 1400
14	3 0 38.86	+ 4.21	+53 57 58.6	— 1.8	" " " 1385
15	3 51 2.88	+ 6.60	+74 21 45.1	— 9.6	Lewetzky, DM. 74°186
16	4 4 19.42	+ 7.83	+77 52 58.3	—10.9	Kasan, A.G.C. 655
17	4 45 31.85	+10.11	+82 22 1.6	—14.4	Carrington 687
18	5 11 17.06	+11.15	+83 46 31.9	—16.3	Greenwich Observations, 1896, No. 736
19	5 55 50.58	+ 9.38	+84 11 18.0	—18.9	Romberg 1355
20	6 37 18.29	+ 8.19	+85 19 46.6	—20.4	Carrington 941
21	8 41 50.78	— 1.76	+85 57 13.8	—21.3	Radcliffe 881
22	11 26 10.33	— 2.58	+69 29 30.0	— 3.0	Fearnley, Christiania, A.G.C. 2156

University Park, Colorado, 1901 *Jan.* 26.

ON THE ASSIGNMENT OF THE NOMENCLATURE AND THE FORMATION OF A NEW CATALOGUE OF VARIABLE STARS.

No one recognizes more fully or with more regret than the undersigned the inconvenience and danger of confusion attending the prolonged delay in the assignment of the nomenclature to the rapidly increasing list of new unnamed variables, and in the publication of a new catalogue; or the personal responsibility for the blame of allowing these matters to fall into desuetude. It has been his hope to continue the work by the preparation of a Fourth Catalogue long ere this, but the pressure of other tasks and avocations have been such as to prevent its fulfillment up to the present time, and would postpone it now to an extent that cannot for an instant be considered or tolerated. He must therefore resign the task, reluctantly in one sense, but cheerfully in another, into the hands of some one who is willing and competent to take it up. It is a work demanding such punctiliousness and complete and instant conversance with detail that no one who is distracted by other subjects to a large extent can perform it properly; far less can it be done at second hand, or by superintendence, or by committee, except in a slovenly manner, worse than its not being done at all.

Dr. Hartwig has shown such unremitting interest in the subject, has the requisite details so fully in hand and mind, and is so eminently the one to whom we can look for the continuance of this work, and its maintenance at the high standard set by Schönfeld, that it is to be hoped he will feel disposed to take it upon his hands. If so, the *Journal* will unhesitatingly adopt any notation that he may assign, to variables at present unnamed or hereafter discovered, according to established rubrics; accepting his interpretation of the same as authoritative, without demur or criticism; and in fact, cheerfully extend to him the courtesy in such matters that has so long been extended by him and others to the undersigned, who is deeply sensible of and grateful for it.

S. C. Chandler.

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SECULAR PERTURBATIONS OF THE *EARTH* BY THE ACTION OF *VENUS*,

By ERIC DOOLITTLE.

The computation by GAUSS's method of the secular perturbations of the *Earth* arising from *Venus* is of special interest on account of the slow convergence of the perturbing function for this case. Mr. R. T. A. INNES has effected the computation by Dr. HILL's second modification of GAUSS's method, using tables prepared by himself (*Monthly Notices*, Vol. LIII, p. 353). Though Dr. HILL's second method is perhaps somewhat shorter, I have preferred in all cases to use the first. This is because the auxiliary tables both of Mr. INNES and of M. CALLANDREAU for use with the second method have been computed with but seven-place logarithmic tables, so that the seventh decimal of these tables is necessarily uncertain,* while on the contrary those of Dr. HILL for use with the first method were computed with ten-place logarithms and published to eight, thus making the eighth decimal entirely accurate.

It is of interest to compare the results of so long a computation with those obtained by Mr. INNES from the same elements. It will be seen that the final values of dp and dq differ by one unit, and $ed\pi$ by eight units in the sixth decimal, while the values of dL differ by two in the sixth significant figure. There is an error in the latter part of the work of Mr. INNES, which arises from errors in $A_1^{(c)}$ and $B_0^{(c)}$ in the computation of de. I have computed from my own results the quantities which Mr. INNES designates by $\frac{a}{r}R_0$ and $\frac{a}{r}S_0$, and find them substantially correct, though the last two significant figures differ, probably on account of the inaccuracy of the auxiliary tables to the second method. The resulting values of $\frac{1}{2}A_1^{(c)}$ and $B_0^{(c)}$ are approximately $+0.010\,173\,7833$ and $-0.000\,170\,65835$, respectively, which very accurately satisfy the test equations arising from the constancy of the major axis.

I have also repeated, by Dr. HILL's second method, the latter part of Mr. INNES's computation, from $\frac{a}{r}R_0$ to the end, using the values of the auxiliaries given by Mr. INNES. The resulting values of dp, dq, $d\pi$ and dL are substantially as he has given them, but de is found to have the value $+0''.013476$, which agrees quite well with that given below.

The elements are from Dr. G. W. HILL's "*New Theory of Jupiter and Saturn*," pages 192 and 554.

The Earth.		*Venus.*	
$\pi =$	$100°\ 21'\ 39''.73$	$\pi' =$	$129°\ 27'\ 42''.83$
$i =$	$0\ \ 0\ \ 0.00$	$i' =$	$3\ 23\ 35.01$
$\Omega =$	– – –	$\Omega' =$	$75\ 19\ 53.08$
$e =$	0.01677114	$e' =$	0.00684311
$n =$	$1295977''.416$	$n' =$	$2106641''.357$
$\log a =$	0.0000000	$\log a' =$	9.8593378
$m =$	$1 \div 327\,000$	$m' =$	$1 \div 408134$

Epoch, 1850.0 Greenwich Mean Time.

The values of the preliminary constants are as follows:

$I =$	$3°\ 23'\ 35''.01$	$\log k =$	$n9.9994999$
$II =$	$25\ \ 1\ \ 46.65$	$\log k' =$	$n9.9997387$
$II' =$	$54\ \ 7\ \ 49.75$	$\log c =$	$n5.3891826$
$K =$	$330\ 56\ 48.79$	$1000 \times c =$	$+0.024\,500\,932$
$K' =$	$330\ 54\ \ 5.12$		

The orbit of the *Earth* was divided into twelve parts with regard to the eccentric anomaly, and the functions computed as in the following tables. The roots G, G' and G'' were computed by the formulae of Dr. HILL's second method, to make the comparison with the work of Mr. INNES more convenient, and they were tested by the equations of the first method. This work was not repeated, but the rest of the computation has been duplicated from the beginning.

* The tables of CALLANDREAU contain but six decimals.

E	v	r	A	$\log B$	ϵ	p	q^2
0	0 0 0.00	0.983 228 860	1.498 447 49	9.853 7612	331 0 3.89	0.499 47 433	0.079 50558
30	30 29 2.39	0.985 475 766	1.504 11382	9.854 6442	1 19 46.170	0.501 36310	0.080 70333
60	60 50 8.59	0.991 614 430	1.514 84369	9.856 7644	31 34 37.20	0.504 93973	0.082 62793
90	90 57 39.46	1.000 000 000	1.527 86583	9.859 7314	61 38 57.20	0.509 28011	0.084 65948
120	120 49 43.50	1.008 385 570	1.539 74201	9.862 6537	91 29 35.875	0.513 23917	0.086 34022
150	150 28 37.29	1.014 524 234	1.547 23847	9.864 4870	121 8 13.24	0.515 73799	0.087 44009
180	180 0 0.00	1.016 771 140	1.548 24407	9.864 5724	150 43 29.14	0.516 07319	0.087 68573
210	209 31 22.71	1.014 524 234	1.542 43719	9.862 9682	180 22 35.176	0.514 13756	0.087 00647
240	239 10 16.50	1.008 385 571	1.531 42595	9.860 3568	210 12 5.09	0.510 46715	0.085 36569
270	269 2 20.54	1.000 000 000	1.518 26327	9.857 6205	240 13 17.66	0.506 07959	0.083 09965
300	299 9 51.41	0.991 614 429	1.506 52764	9.855 4195	270 23 50.222	0.502 16772	0.080 90038
330	329 30 57.61	0.985 475 766	1.499 31255	9.854 0767	300 40 21.95	0.499 76268	0.079 54648
Σ_1	900 0 0.00	6.000 000 000	9.139 23085	9.153 5280	1085 23 41.417	3.046 36129	0.502 42553
Σ_2	1080 0 0.00	6.000 000 000	9.139 23113	9.153 5280	905 23 41.396	3.046 36136	0.502 42550

E	r_1	θ_1	G	G'	G''	$1000 \times g$	θ
0	0.002 738 0635	7 0 55.62	0.975 94422	0.522 481 505	0.000 00579	0.002 936 418	47 1 40.84
30	0.002 320 0483	5 18 28.79	0.983 53171	0.520 557 558	0.000 00004	0.000 006 753	46 10 41.61
60	0.001 789 3068	4 19 13.62	0.995 45723	0.519 388 564	0.000 00667	0.003 473 631	46 14 52.33
90	0.001 377 0128	3 12 16.55	1.007 73136	0.520 128 988	0.000 01898	0.009 946 120	45 55 31.27
120	0.001 134 0363	2 33 43.11	1.017 74388	0.521 998 108	0.000 02454	0.013 007 536	45 41 23.02
150	0.000 973 2049	2 9 29.14	1.024 06979	0.523 162 060	0.000 01795	0.009 615 238	45 37 23.13
180	0.000 846 4697	1 52 5.43	1.025 71566	0.522 509 836	0.000 00592	0.003 140 184	45 32 21.65
210	0.000 852 9468	1 51 16.46	1.021 73849	0.520 674 148	0.000 00000	0.000 000 562	45 32 59.47
240	0.001 144 9685	2 37 52.10	1.011 99593	0.519 111 649	0.000 00609	0.003 259 143	45 45 34.89
270	0.001 729 9249	1 8 28.42	0.998 28868	0.519 968 638	0.000 01853	0.009 581 104	46 11 45.77
300	0.002 384 3853	5 56 51.93	0.984 67983	0.521 847 851	0.000 02449	0.012 589 278	46 43 7.27
330	0.002 784 1227	7 7 42.75	0.976 15654	0.523 149 586	0.000 01807	0.009 256 115	47 3 38.83
Σ_1	0.010 037 2301	24 20 41.81	6.011 51675	3.127 640 513	0.000 07350	0.038 406 190	277 2 0.00
Σ_2	0.010 037 2604	24 20 42.41	6.011 51657	3.127 640 978	0.000 07357	0.038 406 192	277 2 0.08

E	$\log \mathfrak{K}$	$\log \mathfrak{L}'$	$\log \mathfrak{N}$	$\log N$	$\log P$	$\log Q$	$\log V$	J'_1
0	0.261 47483	0.607 79696	0.547 75634	0.262 6427	0.891 5846	0.820 9715	0.820 9685	0.522 862 677
30	0.256 83878	0.602 07812	0.541 49917	0.254 9482	0.871 4495	0.803 6590	0.803 6589	0.521 939 851
60	0.251 22074	0.595 13848	0.533 90180	0.246 8811	0.845 9860	0.782 7661	0.782 7628	0.521 368 465
90	0.247 07162	0.590 00660	0.528 28039	0.242 0422	0.825 3429	0.766 9696	0.766 9602	0.521 723 193
120	0.244 70700	0.587 07937	0.525 07272	0.240 4872	0.812 2693	0.757 9113	0.757 8992	0.522 632 205
150	0.243 22990	0.585 24987	0.523 06751	0.240 2489	0.804 8244	0.752 9792	0.752 9704	0.523 191 641
180	0.242 17345	0.583 94092	0.521 63264	0.240 0756	0.804 9574	0.750 6787	0.750 6758	0.522 862 807
210	0.242 30579	0.584 10492	0.521 81242	0.240 8209	0.806 2462	0.753 2935	0.753 2935	0.521 968 683
240	0.244 96050	0.587 39327	0.525 41673	0.244 4416	0.824 4721	0.764 6769	0.764 6739	0.521 377 068
270	0.250 55055	0.594 30993	0.532 99440	0.251 6543	0.847 4359	0.785 3845	0.785 3752	0.521 675 619
300	0.257 37182	0.602 73603	0.542 21917	0.260 0985	0.876 2226	0.809 0118	0.808 9994	0.522 582 150
330	0.261 94246	0.608 33646	0.548 34645	0.264 9132	0.894 1947	0.823 7321	0.823 7229	0.523 186 451
Σ_1	1.501 90834	3.564 08503	3.195 99940	1.494 6267	5.049 4920	4.686 0163	4.685 9796	*3.133 611 871
Σ_2	1.501 90940	3.564 08590	3.196 00034	1.494 6277	5.049 4936	4.686 0179	4.685 9811	*3.133 611 870

* The term in G'' has been removed.

E	J_2	J_3	F_2	F_3	$-R_0$	S_0
0°	−0.003 063 2731	+0.012 629 049	+0.001 237 2987	+0.000 065 35471	1.828 3274	−0.010 644 421
30	−0.000 733 6503	+0.024 877 829	−0.000 059 3369	−0.000 001 96302	1.789 8617	−0.005 109 526
60	+0.002 450 1101	+0.030 338 229	−0.001 345 7284	−0.000 005 70449	1.753 5729	+0.005 420 211
90	+0.005 083 5797	+0.027 547 130	−0.002 277 1885	+0.000 059 16590	1.736 7356	+0.014 494 077
120	+0.005 834 5100	+0.017 252 379	−0.002 604 1325	+0.000 128 85901	1.736 2036	+0.016 510 300
150	+0.004 423 4178	+0.002 212 471	−0.002 238 9562	+0.000 134 26042	1.738 6892	+0.010 760 758
180	+0.001 738 6808	−0.013 542 647	−0.001 279 5084	+0.000 069 88985	1.736 1351	+0.001 682 926
210	−0.000 906 9972	−0.025 791 387	+0.000 017 1274	−0.000 000 59748	1.733 7538	−0.005 029 615
240	−0.002 685 9269	−0.031 251 723	+0.001 303 5189	−0.000 007 87443	1.744 2502	−0.006 981 624
270	−0.003 598 7100	−0.028 460 585	+0.002 231 9789	+0.000 054 04190	1.774 7204	−0.006 225 129
300	−0.004 033 1250	−0.018 165 873	+0.002 561 9225	+0.000 122 15392	1.815 4792	−0.006 714 245
330	−0.004 026 3352	−0.003 126 030	+0.002 196 7464	+0.000 127 77090	1.840 2326	−0.009 613 070
Σ_1	+0.000 241 3059	−0.002 740 583	−0.000 126 6292	+0.000 372 67857	10.613 9684	−0.000 726 856
Σ_2	+0.000 241 3048	−0.002 740 574	−0.000 126 6289	+0.000 372 67861	10.613 9933	−0.000 722 505

E	W_0	$\frac{1}{a} \sin E . R^{(0)}$	$\frac{1}{a} . S^{(0)}$	$R_0 \sin v + (\cos v + \cos E) . S_0$	$-R_0 \cos v + \left(\frac{r}{a} \sec^2\varphi + 1\right) . \sin v . S_0$	$\sin(v+\pi) . W_0$	$\cos(v+\pi) . W_0$	$-\frac{r}{a} . R_0$
0	+0.084 13477	. . . . . .	−0.010 825 990	−0.021 2888	+1.828 3274	+0.082 76286	−0.015 13166	1.797 6643
30	+0.158 28193	−0.908 1206	−0.005 184 831	−0.916 8211	+1.537 3039	+0.119 73727	−0.103 51881	1.763 8656
60	+0.183 93250	−1.531 4810	+0.005 466 048	−1.525 9137	+0.863 9706	+0.059 28500	−0.174 11620	1.738 8680
90	+0.161 47390	−1.736 7356	+0.014 494 077	−1.736 7344	−0.000 1389	−0.031 70097	−0.158 33153	1.736 7356
120	+0.099 63133	−1.491 0928	+0.016 373 004	−1.507 5995	−0.861 2810	−0.065 61473	−0.074 97804	1.750 7629
150	+0.013 38361	−0.856 8990	+0.010 606 704	−0.875 4611	−1.502 2509	−0.012 64209	−0.004 39303	1.763 9423
180	−0.075 83153	. . . . . .	+0.001 655 167	−0.003 3658	−1.736 1351	+0.074 59502	−0.013 63832	1.765 2520
210	−0.146 11352	+0.854 4665	−0.004 957 609	+0.863 0786	−1.503 6464	+0.112 14236	−0.093 71241	1.758 9352
240	−0.181 83297	+1.498 0034	−0.006 923 567	+1.504 8615	−0.881 8403	+0.063 58327	−0.170 35373	1.758 8768
270	−0.173 24740	+1.774 7204	−0.006 225 129	+1.774 5751	−0.017 3138	−0.028 29602	−0.170 92102	1.774 7204
300	−0.116 10019	+1.585 5471	−0.006 771 025	+1.578 6949	+0.896 3895	−0.073 88837	−0.089 55312	1.800 2552
330	−0.019 82985	+0.933 6770	−0.009 754 749	+0.916 9364	+1.595 5126	−0.018 61936	−0.006 82218	1.813 5046
Σ_1	−0.006 06309	+0.060 9767	−0.001 026 363	+0.025 3885	+0.109 4311	+0.140 72305	−0.537 77107	10.611 6792
Σ_2	−0.006 08133	+0.061 1087	−0.001 021 537	+0.025 5735	+0.109 4965	+0.140 62119	−0.537 69898	10.611 7037

The equation, $\sin q . \frac{1}{2} A_1^{(c)} + \cos q . R_0^{(c)} = 0$ is found to give the residual,

$$-0.000\,000\,008\,3$$

If the value of m' is left indefinite, the values of the differential coefficients are as follows:

		log coeff.
$\left[\frac{de}{dt}\right]_{00} =$	$+\ 5503.0089''\ m'$	$p3.740\,6002$
$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} =$	$+1409586.4\ m'$	$p6.149\,0917$
$\left[\frac{dp}{dt}\right]_{00} =$	$+\ 30388.832\ m'$	$p4.482\,7140$
$\left[\frac{dq}{dt}\right]_{00} =$	$-\ 116164.73\ m'$	$n5.065\,0743$
$\left[\frac{dL}{dt}\right]_{00} =$	$+4584354.6\ m'$	$p6.661\,2782$

If the above value is substituted for m', there finally results:

$$\left[\frac{de}{dt}\right]_{00} = +\ 0.013\,483\,339''$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +\ 3.453\,7341$$

$$\left[\frac{dp}{dt}\right]_{00} = +\ 0.074\,457\,966$$

$$\left[\frac{dq}{dt}\right]_{00} = -\ 0.284\,623\,99$$

$$\left[\frac{dL}{dt}\right]_{00} = +11.232\,473$$

The comparison with the results of LeVerrier, Newcomb, Hill and Innes is as follows, the results all being reduced to the above value of m':

	LeVerrier (1)	Newcomb (2)	Hill (3)	Innes	Method of Gauss
$\left[\frac{de}{dt}\right]_{00}$	+ 0.013 41	+0.013 48	. . . .	[+ 0.013 456]	+ 0.013 4833
$e\left[\frac{d\pi}{dt}\right]_{00}$	+ 0.057 96	0.057 92	. . . . . .	+ 0.057 915	+ 0.057 9231
$\left[\frac{dp}{dt}\right]_{00}$	+ 0.074 50	0.074 46	+0.074 4329	+ 0.074 459	+ 0.074 4580
$\left[\frac{dq}{dt}\right]_{00}$	− 0.284 51	−0.284 62	−0.284 5280	− 0.284 623	− 0.284 6240
$\left[\frac{dL}{dt}\right]_{00}$	+11.229 8	. . . .	. . . . . .	+11.232 490	+11.232 473

(1) *Annales de l'Observatoire de Paris*, Vol. II, p. 59, and IV, pp. 11–12. (2) "*New Theory*," pp. 511–512.
(3) "*Secular Variations of the Orbits of the Four Inner Planets*," pp. 336 and 377.

The Flower Observatory, 1901 *January* 1.

HANSEN'S LUNAR TABLES,

BY A. HALL.

The numerical work of these tables appears to have been done with remarkable accuracy, and the author must have been an accomplished computer. It is surprising, therefore, that after a few years the moon should have departed so decidedly from this theory. The coefficients stand the test of revision so well, and their agreement with those of other theories is so good, that one is inclined to look elsewhere for the trouble. Perhaps an astronomer who had access to HANSEN's manuscripts might discover some oversight. Apparently it needs but a small change to make these tables right. The method of deducing values by comparison with observations may be justified, since we cannot prove the convergence of all the series.

It may be worth while to point out a small discrepancy indicated by recent investigations. This is in HANSEN's determination of the mass of the earth. If this mass were known we could determine the solar parallax by the formula,

$$\pi = 609''.50 \times \sqrt[3]{M}$$

Denote by M, m, m', the masses of the earth, moon and sun; and by a and a', the semi-major axes of the orbits of the moon and earth, we have

$$\frac{a}{a'} = \sqrt[3]{n^2 \cdot \frac{M+m}{M+m'}}$$

where n is the ratio of the mean motions of the earth and moon, or

$$n = \frac{n'}{n} = 0.07543826$$

In his tables HANSEN finds that the ratio $\frac{a}{a'}$ must be multiplied by 1.03573, in order to satisfy observations. Hence $\log \frac{a}{a'} = 7.4031757$, becomes 7.4187223.

Cambridge, 1901 *February* 5.

From the above value of this ratio we have

$$\frac{m'}{M} = \frac{n^2\left(1+\frac{m}{M}\right)}{\left(\frac{a}{a'}\right)^3} - 1$$

Assuming $\frac{m}{M} = \frac{1}{80}$, we find

$$\frac{m'}{M} = 319455$$

If m' be taken for the unit of mass, the earth's mass is

$$\frac{1}{319455}$$

a result which differs from the value given by the secular perturbations of the planets. Computing the solar parallax from this value we have

$$\pi = 8''.9159$$

a value which was soon confirmed by other determinations. If the mass of the earth from the secular perturbations is

$$\frac{1}{333500}$$

the solar parallax is

$$\pi = 8''.789$$

agreeing with recent determinations.

Another point, and perhaps an essential one, concerning which HANSEN was led into error, relates to the records of the ancient eclipses. Thus the eclipse of Stiklastad, in 1030, has been shown by recent historical writers to have happened a month after the battle to which HANSEN referred it. The equations of condition designed to adjust the tables to observations should be examined.

It is to be hoped that the error in HANSEN's Lunar Tables may be discovered, and that these tables will remain a monument to one who devoted his life so completely to theoretical astronomy.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENNA., WITH A 4½-INCH REFRACTOR,

By A. W. QUIMBY.

1900	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
July 1	8	–	1	1	1	fair
2	5	2	2	7	2	fair
3	7	–	2	11	3	fair
4	8	–	1	5	2	fair
5	8	–	–	–	–	fair
6	8	–	–	–	–	poor
7	8	–	–	–	1	fair
8	7	–	–	–	–	poor
9	7	–	–	–	–	fair
10	7	–	–	–	–	fair
11	7	–	–	–	1	fair
12	7	–	–	–	–	poor
13	7	–	–	–	–	fair
14	7	–	–	–	–	fair
15	7	1	1	3	1	fair
16	7	–	1	8	1	fair
17	7	–	1	28	1	fair
18	4	–	1	32	1	fair
19	7	–	1	12	2	fair
*20	7	–	1	4	–	poor
*21	7	–	1	6	–	fair
22	8	1	2	10	–	fair
23	9	–	2	3	–	poor
24	8	–	1	2	–	poor
25	9	–	–	–	–	poor
26	3	–	–	–	–	poor
27	7	–	–	–	–	fair
28	7	–	–	–	–	fair
29	8	–	–	–	–	good
30	2	–	–	–	–	poor
31	6	–	–	–	–	fair
Aug. 1	7	–	–	–	–	fair
2	7	–	–	–	–	fair
3	6	–	–	–	–	fair
4	7	–	–	–	–	fair
5	7	–	–	–	–	fair
6	7	–	–	–	–	fair
* 7	6	–	–	–	–	fair
* 8	6			–	–	poor
* 9	7	–	–	–	–	poor
*10	7	2	2	3	1	poor
11	6	–	2	8	1	fair
12	8		2	11	1	fair
*13	1	–	1	8	–	poor
*14	9	–	–	–	–	poor
*15	1	–	–	–	–	poor
16	9	–	–		1	fair
17	9	–	–		1	fair
18	9	–	–		–	fair
19	5		–	–	–	fair
*20	3		–	–	–	poor
*21	4	–	–	–	–	poor
*22	10	–	–	–	–	fair
*23	8	–	–		–	fair
*24	12	–	–	–	–	poor
25	6	–	–	–	–	poor
26	8	–	–	–	–	fair
27	8	–	–	–	–	poor
Aug. 28	8	1	1	1	1	fair
29	3	–	1	5	–	fair
30	8	–	–	–	1	fair
31	8	–	–	–	1	fair
Sept. 1	8	–	1	3	1	fair
2	8	–	1	4	2	fair
3	8	1	2	8	2	fair
4	3	1	2	8	2	fair
5	8	–	2	12	–	fair
6	8	–	2	15	–	fair
7	9	–	2	7	–	fair
8	11	1	2	2	1	poor
9	4	–	1	3	1	poor
10	3	–	1	3	–	fair
11	7	–	1	3	–	poor
12	8	–	1	4	–	fair
13	8	–	–	–	2	fair
14	11	–	–	–	–	poor
15	2	–	–	–	1	poor
16	1	–		–	1	poor
17	8	–	–	–	2	fair
18	8	–	–	–	1	fair
19	8	–	–	–	–	fair
20	8	–	–	–	–	fair
21	8	–	–	–	–	fair
22	1	–	–	–	1	fair
23	9	–	–	–	–	poor
24	8	–	–	–	–	fair
25	8	–	–	–	–	fair
26	8	–	–	–	–	fair
27	10	–	–	–	1	good
28	10	–		–	1	fair
29	3	–	–	–		poor
Oct. 2	9	–	–		1	poor
3	3	–	–	–	1	fair
4	10	–	–		–	poor
5	12	–	–	–		poor
6	9	–	–		1	good
8	11	1	1	13	1	good
10	8	–	1	8	1	poor
11	7	–	1	8	1	fair
12	8	–	1	2	1	poor
13	12			–	–	v. poor
15	8	–		–		good
16	3	1	1	4	1	fair
17	3	–	1	11	1	fair
18	9	–	1	20	1	fair
19	8		1	32	1	good
20	8	–	1	29	–	good
21	8	–	1	26	–	good
22	8	–	1	25	–	good
23	8	–	1	19	–	fair
24	8	–	1	12	–	good
25	8	–	1	9	–	good
26	8	–	1	6	–	fair
27	11	1	2	4	2	fair
28	8	–	2	4	2	fair
29	10	–	1	4	2	fair
Oct. 30	8	–	1	2	1	fair
Nov. 1	8	–	1	1	–	poor
2	2	–	1	1	–	poor
4	8	–	–	–	–	poor
5	8	–	–	–	1	fair
6	8	–		–	1	fair
7	8	–	–	–	1	fair
8	11	–	–	–	1	good
9	10	–	–	–	–	fair
10	7	–	–	–	–	poor
11	7	–	–	–	–	poor
12	8	–	–	–	1	fair
13	10	1	1	1	1	poor
14	9	–	1	1	1	fair
15	3	–	1	1	–	poor
16	9	–	1	1	–	poor
17	10	–	1	3	–	poor
18	2	–	1	4	–	poor
19	9	–	1	8	–	fair
20	8		1	4	–	poor
21	11	–	1	4	1	poor
22	10	–	1	5	1	poor
23	8	–	–	–	1	fair
25	1	–	–	–	–	poor
26	8	–	–	–	–	poor
27	8	–	–	–	–	poor
28	8	–	–	–	–	poor
29	2	–	–	–	–	fair
30	9	–	–	–	–	fair
Dec. 1	7	–	–	–	–	poor
2	8	–	–	–	–	fair
3	9	–	–	–	–	poor
5	8	–	–	–	–	fair
6	11	–	–	–	–	poor
7	8	–	–	–	–	fair
8	8	–		–	–	fair
9	7	–	–	–	–	poor
10	8	–	–	–	2	fair
11	8	–	–		1	fair
12	8	–	–	–	–	fair
13	8	–	–	–	–	fair
14	8	–	–	–	–	fair
15	8	–	–			fair
16	9	–	–		–	fair
17	8		–		–	fair
18	8		–	–	–	poor
19	8	–	–	–	–	fair
20	3		–	–	1	good
22	3	–			–	fair
23	8	–	–	–	–	fair
24	11	–	–	–		fair
25	8		–	–		fair
26	8	–	–	–	–	fair
27	8	–	–	–	–	fair
28	3		–	–	–	fair
29	8		–			fair
30	9		–	–	–	poor
31	11		–			poor

* Made with 2½-inch refractor.

Berwyn, Penn.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 18-INCH EQUATORIAL, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

By HENRY B. EVANS.

Greenwich Mean Time	✳	No. Comp.	Planet — ✳ $\Delta\alpha$	Planet — ✳ $\Delta\delta$	Planet's apparent α	Planet's apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			(9) *Metis.*					
1898			m s	′ ″	h m s	° ′ ″		
Feb. 20 20h 22m 17s	1	8	−2 15.76	−2 30.4	10 18 11.76	+20 57 8.5	9.551	0.566
21 13 56 8	1	6 , 8	−3 29.23	+1 24.2	10 18 1.30	+20 58 3.1	*n*9.597	0.593
27 13 41 18	5	8 , 11	−1 16.29	−0 41.9	10 12 1.99	+21 26 54.8	*n*9.542	0.553
			(92) *Undina.*					
1898								
Feb. 20 21 42 40	2	8	−1 30.77	. . .	10 20 7.01	. . . .	9.652	. .
20 21 27 9	2	5	. . . .	−0 12.6	. . . .	+21 18 14.2	. . .	0.623
27 13 8 11	3	8	−1 13.20	+1 27.1	10 15 7.26	+21 52 34.3	*n*9.601	0.583
27 13 8 14	4	8	−1 5.86	. . .	10 15 7.31	. . . .	. . .	. .
Mar. 16 16 26 58	6	8 , 12	−3 14.55	+3 53.9	10 3 35.03	+22 55 43.3	9.108	0.430
			(375) 1893 *AL.*					
1899								
Nov. 5 14 48 37	7	*d*12	−0 2.90	−1 3.5	2 52 20.09	+39 39 20.2	*n*9.485	9.830
6 14 22 11	7	9 , 10	−1 4.47	−2 40.7	2 51 18.54	+39 37 43.2	*n*9.501	9.863
7 12 15 59	7	9 , 8	−2 1.18	−4 23.2	2 50 21.85	+39 36 0.9	*n*9.736	0.432
8 13 49 19	8	9 , 8	−2 35.62	+4 58.9	2 49 15.25	+39 33 51.3	*n*9.599	0.076
12 12 9 18	9	*d*10	+0 5.48	−7 48.4	2 45 11.95	+39 23 35.3	*n*9.718	0.383
13 13 0 6	10	9 , 10	+1 46.40	−2 31.6	2 44 8.89	+39 20 24.3	*n*9.645	0.192
			(336) *Lacadiera.*					
1899								
Nov. 7 12 58 55	11	9 , 8	+1 8.79	−5 38.8	2 26 44.56	+16 46 9.5	*n*9.563	0.626
12 12 39 42	13	*d*10	−0 9.02	−3 29.1	2 21 51.26	+16 8 23.6	*n*9.554	0.630
13 13 32 8	14	9 , 10	−0 15.12	−4 53.2	2 20 56.91	+16 0 38.4	*n*9.430	0.598
20 14 41 16	15	9 , 11	−1 22.46	+6 10.9	2 14 58.34	+15 10 2.7	*n*8.861	0.577
21 12 34 19	15	8	−2 4.18	−0 3.5	2 14 16.63	+15 3 48.4	*n*9.479	0.621
			(256) *Walpurga.*					
1899								
Nov. 26 16 13 3	16	*d*10	+0 14.83	+2 51.9	3 37 36.66	+ 3 42 11.2	*n*7.643	0.724
27 14 52 49	16	*d*11 , 12	−0 28.68	−0 26.1	3 36 53.15	+ 3 38 50.1	*n*8.185	0.726
			(324) *Bamberga.*					
1899								
Dec. 13 16 3 45	17	8 , 12	−2 26.37	−1 39.5	5 17 45.61	+42 10 52.8	*n*9.079	*n*9.417
15 15 0 30	18	11	+0 24.82	. . .	5 15 14.55	. . . .	*n*9.407	. .
15 15 0 30	19	11	+0 15.16	. . .	5 15 14.65	. . . .	. . .	. .
			1899 *EY.*					
1900								
Mar. 21 14 32 39	20	12 , 10	+0 44.63	+6 17.3	4 48 18.26	+23 15 16.2	9.662	0.631
22 14 13 52	21	12 , 10	+1 43.27	−3 7.0	4 49 25.76	+23 20 12.9	9.650	0.612
24 14 15 5	22	12	+2 35.21	−0 20.5	4 51 54.30	+23 30 18.1	9.656	0.618
Apr. 6 14 53 4	23	*d*12	−0 24.84	+6 3.2	5 8 52.30	+24 30 12.8	9.692	0.693
7 12 46 22	24	*d*10	+0 8.08	−2 47.9	5 10 8.21	+24 34 2.9	9.605	0.546
8 12 58 36	25	12	+1 46.53	−5 17.5	5 11 32.29	+24 38 17.4	9.624	0.563
9 12 47 3	25	12 , 6	+3 9.78	−1 10.0	5 12 55.52	+24 42 24.8	9.613	0.551
			(308) *Polyxo.*					
1900								
Mar. 24 15 4 21	26	*d*10	−0 18.27	−4 22.2	11 34 59.64	+ 2 4 30.2	*n*9.203	0.731
Apr. 6 15 44 55	27	12	+2 19.64	−0 7.5	11 26 1.36	+ 3 23 30.5	8.540	0.719
7 14 42 53	27	12	+1 46.22	+4 59.9	11 25 27.94	+ 3 28 37.9	*n*8.894	0.719
8 14 26 2	28	12	−1 23.71	−2 45.7	11 24 54.09	+ 3 33 45.5	*n*9.009	0.718
9 15 21 54	29	*d*10	−0 2.01	−1 26.7	11 24 19.38	+ 3 39 2.8	8.200	0.716
			(434) *Hungaria.*					
Mar. 24 16 19 22	31	*d*10 , 11	−0 4.39	+0 9.7	12 1 25.10	+ 5 35 37.7	*n*8.830	0.696
Apr. 5 16 29 22	33	12	+1 8.77	+2 56.2	11 51 26.61	+ 9 50 21.1	8.814	0.646

Mean Places for 1898.0, 1899.0, 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	10 21 27.46	+3.06 +3.07	+20 56 55.0	−16.1	Becker, Berlin A.G.C. 4012
2	10 24 34.72	+3.06	+21 19 13.0	−16.2	" " " 4032
3	10 16 17.35	+3.11	+21 51 22.5	−15.3	" " " 4000
4	10 16 10.06	+3.11	+21 54 10.4	−15.3	" " " 3999
5	10 13 45.18	+3.10	+21 27 54.9	−15.2	" " " 3988
6	10 6 16.47	+3.11	+22 52 3.0	−13.6	" " " 3958
7	2 52 16.81	+6.18 +6.20 +6.22	+39 40 2.4	+21.3 +21.5 +21.7	Lund A.G. Zones
8	2 51 44.65	+6.22	+39 28 30.4	+22.0	" " "
9	2 45 0.25	+6.22	+39 31 0.2	+23.5	" " "
10	2 42 16.29	+6.20	+39 22 34.9	+24.0	" " "
11	2 25 30.54	+5.23	+16 51 23.6	+24.7	Micrometer comparison *12
12	2 22 46.93	+5.22	+16 54 48.6	+24.9	Auwers, Berlin A.G.C. 681
13	2 21 58.05	+5.23	+16 11 27.5	+25.2	" " " 676
14	2 21 36.81	+5.22	+16 5 6.4	+25.2	Micrometer comparison *13
15	2 16 15.59	+5.21 +5.22	+15 3 26.3	+25.5 +25.6	Auwers, Berlin A.G.C. 650
16	3 37 16.66	+5.17	+ 3 38 59.6	+16.7 +16.6	Boss, Albany A.G.C. 1082
17	5 20 4.69	+7.32	+12 12 28.4	+ 3.9	Deichmüller, Bonn A.G.C. 4449
18	5 14 42.40	+7.33	+41 57 22.1	+ 5.0	" " " 4375
19	5 14 51.86	+7.33	+41 57 33.9	+ 5.0	" " " 4380
20	4 47 32.07	+1.56	+23 8 57.7	+ 1.2	Becker, Berlin A.G.C. 1549
21	4 47 40.93	+1.56	+23 23 18.7	+ 1.2	" " " 1551
22	4 49 17.55	+1.54	+23 30 37.5	+ 1.1	" " " 1565
23	5 9 12.68	+1.46	+24 24 9.4	+ 0.2	" " " 1687
24	5 9 58.68	+1.45	+24 36 50.6	+ 0.2	Micrometer comparison *25
25	5 9 44.32	+1.45 +1.44 +1.42	+24 43 31.6	+ 0.3 + 0.3 + 0.2	Becker, Berlin A.G.C. 1691
26	11 35 14.95	+2.96	+ 2 9 11.8	−19.4	Boss, Albany A.G.C. 4326
27	11 23 38.79	+2.93	+ 3 23 57.2	−19.2	" " " 4289
28	11 26 14.87	+2.93	+ 3 36 50.4	−19.2	" " " 4297
29	11 24 18.47	+2.92	+ 3 40 18.6	−19.1	Micrometer comparison *30
30	11 23 52.55	+2.92	+ 3 44 50.7	−19.1	Boss, Albany A.G.C. 4292
31	12 1 26.52	+2.97	+ 5 35 47.2	−19.2	Micrometer comparison *32
32	11 59 1.82	+2.97	+ 5 32 38.9	−19.2	Bruns & Peter, Leipzig II, A.G.C. 6006
33	11 50 14.85	+2.99	+ 9 47 46.3	−18.4	Micrometer comparison *34
34	11 50 19.66	+2.99	+ 9 41 13.5	−18.5	Bruns & Peter, Leipzig II, A.G.C. 5960

NOTE REGARDING SEVERAL LALANDE STARS,

BY HERMAN S. DAVIS.

BAILY's reductions, by aid of SCHUMACHER's tables, of the observations of right-ascension on page 339 of the *Histoire Céleste* are in error. As this error increases with the right-ascension, it may be due to an increment of error in the value of k in the tables for this zone. If VON ASTEN's tables be used a better value of right-ascension for all stars recorded on page 339 of the *Histoire Céleste* is obtained. Moreover, the identity of the following stars, among others, is established: 25422 = 25423, 25441 = 25442, 25519 = 25522, 25523 = 25525, 25538 = 25512, (BAILY).

It is regarding the confusion which exists in different catalogues respecting the last two numbers to which attention is here more particularly called.

(1) LALANDE's zenith-distance for 25542 in the *Hist. Cél.*, page 61, is 13° 21′ 5″, this should be 35″, and in BAILY the N.P.D. should have a corresponding increase.

(2) This star is B.A.C. 4632 whose N.P.D. should therefore be increased by 30″, since the note on page 412 of the B.A.C. says: "The position of this star depends entirely on the observation at page 61 of *Hist. Cél.*"

(3) The Paris Catalogue, tome iii, page [103], notes the identity of 25422 = 25423, and 25538 = 25542, but to establish the former assumes arbitrarily a correction of 2″ to the right-ascension of 25422, and ignores any other identities arising from the cause stated above. It calls attention to correcting 25542 by 30″, however (though by

misprint says apply it to LALANDE'S *A* instead of *P*), and on page 91 gives the epoch 1794.3 instead of 1796.3.

(4) SMYTH, in the Edinburgh Catalogue, says: "In N.P.D. there is an awful error of tabular place, and much discrepance of observation, the R.A. place being fair enough," neglecting to trace the discrepance to any cause.

(5) RESPIGHI, for his catalogue of epoch 1875, observed this star twenty-eight times, and records the consequent declination thrice, and yet each of the records of his catalogue is in error by 30″. He gives the difference between his own declination and that of the B.A.C. reduced to 1875 as −3″.10. As the B.A.C. has an error of 30″, it seems not improbable that RESPIGHI arbitrarily changed his own results by this amount, as he could hardly have made the same error of observation on twenty-eight occasions.

The mean position of this star for 1890 is:

$$\alpha = \quad 13^h\ 16^m\ 56^s \qquad +\ 2^s.65$$
$$\delta = +34^\circ\ 59'\ 21'' \qquad -17''.91$$

NOVA PERSEI,

BY EDWIN B. FROST.

This brilliant object attracted my attention at eleven o'clock on the evening of February 22d, before the receipt of the announcement of its discovery by Dr. ANDERSON. It was at that time to my eye brighter than a standard first-magnitude star, and showed a distinct yellowish color, recalling to my mind the shade of *Nova Aurigae*.

It was cloudy here on the 23d, and the spectrum was first examined, between clouds, on the 24th, from $6^h\ 30^m$ to $10^h\ 30^m$ E.S.T. The observations were made with a McClean direct-vision star spectroscope attached to the nine-inch refractor of the Dartmouth Observatory. The general appearance of the visual spectrum was quite similar to that of *Nova Aurigae*, with the bright components of the doubled lines on the less refrangible side (toward red). The dark components appeared relatively more intense, however, than in case of *Nova Aurigae*, probably in great part a result of the superior brightness of the present star. The dark band on the more refrangible side of *C* was especially broad, much more so than in CAMPBELL'S drawing* of the visual spectrum of *Nova Aurigae*.

Although the spectroscope employed does not permit micrometer settings to be made, the identification would seem to be sufficiently exact of the hydrogen lines $H\alpha$ and $H\beta$, the sodium lines at *D*, the magnesium group *b* (in whole or part), and probably the strong line at λ 5016,—all these being represented by dark and bright components. Numerous other lines were seen which cannot yet be identified. Singularly enough, the helium line D_3 was very faint or absent (the identification of the sodium lines being assumed). This was also the case with *Nova Aurigae*.

Dartmouth College, Hanover, N.H., 1901 *Feb.* 25.

* SCHEINER'S *Astronomical Spectroscopy*, Fig. 69, p. 285. This was probably the best drawing made of the visual spectrum.

LINDEMANN FUND FOR THE COMPUTATION OF COMETARY ORBITS.

Mr. A. F. LINDEMANN of Sidmouth, England, has placed means at the disposition of the *Astronomische Gesellschaft* to be administered by a committee consisting of H. SEELIGER, E. WEISS, G. MÜLLER and H. KREUTZ, for the purpose of accelerating the work of calculation of comet-material from ancient times to the middle of the nineteenth century. The conditions are briefly as follows. An average amount of 100 marks (about $24) will be paid for definitive calculation of the orbit of one of these comets; the award being lower for those requiring a relatively small amount of time and higher for those presenting especial difficulty. The award will be made to the first calculation which sufficed the requirements of a definitive computation, but may be divided in case of simultaneous determinations. The committee will decide in each case whether the requirements are met, and the amount of the payment. Following is a list of the comets between 1750 and 1852 whose orbits require to be newly determined:

1757	1774	1790 I	1804	1822 III	1827 III
1758	1779	1790 III	1806 II	1823	1830 II
1759 II	1780 I	1792 I	1808 II	1824 II	1843 II
1759 III	1781 I	1792 II	1811 II	1825 I	1844 II
1762	1781 II	1793 I	1813 I	1825 II	1844 III
1763	1784	1796	1813 II	1826 II	1845 II
1764	1785 I	1797	1818 II	1826 III	1845 III
1766 I	1786 II	1798 I	1818 III	1826 IV	1846 VII
1766 II	1787	1798 II	1819 II	1826 V	1849 II
1770 II	1788 I	1799 II	1819 IV	1827 I	1849 III
1773	1788 II	1802	1822 I	1827 II	1852 IV

and the older appearances of BRORSEN'S Comet.

It is desirable that intending computers, in making their selections from the list, should communicate with Dr. KREUTZ in order that duplicate calculations may be so far as possible avoided.

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OBSERVATIONS OF ONE HUNDRED DOUBLE STARS.

THIRD CATALOGUE.

BY WILLIAM J. HUSSEY.

This list of new double stars which I have discovered with the 12 and 36-inch telescopes may be regarded as a continuation of those already printed in *A.J.* 480 and 485. The arrangement of the lists is the same. The positions of the stars are for the epoch 1900.

With respect to the distance between the components the stars of the present list have the following classification:

0″.25 or less,	5 pairs
0″.26 to 0″.50,	12 "
0.51 to 1.00,	20 "
1.01 to 2.00,	25 "
2.01 to 5.00,	38 "

201. DM. −11°152.

$\alpha = 0^h\,47^m\,7^s \;:\; \delta = -13°\,46'.1.$

	°	″	M	M
1900.818	130.0	0.69	9.0	9.8
.896	127.9	0.49	8.8	9.5
.920	129.7	0.60	8.8	9.2
1900.88	129.2	0.59	8.9	9.5

202. DM. −11°188.

$\alpha = 0^h\,56^m\,36^s \;;\; \delta = -10°\,46'.0.$

1900.740	241.1	2.22	8.5	13.5
.808	239.0	1.96	8.5	13.0
.896	240.0	2.32	8.5	12.5
1900.81	240.0	2.17	8.5	13.0

203. DM. +52°599.

$\alpha = 2^h\,27^m\,52^s \;:\; \delta = +52°\,20'.1$

1900.818	68.0	0.58	9.5	9.5
.830	69.2	0.72	9.5	9.5
.866	70.0	0.80	9.5	9.5
1900.84	69.1	0.70	9.5	9.5

204. DM. +49°770.

$\alpha = 2^h\,40^m\,9^s \;;\; \delta = +49°\,42'.2.$

1900.860	144.5	3.18	8.6	8.6
.866	141.2	3.23	8.5	8.5
.907	144.6	3.11	8.5	8.5
1900.87	143.4	3.17	8.5	8.5

205. DM. +49°773.

$\alpha = 2^h\,41^m\,5^s \;:\; \delta = +49°\,39'.6.$

1900.860	156.8	1.51	9.0	11.5
.866	154.9	1.54	9.5	11.5
.907	156.1	1.53	9.0	11.5
1900.88	155.9	1.53	9.2	11.5

206. DM. +18°765.

$\alpha = 2^h\,43^m\,10^s \;:\; \delta = +18°\,23'.2.$

	°	″	M	M
1900.860	339.0	1.81	7.8	12.0
.866	337.1	1.83	7.5	11.5
.907	335.5	1.80	8.0	12.5
1900.88	337.2	1.81	7.8	12.0

207. DM. −13°681.

$\alpha = 3^h\,28^m\,41^s \;:\; \delta = -13°\,21'.0.$

1900.070	312.0	0.85	8.5	9.0
.128	310.4	0.93	8.5	10.0
1900.10	311.2	0.89	8.5	9.5

208. DM. −10°738.

$\alpha = 3^h\,40^m\,6^s \;:\; \delta = -10°\,40'.7.$

1900.070	159.0	2.58	9.0	9.8

209. DM. +49°1032.

$\alpha = 3^h\,41^m\,51^s \;:\; \delta = +50°\,4'.8.$

1900.201	87.1	1.26	8.8	9.5
.768	87.7	1.23	8.8	9.0
.848	88.0	1.32	8.5	10.5
1900.61	87.6	1.27	8.7	9.7

210. DM. +51°835.

$\alpha = 3^h\,55^m\,6^s \;:\; \delta = +51°\,52'.0.$

1900.848	193.6	0.33	9.0	10.0
.866	191.8	0.34	9.0	9.5
1900.86	192.7	0.33	9.0	9.8

211. DM. +19°1106.

$\alpha = 4^h\,1^m\,14^s \;:\; \delta = +50°\,0'.3.$

1900.768	270.4	1.56	8.5	9.5
.818	271.9	1.63	8.5	11.0
.866	270.4	1.63	8.5	10.0
.907	270.4	1.80	8.8	10.5
1900.85	270.8	1.65	8.6	10.3

212. DM. +51°883.

$\alpha = 4^h\,6^m\,33^s \;:\; \delta = +51°\,34'.3.$

A and *B*.

	°	″	M	M
1900.818	6.8	0.36	8.5	9.5
.866	9.3	0.30	9.5	10.5
1900.86	8.0	0.33	9.0	10.0

A and *C*.

1900.818	192.1	1.34	8.5	10.0
.866	191.2	1.28	9.5	12.0
1900.86	191.6	1.31	9.0	11.0

213. DM. +50°980.

$\alpha = 4^h\,14^m\,30^s \;:\; \delta = +50°\,46'.7.$

1900.768	28.9	0.87	8.5	12.0
.818	29.3	0.86	8.5	12.0
.866	29.5	0.87	8.8	13.0
1900.83	29.2	0.87	8.6	12.3

214. DM. −10°1013.

$\alpha = 4^h\,43^m\,4^s \;:\; \delta = -10°\,53'.1.$

1900.128	234.9	4.93	8.8	10.5

215. DM. −11°1011.

$\alpha = 4^h\,52^m\,3^s \;;\; \delta = -11°\,6'.2.$

1900.163	285.4	0.98	8.5	9.0

216. DM. −10°1101.

$\alpha = 5^h\,3^m\,0^s \;:\; \delta = -9°\,59'.8.$

1900.163	229.9	2.40	8.5	13.5

217. DM. +35°1137.

$\alpha = 5^h\,23^m\,2^s \;:\; \delta = +35°\,17'.6.$

1900.932	255.5	0.61	7.0	8.5
.950	258.6	0.51	7.0	8.5
1900.94	257.1	0.56	7.0	8.5

218. DM. −11°1193.

$\alpha = 6^h\ 22^m\ 36^s\ ;\ \delta = -11°\ 46'.4.$

	°	″	M	M
1900.220	11.9	1.40	8.8	13.0
.243	15.3	1.21	8.5	13.0
1900.23	13.6	1.35	8.6	13.0

219. DM. +61°895.

$\alpha = 6^h\ 29^m\ 24^s\ ;\ \delta = +61°\ 6'.6.$

1900.768	313.5	0.59	8.5	12.0
.776	317.0	0.77	8.8	12.0
.818	316.0	0.71	8.2	11.0
1900.80	315.5	0.69	8.5	11.7

220. DM. −13°1553.

$\alpha = 6^h\ 30^m\ 27^s\ ;\ \delta = -13°\ 55'.7.$

1900.163	76.1	1.00	9.0	10.5
.246	77.8	. .	9.0	11.5
1900.20	77.0	1.00	9.0	11.0

221. DM. +61°951.

$\alpha = 7^h\ 7^m\ 49^s\ ;\ \delta = +61°\ 33'.2.$

A and *B*.

1900.768	151.4	0.59	9.2	10.5
.932	152.4	0.59	9.2	11.5
1900.85	151.9	0.59	9.2	11.0

A and *C*.

1900.932	285.5	4.56	9.2	13.0

222. DM. −12°2259.

$\alpha = 7^h\ 54^m\ 45^s\ ;\ \delta = -13°\ 3'.4.$

1900.226	281.0	3.15	8.5	12.0

223. DM. −13°2343.

$\alpha = 7^h\ 56^m\ 18^s\ ;\ \delta = -13°\ 29'.8.$

1900.207	214.0	0.87	8.8	12.0
.213	210.1	0.76	8.5	13.0
1900.23	212.1	0.82	8.6	12.5

224. DM. +47°1565.

$\alpha = 8^h\ 13^m\ 52^s\ ;\ \delta = +47°\ 44'.0.$

1898.271	315.3	1.28	7.0	12.0
.282	314.2	1.32	. .	. .
1900.204	314.9	1.37	. .	. .
1898.92	314.8	1.32	7.0	12.0

As a wide pair this is $O\Sigma$ 190.

225. DM. −11°2520.

$\alpha = 8^h\ 55^m\ 19^s\ ;\ \delta = -12°\ 4'.4.$

1900.240	99.7	0.34	8.5	8.6
.377	280.8	0.34	8.2	8.5
1900.31	280.2	0.34	8.3	8.5

226. DM. −13°2757.

$\alpha = 9^h\ 0^m\ 2^s\ ;\ \delta = -13°\ 19'.0.$

1900.240	122.5	3.21	9.0	13.0

227. DM. −13°2773.

$\alpha = 9^h\ 4^m\ 17^s\ ;\ \delta = -13°\ 46'.8.$

	°	″	M	M
1900.220	216.2	2.25	7.5	11.5
.246	215.1	2.11	8.2	10.5
.273	216.2	2.13	7.5	12.0
1900.25	215.8	2.26	7.7	11.3

228. DM. +62°1077.

$\alpha = 9^h\ 25^m\ 50^s\ ;\ \delta = +62°\ 42'.9.$

1900.112	81.0	0.53	8.5	13.0

229. DM. +60°1201.

$\alpha = 9^h\ 35^m\ 20^s\ ;\ \delta = +60°\ 43'.4.$

1900.112	186.0	1.04	9.5	10.0

230. DM. −11°2756.

$\alpha = 9^h\ 49^m\ 44^s\ ;\ \delta = -11°\ 35'.1.$

1900.240	85.5	0.30	9.0	9.2

231. DM. −11°3004.

$\alpha = 10^h\ 57^m\ 10^s\ ;\ \delta = -11°\ 18'.5.$

1900.240	49.0	4.86	8.5	13.5

232. DM. −13°3433.

$\alpha = 11^h\ 39^m\ 23^s\ ;\ \delta = -13°\ 33'.3.$

1900.377	109.8	0.87	8.5	8.8

233. DM. −11°4235.

$\alpha = 16^h\ 49^m\ 55^s\ ;\ \delta = -11°\ 43'.4.$

1900.491	63.0	4.88	8.5	11.5
.494	64.8	5.01	8.5	11.5
.502	60.7	5.09	8.5	11.5
.510	62.0	4.96	8.5	11.5
1900.50	62.6	4.98	8.5	11.5

234. DM. −12°4757.

$\alpha = 17^h\ 22^m\ 12^s\ ;\ \delta = -12°\ 6'.2.$

A and *B*.

1900.442	168.2	0.96	7.0	12.5
.470	168.3	1.12	8.5	12.0
.494	165.1	0.98	8.5	11.5
1900.47	167.2	1.02	8.0	12.0

A and *C*.

1900.418	306.3	5.47	8.0	9.0
.442	306.6	5.29	7.0	11.0
.470	305.5	5.35	8.5	9.0
1900.44	306.1	5.37	7.8	9.7

235. DM. +45°2629.

$\alpha = 17^h\ 54^m\ 15^s\ ;\ \delta = +45°\ 52'.0.$

1900.691	264.2	1.48	6.5	9.5
.713	266.1	1.49	6.5	9.5
.745	265.5	1.62	7.0	8.8
1900.71	265.4	1.53	6.7	9.3

236. DM. −10°4581.

$\alpha = 17^h\ 55^m\ 23^s\ ;\ \delta = -10°\ 11'.6.$

1900.513	119.0	1.20	9.0	12.5

237. DM. −17°5172.

$\alpha = 18^h\ 17^m\ 19^s\ ;\ \delta = -17°\ 6'.3.$

	°	″	M	M
1900.620	23.6	0.44	8.5	9.5
.626	23.3	0.42	8.6	9.5
1900.62	23.5	0.43	8.5	9.5

238. DM. +9°3680.

$\alpha = 18^h\ 17^m\ 40^s\ ;\ \delta = +9°\ 54'.1.$

1900.571	163.7	0.90	8.8	9.0
.588	163.5	0.91	8.5	9.5
.591	163.6	1.06	8.5	9.0
1900.58	163.6	0.96	8.6	9.2

239. DM. −21°5005.

$\alpha = 18^h\ 21^m\ 20^s\ ;\ \delta = -21°\ 58'.1.$

1900.560	182.7	2.94	9.0	9.0
.562	184.4	3.28	9.0	9.5
.568	185.8	3.18	9.0	9.2
1900.56	184.3	3.13	9.0	9.2

240. DM. −21°5010.

$\alpha = 18^h\ 22^m\ 0^s\ ;\ \delta = -21°\ 39'.0.$

1900.560	34.1	4.76	8.5	10.5
.562	35.0	4.88	8.5	11.0
.568	35.5	4.84	8.5	10.5
1900.56	34.9	4.83	8.5	10.7

241. DM. −21°5019.

$\alpha = 18^h\ 22^m\ 52^s\ ;\ \delta = -21°\ 26'.3.$

1900.560	35.4	4.12	9.0	10.5
.562	35.3	4.33	8.8	11.0
.568	36.3	4.34	8.5	10.0
1900.56	35.7	4.26	8.8	10.5

242. DM. −21°5024.

$\alpha = 18^h\ 23^m\ 46^s\ ;\ \delta = -21°\ 47'.7.$

1900.560	276.0	0.78	10.0	11.0
.562	275.8	0.76	10.0	10.5
.568	273.1	0.91	10.0	10.5
1900.56	275.0	0.82	10.0	10.7

243. DM. −17°5225.

$\alpha = 18^h\ 25^m\ 57^s\ ;\ \delta = -17°\ 1'.7.$

1900.626	353.9	1.38	9.2	9.5
.666	355.1	1.30	9.5	10.0
1900.65	354.5	1.34	9.4	9.8

244. DM. +11°3494.

$\alpha = 18^h\ 26^m\ 55^s\ ;\ \delta = +11°\ 58'.3.$

1900.415	254.6	1.14	9.0	12.5
.418	256.4	1.11	8.8	12.0
.470	255.8	1.00	8.8	11.5
.571	255.6	1.12	9.0	13.0
1900.47	255.6	1.09	8.9	12.2

245. DM. +11°3504.

$\alpha = 18^h\ 28^m\ 8^s\ ;\ \delta = +11°\ 43'.0.$

1898.269	53.2	2.10	7.5	8.8
.515	52.1	2.08	8.0	9.0
1900.412	51.9	2.00	9.0	9.8
1899.07	52.4	2.06	8.2	9.2

246. DM. −21°5056.

$\alpha = 18^h\ 29^m\ 11^s$; $\delta = -21°\ 45'.2$.

	°	″	M	M
1900.568	70.0	2.67	9.5	10.2
.740	68.0	2.70	9.0	10.0
.743	70.1	2.78	9.0	10.5
1900.68	69.4	2.72	9.2	10.2

247. DM. +10°3588.

$\alpha = 18^h\ 32^m\ 17^s$; $\delta = +10°\ 10'.8$.

1900.412	47.4	0.43	9.0	9.5
.415	45.3	0.50	9.0	9.2
.418	44.6	0.46	9.0	9.2
1900.42	45.8	0.46	9.0	9.3

248. DM. +9°3800.

$\alpha = 18^h\ 34^m\ 6^s$; $\delta = +9°\ 3'.4$.

1900.418	113.6	1.96	9.5	9.5
.470	113.6	2.00	9.5	9.6
.571	113.2	2.12	9.5	9.8
1900.49	113.5	2.03	9.5	9.6

249. DM. −14°5157.

$\alpha = 18^h\ 36^m\ 41^s$; $\delta = -14°\ 43'.0$.

1900.626	224.9	3.53	8.8	13.5
.666	224.0	3.21	8.8	13.5
.743	223.8	3.40	8.8	14.5
1900.68	224.2	3.38	8.8	13.8

250. DM. −15°5068.

$\alpha = 18^h\ 36^m\ 59^s$; $\delta = -15°\ 53'.1$.

1900.626	296.0	2.02	9.0	15.5
.666	296.4	2.11	9.1	14.5
.743	296.0	2.25	9.0	14.5
1900.68	296.1	2.13	9.0	14.8

251. DM. −15°5086.

$\alpha = 18^h\ 40^m\ 22^s$; $\delta = -15°\ 34'.8$.

1900.620	309.8	2.31	8.0	13.0
.666	311.3	2.28	8.0	12.5
.740	307.3	2.54	8.0	13.0
1900.68	309.5	2.38	8.0	12.8

252. DM. +9°3873.

$\alpha = 18^h\ 42^m\ 59^s$; $\delta = +9°\ 9'.2$.

1900.597	190.5	0.19	9.0	9.5
.626	193.0	0.20	9.0	9.5
1900.61	191.8	0.20	9.0	9.5

253. DM. +8°3853.

$\alpha = 18^h\ 43^m\ 14^s$; $\delta = +8°\ 34'.5$.

1900.597	321.5	0.61	8.8	13.0
.626	324.0	0.72	9.0	12.0
1900.61	322.8	0.66	8.9	12.5

254. DM. +7°3861.

$\alpha = 18^h\ 45^m\ 28^s$; $\delta = +8°\ 0'.4$.

1900.597	158.1	1.21	8.8	13.5
.626	156.4	1.17	9.0	13.5
1900.61	157.2	1.19	8.9	13.5

255. DM. −17°5350.

$\alpha = 18^h\ 45^m\ 48^s$; $\delta = -17°\ 25'.4$.

	°	″	M	M
1900.620	168.5	1.51	8.5	9.0
.666	170.8	1.66	8.0	9.0
.740	169.1	1.62	8.5	9.0
1900.68	169.5	1.60	8.3	9.0

256. DM. +8°3866.

$\alpha = 18^h\ 46^m\ 0^s$; $\delta = +8°\ 35'.2$.

1900.597	43.1	4.46	8.5	12.5
.626	43.4	4.43	8.5	13.0
1900.61	43.2	4.45	8.5	12.8

257. DM. −17°5359.

$\alpha = 18^h\ 46^m\ 57^s$; $\delta = -17°\ 32'.5$.

1900.620	341.3	2.16	9.1	12.0
.666	340.5	2.39	9.0	12.0
.740	341.3	2.31	9.0	11.5
1900.68	341.1	2.29	9.0	11.8

258. DM. +11°3651.

$\alpha = 18^h\ 48^m\ 20^s$; $\delta = +11°\ 29'.8$.

1900.591	217.4	2.57	8.8	9.2
.597	217.7	2.51	8.8	9.5
.626	213.3	2.50	9.0	9.5
1900.60	216.1	2.53	8.9	9.4

259. DM. +8°3896.

$\alpha = 18^h\ 50^m\ 7^s$; $\delta = +8°\ 22'.6$.

1900.597	7.3	0.19	9.2	9.5
.626	3.0	0.23	9.5	9.5
1900.61	5.2	0.21	9.3	9.5

260. DM. −16°5113.

$\alpha = 18^h\ 55^m\ 13^s$; $\delta = -16°\ 21'.2$.

1900.740	308.0	3.45	8.5	14.0
.743	306.8	3.31	8.8	14.0
1900.74	307.4	3.38	8.7	14.0

261. DM. −21°5233.

$\alpha = 18^h\ 58^m\ 21^s$; $\delta = -21°\ 40'.7$.

1900.740	124.5	0.42	7.0	7.0
.743	121.6	0.42	8.0	8.0
.776	123.4	0.35	8.0	8.0
1900.75	123.2	0.40	7.8	7.8

262. DM. −17°5552.

$\alpha = 19^h\ 10^m\ 53^s$; $\delta = -17°\ 58'.9$.

1900.533	359.5	1.85	9.0	9.2
.538	360.0	1.80	9.0	9.2
.549	358.2	1.89	9.0	9.2
1900.54	359.2	1.85	9.0	9.2

263. DM. −15°5302.

$\alpha = 19^h\ 11^m\ 57^s$; $\delta = -15°\ 8'.7$.

1900.702	19.5	2.19	9.0	12.0
.740	18.4	2.22	9.0	12.0
1900.72	19.0	2.20	9.0	12.0

264. DM. −16°5260.

$\alpha = 19^h\ 12^m\ 34^s$; $\delta = -16°\ 0'.7$.

	°	″	M	M
1900.538	290.0	1.46	8.5	13.5
.740	289.5	1.45	8.2	13.5
1900.64	289.8	1.46	8.4	13.5

265. DM. −17°5601.

$\alpha = 19^h\ 16^m\ 33^s$; $\delta = -17°\ 30'.4$.

1900.549	89.0	0.85	9.0	9.5
.571	91.7	1.00	9.5	9.8
.740	88.9	0.97	9.5	9.6
1900.62	89.9	0.94	9.3	9.6

266. DM. −16°5291.

$\alpha = 19^h\ 17^m\ 44^s$; $\delta = -16°\ 41'.9$.

1900.571	190.7	4.30	8.8	12.5
.740	187.9	4.21	8.8	12.5
1900.66	189.3	4.26	8.8	12.5

267. DM. −17°5785.

$\alpha = 19^h\ 48^m\ 5^s$; $\delta = -16°\ 54'.4$.

1900.442	347.3	1.60	8.0	14.0
.491	348.9	1.94	8.5	14.0
.571	354.2	1.62	8.5	14.5
.588	350.7	1.88	8.0	15.0
1900.52	350.3	1.76	8.3	14.4

268. DM. −15°5609.

$\alpha = 20^h\ 11^m\ 41^s$; $\delta = -15°\ 28'.6$.

1900.571	29.8	2.95	10.0	10.0
.666	29.7	3.23	9.0	9.0
.743	30.2	3.16	9.0	9.0
1900.66	29.9	3.11	9.3	9.3

269. DM. −18°5718.

$\alpha = 20^h\ 30^m\ 13^s$; $\delta = -18°\ 18'.6$.

1900.560	340.6	2.64	9.0	12.0
.741	340.3	2.80	9.0	12.5
1900.65	340.5	2.72	9.0	12.2

270. DM. −19°5902.

$\alpha = 20^h\ 37^m\ 48^s$; $\delta = -19°\ 28'.2$.

1900.560	91.1	1.94	9.5	10.0
.743	92.7	2.05	9.0	9.2
1900.65	91.9	2.00	9.2	9.6

271. DM. −17°6079.

$\alpha = 20^h\ 40^m\ 46^s$; $\delta = -17°\ 14'.7$.

1900.571	5.7	0.46	8.8	9.0
.626	5.6	0.47	9.0	9.2
.776	5.5	0.58	9.0	9.5
1900.66	5.6	0.50	8.9	9.2

272. DM. −14°5873.

$\alpha = 20^h\ 47^m\ 30^s$; $\delta = -14°\ 38'.7$.

1900.626	186.5	3.73	9.0	11.5
.666	187.3	3.55	9.0	12.0
.743	186.8	3.64	9.0	12.5
1900.68	186.9	3.64	9.0	12.0

273. DM. −16°5792.

$\alpha = 21^h\ 1^m\ 15^s\ :\ \delta = -15°\ 58'.4.$

	°	″	M	M
1900.571	117.1	1.12	8.0	13.0
.666	116.9	1.32	8.5	13.5
.740	117.6	0.94	8.0	13.5
1900.66	117.3	1.13	8.2	13.3

274. DM. +4°4612.

$\alpha = 21^h\ 12^m\ 14^s\ :\ \delta = +4°\ 12'.6.$

1900.491	149.1	1.14	8.5	15.5
.623	149.3	1.18	8.5	16.0
1900.56	149.2	1.16	8.5	15.8

275. DM. +7°4670.

$\alpha = 21^h\ 17^m\ 34^s\ :\ \delta = +8°\ 1'.7.$

1900.597	69.3	0.33	8.8	8.8
.623	65.7	0.35	8.8	9.0
.647	62.2	0.31	8.8	8.8
1900.62	65.7	0.33	8.8	8.9

276. DM. +7°4698.

$\alpha = 21^h\ 23^m\ 44^s\ :\ \delta = +7°\ 17'.0.$

1900.597	28.9	0.99	9.0	9.2
.623	26.1	0.89	9.5	10.0
.647	26.7	0.85	9.5	10.0
1900.62	27.3	0.91	9.3	9.7

277. DM. +6°4842.

$\alpha = 21^h\ 25^m\ 39^s\ :\ \delta = +6°\ 44'.1.$

1900.597	107.5	0.95	8.0	12.5
.623	108.8	1.05	8.5	11.5
.647	109.5	1.28	8.5	12.5
1900.62	108.6	1.09	8.3	12.2

278. DM. +5°4847.

$\alpha = 21^h\ 36^m\ 44^s\ :\ \delta = +5°\ 57'.9.$

1900.623	222.0	3.90	8.3	11.0
.647	221.8	3.82	8.3	13.0
1900.64	221.9	3.86	8.3	12.0

279. DM. +6°4884.

$\alpha = 21^h\ 37^m\ 17^s\ :\ \delta = +6°\ 47'.7.$

1900.623	357.0	2.48	9.0	9.1
.860	358.5	2.62	9.0	9.0
1900.74	357.8	2.55	9.0	9.1

280. DM. +5°4851.

$\alpha = 21^h\ 37^m\ 49^s\ :\ \delta = +5°\ 27'.2.$

1900.597	132.7	0.18	8.0	8.2
.623	140.7	0.20	7.5	8.0
.647	141.1	0.18	8.0	8.0
1900.62	138.2	0.19	7.7	8.1

281. DM. +4°4749.

$\alpha = 21^h\ 43^m\ 53^s\ :\ \delta = +4°\ 54'.9.$

1900.623	328.3	1.59	9.0	9.2
.647	326.5	1.56	9.0	10.0
1900.64	327.4	1.58	9.0	9.6

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282. DM. −14°6188.

$\alpha = 21^h\ 55^m\ 16^s\ :\ \delta = -14°\ 15'.2.$

	°	″	M	M
1900.647	29.1	0.77	7.5	9.0
.741	34.2	0.70	7.5	8.5
1900.69	31.6	0.74	7.5	8.8

283. DM. −17°6423.

$\alpha = 21^h\ 57^m\ 18^s\ :\ \delta = -16°\ 55'.0.$

1900.626	316.1	1.20	9.2	11.5
.743	316.0	1.17	9.2	10.5
1900.68	316.1	1.19	9.2	11.0

284. DM. −19°6230.

$\alpha = 22^h\ 3^m\ 50^s\ :\ \delta = -19°\ 28'.4.$

1900.745	112.3	3.27	9.0	9.2
.776	111.7	3.50	8.8	9.0
1900.76	112.0	3.38	8.9	9.1

285. DM. −15°6158.

$\alpha = 22^h\ 5^m\ 50^s\ :\ \delta = -15°\ 19'.4.$

1900.626	116.9	2.02	9.0	9.5
.743	116.5	1.95	9.0	9.2
1900.68	116.7	1.98	9.0	9.3

286.* DM. +4°4824.

$\alpha = 22^h\ 9^m\ 2^s\ :\ \delta = +5°\ 6'.7.$

1900.606	270.0	1.53	9.0	13.5

287. DM. +7°4836.

$\alpha = 22^h\ 12^m\ 52^s\ :\ \delta = +7°\ 47'.2.$

1900.606	67.7	1.55	8.2	13.5

288. DM. −16°6125.

$\alpha = 22^h\ 34^m\ 47^s\ :\ \delta = -16°\ 28'.5.$

1900.647	255.6	0.20	8.5	8.6
.743	254.0	0.21	8.5	8.5
1900.69	254.8	0.21	8.5	8.6

289. DM. −16°6142.

$\alpha = 22^h\ 38^m\ 11^s\ :\ \delta = -16°\ 39'.6.$

1900.713	102.5	1.57	8.8	9.0
.743	103.8	1.74	8.6	8.6
.806	103.6	1.62	8.5	8.5
1900.75	103.3	1.64	8.6	8.7

290. DM. −16°6150.

$\alpha = 22^h\ 40^m\ 39^s\ :\ \delta = -16°\ 6'.5.$

1900.713	356.4	3.10	9.0	11.0
.743	355.5	3.54	9.0	10.5
.806	354.4	3.17	9.0	11.5
1900.75	355.4	3.27	9.0	11.0

291. DM. −16°6152.

$\alpha = 22^h\ 41^m\ 57^s\ :\ \delta = -16°\ 40'.2.$

1900.713	6.7	2.24	7.0	10.5
.743	6.4	2.12	7.0	9.0
.806	4.9	2.01	7.2	10.0
1900.75	6.0	2.12	7.1	9.8

292. DM. −21°6409.

$\alpha = 23^h\ 16^m\ 17^s\ :\ \delta = -20°\ 50'.7.$

	°	″	M	M
1900.776	35.9	0.42	8.5	12.0
.836	38.6	0.42	8.5	11.0
1900.81	37.3	0.42	8.5	11.5

293. DM. −17°6737.

$\alpha = 23^h\ 16^m\ 38^s\ :\ \delta = -17°\ 15'.1.$

1900.711	291.3	1.00	9.0	11.0
.743	295.1	1.01	9.0	10.0
1900.71	293.2	1.00	9.0	10.5

294. DM. +4°4999.

$\alpha = 23^h\ 17^m\ 23^s\ :\ \delta = +4°\ 57'.1.$

1900.591	141.1	1.94	8.8	13.0
.776	140.7	1.83	8.8	13.5
1900.68	140.9	1.89	8.8	13.2

295. DM. −15°6406. 97 *Aquarii.*

$\alpha = 23^h\ 17^m\ 24^s\ :\ \delta = -15°\ 35'.3.$

1900.710	85.4	0.39	5.5	6.5
.713	83.4	0.34	5.5	7.0
1900.71	84.4	0.37	5.5	6.8

296. DM. −17°6742.

$\alpha = 23^h\ 17^m\ 59^s\ :\ \delta = -17°\ 5'.1.$

1900.740	191.6	4.09	8.8	11.0
.743	190.8	4.09	9.0	12.0
1900.74	191.2	4.09	8.9	11.5

297. DM. −16°6291.

$\alpha = 23^h\ 21^m\ 44^s\ :\ \delta = -15°\ 47'.9.$

1900.740	308.8	0.35	7.0	9.0
.743	315.9	0.35	7.0	9.0
1900.74	312.3	0.35	7.0	9.0

298. DM. +6°5168.

$\alpha = 23^h\ 27^m\ 7^s\ :\ \delta = +6°\ 32'.1.$

1900.606	90.8	0.16	7.0	7.5
.776	90.7	0.18	7.0	7.2
.920	93.4	0.18	6.5	7.5
1900.76	91.6	0.17	6.8	7.4

299. DM. −20°6612.

$\alpha = 23^h\ 27^m\ 45^s\ :\ \delta = -20°\ 15'.4.$

1900.745	72.8	0.54	9.0	9.2
.836	79.0	0.51	8.5	8.5
1900.79	75.9	0.52	8.8	8.9

300. DM. +5°5219.

$\alpha = 23^h\ 40^m\ 24^s\ :\ \delta = +5°\ 55'.8.$

1900.606	122.7	1.09	8.8	9.0
.776	124.6	1.12	8.8	9.0
.920	122.7	1.12	8.5	9.0
1900.77	123.3	1.11	8.7	9.0

ON THE NEW COMPONENT OF THE POLAR MOTION.

By S. C. CHANDLER.

Following are the facts, condensed as much as is consistent with clearness, which appear to support the conclusions that were briefly stated in *A.J.* 490. Table 1 gives all the essentials needed for the discussion, and embraces the results of all the series of observations that are suitable for the purpose.

TABLE 1.

Series	Observatory	Instrument	Dates	r_1	p	T_1	p	E	Reference
1	Kew and Wanstead	Zenith-Tube and Sector	1726–31	0.200	½	235 2250	½	−124	*A.J.* 250-251
2	Königsberg	Reichenbach Circle	1820–27	.171	1	238 6920	1	43	
3	Dorpat	Reichenbach Circle	1822–26	.145	½	7438	½	42	
4	Greenwich	Mural Circles	1825–30	.141	3	238 8646	3	39	*A.J.* 315
5	Greenwich	Mural Circles	1830–36	.134	2	239 0834	2	34	*A.J.* 315
6	Greenwich	Mural Circles	1836–43	.049	1	2776	½	29	*A.J.* 320
7	Pulkowa	Prime Vert. Transit	1840–55	.035	1	4112	½	26	*A.J.* 296
8	Pulkowa	Vertical Circle	1842–49	.057	1	4915	½	24	*A.N.* 3285
9	Greenwich	Mural Circles	1843–51	.092	1	5462	½	23	*A.J.* 320
10	Greenwich	Reflex Zenith-Tube	1852–56	.292	½	8459	½	16	
11	Greenwich	Transit Circle	1851–58	.069	1	239 8358	½	16	*A.N.* 3261
12	Greenwich	Reflex Zenith-Tube	1857–63	.246	1	240 0541	1	11	
13	Greenwich	Transit Circle	1858–65	.175	1	0976	1	10	*A.N.* 3261
14	Washington	Prime Vert. Transit	1862–67	.127	2	2295	2	7	*A.J.* 307
15	Pulkowa	Vertical Circle	1863–70	.235	2	2815	2	6	*A.J.* 293
16	Leyden	Meridian Circle	1864–74	.157	2	3176	2	5	Amst. Akad.
17	Greenwich	Reflex Zenith-Tube	1864–70	.222	1	3238	1	5	
18	Greenwich	Transit Circle	1865–72	.233	1	3665	1	4	*A.N.* 3261
19	Washington	Transit Circle	1866–71	.370	1	3592	1	− 1	
20	Pulkowa	Vertical Circle	1871–75	.181	1	5366	1	0	*A.J.* 293
21	Greenwich	Transit Circle	1872–79	.088	1	5794	½	+ 1	*A.J.* 322
22	Washington	Transit Circle	1871–78	.280	1	6183	1	2	
23	Pulkowa	Prime Vert. Transit	1875–82	.236	4	7075	1	4	*A.J.* 297
24	Greenwich	Transit Circle	1879–85	.162	1	8355	1	7	*A.J.* 322
25	Washington	Transit Circle	1879–86	.290	1	8779	1	8	
26	Pulkowa	Vertical Circle	1882–89	.150	2	240 9654	4	10	*A.J.* 426
27	Madison	Meridian Circle	1883–90	.152	2	241 0489	2	12	*A.J.* 307
28	Greenwich	Transit Circle	1884–91	.171	1	0520	1	12	*A.J.* 322
29	Lyons	Meridian Circle	1885–93	.175	1	0936	1	13	*A.J.* 334
30	Combination of series at many observatories	Principally Zenith-Telescope and Talcott's method	1890.0–90.9	.173	4	1371	2	14	
31			1891.5–92.4	.155	4	1790	2	15	
32			1892.5–93.4	.160	4	2215	2	16	
33			1893.5–94.7	.141	4	2654	2	17	
34			1894.8–95.9	.151	4	3080	2	18	
35			1896.0–97.1	.141	1	3521	2	19	
36			1897.2–98.2	.116	1	3963	2	20	
37			1898.3–99.8	0.114	1	241 4392	2	+ 21	

Following the serial number in the first column the next three columns sufficiently describe the nature and extent of the observations employed. Then come the values, with their weights, of the observed semi-amplitude r_1, and the observed epoch T_1 of minimum latitude referred to the meridian of Greenwich, after the elimination of the annual term of the latitude-variation; then the number of the epoch; and finally a reference to the place where the discussion will be found. In cases where no such reference is given the investigations are yet unprinted. It is regretted that space does not allow a full account here of the more important, at least, of these unpublished results, such as those for Kew and Wanstead, Königsberg, Dorpat and the Greenwich Reflex Zenith-Tube. But this must be postponed until it can be comprehensively done.

With regard to the selection of the data herein employed the governing condition is that each series should in itself contain the means of eliminating entirely or very nearly the annual component. A glance down the fourth column will give assurance that this could be effected in most cases, since in general the limiting dates comprise a harmonic seven-year cycle. Where this desirable condition was not entirely attainable the method of treatment has been adapted to remedy the defect as far as possible, and any remaining error from this source guarded against by reducing the weights. The choice of the weights depends

on various elements and, as in all like investigations, there is room for difference of judgement in allotting them, but not enough to influence materially the numerical values of the constants deduced, far less to alter the general conclusions. It is easy to discern shades of difference not recognized by the simple scale of weights here adopted, but over-elaboration may still easier degenerate into priggishness and the substance of precision be lost by grasping at its shadow.

The study of the subject can be pursued most conveniently by normals formed from Table I by combining near-lying epochs, as given below in Table II, in the columns "Observed r_1" and "Observed T_1". The subsequent columns will be presently explained.

TABLE II.

Series	E	p	Obs'd r_1	Comp. r_1	Obs'd T_1	p	T	Obs'd V	Comp. V
			″	″					
1	−124	½	0.200 ::	0.220	235 2250 ::	½	235 2197.0	+53 ::	−22
2, 3	43	1½	.162	.160	238 6949	1½	238 6905.5	+43	+44
4	39	3	.144	.127	238 8646	3	238 8619.5	+27	+18
5, 6	33	2½	.117	.070	239 1226	2½	239 1190.5	+35	+40
7–9	24	3	.061	.077	1972	1½	2047.0	−75	−45
10, 11	16	1½	.143	.150	239 8409	1	239 8475.0	−66	−43
12–14	9	4	.169	.205	240 1419	1	240 1474.5	−55	−27
15–19	− 5	7	.230	.223	3211	7	3188.5	+22	−15
20–22	+ 1	3	.202	.229	5778	2½	5759.5	+18	+ 3
23	4	4	.236	.225	7075	4	7045.0	+30	+12
24–26	9	4	.188	.193	240 9220	6	240 9187.5	+33	+26
27–29	12	4	.163	.184	241 0501	4	241 0473.0	+28	+36
30	14	4	.173	.170	1371	2	1330.0	+41	+40
31	15	4	.155	.160	1790	2	1758.5	+32	+42
32	16	4	.160	.152	2215	2	2187.0	+28	+43
33	17	4	.141	.143	2654	2	2615.5	+39	+45
34	18	4	.154	.135	3080	2	3044.0	+36	+47
35	19	4	.141	.125	3521	2	3472.5	+49	+47
36	20	4	.116	.116	3963	2	3901.0	+62	+48
37	+ 21	4	0.114	0.107	241 4392	2	241 4329.5	+62	+48

From the above normal values of T_1 we get the subjoined Table III of differences and corresponding observed periods.

TABLE III.

	Interval		Observed Period	Middle Epoch	Obs'd θ_1
					°
−43 to −33	4277	= 10 ×	427.7	−38	0.842
−39 ·· −24	6326	15	421.7	31½	.854
−33 ·· −24	3746	9	416.2	28½	.865
−33 ·· −16	7183	17	422.5	24½	.852
−24 ·· − 9	6447	15	429.8	16½	.837
−16 ·· − 5	4802	11	436.5	10½	.825
− 9 ·· + 1	4359	10	435.9	4	.826
− 5 ·· + 4	3864	9	429.3	− 0½	.838
+ 1 ·· + 9	3442	8	430.2	+ 5	.837
+ 4 ·· +12	3426	8	428.3	8	.840
+ 9 ·· +18	3860	9	428.9	13½	.839
+12 ·· +21	3891	= 9 ×	432.4	+16½	0.832

From the last column, of observed daily angular velocities, it is clear that this element is variable. But accepted dynamical theory requires it to be regarded as constant. What renders the contradiction more emphatic is the curious fact that strikingly appears from comparison of the observed values of r_1 and θ_1, namely, that the angular velocity and radius of motion have some inverse relation.

In view of this apparent deadlock between theory and observation what are we to do? As in similar circumstances twice before, I shall proceed as if there were no dynamical theory, inadmissible as this may seem to those who accept its precepts. I do not presume to meddle with the theory, but simply fail to recognize it as of any use in interpreting observation. For in truth the theory is beyond dispute in a peculiar sense of that phrase. It prescribes that the pole of rotation moves uniformly in a circle about the pole of figure wherever it may be at any instant, and that, for any displacement of the latter, the former will at once take the new point as the center of its circle, maintaining its original angular rate. Aside from displacements of a periodical kind like those producing the annual elliptical motion that is demonstrated by the observations, and to which theory seems reconciled, any other disturbances will have no effect on the rate of motion. Theory does not now, as it once erroneously did, undertake to assign this angular rate, but accepts the value of observation, namely, about 428 days for a complete revolution; but it still requires this to be invariable, and that the effect of displacements must be regarded as abrupt changes of phase and not of rate, and that any sequence of such changes can only accidentally produce a merely apparent change of rate during its brief operation. Any such prevailing alterations as those above brought out are consequently impossible.

It is manifest that this teaching of theory can never be disproved, for whenever and soever often as it is brought to face with an apparent alteration of period it can con-

sistently summon to its aid the hypothesis of an abrupt displacement or series of displacements of the pole, with their consequent derangements of phase. This makes it intangible and safe forever from destruction by any accumulation of hostile facts. In this attitude it stands curiously apart from other theories relating to natural phenomena. But, being thus insusceptible of correction by observed facts, it cannot be allowed to corrupt our construction of them. That it may do so is easily shown. By this doctrine any assumption as to the length of the period may be maintained, and all assumptions with the same facility. Take for example the interpretation of the above data according to any assumed period, it matters little what, say 432 days. After 1850 this agrees well enough with the observations, but before it not at all. Introduce the hypothesis of abrupt derangement at this point; alter, if you please, the enumeration of the elapsed revolutions between 1727 and 1825; or, if this does not suffice, imagine another abrupt displacement. Work similarly with the developments of future observation, and the adopted hypothesis of 432 days is on a permanently sure footing. And so of any other that fancy may choose. If this statement of it should appear to burlesque the comfortable elasticity of the present dynamic theory, it is not intentionally unfair or frivolous. Under this theory the dogma of chance accumulations of motion may be called upon for any service without limit.

This is why it is above said that the theory in question is beyond dispute but, by the same token, is useless for the purpose of drawing practical conclusions. It paralyzes one and sterilizes the other. We can never get anywhere with it, since it stalemates all attempts at advance. Undue deference to it has led at least one investigator astray. Thus, for an instance, the finest and most useful piece of evidence on this subject in the whole range of astronomy stands square across the path of the assumption of a constant period. I allude to POND's observations. Without actually presuming to discredit this evidence, it has recently been conveniently disposed of by the ingenious device above described. Yet this apparently refractory testimony in reality furnishes the key that unlocks the whole mystery. Without the help of POND's series, even with the mutually corroborative results afforded by BESSEL's and STRUVE's, we might easily be left at the end of the coming quarter of a century still groping for its true solution in spite of the quantity of new material of observation that will accumulate during that time.

This digression has seemed necessary to make plain the reason for ignoring the view that the earth's axis is jumping about in the disorderly fashion permitted by such an interpretation of dynamic theory, and for preferring to conduct the inquiry on the hypothesis that nature proceeds in a consequent manner in this as in other phenomena, until the facts, and not theory, compel us to relinquish it. The remarks may also be of service if they should in the future deter any investigator from following the delusive beckoning of this specter of theory into wrong paths.

Let us now try to get a true insight into the meaning of the facts. The first thing to be settled is the true average length of the period irrespective of its perturbations. The best that can be done, provisionally, is to deduce this from the longest intervals we can command in the whole range of 170 years covered by the available observations. Fortunately we have two intervals, of 100 years and 70 years — namely, between MOLYNEUX's and BRADLEY's epoch and that of BESSEL, STRUVE and POND, and between the latter and the latest observations — which enable us to define this element very nearly, without possible dispute as to the proper enumeration of the elapsed periods. For the middle epoch we find, by reducing series 2, 3, 4 and 5 to E = −39,

	Obs'd T_1	p	O−C
Bessel	238 8636	1	−27
Struve	8725	½	+62
Pond I	8646	3	−17
Pond II	8685	2	+22
Mean	238 8663		

Placing this mean value together with series I, and the mean of series 30–37, and making a rough estimate of the uncertainty of the several dates we have,

E	T_1
−124	235 2250 ± 50
− 39	238 8663 ± 20
+ 18	241 3090 ± 5

the intervals of which furnish the observed average period.

$$36413 \div 85 = 428^{d}.40 \pm 0^{d}.63$$
$$24427 \div 57 = 428.55 \pm 0.37$$

The agreement is quite satisfactory, and shows that whatever the inequality of the revolution may be it has not affected appreciably the differences of these epochs. To demonstrate that there can be no error in the numeration of the epochs we have, by different hypotheses,

$36413^{d} \div 86 = 423^{d}.4$	$24427^{d} \div 58 = 421^{d}.2$
$36413 \div 85 = 428.4$	$24427 \div 57 = 428.5$
$36413 \div 84 = 433.5$	$24427 \div 56 = 436.2$

The values in the first and third lines are incompatible with each other, as well as with all the other data. We may, therefore, adopt the mean value of the period, 428.5 days, as the best provisionally attainable, and as with reasonable certainty correct within a small fraction of a day.

To ascertain the nature of the deviations from this mean period we compare it with all the values of T_1 in Table II, using the value 240 5331 for the principal epoch, or

$$T = 240\,5331^{d} + 428^{d}.5\,E$$

as adopted in *A.J.* 490. The differences $T_1 - T = U$ are given in the column "Observed U." These, together with the "Observed r_1" of the same table, and "Observed Period" of Table III, may be perspicuously and comparatively studied by means of the accompanying diagrams.

where the broken lines represent the observed data, and the fainter continuous curves the theory propounded in *A.J.* 490, according to the numerical values of the constants and the formulas there given, from which also are derived the "Computed" values in Table II. It is needless to say that the small term in eq. (5) depending on $\theta - \theta''$ may be neglected.

This completes the statement necessary to enable an independent opinion to be formed on the reality of these twin-terms. It would be of no sort of scientific value or interest to recount the reasoning by which this simple explanation of the phenomena was arrived at. A close comparative study of the three curves in the diagram will suffice, I think, to carry conviction of the correctness of this explanation. But it is of importance to draw attention to the deviations of the observed from the computed curves, some of which appear to have a systematic character. While these may be due in part to sequences of accidental errors, I do not think they are wholly so. On the other hand it is not at present prudent to ascribe to them an objective existence. For one thing, it is to be remembered that the observed dates T_1 are derived through an elimination of the annual term which, for convenience, assumes the period of this term to be exactly a year. But the research in *A.J.* 489 upon the elements of the annual ellipse shows that this period is variable. The errors from this source must in any event be too small to affect the general nature of the curves here laid down. But it is possible that they may be large enough to be responsible in part for the systematic deviations of which we are speaking, especially as regards the deviations in the anomaly of the epoch between 1890 and 1899, which depend severally on observations covering a range of little more than a year each. It is proposed to make this subject a matter of special inquiry. Until this is done further speculation is idle. Nevertheless, in a subject where dynamical theory is so much beclouded, and at a loss as to the direction in which to push its search for causes, it seems permissible to utter a vague hint, that may be of service, without incurring the charge of undue venturesomeness. Therefore I beg to remark that, if the residual phenomena under discussion should prove, after sufficient scrutiny and confirmation by future observation, to have any objective reality at all, their course is such as to lead to the suspicion that there may still be a smaller term of shorter period, say of 112 days, coexisting with the others of 428.5 and 436.0 days above demonstrated. I know of nothing that makes this suggestion of possibility absurd, in the nature of things. It is manifest that we stand in the presence of a very complex phenomenon, and, although every step in its disentanglement must be taken with caution, all possibilities must be examined, with observation as our sole guide until dynamic theory has caught the clue, and may be used as an assistance instead of a hindrance to investigation.

OBSERVATIONS OF COMETS.

By C. D. PERRINE.

1900 Mt. Hamilton M.T.	✱	No. Comp.	☌ – ✱ $\Delta\alpha$	☌ – ✱ $\Delta\delta$	☌'s apparent α	☌'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			Comet *a* 1900 (Giacobini).					
July 28 11h 1m 15s	8	10 , 8	–1m 23s.31	+1′ 24″.6	20h 3m 11s.22	+43° 39′ 59″.0	9.582	*n*8.845
29 10 39 31	7	10 , 6	–1 15.04	–1 50.7	19 56 16.14	+43 8 18.1	*n*9.117	*n*9.924
Aug. 1 15 26 18	6	10 , 6	–1 6.78	–3 52.1	19 12 11.76	+38 38 35.5	9.768	0.502
17 12 6 1	5	10 , 8	+0 8.04	+0 11.8	18 12 50.80	+28 11 14.1	9.637	0.411
			Comet *b* 1900 (Borrelly–Brooks).					
July 30 11 11 16	1	10 , 8	–0 12.92	+0 40.0	2 51 39.16	+33 50 31.0	*n*9.655	0.320
Aug. 3 11 3 20	2	10 , 8	–0 8.25	+1 44.5	2 58 0.30	+45 58 32.5	*n*9.773	0.107
4 16 25 6	3	8 , 6	–1 15.69	–2 2.3	3 0 9.07	+49 11 50.1	*n*9.483	*n*0.130

✱	α	Red. to app. place	δ	Red. to app. place	Authority
1	2h 51m 18s.78	+3s.30	+33° 49′ 16″.7	+ 4″.3	Leiden A.G. Zones, 239, 121; 333, 150
2	2 58 1.81	+3.74	+45 56 47.6	+ 0.4	Deichmüller, Bonn A.G. Catal. 2590
3	3 1 50.78	+3.98	+49 13 53.1	– 0.7	*ι Persei*, B.J.
4	18 10 12.02	+3.22	+28 11 16.8	–17.2	Graham, Cambridge A.G. Catal. 8753
5	18 12 39.52	+3.24	+28 13 15.3	+17.3	Micrometer comparison with (4)
6	19 13 17.96	+3.58	+38 42 9.8	+18.1	Lund A.G. Zones 426; 428
7	19 57 57.71	+3.71	+43 9 51.8	+17.0	Deichmüller, Bonn A.G. Catal. 13665
8	20 4 30.80	+3.76	+43 38 17.9	+16.5	" " " " 13788

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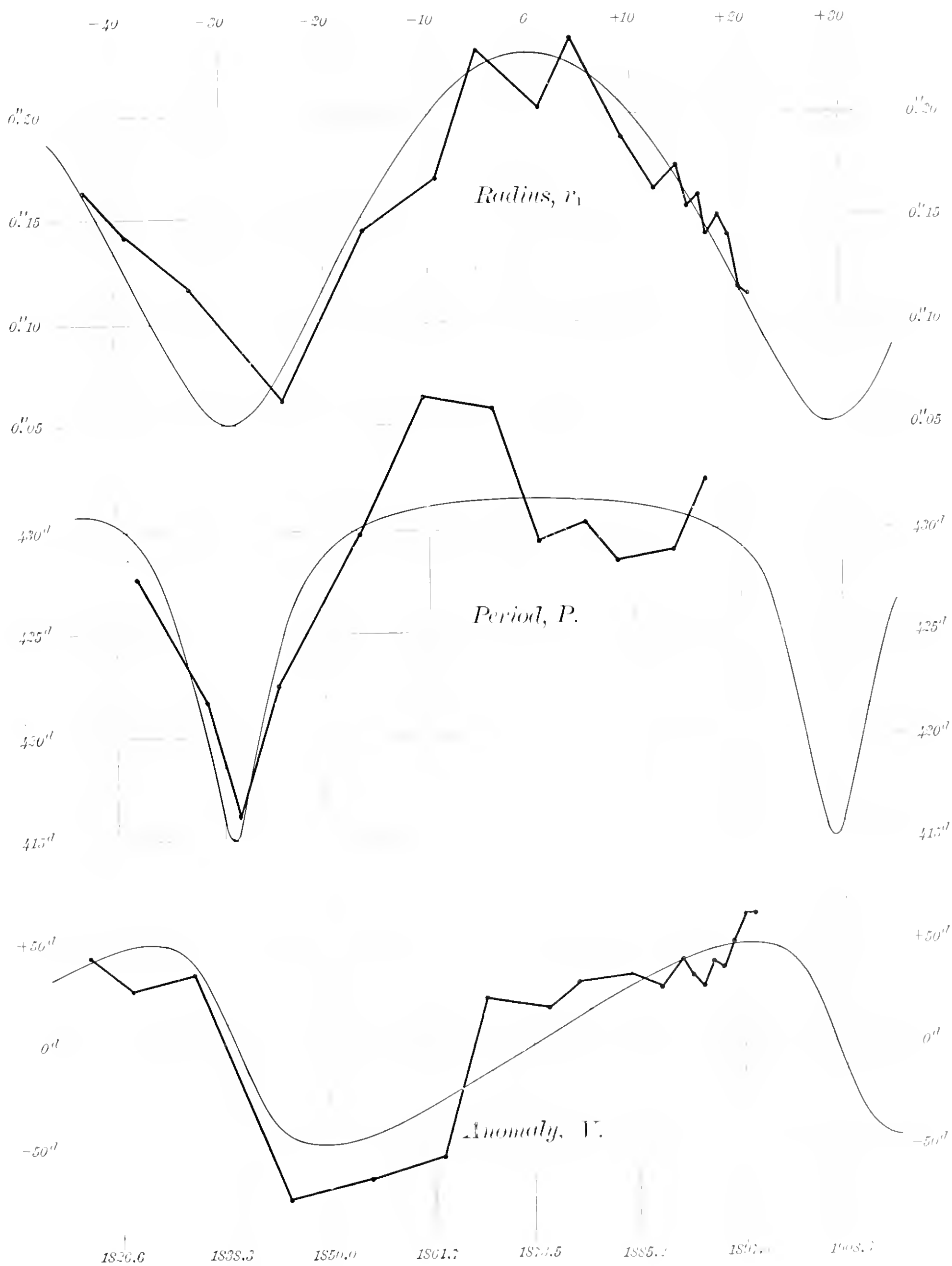
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−30
−20
−10
0
+10
+20
+30
0.″20
0.″15
0.″10
0.″05
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425^d
420^d
415^d
Period, P.
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0^d
-50^d
Anomaly, V.
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1838.3
1850.0
1861.7
1885.

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THE PROBLEM OF THREE BODIES.

By A. Hall.

Half a century after the publication of Newton's *Principia* the investigations of theoretical astronomy were taken up in earnest by the European mathematicians, Clairaut, d'Alembert, and Euler. It was easy for them to solve the problem of two bodies, but when they came to consider the motions of the moon disturbed by the action of the sun, although the differential equations of motion could readily be formed, the integrations could not be made. Clairaut says, "integrate who can." The equations of motion were reduced to an elegant and natural form, especially by Euler, but for the integration resort was had to expansion of the disturbing forces into series. In 1772 Lagrange investigated the problem of three bodies in a very remarkable memoir. This work has never been superseded. Lagrange found the problem insoluble except under certain special conditions. These conditions are that the bodies shall be on a right line, or are at the vertices of an equilateral triangle. Laplace proves these special results in a simple manner in his *Mécanique Céleste*, Livre X. The three bodies describe similar conic sections around their center of gravity, forming constantly an equilateral triangle whose sides vary continuously, and may become infinite if the conic is a parabola or an hyperbola. If the sides are not equal in order that the problem be soluble the three bodies must lie on a right line. But it was shown by Liouville in 1845 that these solutions are only mathematical fictions, and cannot exist in nature, since the systems are unstable.

A discussion of the general equations of motion of n material particles, which mutually attract each other, and are not acted on by external forces, shows that we can make ten integrations. Six of these relate to the motion of the center of gravity of the system, which is rectilinear and uniform; three belong to the conservation of areas; and one is the equation of living force. Since in this question we shall have $3n$ linear differential equations of the second order, we shall have to perform $6n$ integrations for the complete solution. In the case of three bodies there will be 18 integrations, and hence 8 integrations that cannot be made. This consideration is sufficient to point out the course the solution must take; that is, the expansion into series. It is fortunate for the astronomer that our solar system is arranged so that the motions of the planets can be determined by series. The mass of our sun is so great that the solution for two bodies gives a good approximation, which can be extended to the general problem by successive approximations. Here it is assumed that the series employed are convergent.

Any special solution of the problem of three bodies is interesting, since it may lead to better methods of treating the general question. G. W. Hill has applied the integral of Jacobi to the lunar theory. The derivation of this integral is given by Jacobi in his "*Vorlesungen über Dynamik*." It results from a combination of the equation of living force with the principle of the conservation of areas. The assumption is made that the two attracting bodies move in circles around their common center of gravity, with equal angular velocities. This is not of course a real case in nature, but in some respects it is a good approximation, and such a solution is valuable. Mr. Hill employs also Euler's idea of revolving axes. The equations for such axes are given by Laplace (*Méc. Cél.*, Livre II). G. H. Darwin has made an interesting application of Hill's formulas to an imaginary case of three bodies.

In his recent work, "*Les Methodes Nouvelles de la Mécanique Celeste*," M. Poincaré has made a very complete discussion of the various methods proposed for solving the problem of three bodies. His conclusion is that the series employed by astronomers for expressing the disturbing forces are divergent, speaking in the general and strict mathematical sense. It follows that the astronomical methods, although good for a few centuries, will not serve for an indefinite time. Laplace points out that most of the questions of astronomy depend on the solution of the differential equation,

$$\frac{d^2y}{dt^2} + a^2y + aQ = 0$$

In this equation y is any coordinate, such as a radius

vector, a longitude, or a latitude; a is a constant, α is a small parameter, and Q is a rational and periodical function of y and t. This equation is solved in the usual manner by first omitting the term αQ, and afterwards extending the solution to the general equation by varying the constants of integration of the first solution. When this is done it is found that in certain terms the time appears outside the signs *sine* and *cosine*. LAGRANGE met this difficulty in computing the secular perturbations of the planets, and he overcame it by the introduction of a superfluous constant, and imposing on this constant the condition of making the undesirable terms disappear. This method is employed by LAPLACE. In his second volume POINCARÉ gives a full discussion of the methods proposed by LINDSTEDT, NEWCOMB, GYLDÉN, DELAUNAY, and BOHLIN, for obtaining a solution of the above equation which shall be free from secular terms. Of these methods that of LINDSTEDT is the most direct. It depends on a change of the variable, and is explained best in the memoirs of the author. As LAPLACE has said, it often happens that an editor or critic injures the clearness of the original writer. POINCARÉ shows that the method of LINDSTEDT has a general application, but the series of this method, as well as those of the others, are divergent, in the strict mathematical sense. This result appears to be correct, and hence we cannot draw general conclusions, such as those of LAGRANGE and POISSON, from our astronomical formulas.

But one may doubt, I think, whether the instability of our solar system is proved. If we denote by i and i' two whole numbers, and by μ the ratio of the mean motions of the disturbed and disturbing bodies, the form $i - i'\mu$ will enter as a divisor into the expressions of the perturbations, and the square of this quantity will appear as a divisor in terms of the longitude. Since μ is a decimal fraction without limit we can find values of i and i' which will render $i - i'\mu$ as small as we please. Thus for *Egeria* and *Jupiter* we have

$$\mu = \frac{299''.1286}{857.9361} = 0.348660577$$
$$\begin{aligned} 1 - 2\mu &= +0.302679 \\ 1 - 3\mu &= -0.045982 \\ 7 - 20\mu &= +0.026788 \\ 8 - 23\mu &= -0.019193 \\ 15 - 43\mu &= +0.007595 \\ 38 - 109\mu &= -0.004003 \end{aligned}$$
etc.

The order of the disturbing terms is therefore very great as we approach commensurability of motions. So far as observations indicate *Egeria* has no sensible disturbance from these terms. May not the trouble arise rather from the form of solution, than from any defect in nature. One gets the conviction that our solar system is stable.

We must be thankful to M. POINCARÉ for his labor, and for the great mathematical skill shown in his work. The reader will feel a kind of negative result, as though he were left in darkness. Not a single numerical example is given of the use of the formulas, and in fact the application of most of them appears to be extremely difficult. Perhaps we shall be obliged to return to the methods of HANSEN, which are the best now known. An astronomer will not feel much interest in an "*intermediare Bahn*," or an "*absolute Bahn*," if he can never find one.

Cambridge, 1901 *March* 11.

NOTES ON THE VISUAL SPECTRUM OF *NOVA PERSEI*,

BY EDWIN B. FROST.

These observations have followed in a qualitative way the changes in the spectrum noted with a direct vision ocular spectroscope and nine-inch refractor since my last communication.

In the early evening of Feb. 24, at about 7^h Eastern Standard Time, the magnitude of the *Nova* was estimated as about 0.3, only slightly fainter than *Capella*, which it closely resembled in color. At $10^h\ 30^m$ its brightness had evidently diminished, and was about 0.5 mag.

February 25, 9^h. The magnitude had fallen to about the first, or a little below, and a glance at the spectrum showed that this loss in brightness was due to a diminution in the intensity of the continuous spectrum. The bright lines and bands were considerably more intense than on the previous evening, particularly in case of $H\alpha$, which also seemed broader. The dark lines seemed relatively less intense. Two faint bright lines were suspected beyond $H\alpha$ toward A. A fine bright line was seen adjacent to dark D on its violet side, and seemed to have a faint dark companion on its own violet side. This bright line would fall near the position of helium D_3, but might be due to the uncovering of a part of bright D by a superposed dark D. Dark D is sharp, but not seen as double. $H\alpha$ considerably broader than last night; $H\gamma$ also more conspicuous. The continuous spectrum, both last night and to-night, is relatively weak in the blue and violet as compared with the yellow and green. The maximum intensity is estimated to fall midway between D and b. The strong dark components on the violet sides of the principal bright bands shade off toward the violet.

Twenty bright lines were observed in the visual spectrum to $H\gamma$.

The relative intensities of the principal bright bands were roughly estimated as follows:

$H\alpha = 120$, $D = 70$, $b = 100$, $\lambda\,4924 = 10$, $H\beta = 50$.

February 26, $8^h\ 30^m - 10^h$. Magnitude estimated as about 1.3. The continuous spectrum has diminished somewhat in intensity. The very broad dark band adjacent to the violet side of $H\alpha$ suggests the possibility of its resolution into fine lines. Bright D appears to be rendered double by a fine dark central line of separation, while dark D is sharp and narrow. The bright line toward violet of dark D, which may be D_3, is distinct. The most important change is the presence of dark fringes on the side toward *red* of bright $H\beta$ and b. These dark lines are clearly seen, but are less conspicuous than the dark components on the violet side which shade off toward violet after the general manner of the bands of fourth-type spectra. A broad dark band, apparently without a bright companion, was seen at some distance toward red from $H\gamma$, in the vicinity of $\lambda 445$.

Estimates of intensities of bright lines:

$H\alpha = 120$, $D = 50$, $b = 100$, $\lambda 5016 = 35$, $H\beta = 75$.

February 27, 8^h and 10^h. Estimate of magnitude about 1.7. Dark lines again visible on both sides of b and $H\beta$, those on the red side being the narrower. A similar dark line now visible on red side of bright D. The most marked feature to-night is the extreme blackness of the two dark bands between b and $H\beta$, of which that on the violet side of $\lambda 5016$ is the stronger. Both bright $H\beta$ and b appear to be doubled by a dark line of separation. (It will be recalled that such duplications (reversals?), and more complicated effects, were noticed by many observers of *Nova Aurigae*).

$H\alpha = 150$, $D = 40$, $b = 80$,
$\lambda 5016 = 25$, $\lambda 4924 = 5$, $H\beta = 70$.

February 28, 8^h. Magnitude about 2.0. Bright $H\beta$, b and D are double, and dark lines are seen on the *red* sides of each (in addition to the conspicuous dark bands on the violet sides). No duplication of bright $H\alpha$ can be detected. Bright D_3(?) is faint. Great deficiency in the blue and violet parts of the continuous spectrum is noted on comparison with spectra of other stars.

$H\alpha = 100$, $D = 35$, $b = 60$,
$\lambda 5016 = 15$, $\lambda 4924 = 5$, $H\beta = 50$.

March 2, $7^h\ 30^m$. Estimated magnitude = 2.5; color seemed whiter than heretofore. $10^h\ 30^m$. $H\alpha$ and $H\beta$ seem to have lost little in intensity. Dark lines on *red* sides of bright bands are somewhat less conspicuous. Bright $H\beta$ doubled, and now more intense than b.

$H\alpha = 100$, $D = 40$, $b = 50$,
$\lambda 5016 = 15$, $\lambda 4924 = 5$, $H\beta = 80$.

March 6, $9^h\ 30^m$. Estimated magnitude = 3.1. Continuous spectrum has fallen off decidedly. Bright D very faint. Dark band on violet side of bright $H\alpha$ less conspicuous than usual, and the background of continuous spectrum between $H\alpha$ and D is very weak. The bright line near position of D_3 cannot now be seen. Duplicity of bright b and $H\beta$ suspected, but difficult.

Rough estimates of intensity of bright bands:

$H\alpha = 80$, $D = 15$, $b = 25$, $\lambda 5016 = 10$, $H\beta = 50$.

I have to thank Professors Nichols and Hull, and Mr. A. A. Bacon, for confirming my observations as to delicate details noted above.

Dartmouth College, Hanover, N.H., 1901 March 8.

OBSERVATIONS OF COMET *b* 1900 (*BORRELLY-BROOKS*),

MADE AT SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS., WITH THE 11-INCH REFRACTOR AND FILAR MICROMETER,

By MARY E. BYRD.

1900 Greenwich M.T.	*	No. Comp.	☌ – * Δα	☌ – * Δδ	☌'s apparent α	☌'s apparent δ	log pΔ for α	log pΔ for δ
Aug. 21 16 8 41 (h m s)	1	9 , 6	−1 38.80 (m s)	−0 20.0 (′ ″)	5 18 3.35 (h m s)	+83 20 12.9 (° ′ ″)	*n*0.512	0.593
25 16 18 1	2	9 , 6	−8 17.26	−6 8.2	8 38 13.56	+85 50 10.2	*n*0.223	0.824

Mean Places for 1900.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
1	5 49 32.14 (h m s)	+10.01 (s)	+83 21 19.5 (° ′ ″)	−16.6 (″)	Carrington 755
2	8 17 2.98	−2.16	+85 57 9.6	−21.2	Carrington 1259

OBSERVATIONS OF MINOR PLANETS AND COMETS,

MADE WITH THE 12-INCH TELESCOPE OF THE VASSAR COLLEGE OBSERVATORY,

By MARY W. WHITNEY AND CAROLINE E. FURNESS.

1900 Greenwich M.T.	*	No. Comp.	Planet – * $\Delta\alpha$	Planet – * $\Delta\delta$	Planet's Apparent α	Planet's Apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ	Obs.
			(148) *Gallia.*						
Jan. 22 15^h 14^m 28^s	1	8 , 8	+0^m 20.50^s	+1′ 57.5″	7^h 11^m 12.71^s	+ 2° 22′ 52.8″	n9.148	0.747	W
23 13 31 10	2	8 , 8	+0 25.17	−7 20.4	7 10 24.06	+ 2 34 32.9	n9.180	0.750	W
27 11 17 24	3	10 , 10	−0 4.45	−5 54.2	7 36 58.93	+ 3 26 33.9	n9.301	0.739	W
			(19) *Fortuna.*						
Feb. 20 15 16 43	4	5 , 5	−2 1.41	−2 52.1	11 17 35.24	+ 2 35 31.5	n9.502	0.751	W
			(451) [1899 *EY*].						
Mar. 3 15 50 53	5	8	−0 19.27	−8 41.6	4 29 21.95	+21 40 11.8	9.667	0.693	W
			(382) [1894 *AT*].						
Apr. 2 14 41 37	6	8*	−0 33.38	−2 21.8	11 15 40.51	− 3 51 51.4	n8.942	0.798	W
3 13 39 41	6	8*	−1 11.19	+0 41.8	11 15 2.70	− 3 48 44.9	n9.284	0.795	W
			(105) *Artemis.*						
Apr. 7 14 56 42	7	10*	+1 1.12	+3 41.6	13 2 7.84	− 0 2 46.4	n9.579	0.768	F
8 15 36 15	8	10*	−0 24.32	−2 1.4	13 1 25.39	+ 0 18 23.5	n9.140	0.765	F
9 15 41 24	9	8*	−2 8.88	+3 7.4	13 0 44.38	+ 0 39 6.0	n9.059	0.762	F
10 15 39 9	10	9*	+0 24.62	+9 43.9	13 0 5.93	+ 0 59 31.0	n9.064	0.759	F
			(11) *Parthenope.*						
May 7 15 45 38	11	9	−0 38.75	+6 41.8	16 6 9.42	−13 22 13.9	n9.398	0.843	F
			Comet *b* 1900 (Brooks).						
Sept. 26 13 49 55	12	9 , 6	−3 53.87	−1 36.5	14 21 44.28	+69 51 55.6	0.096	0.540	W
27 15 39 44	13	12 , 7	−2 47.74	−5 23.6	14 23 34.22	+69 36 30.0	9.976	0.790	W
			(241) *Germania.*						
Sept. 26 15 56 58	14	15 , 10	−0 23.33	+8 31.3	0 0 36.54	+ 9 27 21.1	n8.861	0.672	W
Oct. 5 15 39 45	15*	6	+0 17.55	−2 45.1	23 54 0.41	+ 8 39 2.1	n8.384	0.681	W
10 14 18 2	16	7 , 9	+0 5.31	−1 15.5	23 50 43.16	+ 8 11 29.9	n9.135	0.690	W
11 13 27 55	17	6 , 12	+0 9.72	+6 18.1	23 50 6.95	+ 8 6 12.2	n9.338	0.697	W

Mean Places for 1900 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
1	7^h 10^m 49.57^s	+2.61^s	+ 2° 18′ 5.0″	− 9.7″	Boss, Albany A.G. Catal. 3007
2	7 39 56.24	+2.65	+ 2 42 3.1	− 9.8	" " " " 3000
3	7 37 0.71	+2.67	+ 3 32 38.4	−10.3	" " " " 2970
4	11 19 36.96	+2.72	+ 2 38 44.1	−17.5	" " " " 4265
5	4 29 39.45	+1.77	+21 48 51.1	+ 2.3	Becker, Berlin B, A.G. Catal. 1468
6	11 16 10.98	+2.91	− 3 49 6.5	−20.1	Radcliffe 1890, 2935
7	13 1 3.64	+3.08	− 0 6 9.4	−18.6	½ [Munich 1 + Munich II]
8	13 1 46.63	+3.08	+ 0 20 43.5	−18.6	Munich I
9	13 2 50.18	+3.08	+ 0 36 17.2	−18.6	Boss, Albany A.G. Catal. 4649
10	12 59 38.23	+3.08	+ 0 50 5.7	−18.6	" " " " 4638
11	16 6 44.63	+3.54	−13 28 47.4	− 8.3	Paris 20262
12	14 25 40.76	−2.61	+69 56 34.7	− 2.6	Fearnley & Geelmuyden, Christiania A.G.C. 2149
13	14 26 24.55	−2.59	+69 41 56.3	− 2.7	" " " " 2152
14	0 0 55.35	+4.52	+ 9 18 20.8	+29.0	Bruns and Peter, Leipsic II, A.G. Catal. 11873
15	23 53 38.34	+4.52	+ 8 41 17.2	+30.0	" " " " " " 11835
16	23 50 33.35	+4.50	+ 8 12 15.2	+30.2	" " " " " " 11814
17	23 49 52.73	+4.50	+ 7 59 23.8	+30.3	" " " " " " 11812

* Observations made with square-bar micrometer.

OBSERVATIONS OF *EROS*,

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY OF THE UNIVERSITY OF VIRGINIA,

By HEBER D. CURTIS.

In the following observations the sets were usually $\Delta\delta$ 6, $\Delta\alpha$ 12, $\Delta\delta$ 6, with symmetrical reversals of the movable wire and of the micrometer box. The recommendations of the Paris Astrophotographic Conference were followed as closely as possible. For the purpose of identifying the stars used, the approximate apparent place has been formed by applying the observed $\Delta\alpha$ and $\Delta\delta$ with contrary sign to the position of the planet calculated for the time of observation from the ephemeris of E. MILLOSEVICH (*A.N.* 3660–61). The observations are corrected for refraction, but not for aberration.

Date	Char. M.T.	$\Delta\alpha$	$\Delta\delta$	log $p\Delta\alpha$	log $p\Delta\delta$	No. of Settings	Star	Approx. app. place of Star α	δ	
1900	h m s	s	′ ″				M	h m s		
Sept. 23	11 50 5	. . .	−0 13.6	8.719	n9.801	10	10.0	2 11 42	42 1.4	
24	14 6 39	− 3.23	+1 46.6	n8.754	n9.850	20, 10	10.0	2 42 9	42 24.5	
25	13 31 49	+ 9.31	−0 54.7	n9.152	n9.758	20, 20	10.6	2 42 21	42 49.2	
26	12 58 15	+ 7.36	+3 19.2	n9.347	n9.631	16, 16	11.5	2 42 45	43 7.3	
27	15 59 51	+10.34	+1 31.4	9.445	n9.608	16, 10	11.0	2 43 5	43 35.1	
Oct. 7	12 52 49	+ 5.40	−3 9.5	n9.133	n0.024	10, 10	9.8	2 43 26	47 18.3	
9	13 6 28	− 5.54	−1 36.6	n8.841	n0.074	10, 10	9.0	2 42 56	47 59.2	A.G. Bonn 2104
14	12 20 32	− 7.56	+0 41.2	n9.156	n0.240	12, 12	10.0	2 40 6	49 37.4	
15	13 44 11	+10.03	+1 46.2	9.122	n0.211	12, 12	10.0	2 38 58	49 56.5	
16	12 18 24	+ 5.37	−2 35.5	n9.087	n0.258	12, 12	9.5	2 38 13	50 18.5	
17	7 29 5	+ 1.91	+3 2.2	n9.857	0.418	21, 12	9.5	2 37 27	50 27.5	DM. 50°616
17	8 9 39	+31.77	+0 13.0	n9.838	0.229	21, 12	10.5	2 37 2	50 31.1	
18	8 4 57	+ 5.95	+2 49.7	n9.810	0.215	21, 12	9.4	2 36 21	50 46.0	DM. 50°612
19	7 18 29	+ 2.07	−3 39.7	n9.862	0.409	24, 12	11.3	2 35 26	51 9.0	
19	11 15 2	−10.05	−0 42.6	n9.439	n0.213	24, 12	11.3	2 35 26	51 9.0	
20	6 58 50	+ 4.19	−3 27.9	n9.868	0.460	24, 12	10.0	2 34 14	51 25.8	
20	12 43 6	−14.74	+0 40.9	8.288	n0.314	12, 12	10.0	2 34 14	51 25.8	
21	9 13 32	+10.22	+0 15.5	n9.746	n9.234	21, 12	11.5	2 32 48	51 39.8	
24	13 4 56	+13.19	+3 10.2	9.208	n0.320	12, 12	11.0	2 28 19	52 24.2	
29	7 14 19	+13.53	−3 0.5	n9.872	0.215	21, 12	10	2 20 23	53 27.9	
Nov. 5	7 30 56	+29.95	+3 20.7	n9.808	n9.144	21, 12	10.0	2 6 32	54 11.1	
13	9 53 17	+28.06	−0 7.5	n8.955	n0.387	12, 12	10.5	1 50 3	54 11.6	
15	9 47 45	− 5.37	+0 32.4	n8.836	n0.385	12, 12	10.5	1 16 47	53 59.9	
16	10 0 48	− 5.41	−0 15.1	n7.589	0.385	12, 12	11.0	1 14 56	53 53.5	
17	8 35 6	+ 4.26	+0 27.6	n9.435	n0.314	12, 12	10.5	1 43 7	53 15.2	
21	9 22 44	−20.80	−0 21.9	n8.585	n0.361	12, 12	11.5	1 37 1	53 3.8	
27	8 0 3	+18.62	−4 3.9	n9.311	n0.274	12, 12	7.0	1 29 30	51 39.5	Camb. A.G. 709
29	7 15 54	− 9.86	−1 18.4	n9.487	n0.179	12, 12	10.0	1 28 29	51 1.9	
Dec. 1	11 9 3	+ 4.86	−2 59.6	9.639	n9.944	12, 12	9.3	1 27 10	50 22.5	DM. 50°304
1	12 14 31	0.00	0 0.0	9.759	9.273	see note	12.0	1 27 11	50 18.6	
2	6 18 15	− 2.57	−1 47.6	n9.625	n9.931	12, 12	10.3	1 26 58	50 5.5	
2	6 55 44	+ 0.91	+1 34.0	n9.490	n0.127	12, 12	11.3	1 26 57	50 1.6	
8	8 22 38	+ 6.68	+0 4.7	8.160	n0.185	12, 12	10.7	1 27 4	47 52.9	
9	9 17 19	+ 7.22	−0 5.2	9.270	n0.107	12, 12	11.0	1 27 31	47 29.3	
10	7 21 56	− 1.24	+0 19.8	n9.160	n0.075	12, 12	10.8	1 28 8	47 7.5	
11	6 54 55	+ 2.38	−1 12.5	n9.324	n0.014	12, 12	10.8	1 28 43	46 46.0	
12	7 28 33	− 7.57	−0 35.2	n9.020	n0.091	12, 12	10.0	1 29 38	46 20.9	
13	9 19 19	+11.72	+2 35.9	9.317	n9.976	12, 12	9.5	1 30 14	45 51.8	
14	9 14 32	− 1.34	−0 21.0	9.334	n9.953	12, 12	11.0	1 31 24	45 30.5	
16	7 46 38	. . .	−1 35.6	. .	n0.020	6	10.0	1 33 3	44 43.8	
16	7 53 31	+29.34	. . .	7.356	. .	6				
27	6 28 59	. . .	+0 3.3	. .	n9.285	6	8.5	1 52 28	40 3.5	A.G. Bonn 1683
27	6 41 36	−16.36	. . .	n9.018	. .	9				
29	7 7 10	+17.18	+3 42.3	n8.661	n9.282	12, 12	9.5	1 56 27	39 7.6	
1901 Jan. 4	6 20 18	+ 3.29	+0 4.0	n9.114	9.493	12, 12	9.1	2 11 43	35 37.3	DM. 36°155
5	7 40 50	+ 7.72	−1 59.0	8.809	9.455	12, 12	10.0	2 14 32	36 12.2	
8	7 31 45	− 8.46	+1 38.4	8.692	9.671	12, 12	10.0	2 23 29	34 51.5	
9	6 58 50	+ 0.76	+2 31.2	n8.463	9.719	12, 12	10.0	2 26 2	31 25.5	
29	7 18 11	− 3.09	−3 14.5	8.648	0.257	12, 12	10.0	3 33 12	26 7.6	

NOTES.

Oct. 7th, Cloudy: seeing 3. 16th, High wind; woolly images. 20th, Seeing poor. 29th, Clear, but images dancing. — *Nov.* 5th, Quite windy. 13th, Windy; large, woolly images. — 17th, Images poor and jumping violently. 27th, A very poor night. — *Dec.* 1st, Air very steady; seeing 1. Tried powers of 850, 1300, and 2000. *Eros* seems quite yellow as compared with DM. 50 301, and the focus is different for best definition on star and planet. At times fancied a very minute disc could be glimpsed, but probably illusory. *Eros* occulted a 12M star which forms the second observation on Dec. 1. Powers 165 and 500 were used, and the object appeared single for about 1m 30s. — *Dec.* 8th, Seeing 4; slightly hazy. — 11.3, Conditions poor. 13th, Windy; seeing poor. — 27th, Stopped by clouds. — *Jan.* 8th, Hazy. — 9th, The n.f. star of a pair of close 10M stars was used. — 29th, Light clouds.

LATITUDE-OBSERVATIONS MADE AT THE IMPERIAL ASTRONOMICAL OBSERVATORY AT KASAN,

By M. A. GRATCHOF, Observer at the Observatory.

[Communicated by Prof. Dubiago, Director.]

Series VI*

Date	φ	Pairs
1899 Nov. 4	55° 47′ 23″.24	17
22	23.03	15
24	23.12	9
27	23.18	2
Dec. 11	23.37	13
17	23.47	17
27	22.95	8
29	23.24	17
30	23.20	17
1900 Jan. 5	23.57	17
9	23.13	17
10	23.24	11
11	23.31	17
19	23.33	17
31	23.20	8
Feb. 6	23.18	16
7	23.33	16
8	23.12	16
9	23.16	8
19	23.15	16
21	23.27	16
22	23.30	16
Mar. 7	23.42	8
17	23.10	2
18	23.54	15
19	23.35	15
21	23.21	15
22	23.09	15
25	23.12	15
30	23.40	15
Apr. 5	23.11	15
6	55 47 23.20	15
1900 Apr. 11	55° 47′ 23″.21	15
12	23.31	15
28	23.39	15
30	23.32	13
May 2	23.18	11
3	23.46	4
6	23.35	6
7	23.34	8
8	23.31	15
15	23.27	8
17	23.36	5
18	23.38	11
19	23.35	16
20	23.39	16
21	23.38	15
June 16	23.25	14
17	23.20	14
19	23.18	14
July 7	23.21	3
8	23.28	3
10	23.20	16
16	23.26	4
17	23.11	13
21	22.99	13
23	23.11	14
Aug. 1	23.18	13
2	23.18	3
7	23.20	13
12	23.10	21
13	23.10	11
14	23.06	20
20	55 47 22.97	7
1900 Aug. 24	55° 47′ 22″.94	17
Sept. 21	23.05	15
26	23.02	15
27	23.01	1
28	22.92	14
29	22.95	14
Oct. 1	23.03	13
6	22.99	13
15	22.94	4
18	23.07	16
22	23.08	17
24	23.08	12
26	22.98	17
Nov. 3	23.12	17
20	23.07	7
25	55 47 23.10	2

Provisional Reduction.

Date	φ	Pairs
1901 Jan. 2	55 47 23.02	8
8	23.20	9
15	23.07	17
16	23.02	17
Feb. 1	22.96	4
9	22.96	16
11	23.06	16
13	23.07	14
14	22.81	11
18	23.43	8
21	22.97	7
26	23.11	8
27	55 47 23.02	16

Monthly Means.

Date		φ	Pairs
1899 Nov. 15	1899.87	55 47 23.139	13
Dec. 23	.97	23.276	72
1900 Jan. 13	1900.04	23.308	90
Feb. 14	.12	23.266	101
Mar. 21	.22	23.292	100
Apr. 15	.29	23.265	88
May 14	.37	23.339	115
June 17	.46	23.210	42
1900 July 16	1900.54	55 47 23.136	66
1900 Aug. 13	1900.62	55 47 23″.082	105
Sept. 26	.74	22.987	59
Oct. 17	.80	23.034	92
Nov. 9	1900.86	55 47 23.105	26

Provisional Reduction.

Date		φ	Pairs
1901 Jan. 12	1901.03	55 47 23.068	51
Feb. 16	1901.13	55 47 23.035	100

* Continued from Series V, A.J. 476.

DEFINITIVE FORMULAS FOR COMPUTING VARIATIONS OF LATITUDE,

By S. C. CHANDLER.

The observations of DOOLITTLE in *A.J.* 490 conjoined with those of GRATCHOF in this number, add cumulative testimony as to the changes going on in the annual component; and I think the fact of these changes may now safely be incorporated in the numerical theory. Accordingly I give below formulas which can be employed with safety for correcting observations made during, say, the past thirty years, and passably for the whole century. The principal uncertainty attending their use for earlier dates arises from uncertainty in the law of the changes in L and ω. For these two elements I adopt provisionally

$$L = 51^\circ\text{–}45^\circ \cos(t-1887.0)\,6^\circ \qquad , \qquad \omega = (1898.0-t)\,6^\circ$$

Observations during the next few years will doubtless improve them. The coordinates of the pole are

$$x = r_1 \sin(t-T_1)\,\theta' + \rho \sin(\beta+\omega)$$
$$y = r_1 \cos(t-T_1)\,\theta' + \rho \cos(\beta+\omega)$$

where r_1 and T_1 are to be found by the formulas in *A.J.* 490; and ρ and β by eq. (19) and (20) of *A.J.* 406, or

$$\rho = \frac{a}{2}\left[1-e^2\sin^2(\odot-L)\right]^{\frac{1}{2}} \qquad , \qquad \tan\beta = \frac{b}{a}\tan(\odot-L)$$

where $a = 0''.275$, $b = 0''.085$, $e^2 = 0.906$

Then the variations of latitude for a station in longitude λ (reckoned west from Greenwich) are given by

$$\varphi-\varphi_0 = x\sin\lambda - y\cos\lambda$$

The following tables will facilitate the application of these equations.. The Julian date is $t = t_0+\tau$ (adding 1d after February in leap years). L, ω, $\odot$ and β need be interpolated only to the nearest degree. In using Table IV when the argument $\odot-L$ is over 180° it may be diminished by 180°, but the value of β so taken out is to be increased by 180°; in other words β is in same quadrant with $\odot-L$.

I				II				III		IV				V		
t_0	Beg. of Yr.	L	ω	τ	Date		$\odot$	T_1	r_1	$\odot-L$	β	ρ	$t-T_1$	sin	cos	$\beta+\omega$
		°	°				°		″	°	°	″	d			°
240 4063 =	1870.0	60.4	168.0	0	Jan.	0	280.0	240 1880	0.210	0	0.0	0.137	0.0	0.00	+1.00	0
4428	71.0	55.7	162.0	10		10	289.8	2312	.214	5	1.5	.136	11.9	+ .17	0.98	10
4793	72.0	51.0	156.0	20		20	299.7	2743	.219	10	3.1	.135	23.8	.34	.91	20
5159	73.0	46.3	150.0	30		30	309.5	3174	.223	15	4.7	.133	35.7	.50	.87	30
5524	74.0	41.6	144.0	40	Feb.	9	319.4	3605	.225	20	6.4	.130	47.6	.64	.77	40
5889	75.0	37.1	138.0	50		19	329.3	4037	.227	25	8.2	.126	59.5	.77	.64	50
6254	76.0	32.7	132.0	60	Mar.	1	339.1	4468	.228	30	10.1	.121	71.4	.87	.50	60
6620	77.0	28.5	126.0	70		11	349.0	4900	.229	35	12.2	.115	83.3	.94	.34	70
6985	78.0	24.5	120.0	80		21	358.8	5331	.230	40	14.5	.109	95.2	0.98	+ .17	80
7350	79.0	20.9	114.0	90		31	8.7	5763	.229	45	17.2	.102	107.1	1.00	.00	90
7715	80.0	17.6	108.0	100	Apr.	10	18.6	6194	.228	50	20.2	.094	119.0	0.98	− .17	100
8081	81.0	14.6	102.0	110		20	28.4	6626	.227	55	23.8	.086	130.9	.94	.34	110
8446	82.0	12.0	96.0	120		30	38.3	7057	.225	60	28.1	.078	142.8	.87	.50	120
8811	83.0	9.9	90.0	130	May	10	48.2	7488	.223	65	33.5	.069	154.7	.77	.64	130
9176	84.0	8.2	84.0	140		20	58.0	7919	.219	70	40.4	.061	166.6	.64	.77	140
9542	85.0	7.0	78.0	150		30	67.8	8351	.214	75	49.0	.054	178.5	.50	.87	150
240 9907	86.0	6.2	72.0	160	June	9	77.7	8782	.210	80	60.3	.048	190.1	.34	.94	160
241 0272	87.0	6.0	66.0	170		19	87.5	9213	.201	85	74.2	.041	202.3	+ .17	0.98	170
0637	88.0	6.2	60.0	180		29	97.4	240 9644	.198	90	90.0	.042	214.2	.00	1.00	180
1003	89.0	7.0	54.0	190	July	9	107.2	241 0075	.192	95	105.8	.044	226.1	− .17	.98	190
1368	90.0	8.2	48.0	200		19	117.1	0507	.184	100	119.7	.048	238.0	.34	.91	200
1733	91.0	9.8	42.0	210		29	126.9	0938	.178	105	131.0	.054	249.8	.50	.87	210
2098	92.0	12.0	36.0	220	Aug.	8	136.8	1369	.168	110	139.6	.061	261.6	.64	.77	220
2464	93.0	14.5	30.0	230		18	146.6	1800	.160	115	146.5	.069	273.5	.77	.64	230
2829	94.0	17.5	24.0	240		28	156.5	2230	.152	120	151.9	.078	285.6	.87	.50	240
3194	95.0	20.8	18.0	250	Sept.	7	166.1	2660	.143	125	156.2	.086	297.5	.94	.34	250
3559	96.0	24.5	12.0	260		17	176.2	3090	.135	130	159.8	.091	309.4	0.98	− .17	260
3925	97.0	28.5	6.0	270		27	186.1	3520	.125	135	162.8	.102	321.3	1.00	.00	270
4290	98.0	32.7	0.0	280	Oct.	7	195.9	3949	.116	140	165.5	.109	333.2	0.98	+ .17	280
4655	1899.0	37.1	354.0	290		17	205.8	4378	.107	145	167.8	.115	345.1	.94	.34	290
5020	1900.0	41.6	348.0	300		27	215.7	4806	.097	150	169.9	.121	357.1	.87	.50	300
5385	01.0	46.3	342.0	310	Nov.	6	225.5	5233	.087	155	171.8	.126	369.0	.77	.64	310
5750	02.0	51.0	336.0	320		16	235.4	5659	.077	160	173.6	.130	380.9	.64	.77	320
6115	03.0	55.7	330.0	330		26	245.2	6083	.069	165	175.3	.133	392.8	.50	.87	330
241 6480 =	1904.0	60.3	324.0	340	Dec.	6	255.1	241 6506	0.062	170	176.9	.135	404.7	.34	.94	340
				350		16	265.0			175	178.5	.136	415.6	− .17	0.98	350
				360		26	274.8			180	180.0	0.137	428.5	0.00	+1.00	360

OBSERVATIONS OF COMET *c* 1900 (*GIACOBINI*),

BY R. G. AITKEN.

1901 Mt. Hamilton M.T.	*	No. Comp.	☌ — * $\Delta\alpha$	☌ — * $\Delta\delta$	☌'s apparent α	☌'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			s	′ ″	h m s	° ′ ″		
Jan. 13 7 10 31	2	10, 8	−15.35	+5 31.1	0 52 27.97	−22 22 18.6	9.369	0.865
15 6 54 9	3	8, 8	+ 8.88	−1 4.7	1 2 31.42	−22 1 23.9	9.292	0.869
Feb. 15 8 9 44	6	8, 8	− 9.83	+2 0.1	3 10 14.05	−14 11 0.6	9.497	0.815

Mean Places for 1901.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	0 50 27.11	+0.68	−22 28 2.3	−4.9	Argelander's Southern Zones 416
2	0 52 42.63	+0.69	−22 27 44.7	−5.0	Micrometer comparison with (1)
3	1 2 21.82	+0.72	−22 3 14.0	−5.2	Micrometer comparison with (4)
4	1 3 15.19	+0.73	−22 8 10.9	−5.3	Porter, Cinn. Zone Catal. 155
5	3 10 11.03	+1.03	−14 18 39.7	−9.0	Paris Catal. 3888
6	3 10 22.85	+1.03	−14 15 51.7	−9.0	Micrometer comparison with (5)

The first two measures were made with the 12-inch, the last with the 36-inch equatorial, $\Delta\alpha$ in every case being measured directly with the micrometer. On February 15, the comet was very faint. In fact, the 7.5 magnitude star (No. 5) had to be placed outside the field to make measure of the comet possible. Careful search with the 36-inch on March 8, under good conditions, failed to show any trace of the comet. The positions given above are in good agreement with those computed from KREUTZ's elliptic elements.

Lick Observatory, University of California, 1901 *March* 14.

THE BENJAMIN APTHORP GOULD FUND.

Since the appropriations announced in *A.J.* 477 the following additional grants have been made: to Mr. JOHN A. PARKHURST, $30; to Dr. HERMAN S. DAVIS, $500; to Mr. PAUL S. YENDELL, $225; to Prof. SIMON NEWCOMB, $25. A considerable additional amount of income has accrued, for the distribution of which applications are awaited. These applications may be made by letter to any of the Directors undersigned stating the amount desired, the nature of the proposed investigation, and the manner in which the money is to be expended. The following information is given for the guidance of applicants.

The BENJAMIN APTHORP GOULD FUND was established in 1897 by Miss ALICE BACHE GOULD, to advance the science of astronomy, and to honor the memory of her father by ensuring that his power to accomplish scientific work shall not end with his death. The principal is $20,000, vested in the National Academy of Sciences as Trustee. The income is to be administered by the undersigned and their successors to assist the prosecution of researches in astronomy.

In recognition of the fact that during Dr. GOULD's lifetime his patriotic feeling and ambition to promote the progress of his chosen science were closely associated, it is preferred that the Fund should be used primarily for the benefit of investigators in his own country or of his own nationality. But it is further recognized that sometimes the best possible service to American science is the maintenance of close communion between the scientific men of Europe and of America, and that therefore, even while acting in the spirit of the above restriction, it may occasionally be best to apply the money to the aid of a foreign investigator working abroad.

In all cases work in the astronomy of precision will be preferred to work in astrophysics, both because of Dr. GOULD's especial predilection and because of the present existence of generous endowments for astrophysics.

Finally, the BENJAMIN APTHORP GOULD FUND is intended for the advancement and not for the diffusion of scientific knowledge, and is to be used to defray the actual expenses of investigation, rather than for the personal support of the investigator during the time of his researches, without absolutely excluding the latter use under exceptional circumstances.

In addition to the above call for applications the Directors, desiring to stimulate the participation of American astronomers in the attempt to bring up the arrears of cometary research, offer to them the sum of $500 for computation of the "definitive" orbits of comets (see list in *A.J.* 493, p. 101); this sum to be distributed at the average rate of $100 for each computation, the amount to vary according to the relative difficulty of the computation, and to be determined by the Directors of the GOULD FUND. Computers should promptly notify the Directors of their participation or desire to participate, and manuscripts should be submitted not much later than July 1, 1902.

LEWIS BOSS, SETH C. CHANDLER, ASAPH HALL.

1901 *March*.

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DEFINITIVE ORBIT OF COMET 1894 II.

By HENRY A. PECK.

The second comet of 1894 was discovered by Mr. WALTER F. GALE of Sydney on the morning of April 1, and was observed for four months at all the principal observatories. When discovered it showed a strong central condensation, but no nucleus. About the time of passing perihelion a short tail 7′ to 10′ long appeared, and as the comet approached the earth it became quite a conspicuous object. It was photographed at Sydney April 5, and by WOLF at Heidelberg on May 6. Photographs made by BARNARD have been reproduced in a number of publications. When nearest the earth, the coma appeared from 10′ to 15′ in diameter, and about the same time it developed a star-like nucleus. It was last seen at Nice, August 21, nearly a month after observations had ceased at other places.

Approximate elements for this comet have been published by a number of computers, but none of them are exact enough to furnish a basis for a definitive orbit. To remedy this defect, an ephemeris was computed from KOHLSCHÜTTER's elements found in *A.N.* 3231, and this was compared with observations at intervals throughout the period of the comet's visibility. From a curve obtained by plotting the results of this comparison, normal places were obtained which yielded the following elements:

$$T = \text{April } 13.40693 \text{ Greenwich M.T.}$$

$$\left.\begin{aligned} \omega &= 324^\circ\ 12'\ 23.''0 \\ \Omega &= 206\ 23\ 54.3 \\ i &= 86\ 59\ 19.1 \end{aligned}\right\} 1894.0$$

$$\log q = 9.9925918$$

$$\log e = 9.9961180$$

EQUATORIAL COORDINATES.

$$\begin{aligned} x &= [9.9523218]\, r \sin(235^\circ\ 42'\ \ 0.''40 + v) \\ y &= [9.7784575]\, r \sin(186\ 59\ 48.97 + v) \\ z &= [9.9612655]\, r \sin(313\ \ 3\ \ 5.58 + v) \end{aligned}$$

In forming the list of comparison-stars, I have been hampered by the absence from the local library of the older star catalogues, as well as many of the principal ones for the southern heavens. This lack has been met in part through the courtesy of Professor BOSS and Mr. ROY of the Dudley Observatory at Albany. I am also under obligations to Professors HARKNESS, PICKERING, DE BALL, BECKER, KORTAZZI, VAN DE S. BAKHUYZEN, BRUNS, and CHARLIER, who have furnished in manuscript, star places from the sections of the A.G. Catalogues prepared under their direction. In addition to these lists the following catalogues have been consulted: Albany A.G., B.B. VI, Bonn A.G., Berlin A, Berlin B, BRISBANE's Paramatta, the Cape of Good Hope Catalogues for 1820, 1840, 1850, 1860, 1885, 1890, 1894; Cambridge A.G., Cordoba General, Cordoba Zones, the first and second Glasgow, COPELAND and B. GÖTTINGEN, GILL's list of comparison-stars in *A.N.* 3308, Greenwich 12, 6, 7, 9, and 10-year Catalogues, Paris, PORTER's Cincinnati Catalogues of Proper Motion Stars, Radcliffe, SCHJELLERUP, STONE's Cape, TAYLOR, WEISS's ARGELANDER, WEISSE's BESSEL, Washington 2d Catalogue, and YARNALL. In estimating the comparative weights of the southern catalogues, use has been made of BOSS's paper in the *Astronomical Journal*. So far as possible all positions have been reduced to the A.G. system. For stars contained in the various A.G. Catalogues, the positions there given have only been modified when observations of approximately the same relative weight could be obtained elsewhere. After thorough search in the various catalogues a residuum of fifteen stars remained for which no meridian observations were obtained. Through the coöperation of Professor Boss, these were observed on the meridian circle at Albany, by Mr. ARTHUR J. ROY. They are for the most part stars that lie outside the program of the A.G. Zones, on account of their faintness, and are designated in the following list by an *. Each position is the mean of two observations, and the stars correspond to the following DM. numbers:

$-\ 3^\circ.2390$	$+33^\circ.2008$	$+35^\circ.2184$	$+43^\circ.2202$
$+21.2085$	$+34.2146$	$+36.2127$	$+43.2222$
$+26.2038$	$+34.2149$	$+38.2204$	$+43.2236$
$+30.1987$	$+35.2180$	$+43.2174$	

Comparison-Stars.

No.	α 1894.0	δ 1894.0
	m s	° ′ ″
	2h	
1	35 39.06	−55 17 36.1
2	44 38.75	55 12 32.8
3	46 46.79	55 31 18.1
4	48 13.94	55 30 40.1
5	54 32.14	55 26 22.7
6	59 33.63	55 23 2.7
	3h	
7	27 35.52	−55 10 15.7
8	31 24.12	54 51 56.5
9	31 55.71	54 40 35.7
10	40 22.85	54 18 23.2
11	41 31.91	54 48 49.6
12	41 51.83	54 36 27.7
13	50 22.28	54 25 11.2
14	59 32.89	53 54 48.1
	4h	
15	15 38.38	−53 9 41.9
16	16 3.00	53 7 5.0
17	22 44.71	52 10 32.4
18	32 25.50	52 55 48.2
19	35 8.68	52 15 53.3
20	36 26.66	51 52 50.4
	5h	
21	5 21.45	−49 26 1.2
22	6 25.74	49 6 45.4
23	9 17.48	49 11 12.0
24	17 36.90	48 7 58.4
25	29 28.27	47 33 5.0
26	33 10.11	47 22 42.9
27	40 16.82	46 30 4.1
28	40 40.61	45 52 53.6
29	43 56.07	45 11 21.5
30	46 16.61	44 42 52.9
31	50 16.43	43 34 39.8
32	51 38.54	43 36 41.3
33	57 55.79	43 51 18.8
34	59 49.67	43 1 25.3
	6h	
35	3 21.59	−42 17 7.6
36	4 2.86	41 33 6.0
37	10 52.35	41 37 17.8
38	13 5.29	40 29 57.8
39	13 25.57	41 29 35.2
40	22 17.91	38 52 26.6
41	31 12.03	37 21 11.8
42	31 51.53	37 16 13.5
43	32 10.80	35 59 53.6
44	33 33.49	36 51 1.1
45	34 6.49	36 1 8.5
46	36 44.21	36 2 8.2
47	43 28.89	34 37 56.0
48	47 11.33	34 9 4.9
49	50 22.77	34 5 22.8
50	50 45.19	32 39 22.7
51	55 2.74	32 31 37.3
	7h	
52	0 57.81	−31 21 57.6
53	3 6.37	30 37 45.4
54	12 19.98	−27 44 38.4
55	14 12.08	−27 45 39.5
56	16 36.72	26 45 55.5
57	22 12.75	23 30 1.1
58	27 27.54	22 44 34.8
59	28 31.01	23 42 23.9
60	29 49.94	23 14 34.6
61	31 6.89	22 36 21.0
62	39 1.15	19 39 57.5
63	39 21.79	19 54 25.6
64	41 14.54	18 38 18.0
65	44 1.99	18 54 37.3
66	45 2.68	18 44 23.1
67	45 1.85	18 39 59.0
68	47 25.09	17 30 35.1
69	50 45.29	15 17 53.8
70	51 13.66	15 36 37.3
71	51 21.90	15 13 46.4
72	53 12.95	14 21 27.2
73	53 33.36	15 27 59.3
74	53 51.80	15 22 6.2
75	56 44.66	14 28 1.2
76	57 11.31	14 35 20.4
	8h	
77	0 35.31	−13 16 15.6
78	3 5.19	13 11 59.1
79	3 55.06	11 1 48.3
80	5 31.61	11 13 54.5
81	6 0.61	11 36 8.3
82	6 6.50	11 26 55.8
83	7 21.07	10 30 54.5
84	7 36.37	10 13 16.3
85	11 35.57	8 43 37.9
86	11 40.34	8 41 11.5
87	12 16.88	8 46 15.0
88	17 36.76	6 21 19.6
89	17 43.41	7 12 15.6
90	17 47.07	7 40 7.2
91	18 22.55	6 30 48.8
92	18 57.13	5 20 13.9
93	19 17.95	3 24 27.9
94	20 21.50	5 8 17.2
95	20 21.81	3 33 38.8
96	21 9.74	6 7 34.9
97	22 4.27	6 3 37.2
98	22 20.22	4 17 48.9
99	27 29.95	3 41 56.3
100	27 33.61	3 29 33.4
101	27 55.09	4 51 42.7
102	28 1.50	4 57 56.6
103*	28 21.43	3 17 25.8
104	28 40.99	1 12 13.1
105	29 6.71	2 21 16.0
106	29 17.05	2 23 28.1
107	30 10.90	2 20 17.9
108	31 13.88	3 8 38.3
109	32 21.35	− 1 11 1.1
110	39 15.92	+ 0 41 17.3
111	39 50.14	0 5 9.1
112	39 57.02	0 31 56.2
113	39 59.04	+ 0 10 0.6
114	42 8.60	+ 0 27 59.2
115	43 41.21	3 52 28.1
116	43 50.51	4 4 33.8
117	46 48.37	5 44 19.1
118	47 9.29	4 2 46.2
119	47 20.86	3 26 8.6
120	47 49.26	4 18 30.1
121	48 4.74	2 18 48.9
122	48 22.47	2 13 35.2
123	48 27.16	4 2 22.9
124	48 54.91	5 52 0.0
125	49 21.46	4 32 59.7
126	49 50.13	5 58 11.0
127	50 45.99	3 58 27.3
128	54 15.75	7 32 55.6
129	57 1.09	7 16 0.1
130	57 1.47	7 29 44.9
131	57 6.00	7 42 55.5
132	57 36.63	7 14 0.7
133	59 18.14	8 14 47.3
134	59 48.33	7 14 13.5
	9h	
135	0 25.12	+ 7 36 46.7
136	1 12.57	10 17 53.0
137	3 22.97	7 56 31.9
138	4 1.68	10 11 37.7
139	4 3.08	10 37 47.6
140	6 15.27	10 11 8.7
141	6 16.59	10 16 10.6
142	6 40.20	10 24 19.1
143	6 54.88	10 18 17.4
144	7 12.41	11 6 16.2
145	7 40.34	10 44 36.4
146	12 6.45	11 56 42.2
147	14 0.01	13 10 20.4
148	14 1.60	13 22 11.8
149	15 31.40	13 33 51.2
150	16 27.67	13 0 27.6
151	18 12.39	13 50 35.2
152	19 32.99	14 38 50.1
153	20 50.39	15 34 28.6
154	21 8.01	14 45 49.0
155	22 35.50	15 37 38.8
156	22 11.71	13 8 24.0
157	22 18.17	15 53 12.7
158	23 32.48	15 50 4.3
159	24 45.85	15 43 32.5
160	25 48.61	16 11 12.2
161	28 53.38	16 55 4.4
162	30 5.03	20 31 5.9
163	30 16.62	18 13 11.3
164	31 11.69	16 51 15.4
165	33 34.94	18 3 41.8
166	34 48.81	20 17 25.0
167	36 38.83	20 27 0.9
168*	37 39.65	20 50 30.8
169	37 26.15	20 10 38.5
170	42 3.08	20 32 18.5
171	42 51.70	22 32 10.6
172	48 31.25	+21 57 9.3
173	48 34.38	+23 57 21.1
174	50 39.73	23 53 53.6
175	50 44.21	25 33 27.3
176	51 8.07	24 31 17.1
177	53 26.32	23 47 49.3
178	56 21.69	25 48 6.3
179*	58 2.99	25 52 17.3
180	58 15.52	25 41 14.3
181	58 30.71	25 44 39.6
182	59 27.62	27 0 18.2
	10h	
183	1 27.39	+27 18 3.8
184	1 51.09	27 11 7.7
185	1 54.98	27 0 12.3
186	2 9.21	27 18 55.5
187	5 34.04	28 21 57.4
188	10 13.17	29 50 18.5
189*	11 49.68	29 50 48.6
190	14 39.07	29 28 30.9
191	15 6.47	30 41 56.2
192	16 50.60	30 51 59.3
193	17 28.86	30 23 5.8
194	18 2.90	30 44 16.6
195	19 41.05	31 41 14.3
196	21 13.94	31 25 43.5
197	23 21.42	32 32 56.5
198	25 50.32	32 55 24.7
199	25 58.46	32 43 48.0
200	26 14.77	33 23 8.2
201	26 18.99	32 41 57.5
202	26 29.20	32 30 8.6
203	26 31.66	31 57 8.1
204*	29 51.87	33 38 53.2
205	30 47.64	33 35 25.3
206	31 44.48	33 17 3.0
207	31 48.44	33 14 19.1
208	32 45.34	32 31 35.8
209	33 10.16	33 23 11.2
210*	33 18.47	34 24 24.2
211	33 58.73	34 15 43.7
212*	34 52.71	33 55 46.5
213	34 53.88	34 12 0.4
214	35 58.69	34 50 10.6
215	36 3.66	35 3 59.2
216	37 32.89	35 3 57.4
217*	40 33.65	35 28 15.0
218	40 46.37	35 17 20.9
219	40 52.38	34 7 2.0
220*	41 9.00	35 43 37.9
221	41 47.53	34 8 55.9
222	45 10.50	36 24 3.8
223*	43 43.24	36 10 33.6
224	46 0.69	36 5 52.7
225	48 48.26	37 19 27.5
226	48 56.83	36 13 9.1
227	51 2.86	37 44 27.7
228	51 31.36	38 12 53.6
229	52 31.23	37 35 52.3
230	53 35.04	37 5 7.4
231	53 38.26	+38 13 8.3

No.	α 1894.0	δ 1894.0	No.	α 1894.0	δ 1894.0	No.	α 1894.0	δ 1894.0	No.	α 1894.0	δ 1894.0
	m s	° ′ ″		m s	° ′ ″		m s	° ′ ″		m s	° ′ ″
	10h		249	11 42.91	+40 4 22.7	268	50 31.37	+42 51 23.0	286	17 1.58	+43 29 58.8
232*	54 14.50	+38 5 45.5	250	13 43.42	40 11 29.0	269	50 54.07	42 36 12.6	287	17 59.83	43 29 7.2
233	54 36.66	37 38 32.2	251	14 31.54	40 22 59.6	270	51 13.25	43 4 26.6	288	19 11.53	43 40 32.2
234	55 46.13	38 34 38.0	252	15 42.74	40 24 46.3	271	51 48.05	43 5 49.6	289	19 45.31	43 26 27.9
235	55 52.06	37 14 44.7	253	15 47.80	41 0 49.1	272	53 40.85	43 3 24.1	290*	20 22.60	43 34 0.4
236	56 14.81	37 33 7.7	254	16 11.59	40 30 25.7	273*	54 58.00	43 10 45.4	291	23 59.55	43 36 3.0
237	57 19.31	38 21 45.1	255	16 55.60	40 45 23.1	274	55 33.96	42 49 22.0	292*	27 2.94	43 34 52.8
238	58 37.70	38 48 45.0	256	20 22.67	40 45 53.8	275	56 59.44	42 55 15.1	293	35 53.85	43 34 49.5
239	58 50.48	38 19 3.9	257	22 45.60	41 3 47.7		12h		294	45 35.78	43 25 25.5
240	59 59.80	38 1 12.3	258	27 5.36	41 30 54.2	276	1 5.52	+43 12 17.6	295	46 56.56	43 28 6.8
	11h		259	31 33.42	41 41 24.6	277	1 42.53	43 44 59.1	296	47 51.85	43 21 7.1
241	1 0.80	+38 20 33.6	260	32 8.20	41 53 41.7	278	2 36.60	43 17 21.4	297	48 20.44	43 20 53.0
242	1 37.83	39 31 27.0	261	33 19.42	41 43 51.5	279	3 47.72	43 18 2.9	298	54 23.38	43 19 11.4
243	2 23.71	38 59 55.5	262	36 40.23	42 2 37.1	280	3 55.95	43 4 19.5	299	54 47.50	43 10 55.6
244	2 30.27	38 33 43.5	263	38 0.42	42 18 38.4	281	6 34.91	43 30 38.1	300	57 48.18	43 13 56.1
245	4 28.68	39 11 20.2	264	44 51.50	42 33 16.9	282*	10 30.91	43 19 15.6		13h	
246	7 9.09	39 21 41.5	265	46 28.37	42 36 55.6	283	12 4.61	43 23 3.3	301	1 23.44	+43 10 25.1
247	9 56.11	39 23 15.2	266	47 10.33	43 0 58.6	284	13 38.65	43 39 58.5	302	5 6.99	43 7 1.7
248	10 25.96	+40 11 8.7	267	47 56.22	+42 43 45.2	285	15 30.17	+43 35 5.6	303	39 53.58	+42 27 27.1

Stars 11, 16 and 28 have well-defined proper motions in declination of +0″.114, +0″.056 and +0″.088 respectively.

The observations were collected from the usual sources. In the following comparison with the ephemeris only the mean appears, whenever the comet was observed more than once on the same day at any given observatory. Exception to this rule has been made when two series of observations exist which are apparently as distinct as if made at different and widely separated points. Whenever a correction of the time of observation would harmonize otherwise discordant data, this has been done without calling attention to the fact. Obvious typographical errors have been corrected, and the correction for aberration has been applied. The residuals $\Delta\alpha \cos\delta$ and $\Delta\delta$ are in the sense O – C.

Observations of Comet 1894 II.

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
April 2.93927	Windsor	2 30 46.01	0.96	–1.04	–55 34 52.3	–5.0	+ 3.5	1
3.87568	Windsor	36 8.02	0.99	–0.29	36 4.1	–2.1	+ 4.2	1
4.89332	Windsor	42 17.45	1.01	–0.15	36 20.0	–2.8	+ 2.7	3, 4
4.93825	Sydney	42 34.28	1.03	+0.02	36 19.2	–5.2	+ 0.1	2, 3, 4
4.97673	Melbourne	42 49.11	0.94	+0.19	36 13.1	–1.0	+ 9.4	3
5.88724	Windsor	48 39.63	1.00	–0.11	35 20.7	–2.5	+ 3.9	3, 4
6.94997	Melbourne	55 54.72	1.01	+0.17	32 34.1	–2.1	+12.5	5
7.87384	Windsor	3 2 34.74	1.14	–0.15	28 52.8	–2.0	– 0.3	5
7.89146	Melbourne	2 43.60	1.08	+0.07	28 34.1	–2.6	+12.4	6
10.95888	Melbourne	27 50.33	1.20	+0.25	1 8.4	–3.3	+ 3.0	7
11.24817	Cape	30 27.26	1.26	+0.24	54 57 5.4	–2.1	+ 3.4	8
11.57092	Cordoba	33 22.20	1.27	+0.02	52 9.5	–7.0	+ 4.9	9
11.89882	Windsor	36 29.93	1.31	+0.03	46 48.1	–3.2	+ 2.1	9, 10, 12
11.94476	Sydney	36 56.20	1.30	+0.08	46 0.4	–5.8	– 0.5	9, 11, 12
12.26870	Cape	40 3.57	1.32	+0.55	40 11.8	–3.3	+ 0.8	12
12.54693	Cordoba	42 45.81	1.34	+0.06	34 51.5	–7.2	– 4.5	12
13.25240	Cape	49 51.57	1.32	+0.17	19 38.4	–2.1	– 0.5	13
13.49629	Cordoba	52 21.73	1.38	–0.42	13 44.7	–2.6	+ 1.5	13
13.87774	Windsor	56 22.97	1.24	–0.10	4 1.0	–1.5	– 0.1	14
14.48964	Cordoba	4 2 59.78	1.40	0.00	53 46 20.0	–2.0	+14.3	14
15.87689	Windsor	18 49.08	1.36	–0.07	52 58 0.5	–1.2	– 0.3	15, 16
15.95955	Sydney	19 48.29	1.46	+0.28	51 42.2	–6.7	– 7.7	18
16.89392	Sydney	31 7.98	1.43	–0.35	13 6.7	–2.2	– 3.4	19
16.89562	Windsor	31 9.80	1.44	–0.02	12 58.9	–2.2	– 1.2	17
17.24226	Cape	35 30.84	1.36	+0.43	51 55 36.3	–0.9	+ 0.4	20
19.49038	Cordoba	5 5 12.22	1.47	–0.23	49 32 39.9	–1.2	– 8.8	21
19.89633	Windsor	5 10 50.22	1.46	+0.08	–49 0 21.6	–1.8	– 0.4	22, 23

Date	Place	App. α	π	$\Delta\alpha \cos\delta$	App. δ	π	$\Delta\delta$	*
		h m s	s	s	° ′ ″	″	″	
April 19.90873	Sydney	5 11 0.79	1.51	+0.23	−18 59 19.3	− 2.8	− 0.3	22
20.50620	Cordoba	19 25.54	1.53	+0.32	7 46.0	− 2.1	− 0.8	24
20.89643	Windsor	24 59.12	1.45	0.00	47 31 19.8	− 1.8	− 0.5	25, 26
21.53497	Cordoba	34 12.55	1.61	+0.34	46 26 40.2	− 4.1	+ 2.2	27
21.88837	Windsor	39 21.51	1.34	+0.02	45 48 11.9	− 0.9	+ 2.6	28, 29
22.48966	Cordoba	48 11.88	1.41	+0.31	44 37 50.7	− 0.9	+ 6.2	30
22.89205	Windsor	54 8.51	1.36	−0.07	43 47 20.2	− 1.6	+ 4.9	33
22.94746	Sydney	54 57.60	1.60	−0.03	40 8.4	− 5.5	+ 2.1	31, 32
23.24061	Cape	59 19.05	1.22	+0.36	1 22.7	− 0.5	+ 3.9	34
23.49521	Cordoba	6 3 6.72	1.40	+0.50	42 26 35.6	− 1.3	(−17.4)	35
23.90742	Windsor	9 13.45	1.36	+0.09	41 27 0.5	− 2.4	+ 1.0	36, 37
23.91160	Sydney	9 18.31	1.44	+0.25	26 20.2	− 3.2	+ 2.5	39
24.28321	Cape	14 51.16	1.44	+0.77	40 30 12.6	− 3.5	+ 0.5	38
24.88819	Windsor	23 50.07	1.17	−0.40	38 53 9.2	− 1.8	+ 3.9	40
24.90786	Sydney	24 8.06	1.37	+0.12	49 54.6	− 3.3	+ 0.9	40
25.26872	Cape	29 29.90	1.30	+0.51	37 48 33.7	− 3.0	+ 2.5	42
25.52563	Cordoba	33 17.17	1.41	−0.07	8 23.0	− 4.2	+ 2.4	41, 44
25.89354	Windsor	38 42.75	1.23	−0.18	35 56 31.7	− 2.8	+ 4.2	43, 45
25.90971	Sydney	38 57.61	1.33	+0.41	53 36.4	− 3.9	− 0.7	46
26.30264	Cape	44 43.78	1.44	+0.88	34 39 15.3	− 5.7	0.0	47
26.50262	Cordoba	47 38.85	1.29	+0.60	33 0 22.9	− 3.1	− 0.1	48, 49
26.90788	Sydney	53 30.77	1.27	+0.02	32 39 22.9	− 4.6	+ 0.6	50, 51
26.91541	Windsor	53 37.35	1.30	+0.09	40 47.0	− 4.9	+ 4.4	50, 51
27.26686	Cape	58 41.48	1.17	+0.48	31 25 15.5	− 4.4	+ 0.2	52
27.48370	Cordoba	7 1 47.03	1.09	+0.11	30 39 30.6	− 2.9	− 2.7	53
28.26677	Cape	12 50.48	1.11	+0.64	27 47 49.0	− 5.5	+ 2.5	55
28.51343	Cordoba	16 16.45	1.24	+0.99	26 51 47.7	− 5.4	+11.9	56
28.56753	Cincinnati	17 0.73	1.11	+0.33	40 14.2	+20.4	−10.4	56
29.33114	Marseilles	27 25.19	1.01	−0.52	23 41 36.9	+21.2	+13.9	57
29.33500	Algiers	27 28.29	1.01	−0.54	41 6.8	+19.6	−12.7	59
29.49117	Cordoba	29 34.76	1.01	+0.17	3 17.7	− 5.6	− 5.1	60
29.55593	Washington Naval	30 29.13	1.13	+1.11	22 48 11.6	+20.0	− 0.8	58
29.58212	Cincinnati	30 48.26	1.14	+0.70	42 3.9	+20.0	−10.4	61
30.26774	Cape	39 50.21	0.98	+0.96	19 55 2.7	− 8.0	− 0.5	63
30.33336	Algiers	40 40.59	1.01	+0.32	39 23.0	+19.5	+ 2.0	62
30.55325	Washington Naval	43 29.61	1.06	−0.61	18 45 27.9	+19.3	− 9.0	66
30.57766	Cincinnati	43 48.34	1.05	−0.67	39 19.6	+20.0	− 0.8	66, 67
30.58742	Wash. Catholic	43 56.39	1.22	+0.01	36 46.2	+19.0	+ 7.1	64, 67
30.86320	Windsor	47 27.63	0.64	−0.10	17 28 10.3	− 7.8	+ 5.8	68
May 1.27319	Cape	52 36.79	0.95	+0.72	15 46 26.5	− 9.4	+ 2.0	71
1.32151	O'Gyalla	53 11.12	1.00	−0.31	34 18.2	+21.3	+ 3.5	70
1.34145	Vienna	53 27.78	1.06	+0.95	. . . . .	. . .	. . .	69
1.36615	Greenwich	53 14.71	0.92	−0.50	23 58.1	+22.3	− 4.6	74
1.36634	Algiers	53 15.39	1.02	+0.07	23 49.0	+18.2	− 2.5	73
1.55368	Washington Naval	56 4.62	0.98	+0.69	14 37 15.3	+19.5	− 3.5	76
1.57652	Wash. Catholic	56 20.70	1.11	+0.10	31 29.5	+19.0	+ 0.4	72, 75, 76
1.58875	Albany	56 29.61	1.01	−0.08	28 33.1	+19.5	− 2.1	75
1.91562	Windsor	8 0 28.54	0.99	+0.38	13 6 47.4	−10.3	+ 4.6	77, 78
2.27358	Cape	4 15.99	0.89	+0.88	11 38 0.0	−10.7	− 1.7	81
2.31044	Vienna	5 11.98	0.89	+0.70	29 26.5	+21.4	− 3.3	82
2.32675	Vienna B.	5 22.83	0.95	+0.05	25 53.3	+20.9	(−32.6)	82
2.36384	Algiers	5 49.01	1.09	+0.09	16 7.8	+17.6	− 0.4	80, 82
2.38747	Marseilles	6 5.63	1.02	−0.10	10 23.2	+19.0	− 1.9	79
2.55789	Albany	8 5.38	0.91	−0.44	10 28 9.6	+19.5	+ 7.0	83
2.60664	Cincinnati	9 40.03	1.10	+0.23	16 17.1	+18.2	− 2.1	84
2.95772	Windsor	12 43.09	1.19	−0.05	8 49 7.9	−12.1	+ 3.2	85, 86, 87
3.26381	Cape	16 13.36	0.75	+1.40	7 35 21.1	−11.5	− 4.0	90
3.32962	Marseilles	16 56.59	0.84	−0.03	19 51.5	+19.1	− 0.1	89
3.34771	Nice	17 8.73	0.96	+0.13	15 30.4	+18.8	− 0.3	89
3.36138	Rome	8 17 18.14	1.12	+0.50	−7 12 17.0	+18.0	− 5.5	89

Date		Place	App. α	π	$\Delta\alpha \cos\delta$	App. δ	π	$\Delta\delta$	*
			h m s	s	s	° ′ ″	″	″	
May	3.55221	Washington Naval	8 19 25.65	0.85	−0.08	− 6 26 27.1	+17.5	− 9.6	91
	3.57565	Albany	19 41.21	0.98	−0.01	20 41.7	+15.7	− 3.0	88
	3.63683	Northfield	20 22.30	0.98	+0.42	6 6.4	+18.8	− 3.0	96, 97 [102
	3.89555	Windsor	23 12.72	0.71	+0.27	5 4 1.4	−12.3	+ 2.3	92, 94, 98, 101,
	4.33148	Nice	27 54.34	0.83	−0.02	3 22 24.0	+18.0	− 4.8	100
	4.33556	Rome	27 57.44	0.95	+0.59	21 16.3	+17.3	+ 5.3	93
	4.34282	Algiers	28 1.80	0.92	+0.28	19 28.5	+15.7	+10.5	99
	4.36081	Greenwich	28 12.90	0.75	−0.26	15 33.3	+20.1	0.0	103
	4.36555	Liverpool	28 16.67	0.71	+0.41	14 18.7	+20.6	+ 8.7	93, 95
	4.36870	Bordeaux	28 18.29	0.90	+0.25	13 46.9	+18.4	− 5.4	108
	4.37957	Toulouse	28 25.34	0.99	+0.46	11 11.7	+17.7	− 5.0	108
	4.39657	Cambridge	28 37.21	. .	+0.50	6 53.5	. .	+ 2.2	108
	4.59844	Washington Naval	30 43.12	0.89	−0.40	2 20 51.8	+16.1	− 5.2	106, 107
	4.60615	Cincinnati	30 48.95	0.99	+0.51	18 59.1	+16.1	− 2.4	105
	4.90787	Windsor	33 56.88	0.75	−0.21	− 1 9 54.8	−13.2	− 1.7	104, 109
	5.26484	Cape	37 36.90	0.65	+0.79	+ 0 10 44.8	−13.6	− 3.0	113
	5.31887	Krakau	38 8.73	0.80	+0.02	21 7.5	+18.6	− 3.1	111, 112
	5.33738	Vienna	38 20.67	0.87	+0.81	25 6.9	+18.0	− 8.8	114
	5.35379	Strassburg	38 30.04	0.83	+0.21	28 49.9	+18.2	− 2.0	114
	5.35529	Padua	38 30.49	0.93	−0.13	29 8.5	+17.2	− 4.2	112
	5.35845	Göttingen	38 32.54	0.82	−0.13	29 44.1	+18.9	− 8.7	110
	5.35966	Prag	38 33.01	0.89	−0.29	28 47.2	+18.6	(−81.8)	112
	5.36139	Vienna B.	38 34.21	0.78	−0.21	30 27.8	+17.9	− 4.6	112
	5.36315	Bordeaux	38 35.77	0.82	−0.31	30 54.1	+17.2	− 2.2	114
	5.36671	Karlsruhe	38 37.82	0.88	−0.27	31 46.1	+18.3	+ 4.0	112
	5.36926	Toulouse	38 39.86	0.89	+0.77	32 11.1	+16.6	− 6.1	112
	5.37402	O'Gyalla	38 42.31	1.01	+0.37	33 31.5	+17.9	+11.6	112
	5.37417	Marseilles	38 42.00	0.97	−0.03	33 26.3	+16.5	+ 4.0	112
	5.37532	Nice	38 42.55	0.79	−0.29	33 36.3	+16.6	− 1.0	112
	5.39080	Bamberg	38 52.09	0.96	+0.08	36 57.9	+18.4	− 1.4	112
	5.39597	Lyons L.C.	38 55.20	1.01	+0.12	38 9.3	+17.3	+ 0.8	110, 112
	5.40924	Lyons G.	39 3.47	1.04	+0.43	40 55.1	+17.3	− 7.8	110
	5.41455	Jena K.	39 6.39	0.99	+0.10	42 6.3	+18.8	− 1.8	112
	5.86977	Windsor	43 37.36	0.41	−0.15	2 20 59.5	−14.0	+ 3.6	122, 121
	6.32251	Krakau	48 0.31	0.78	+0.85	3 55 41.5	+17.4	(+22.6)	115, 116
	6.33710	Vienna	48 4.90	0.80	+0.60	57 4.0	+16.9	− 2.2	127
	6.34734	Dresden	48 14.10	0.79	+0.45	4 0 23.0	+17.6	− 2.8	115
	6.35462	Munich	48 17.24	0.84	−0.51	2 1.6	+13.8	+ 1.9	123
	6.36582	Kremsmünster	48 21.55	0.91	+0.48	4 9.9	+16.9	− 4.6	118
	6.36668	Berlin	48 24.39	0.88	−0.22	4 25.0	+18.8	+ 1.6	123
	6.37186	Vienna B.	48 26.90	0.94	−0.59	5 13.8	+17.0	−15.2	123
	6.37188	Bamberg	48 27.43	0.86	−0.15	5 26.4	+17.8	− 2.0	119, 125
	6.37488	O'Gyalla	48 27.16	0.97	−0.31	5 21.2	+17.0	− 8.0	118
	6.41036	Jena K.	48 49.10	0.95	−0.31	13 18.8	+17.9	− 3.4	127
	6.41047	Hamburg	48 49.56	0.89	+0.02	13 21.6	+18.5	− 1.5	120
	6.86590	Windsor	53 5.13	0.34	−0.73	5 45 48.3	−14.6	+ 3.2	117, 124, 126
	7.33953	Nice	57 23.89	0.73	−0.48	7 17 31.0	+14.0	+ 2.1	132
	7.34437	Krakau	57 26.38	0.82	−0.48	17 28.6	+16.2	(−53.8)	132
	7.36152	Bordeaux	57 36.23	0.73	+0.04	21 43.0	+14.4	+ 2.3	134
	7.37592	Greenwich	57 43.56	0.70	−0.40	24 30.5	+16.4	+ 7.3	131, 132
	7.37645	Marseilles	57 43.45	0.90	−0.61	24 33.6	+14.1	+ 1.9	132
	7.39078	Hamburg	57 51.62	0.81	−0.23	27 9.9	+17.3	− 2.4	129, 130
	7.39527	Cambridge	57 54.89	. .	−0.19	28 21.0	. .	+ 0.2	131, 132
	7.40504	Liverpool	58 0.04	0.75	+0.47	29 58.7	+17.0	+ 3.3	132
	7.42012	Uccle	58 8.19	0.90	+0.67	31 32.3	+16.7	(−75.2)	135
	7.43455	Padua	58 11.62	1.06	−0.44	35 26.1	+15.6	− 6.1	131
	7.60124	Washington Naval	59 41.83	0.92	+0.66	8 7 0.5	+12.1	− 0.2	137
	7.61609	Cincinnati	59 51.96	0.90	−0.10	9 45.7	+12.4	− 1.6	133
	8.32672	Nice	9 6 1.17	0.63	−0.31	10 18 48.1	+12.5	− 1.3	141, 143
	8.33586	Strassburg	9 6 6.32	0.63	+0.19	+10 20 25.3	+14.1	+ 1.3	143

Date		Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
			h m s	s	s	° ′ ″		″	
May	8.34724	Algiers	9 6 11.91	0.60	+0.13	+10 22 31.3	+10.4	+ 2.7	138
	8.34866	Lyons L.C.	6 12.46	0.69	−0.12	22 44.5	+13.3	+ 4.1	140, 141
	8.36084	Lyons G.	6 19.36	0.74	+0.64	24 54.9	+13.4	+ 6.3	141, 142
	8.36764	Prag	6 22.77	0.78	+0.62	26 9.9	+11.5	+10.5	143
	8.36893	Toulouse	6 22.90	0.76	+0.08	26 17.8	+12.7	+ 3.0	143
	8.38150	Karlsruhe	6 29.50	0.82	+0.37	28 29.2	+14.7	+ 4.0	143
	8.38161	Besançon	6 29.41	0.82	+0.22	28 27.5	+11.1	+ 0.3	143, 145
	8.43313	Jena K.	6 55.33	0.92	+0.15	37 19.8	+16.1	− 6.2	145
	8.43506	Berlin	6 56.48	0.90	+0.30	37 34.6	+16.8	−11.0	139
	8.47940	Cordoba	7 18.47	0.41	−0.58	45 53.7	−11.3	− 6.7	145
	8.62314	Cincinnati	8 30.67	0.85	−0.13	11 10 26.8	+10.9	+ 0.5	144
	8.84620	Melbourne	10 22.64	. .	+1.10	48 52.6	−16.3	+ 1.2	M.T.
	8.94034	Windsor	11 7.62	0.78	−0.18	12 4 38.8	−11.1	+ 1.5	146
	9.32024	Nice	14 13.05	0.56	+0.15	13 6 40.6	+11.2	+ 1.9	147
	9.33802	Algiers	14 21.68	0.66	−0.33	9 30.1	+ 9.1	− 3.0	147
	9.33979	Krakau	14 23.02	0.73	+0.87	9 23.3	+13.8	(−22.3)	148
	9.34399	Kremsmunster	14 24.79	0.72	+0.63	10 23.3	+13.0	− 3.9	147
	9.34447	Munich	14 24.46	0.65	+0.04	10 31.9	+13.7	+ 1.2	147
	9.34729	Strassburg	14 25.97	0.65	+0.39	10 55.5	+12.9	− 3.5	147
	9.35571	Jena W.	14 29.84	0.69	+0.03	12 16.5	+13.9	− 3.3	150
	9.35660	Padua	14 29.67	0.77	−0.47	12 13.0	+12.3	−16.7	147, 148
	9.36382	Bamberg	14 33.65	0.73	0.00	13 38.8	+15.7	+ 2.8	146, 147
	9.36653	O'Gyalla	14 33.91	0.84	−0.87	13 50.6	+13.5	−13.8	147, 148
	9.36714	Dresden	14 36.12	0.75	+0.87	14 5.6	+14.1	− 4.2	156
	9.37073	Besançon	14 37.14	0.74	+0.19	14 44.8	+12.8	− 0.8	147, 148
	9.37183	Berlin	14 37.82	0.75	+0.35	14 51.4	+13.4	− 4.3	147, 148
	9.37352	Kiel	14 37.77	0.69	−0.53	15 4.0	+14.9	− 6.5	147, 149
	9.38917	Vienna	14 46.53	0.88	+0.85	17 28.6	+13.9	−14.1	148
	9.40369	Hamburg	14 52.99	0.78	+0.29	19 59.6	+15.2	− 1.4	148
	9.42032	Greenwich	15 0.72	0.79	+0.10	22 36.2	+14.5	− 5.5	148
	9.42416	Karlsruhe	15 2.52	0.90	+0.16	23 12.0	+14.4	− 6.6	148, 149
	9.42484	Liverpool	15 2.65	0.74	−0.18	23 19.4	+14.9	− 5.3	148, 149
	9.42616	Jena K.	15 3.69	0.88	+0.35	23 33.9	+15.0	− 3.4	149
	9.48139	Göttingen	15 29.41	0.89	−0.25	32 13.9	+16.3	−11.4	149
	9.56250	Washington Naval	16 7.97	0.86	−0.21	45 29.7	+10.2	+ 5.9	149
	9.61913	Wash. Catholic	16 35.51	0.92	+0.41	54 22.0	+10.5	− 0.2	151
	9.89990	Windsor	18 47.06	0.18	+0.07	14 38 19.2	−11.9	− 2.4	152, 154
	10.32985	Rome	22 1.84	0.68	+0.62	15 42 22.3	+ 9.8	−13.0	155
	10.34534	Kiew	22 12.00	0.79	+0.85	44 37.1	+13.3	−11.5	155
	10.35900	Krakau	22 17.28	0.77	−0.04	46 53.2	+13.0	+ 3.8	153
	10.37041	Hamburg	22 23.28	0.66	+0.66	48 18.5	+13.6	−10.8	158
	10.37142	Algiers	22 23.15	0.80	+0.23	48 33.8	+ 9.0	− 8.2	158
	10.37253	Greenwich	22 23.71	0.60	+0.09	48 41.2	+12.6	− 7.8	158
	10.39201	Lyons L.C.	22 32.64	0.80	+0.41	51 33.1	+11.6	− 8.1	155
	10.39592	O'Gyalla	22 34 10	0.88	+0.17	52 3.5	+13.0	−11.0	158
	10.39813	Cambridge	22 36.20	. .	+0.39	52 38.1	. .	− 8.7	158, 159
	10.41824	Liverpool	22 44.24	0.70	+0.05	55 24.4	+13.7	− 5.3	158
	10.47075	Lyons G.	23 7.84	0.97	+0.26	16 3 4.1	+13.3	− 5.4	158
	10.55718	Wash. Catholic	23 47.17	0.58	+0.70	15 53.3	+ 8.3	+ 7.9	160
	10.85133	Melbourne	25 58.01	. .	+0.10	57 19.2	−16.1	−11.3	M.T.
	10.86206	Windsor	26 2.23	0.25	−0.13	59 19.9	−15.2	− 9.8	161, 164
	11.35738	Nice	29 36.72	0.69	−0.06	18 6 12.7	+ 9.5	−10.1	165
	11.38231	Marseilles	29 46.66	0.77	−0.66	10 5.8	+ 9.8	− 6.2	163
	12.37226	Nice	36 38.64	0.72	+0.08	20 16 1.7	+ 8.9	− 5.4	166
	12.39559	Marseilles	36 47.60	0.79	−0.28	18 59.5	+ 9.2	+ 3.1	167
	12.39672	Bordeaux	36 49.03	0.72	+0.56	18 55.8	+ 9.3	− 8.5	170
	12.44317	Jena W.	37 6.11	0.82	−0.87	24 37.6	+12.5	+ 0.1	162, 167
	12.47367	Liverpool	37 19.31	0.76	−0.04	28 18.7	+12.8	+ 1.4	167, 169
	12.49524	Greenwich	37 28.00	0.82	+0.05	30 50.3	+13.2	− 1.7	167
	12.57293	Cincinnati	9 37 59.74	0.19	+0.36	+20 10 12.9	+ 6.4	− 2.5	169

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
May 12.62838	Washington Naval	9 38 20.86	0.85	−0.24	+20 46 50.7	+ 7.9	+ 2.2	169
12.67371	Tacubaya	38 38.89	0.99	−0.11	52 14.9	+ 2.0	− 0.5	168
13.37530	Bordeaux	43 12.76	0.62	+0.08	22 12 3.9	+ 8.0	− 2.1	172
13.58884	Washington Naval	43 33.43	0.66	−0.26	35 19.4	+ 6.2	− 1.8	171
13.85683	Melbourne	46 14.81	. .	−0.01	23 4 28.9	−15.1	(+29.5)	M.T.
14.36769	Bordeaux	49 22.50	0.55	+0.66	56 22.1	+ 7.1	− 3.6	174
14.37268	Padua	49 23.96	0.69	+0.48	56 48.2	+ 8.0	− 6.7	174
14.37673	Nice	49 25.36	0.69	+0.42	57 19.2	+ 7.5	− 0.5	173
14.40152	O'Gyalla	49 32.43	0.78	−1.19	59 44.5	+ 9.8	− 0.8	177
14.40527	Jena W.	49 34.08	0.70	−1.03	59 19.4	+10.1	−18.9	177
14.73206	Tacubaya	51 31.80	1.01	−0.10	24 32 36.6	+ 2.9	+13.0	176
14.85815	Melbourne	52 17.42	. .	−0.09	44 59.3	−14.8	+ 9.9	M.T.
15.37347	Padua	55 16.06	0.67	−0.04	25 32 35.0	+ 7.4	+ 1.9	180, 181
15.37675	Kremsmünster	55 18.16	0.67	+0.84	32 49.5	+ 8.2	− 0.7	175
15.38191	Strassburg	55 19.49	0.62	+0.40	33 15.6	+ 8.1	− 2.9	175
15.39512	Besançon	55 23.90	0.66	+0.32	34 28.2	+ 7.9	− 2.6	181
15.39947	Prag	55 25.04	0.70	+0.05	34 14.5	+ 9.3	− 8.6	178
15.40025	Jena W.	55 26.36	0.67	−0.37	35 8.7	+ 9.2	+11.6	181
15.40211	Hamburg	55 26.44	0.62	+0.44	34 58.6	+ 9.7	− 8.2	181
15.63896	Tacubaya	56 46.19	0.67	−0.35	56 34.9	− 0.6	+ 0.3	179
16.35235	Padua	10 0 44.16	0.58	+0.20	26 57 31.2	+ 6.3	−13.3	182, 184
16.35431	Besançon	0 44.97	0.50	+0.52	57 45.7	+ 6.4	− 7.0	182
16.35923	Strassburg	0 46.60	0.53	+0.56	58 16.1	+ 6.9	− 0.6	182
16.36911	Kremsmünster	0 49.91	0.62	+0.72	59 3.1	+ 7.4	− 2.3	182
16.37885	Jena W.	0 52.16	0.60	−0.12	59 45.0	+ 8.1	− 8.1	182, 184, 185
16.40658	Krakau	1 1.27	0.59	−0.05	27 2 6.6	+10.0	+ 2.3	184
16.44650	Liverpool	1 13.87	0.62	−0.35	5 19.1	+ 9.2	− 8.0	182, 184
16.58047	Cincinnati	1 57.75	0.45	−0.02	16 21.2	+ 3.9	− 8.6	186
16.63794	Tacubaya	2 15.98	0.64	−0.09	21 14.4	− 0.9	+ 0.2	183
16.65233	Northfield	2 21.21	0.61	+0.43	22 13.9	+ 6.1	− 3.1	183, 186
17.34543	Padua	5 59.87	0.53	+0.27	28 16 26.1	+ 5.6	− 4.0	187
17.36580	Besançon	6 5.97	0.53	+0.09	17 52.8	+ 6.1	− 9.3	187
17.36744	Strassburg	6 6.47	0.54	+0.09	18 8.3	+ 6.5	− 0.7	187
17.37565	Nice	6 8.93	0.61	+0.08	18 16.2	+ 5.6	− 1.0	187
17.38643	Greenwich	6 12.02	0.50	−0.24	19 34.4	+ 7.1	− 0.1	187
17.38910	Algiers	6 13.02	0.68	+0.07	19 49.9	+ 4.1	+ 0.2	187
17.39205	Hamburg	6 14.27	0.56	+0.26	19 58.3	+ 8.2	− 0.7	187
17.40139	Cambridge	6 17.21	. .	−0.19	20 52.2	. .	+ 2.8	187
17.40699	Liverpool	6 18.58	0.52	−0.06	21 3.7	+ 6.1	− 4.8	187
18.32947	Nice	10 57.68	0.56	+0.22	29 27 42.4	+ 4.0	− 5.2	190
18.37984	Krakau	11 12.11	0.63	−0.11	31 6.2	+ 7.4	− 6.5	196
18.39507	Algiers	11 17.72	0.68	+0.92	32 16.8	+ 3.8	− 2.5	190
18.40764	Jena W.	11 19.56	0.64	−0.73	32 51.5	+ 7.7	−15.8	188, 190
18.41515	Cambridge	11 23.64	. .	+0.33	33 42.4	. .	− 3.6	190
18.42339	Greenwich	11 23.97	0.58	−0.97	33 54.0	+ 7.4	−20.5	190
18.42861	Besançon	11 26.48	0.68	−0.04	34 28.8	+ 7.1	− 5.4	190
18.43978	Liverpool	11 29.70	0.57	−0.19	35 17.2	+ 7.9	− 2.3	190
18.45642	Leiden	11 35.10	0.67	+0.33	35 19.4	+ 9.4	− 6.8	188
18.63755	Tacubaya	12 27.63	0.98	−0.64	48 49.4	− 1.5	− 6.7	189
19.34567	O'Gyalla	15 48.78	0.53	(−1.62)	30 35 5.1	+ 5.6	+ 8.3	191
19.35721	Krakau	15 54.01	0.44	−0.03	35 33.8	+ 6.4	− 5.9	192
19.37698	Besançon	15 59.64	0.53	+0.14	36 45.1	+ 5.4	−10.4	193, 194
19.37764	Göttingen	15 59.85	0.52	+0.19	36 52.6	+ 6.6	− 3.6	191
19.38300	Strassburg	16 1.67	0.55	+0.45	37 15.0	+ 5.9	− 2.5	191
19.39303	Berlin	16 4.19	0.55	+0.20	37 53.0	+ 5.2	− 2.8	191
19.40426	Hamburg	16 7.39	0.56	+0.23	38 31.9	+ 7.5	+ 2.9	191, 194
19.41388	Algiers	16 9.86	0.72	+0.19	39 11.4	+ 4.0	− 4.5	194
19.45452	Jena W.	16 20.17	0.67	−0.81	41 35.3	+ 8.5	− 8.9	194
19.47029	Kremsmünster	16 25.93	0.72	+0.39	42 44.5	+ 8.7	+ 1.6	194
20.36452	Vienna B.	10 20 34.25	0.56	+0.25	+31 36 17.3	+ 3.6	−10.1	195

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
May 20.39735	Krakau	10 20 38.74	0.62	−0.83	+31 38 22.2	+ 7.0	+ 4.7	196
20.63796	Washington Naval	21 44.94	0.61	+0.88	50 59.2	+15.7	+ 8.5	203
21.30143	Nicolajew	24 37.82	0.49	+0.24	32 28 15.9	+ 4.6	− 4.8	197
21.35311	O'Gyalla	24 48.42	0.52	(−2.00)	31 20.8	+ 5.3	+11.7	197
21.36274	Vienna	24 54.26	0.53	+0.86	31 32.8	+ 5.2	− 2.2	208
21.37960	Krakau	24 57.29	0.64	−0.49	32 22.3	+ 7.2	− 8.2	202
21.38810	Greenwich	24 59.94	0.45	0.00	32 49.0	+ 5.4	− 7.2	197, 198, 202
21.40439	Bordeaux	25 4.35	0.42	+0.58	33 47.6	+ 3.9	+ 3.1	197
21.40027	Hamburg	25 3.26	0.52	+0.28	33 30.8	+ 6.6	− 3.2	197
21.40775	Algiers	25 4.68	0.67	−0.01	33 59.4	+ 3.2	− 0.3	197
21.41151	Liverpool	25 6.38	0.46	+0.44	34 11.8	+ 6.0	− 7.2	197, 202
21.42546	Cambridge	25 10.00	. .	+0.09	34 59.5	. .	+ 0.4	197, 202
21.64189	Cincinnati	26 4.39	0.61	−0.03	46 3.0	+ 3.4	− 9.1	198, 199
21.66245	Tacubaya	26 9.36	0.66	−0.23	47 18.2	− 1.1	− 2.6	199
22.35882	Padua	29 3.90	0.50	+0.11	33 22 10.4	+ 4.0	+ 2.3	206, 207, 209
22.43033	Hamburg	29 21.49	0.55	−0.09	25 25.6	+ 6.9	− 7.5	200
22.62830	Tacubaya	30 9.79	0.49	+0.07	35 10.2	− 2.1	− 1.3	205
23.39208	Greenwich	33 12.55	0.44	−0.17	34 10 4.3	+ 4.8	− 9.1	210, 212
23.41014	Hamburg	33 17.31	0.51	+0.32	10 51.6	+ 6.1	− 8.9	211, 213
23.43414	Cambridge	33 22.73	. .	−0.30	12 15.4	. .	+ 5.5	219, 221
23.61934	Tacubaya	34 6.18	0.51	+0.15	20 23.4	− 2.0	+ 1.3	210
24.38634	Greenwich	37 2.27	0.41	−0.35	52 42.7	+ 4.4	− 1.5	214, 216
24.39712	Hamburg	37 5.49	0.32	+0.22	53 10.9	+ 5.5	+ 1.2	214
24.41706	Göttingen	37 9.54	0.53	−0.02	53 55.9	+ 5.6	− 2.5	218
24.42949	Liverpool	37 12.11	0.59	−0.15	54 29.0	+ 5.4	− 0.4	215, 216
24.43283	Cambridge	37 13.59	. .	−0.03	54 41.5	. .	− 1.4	214
24.48520	Algiers	37 25.81	0.75	+0.92	54 48.7	+ 5.0	+ 2.7	214
24.49557	Dunsink	37 27.25	0.57	−0.14	57 5.4	+ 7.3	− 4.4	216
24.63556	Northfield	37 59.11	0.43	+0.18	35 2 49.2	+ 3.0	− 4.0	215, 216
25.44383	Kiew	40 57.51	0.58	+0.12	34 15.4	+ 7.5	− 1.4	217
25.53495	Dunsink	41 15.30	0.54	−1.00	37 40.1	+ 7.1	+ 3.7	220
26.34887	O'Gyalla	44 13.03	0.47	−0.82	36 8 43.5	+ 4.1	− 0.4	223, 224
26.61712	Cincinnati	45 13.65	0.56	+0.15	17 33.4	+ 2.5	− 2.6	222, 224
27.44791	Greenwich	47 51.68	0.45	−0.16	43 19.4	+ 4.4	+ 3.9	226
27.50513	Liverpool	48 9.27	0.52	−0.15	46 2.4	+ 6.3	+ 5.0	226
27.58260	Albany	48 24.69	0.42	−0.37	48 31.2	+ 2.2	+ 1.2	226
27.64687	Washington Naval	48 37.96	0.59	+0.03	50 30.7	+ 2.9	− 1.0	226
28.39124	Padua	51 5.73	0.51	+0.04	37 13 21.3	+ 3.5	− 4.5	225, 230
28.39322	Jena W.	51 5.55	0.46	−0.42	13 22.3	+ 4.3	− 6.2	230, 235
28.39495	Krakau	51 6.53	0.51	+0.10	13 39.4	+ 4.7	+ 8.1	225
28.41923	Hamburg	51 11.61	0.47	+0.37	14 10.6	+ 5.0	− 3.8	225
28.42896	Liverpool	51 12.80	0.43	−0.27	14 34.1	+ 4.7	+ 2.0	225
28.59423	Cincinnati	51 45.44	0.38	−0.01	19 30.5	+ 1.2	+ 0.8	225
28.63234	Tacubaya	51 52.55	0.45	−0.20	20 30.8	− 2.2	− 9.4	225
28.63263	Washington Naval	51 52.65	0.58	−0.02	20 34.9	+ 2.4	− 2.8	225
29.35021	Kiew	54 9.45	0.47	−0.94	41 11.8	+ 4.2	+ 7.7	229
29.37924	Lyons L.C.	54 16.69	0.43	+0.40	41 55.1	+ 2.7	+ 1.1	232
29.38392	Bordeaux	54 17.80	0.28	+0.50	42 6.4	+ 1.7	+ 4.2	229
29.39417	Lyons G.	54 19.50	0.46	+0.42	42 20.3	+ 3.0	+ 1.8	232
29.40298	Jena W.	54 19.29	0.46	−1.05	42 35.0	+ 4.3	+ 3.1	236
29.46630	Liverpool	54 32.58	0.47	−0.03	44 19.3	+ 5.1	+ 3.3	227, 236
29.55587	Greenwich	54 48.68	0.50	−0.67	46 44.5	+ 7.1	+ 2.9	236
29.60125	Albany	54 57.86	0.46	−0.20	48 1.2	+ 2.3	+ 0.4	227
30.32375	Nicolajew	57 12.58	0.45	−0.03	38 7 9.6	+ 3.2	− 0.5	240
30.38789	Marseilles	57 24.32	0.48	−0.06	8 47.6	+ 2.4	− 2.4	231
30.40949	Jena W.	57 27.50	0.47	−0.69	9 23.1	+ 4.3	+ 1.3	228
30.41424	Prag	57 28.15	0.49	−0.85	9 30.0	+ 4.5	+ 1.4	234
30.42950	Krakau	57 31.50	0.52	−0.39	9 47.6	+ 5.3	− 3.0	241
30.43398	Greenwich	57 32.75	0.46	−0.27	10 8.8	+ 4.3	+ 9.5	231, 233
30.45612	Lyons G.	10 57 38.22	0.55	(+1.07)	+38 10 46.3	+ 4.3	+13.0	237

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
May 30.45845	Liverpool	h m s 10 57 36.90	s 0.39	s −0.41	° ′ ″ +38 10 34.8	+ 3.9	″ − 2.2	231, 241
30.46265	Göttingen	57 38.53	0.51	+0.44	10 46.7	+ 5.3	+ 7.7	237
30.68134	Tacubaya	58 18.92	0.61	+0.79	16 39.2	− 1.0	(−42.0)	237
31.37591	Marseilles	11 0 23.49	0.43	+0.16	33 23.4	+ 2.1	+ 3.9	244
31.37896	Kiew	0 24.03	0.49	+0.22	33 16.4	+ 4.5	− 4.8	244
31.39835	Greenwich	0 27.25	0.38	−0.07	33 52.1	+ 3.4	+ 1.9	244
31.40310	Bordeaux	0 28.64	0.44	+0.10	33 50.9	+ 2.5	− 6.9	244
31.40585	Hamburg	0 29.20	0.42	+0.43	34 1.0	+ 6.8	+ 3.6	244
31.43938	Cambridge	0 35.21	. .	+0.13	34 55.7	. .	+ 3.3	244, 245
31.44501	Liverpool	0 35.50	0.43	−0.08	35 0.2	+ 4.3	+ 4.1	244
31.44986	Besançon	0 36.91	0.52	+0.42	34 58.0	+ 4.2	− 4.9	244
31.47257	Karlsruhe	0 41.26	0.52	+0.72	35 40.8	+ 5.1	+ 6.1	238, 239, 241
31.58845	Cincinnati	1 1.18	0.34	0.00	38 22.7	+ 0.9	+ 0.1	244
June 1.35255	Nicolajew	3 15.96	0.47	+0.76	55 41.7	+ 3.5	− 6.0	243
1.36004	Kiew	3 16.88	0.46	+0.45	56 20.7	+ 4.0	(+23.5)	243
1.37981	Marseilles	3 19.89	0.43	+0.10	56 26.7	+ 2.1	+ 1.1	243
1.38795	Vienna	3 22.07	0.47	+0.76	56 32.1	+ 3.5	− 2.5	243
1.45660	Karlsruhe	3 33.95	0.51	+0.77	58 6.8	+ 4.6	+ 1.8	243
2.33272	Kiew	6 3.65	0.43	+0.73	39 17 2.4	+ 3.3	+13.9	245
2.38255	Marseilles	6 10.87	0.43	−0.21	17 56.9	+ 2.1	+ 5.2	242
2.40192	Greenwich	6 14.62	0.38	+0.01	18 14.9	+ 3.2	− 0.9	245, 247
2.40280	Lyons L.C.	6 15.12	0.45	+0.19	18 16.6	+ 2.8	+ 0.5	246
2.41317	Lyons G.	6 16.97	0.47	+0.56	18 26.9	+ 3.0	− 1.7	245
2.43369	Bordeaux	6 20.50	0.49	+0.64	18 50.5	+ 3.0	− 3.5	242
2.59695	Washington Naval	6 47.41	0.42	+0.17	22 42.7	− 1.2	(+31.9)	247
2.64138	Cincinnati	6 54.67	0.48	+0.07	23 14.5	+ 1.7	+ 5.3	247
3.41582	Karlsruhe	9 3.19	0.45	+0.08	38 20.7	+ 3.5	− 0.5	247
4.37806	Kiew	11 38.40	0.45	−0.16	56 7.0	+ 4.0	− 0.2	249
4.38190	Marseilles	11 39.47	0.42	−0.14	56 24.2	+ 1.9	+10.7	249
4.69260	Tacubaya	12 28.18	0.59	−0.68	40 1 49.3	− 5.8	− 0.8	249
4.70835	Northfield	12 32.20	0.51	+0.45	1 59.8	+ 2.9	+ 1.8	249
5.38937	Marseilles	14 17.63	0.43	−0.95	13 32.6	+ 2.0	− 1.1	252
5.39030	Lyons G.	14 19.25	0.41	+0.05	13 33.5	+ 2.3	− 1.0	251
5.39863	Lyons L.C.	14 21.15	0.42	+0.50	13 47.0	+ 2.4	+ 4.1	250
5.47007	Besançon	14 31.66	0.49	+0.07	14 46.3	+ 4.2	− 5.5	252
5.72269	Northfield	15 11.13	0.50	+0.18	18 58.4	+ 3.4	− 2.4	252
6.37926	Marseilles	16 52.50	0.40	+0.22	29 29.0	+ 1.8	+ 2.8	252
6.40119	Kiew	16 55.09	0.45	+0.08	29 45.0	+ 4.3	+ 0.9	254
6.40465	Nicolajew	16 56.09	0.48	+0.02	29 48.1	+ 4.3	+ 0.8	252
6.42978	Liverpool	16 59.26	0.41	−0.56	30 12.7	+ 4.1	+ 1.3	252
6.47720	Besançon	17 7.10	0.48	−0.04	30 57.6	+ 4.3	+ 3.1	252
7.50309	Greenwich	19 41.52	0.43	−0.29	46 9.5	+ 4.7	+ 3.0	255
7.64208	Cincinnati	20 2.24	0.44	−0.25	48 5.3	+ 1.5	− 4.2	255
7.65797	Ann Arbor	20 4.73	0.45	−0.14	48 18.5	+ 2.2	− 2.8	256
7.71947	Northfield	20 14.06	0.48	+0.03	49 10.5	+ 3.2	− 1.7	256
8.32487	Nicolajew	21 43.62	0.40	+0.08	57 31.1	+ 2.5	− 2.3	253, 257
8.41784	Bordeaux	21 56.71	0.42	−0.21	58 51.7	+ 2.3	+ 2.8	256
8.63214	Tacubaya	22 28.04	0.39	−0.19	41 1 45.7	− 7.8	− 1.3	257
10.36837	Kiew	26 36.93	0.40	−0.03	23 10.1	+ 3.5	− 2.7	258
10.75676	Cincinnati	27 31.46	0.50	+0.04	27 41.8	+ 4.0	+ 1.4	258
11.49568	Cambridge	29 14.79	. .	+0.18	35 57.9	. .	+ 2.7	261
12.38753	Marseilles	31 16.92	0.38	+0.43	45 13.1	+ 1.7	+ 0.6	261
12.39174	Lyons G.	31 16.95	0.37	+0.02	45 8.1	+ 1.9	− 6.8	259
12.46065	Bordeaux	31 26.94	0.45	+0.57	45 57.4	+ 2.9	+ 1.8	261
12.67849	Tacubaya	31 55.59	0.50	−0.06	48 8.1	− 7.1	− 8.5	261
13.42506	Bordeaux	33 36.17	0.40	+0.03	55 8.9	+ 1.2	−14.3	260
14.48927	Besançon	36 56.54	0.42	−0.39	42 4 53.3	+ 3.9	− 1.4	262
15.64028	Washington Naval	38 26.85	0.43	−0.16	14 50.9	+ 1.7	+17.6	263
19.38985	Lyons G.	46 19.63	0.34	−0.07	40 50.7	+ 1.8	+ 6.8	264
19.39713	Bordeaux	11 46 20.41	0.34	−0.15	+42 40 56.8	+ 1.6	+10.1	264

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
June 19.40512	Lyons L.C.	11 46 21.69	0.36	+0.06	+42 40 50.9	+2.0	+ 1.6	265
19.66974	Northfield	46 54.48	0.36	+0.27	42 22.9	+1.8	+ 1.0	270
20.37771	Lyons L.C.	48 21.86	0.33	+0.90	46 33.6	+1.7	+ 5.7	267
20.37914	Marseilles	48 21.87	0.34	+0.79	46 27.3	+1.4	− 1.3	266
20.39737	Toulouse	48 22.90	0.35	0.00	46 35.7	+1.6	+ 1.1	266
20.39758	Bordeaux	48 23.31	0.34	+0.21	46 36.5	+1.6	+ 1.8	269
20.40225	Lyons G.	48 21.05	0.36	+0.64	46 39.2	+2.0	+ 3.4	267
20.43226	Liverpool	48 24.33	0.31	(−2.17)	46 50.8	+2.7	+ 5.7	267
21.35513	Nicolajew	50 19.15	0.36	+0.18	51 40.9	+2.5	− 0.8	268
21.38217	Lyons G.	50 22.52	0.33	+0.23	51 51.4	+1.6	+ 0.1	268
21.38578	Lyons L.C.	50 22.91	0.33	+0.20	51 55.1	+1.7	+ 3.0	268
21.38959	Toulouse	50 23.54	0.33	+0.33	51 52.2	+1.4	− 1.3	271
21.40354	Besançon	50 24.64	0.34	−0.10	51 50.3	+2.1	− 6.8	268
21.41752	Strassburg	50 26.67	0.32	+0.45	52 0.8	+2.5	− 0.2	274
21.42073	Bordeaux	50 26.89	0.36	+0.06	51 58.6	+1.9	− 3.9	274
21.44455	Liverpool	50 29.20	0.33	−0.37	52 9.6	+2.9	+ 0.7	274
21.44925	Greenwich	50 32.74	0.33	(+1.81)	52 37.1	+2.9	(+26.8)	268
21.64515	Albany	50 53.35	0.38	−0.32	53 15.0	+2.2	+ 4.4	274
22.38419	Lyons G.	52 22.30	0.33	+0.06	56 33.1	+1.6	−14.8	271
22.38742	Bordeaux	52 28.09	0.32	(+4.01)	56 53.8	+1.4	+ 4.8	272
22.38745	Marseilles	52 22.98	0.34	+0.29	56 50.8	+1.5	+ 1.9	270
22.39253	Toulouse	52 23.27	0.34	+0.05	56 45.3	+1.5	− 5.0	271
22.39645	Lyons L.C.	52 24.27	0.31	+0.44	56 50.0	+1.8	− 1.1	271
22.40370	Vienna	52 25.17	0.35	+0.47	56 57.5	+2.7	+ 5.2	275
22.41119	Kremsmünster	52 25.59	0.36	−0.14	56 59.1	+2.7	+ 4.7	274
22.41476	Besançon	52 26.35	0.35	+0.31	56 50.1	+2.3	− 5.7	271
22.46928	Greenwich	52 31.51	0.34	−0.61	57 13.0	+3.2	+ 2.7	268, 270
22.46953	Liverpool	52 31.98	0.32	−0.31	57 8.4	+3.1	− 2.1	272
22.60614	Cincinnati	52 48.02	0.32	−0.16	57 48.1	+0.7	− 3.1	272
22.62650	Washington Naval	52 50.54	0.38	−0.35	58 10.5	+1.4	+14.3	272
22.67567	Northfield	52 56.96	0.35	+0.49	58 8.5	+1.9	− 0.9	272
23.38447	Lyons G.	54 20.85	0.32	+0.01	43 1 16.1	+1.6	− 4.3	272
23.44747	Kremsmünster	54 28.46	0.36	+0.16	1 33.3	+3.3	− 1.8	271
23.45073	Bordeaux	54 29.14	0.37	+0.39	1 40.4	+2.5	+ 3.7	271
23.61704	Cincinnati	54 47.90	0.33	−0.25	2 17.2	+0.9	− 3.9	272
23.61866	Wash. Catholic	54 48.68	0.37	+0.21	2 25.1	+1.2	+ 3.9	274
23.63990	Washington Naval	54 49.95	0.39	−0.68	2 27.6	+1.6	+ 1.4	272
24.47275	Greenwich	56 27.96	0.33	−0.50	5 22.0	+3.1	(−27.7)	275, 273
25.41567	Bordeaux	58 19.08	0.34	+0.50	9 31.6	+1.8	+ 6.7	277
25.41601	Besançon	58 17.91	0.34	−0.39	9 21.3	+2.2	− 3.3	277
25.45766	Liverpool	58 22.63	0.31	−0.48	9 31.8	+3.0	− 0.9	273, 277
25.62025	Cincinnati	58 41.47	0.34	−0.46	10 6.4	+0.9	− 3.2	277
26.39083	Bordeaux	12 0 11.79	0.31	+0.52	12 57.8	+1.4	+10.4	277
26.39684	Lyons L.C.	0 12.53	0.33	+0.58	12 46.5	+1.8	− 1.6	276
26.43385	Besançon	0 16.44	0.34	+0.33	12 56.5	+2.5	+ 1.7	277
27.38787	Lyons G.	2 5.53	0.32	+0.15	15 51.1	+1.7	− 3.9	277
27.39776	Lyons L.C.	2 7.08	0.32	+0.46	16 5.3	+1.8	+ 8.7	277
27.41407	Strassburg	2 9.05	0.32	+0.53	15 59.2	+2.4	+ 0.3	277
27.45016	Besançon	2 12.78	0.31	+0.26	16 2.6	+2.8	− 2.0	277
27.45450	Liverpool	2 13.42	0.30	+0.33	16 3.6	+2.8	− 2.2	277
27.61934	Cincinnati	2 31.56	0.33	−0.13	16 40.4	+0.9	+ 3.6	277
28.39286	Bordeaux	4 0.38	0.31	+0.50	18 42.0	+1.5	− 4.0	277
28.39520	Lyons L.C.	4 0.61	0.32	+0.48	18 53.1	+1.8	+ 7.0	279
28.39914	Strassburg	4 1.32	0.31	+0.66	18 45.7	+2.1	− 0.7	277
28.40133	Lyons G.	4 0.66	0.34	+0.03	18 48.5	+1.9	+ 1.5	280
28.42680	Besançon	4 4.04	0.33	+0.37	18 46.5	+2.4	− 4.1	277
28.45497	Liverpool	4 7.13	0.29	+0.27	18 55.6	+2.7	+ 0.7	277
28.46219	Greenwich	4 4.19	0.31	(−2.45)	18 36.1	+2.8	(−19.9)	277, 278
28.59232	Wash. Catholic	4 22.64	0.32	+0.26	19 14.4	+0.8	− 4.1	277
28.60276	Cincinnati	12 4 23.30	0.30	−0.11	+43 19 21.0	+0.7	+ 0.7	278

Date	Place	App. α	π	Δα cos δ	App. δ	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
June 28.65399	Washington Naval	12 4 28.25	0.38	−0.71	+43 19 39.8	+1.9	+12.7	277
29.39251	Lyons G.	5 52.05	0.32	−0.47	21 15.1	+1.7	− 3.3	281
29.39631	Lyons L.C.	5 53.83	0.32	+0.52	21 18.1	+1.8	− 0.7	281
29.41460	Besançon	5 55.38	0.32	+0.15	21 12.1	+2.2	− 8.9	281
29.43822	Greenwich	5 58.22	0.30	+0.27	21 13.5	+2.5	−10.5	281
29.46121	Liverpool	6 0.95	0.29	+0.36	21 20.2	+2.8	− 6.8	277, 281
29.60693	Wash. Catholic	6 15.57	0.33	(−0.90)	21 43.1	+1.0	− 6.2	277
30.38636	Vienna	7 45.06	0.32	+0.33	23 37.2	+2.3	+ 2.1	282
30.39534	Lyons L.C.	7 45.88	0.31	+0.41	23 35.8	+1.8	+ 1.4	281
30.39976	Lyons G.	7 45.18	0.32	−0.45	23 49.8	+1.8	+14.8	281
30.40310	Toulouse	7 46.09	0.32	−0.06	23 37.1	+1.6	+ 1.5	281
30.41925	Besançon	7 48.26	0.32	+0.21	23 39.5	+2.3	+ 2.5	281
30.45294	Liverpool	7 51.43	0.29	−0.25	23 50.4	+2.7	+ 9.6	281, 283
30.46289	Greenwich	7 52.33	0.30	+0.41	23 36.8	+2.9	− 5.0	281
30.58765	Albany	8 6.53	0.30	−0.22	24 2.4	+1.2	+ 3.0	283
30.60857	Cincinnati	8 8.56	0.31	−0.46	24 2.4	+0.8	+ 9.9	281
30.64898	Wash. Catholic	8 12.60	0.37	−0.73	24 3.0	+1.8	− 3.3	277, 281
July 1.60357	Wash. Catholic	10 0.84	0.33	+0.53	26 1.1	+1.0	+ 2.5	281
2.39140	Lyons L.C.	11 28.13	0.31	+0.43	27 23.2	+1.7	+ 3.4	283
2.40144	Lyons G.	11 29.62	0.31	+0.70	27 24.9	+1.9	+ 4.3	283
2.40770	Marseilles	11 29.64	0.33	+0.22	27 28.5	+1.8	+ 7.2	283
2.41319	Besançon	11 30.01	0.31	+0.04	27 22.9	+2.2	+ 1.6	283
2.44648	Greenwich	11 32.00	0.29	(−1.22)	27 6.6	+2.6	(−17.5)	283
2.60442	Cincinnati	11 51.44	0.30	+0.22	27 43.0	+0.8	+ 1.8	283
3.37740	Marseilles	13 16.96	0.30	+0.37	28 54.5	+1.4	+ 4.2	283
3.39780	Bordeaux	13 19.08	0.30	+0.27	28 52.4	+1.6	+ 0.6	283
3.46595	Greenwich	13 24.98	0.29	−0.83	28 55.3	+2.9	− 0.9	283
4.35206	Nicolajew	15 2.20	0.31	−0.99	29 59.4	+2.3	− 6.2	287
4.38513	Algiers	15 6.79	0.33	−0.28	30 9.6	+1.0	+ 0.4	284
4.39633	Toulouse	15 8.81	0.31	+0.28	30 13.2	+1.5	+ 3.6	284
4.39915	Lyons G.	15 8.03	0.31	−0.50	30 8.0	+1.8	− 1.5	285
4.40220	Lyons L.C.	15 9.56	0.31	+0.37	30 7.4	+1.9	− 2.2	285
4.42317	Bordeaux	15 11.85	0.32	+0.37	30 10.3	+1.9	− 0.7	287
4.46212	Besançon	15 16.22	0.31	+0.43	30 18.8	+2.9	+ 6.0	283
4.63468	Albany	15 34.21	0.33	−0.25	30 28.9	+1.9	+ 3.1	287
5.38096	Algiers	16 56.15	0.34	+0.02	31 26.8	+0.9	+12.5	287
5.39771	Lyons G.	16 58.89	0.30	+0.65	31 5.7	+1.8	− 8.6	286
5.40184	Marseilles	16 58.39	0.32	−0.01	31 5.9	+1.7	− 8.8	287
5.41233	Bordeaux	17 0.29	0.31	+0.52	31 20.8	+1.8	+ 5.6	288
5.41353	Lyons L.C.	17 0.21	0.31	+0.39	31 14.4	+2.0	− 0.7	286
5.41548	Toulouse	17 0.10	0.32	+0.15	31 13.3	+1.8	− 2.1	287
5.41690	Besançon	16 58.79	0.30	−0.93	31 13.4	+2.2	− 1.7	289
5.44092	Hamburg	17 3.30	0.27	+0.42	31 2.0	+3.0	−13.7	287
5.60547	Albany	17 20.54	0.31	−0.06	31 28.6	+1.5	+ 1.9	287
6.37363	Algiers	18 44.27	0.32	+0.02	32 23.1	+0.8	+15.6	287
6.39875	Marseilles	18 47.51	0.31	+0.38	32 12.1	+1.5	+ 4.2	289
6.64119	Cincinnati	19 13.06	0.33	−0.17	32 14.2	+1.1	− 5.4	287
7.37247	Nice	20 32.70	0.30	+0.06	32 47.2	+1.4	− 1.6	287
7.39350	Lyons L.C.	20 35.45	0.30	+0.40	32 45.2	+1.7	− 4.0	290
7.44013	Greenwich	20 39.33	0.28	0.00	32 43.6	+2.5	− 6.5	289
7.48667	Bordeaux	20 45.91	0.31	+0.70	32 41.9	+2.9	− 9.5	291
7.64945	Washington Naval	21 0.89	0.34	(−1.21)	32 41.6	+1.8	−16.5	289
8.43638	Hamburg	22 29.72	0.26	(+1.46)	33 10.0	+2.9	−10.0	291
9.36355	Nice	24 7.75	0.28	+0.24	33 39.0	+1.3	− 1.7	291
9.41887	Marseilles	27 13.18	0.29	−0.12	33 48.8	+1.4	+ 7.4	291
11.46187	Greenwich	27 51.96	0.27	−0.14	33 51.5	+2.7	+ 0.2	292
12.36744	Nice	29 29.27	0.28	+0.30	33 45.6	+1.4	0.0	291
19.35926	Nice	41 48.92	0.26	+0.75	29 22.9	+1.3	+ 3.0	293
21.35266	Nice	12 45 17.46	0.26	+0.32	+43 27 6.2	+1.3	+ 2.3	295

Date	Place	App. α	π	$\Delta\alpha\cos\delta$	App. δ	π	$\Delta\delta$	*
		h m s	s	s	° ′ ″	″	″	
July 21.38789	Lyons L.C.	12 45 21.21	0.26	+0.37	+43 27 7.4	+1.7	+ 6.5	294
21.39137	Marseilles	45 20.36	0.27	−0.51	27 6.2	+1.5	+ 5.3	295
22.35525	Nice	47 2.61	0.26	+0.47	25 48.4	+1.3	+ 1.0	295
22.59962	Cincinnati	47 26.31	0.27	−0.87	25 46.4	+1.0	−12.3	297
23.36924	Vienna	48 48.61	0.25	+0.44	24 21.3	+2.0	0.0	296, 297
25.35550	Vienna	52 16.23	0.24	+0.53	21 34.7	+1.8	+ 6.2	298
27.38727	Lyons L.C.	55 47.91	0.25	+0.29	18 10.1	+1.7	− 0.3	298
27.40246	Bordeaux	55 49.98	0.26	+0.67	18 22.2	+1.7	+13.2	298
27.64739	Cincinnati	56 10.79	0.27	−0.51	17 41.9	+1.2	− 5.5	298
28.35683	Vienna	57 29.40	0.24	+0.32	16 34.8	+1.8	+ 5.0	298
28.37237	Lyons L.C.	57 30.64	0.24	+0.26	16 36.7	+1.6	+ 8.3	300
31.36827	Vienna	13 2 43.49	0.23	+0.55	10 57.3	+2.0	+ 0.7	301
31.44445	Bordeaux	2 49.00	0.25	+1.10	11 3.7	+1.9	+12.3	302
Aug. 21.32446	Nice	13 39 20.32	0.13	+1.24	+42 24 27.9	+0.9	− 5.3	303

In general, the observations have each been given the weight unity. If, however, the residual differed too widely from the general mean of those in its immediate vicinity, this weight has been reduced. The ( ) indicates that the observation has been excluded from all subsequent computation. The observations made within a few hours of each other were combined into means, and these means with their proper weights were combined to form the following normal places by making use of an equation of the form

$$A\theta = A + Bt + Ct^2$$

Date	α 1894.0	δ 1894.0
	h m s	° ′ ″
April 10.5	3 23 46.786	−55 7 11.56
25.5	6 32 55.868	37 8 9.33
May 3.5	8 18 51.315	− 6 38 38.17
9.5	9 15 38.630	+13 35 31.88
15.5	9 55 58.849	25 44 5.76
24.5	10 37 27.305	34 57 28.08
June 7.5	11 19 40.408	40 46 13.72
24.5	11 56 30.371	43 6 7.10
July 6.5	12 48 56.421	+43 32 22.03
July 21.5	12 50 44.358	+43 23 2.74
Aug. 21.32446	13 39 18.30	+42 24 44.3

Before forming the normal equations it was thought best to compute the perturbations caused by *Jupiter*, *Saturn* and the *Earth*. The method chosen was that of rectangular coordinates, the osculation being at perihelion and the interval every twelve days, with the following results:

	$\delta\alpha\cos\delta$	$\delta\delta$		$\delta\alpha\cos\delta$	$\delta\delta$
	″			″	
April 10.5	0	0	June 7.5	+1.40	+0.55
25.5	+0.07	+0.03	24.5	1.69	0.33
May 3.5	0.33	0.18	July 6.5	1.85	+0.15
9.5	0.60	0.39	24.5	2.12	−0.22
15.5	0.86	0.63	Aug. 21.3	+2.42	−0.91
24.5	+1.12	+0.67			

After correcting the normal places on account of the perturbations, the following equations of condition were formed, using Schönfeld's notation, and expressing the coefficients by means of their logarithms.

	$\delta\kappa$	$\frac{\kappa\delta T}{\sqrt{p}}$	$\frac{q}{1+e}\delta\frac{1}{a}$	δq	$\delta\lambda$	$\delta\nu$			Weight
1)	$+8.89985\,\delta\kappa$	$-9.02075\,\frac{\kappa\delta T}{\sqrt{p}}$	$+6.93884\,\frac{q}{1+e}\delta\frac{1}{a}$	$+9.89290\,\delta q$	$+0.08890\,\delta\lambda$	$-8.94891\,\delta\nu$	−9.91381	= 0	24.5
2)	9.96539	0.39373	−9.36527	0.26887	+9.92343	+9.40881	−0.63448		41.0
3)	0.04520	0.50910	9.72826	0.20934	−9.38337	−9.10105	−0.50515		66.5
4)	9.92748	0.41833	9.76855	0.01597	9.81498	9.65902	−0.33646		66.0
5)	9.76120	0.28195	9.71188	9.79094	9.84713	9.79766	−9.32222		62.5
6)	9.54628	0.08877	9.69009	9.47830	9.76967	9.85996	+0.02531		51.0
7)	9.49045	9.85290	9.62555	9.03978	9.59086	9.88062	−0.38739		64.0
8)	8.92454	9.64801	9.57340	+8.13632	9.33751	9.88426	+9.56820		74.0
9)	8.86859	9.54068	9.54091	−8.48716	9.10689	9.88484	+9.00000		36.0
10)	8.96418	9.42019	9.48264	8.94946	−8.36014	9.88514	+0.17319		15.0
11)	+9.22940	9.29844	9.32198	9.25089	+8.99429	9.88368	−1.05234		1.0
12)	−9.69468	+9.96227	8.47481	0.08396	9.86868	−8.72869	−0.62737		24.5
13)	+9.94566	−0.13706	8.86012	0.17838	0.33516	+9.82054	−0.28780		40.0
14)	0.39460	0.61030	9.61507	0.37714	0.43494	9.84959	+0.33244		61.0
15)	0.38189	0.56399	9.63652	0.40349	9.60270	9.44674	+0.56937		63.0
16)	0.29344	0.42645	9.54681	0.35478	+8.48860	+8.43913	+0.52763		62.0
17)	0.47528	0.23339	9.26546	0.25787	−8.83474	−8.92200	+0.40993		53.5
18)	0.06499	0.02264	−8.69558	0.44530	8.58979	8.87955	+0.32838		61.0
19)	9.99634	9.85694	+8.66721	0.05494	−7.65865	−8.20540	+0.04439		73.5
20)	9.96772	9.77179	8.92037	0.00874	+7.71439	+8.49234	+0.40824		38.0
21)	9.93924	9.67121	9.47362	9.95478	+7.48438	9.00968	−0.24551		45.0
22)	+9.91013	−9.55236	+9.31520	−9.89054	−8.43484	+9.32679	+0.64246		1.0

Multiplying each equation by the square root of its weight, and making

$$u = 1.28427\,\delta\kappa \qquad x = 1.30286\,\delta q$$

$$v = 1.50297\,\frac{\kappa\delta T}{\sqrt{p}} \qquad y = 1.13669\,\delta\lambda$$

$$w = 0.67832\,\frac{q}{1+e}\,\delta\frac{1}{a} \qquad z = 0.81888\,\delta\nu$$

there results the following homogeneous equations, the logarithm of the unit being 1.4691:

+8.3101 u	−8.2123 v	+6.9550 w	+9.2845 x	+9.6467 y	−8.8245 z	−9.1392	= 0
9.4875	9.6971	−9.4933	9.7724	+9.5931	+9.3963	−9.9718	
9.6723	9.9175	9.9613	9.8179	−9.1581	−9.1936	−9.9474	
9.5530	9.8253	0.0000	9.6229	9.5881	9.7499	−9.7772	
9.3749	9.6769	9.9645	9.3860	9.6084	9.8767	−8.7510	
9.0858	9.4396	9.8656	9.0292	9.4868	9.8949	+9.4100	
8.8093	9.2530	9.8503	8.6400	9.3573	9.9648	−9.8214	
8.5749	9.0797	9.8297	+7.7681	9.1354	0.0000	+9.0337	
8.3625	8.8159	9.6407	−7.9624	8.7483	9.8441	+8.3090	
8.2679	8.5053	9.3924	8.2346	−7.8115	9.6546	+9.2921	
+7.9451	−7.7955	8.6437	7.9480	+7.8546	9.0648	−9.5832	
−9.1049	+9.1538	8.1880	9.4756	9.4265	−8.6043	−9.8528	
+9.4629	−9.4356	8.9833	9.6770	0.0000	+9.8032	−9.6202	
0.0000	0.0000	9.8294	9.9669	9.8879	9.9234	+9.7560	
9.9973	9.9607	9.8579	0.0000	9.3657	9.5275	+0.0000	
9.9054	9.8197	9.7347	9.9451	+8.2481	+8.5164	+9.9547	
9.7552	9.5946	9.4513	9.8192	−8.5592	−8.9673	+9.8050	
9.6734	9.4123	−8.9099	9.7351	8.3458	8.9533	+9.7520	
9.6452	9.2871	+8.9220	9.6852	−7.4551	−8.3197	+9.5054	
9.4733	9.0587	9.0319	9.4958	+7.3676	+8.4633	+9.7290	
9.2430	8.7563	9.0833	9.2400	+6.9357	8.7788	−9.3644	
+8.6259	−8.0494	+8.6369	−8.5877	−7.2981	+8.5079	+9.1734	

Proceeding according to the method of least-squares, the normal equations are,

+0.6147 u	−0.5974 v	−0.5104 w	−0.4745 x	+0.0047 y	+9.8511 z	+0.3446	= 0
−0.5956	+0.6407	+0.6311	+0.2490	−9.8430	+8.0682	−9.9619	
−0.5106	+0.6311	+0.7734	+9.6764	+9.6895	+0.4479	+9.3304	
−0.4796	+0.2490	+9.6764	+0.7088	−0.1941	−0.2638	−0.6394	
+0.0180	−9.8430	+9.6895	−0.1941	+0.4111	+0.4148	+8.7076	
+9.8483	+8.0682	+0.4479	−0.2638	+0.4148	+0.7317	+0.0149	

The weight of z, derived from these equations, is so slight as to make that quantity quite indeterminate. Treating the other variables as functions of this one there results the elimination equations,

$$u - 9.9827\,v - 9.8956\,w - 9.8598\,x + 9.3901\,y = -9.7299 - 9.2365\,z$$
$$v + 0.3108\,w - 0.2821\,x + 9.6839\,y = -0.3268 - 0.0858\,z$$
$$w + 9.5565\,x + 9.8561\,y = +9.7135 - 0.2863\,z$$
$$x - 9.8858\,y = +9.5413 + 9.9742\,z$$
$$y = +9.8587 - 9.1150\,z$$

passing from logarithms

$$u = -0''.358 + 3.4354\,z \qquad \delta\kappa = -0''.548 + 1.176\,\delta\nu$$

$$v = -0.048 + 1.8407\,z \qquad \frac{\kappa\delta T}{\sqrt{p}} = -0.044 + 1.002\,\delta\nu$$

$$w = -0.329 - 2.1130\,z \qquad \frac{q}{1+e}\,\delta\frac{1}{a} = -2.032 - 2.962\,\delta\nu$$

$$x = +0.913 + 0.8420\,z \qquad \delta q = +1.339 + 0.276\,\delta\nu$$

$$y = +0.722 - 0.1303\,z \qquad \delta\lambda = +1.553 - 0.063\,\delta\nu$$

If these values are substituted in the weighted equations, the sum of the squares of the residuals has the form

$$1740''.5 + 25.725\,\partial\nu + 0.32488\,\partial\nu^2.$$

Placing the first derivative equal to zero, and solving

$$\partial\nu = -39''.59$$

and [pvv] sinks from 5873″ to 1231, while [vv], which was 263″.7, is now 59″.7. For the observation made at Nice, Aug. 21, the residuals $\Delta\alpha\cos\delta$ and $\Delta\delta$ become $+3''.19$ and $-2''.90$ respectively, where they were formerly $+11''.3$ and $-1''.4$. Substituting in the equations of condition the residuals are

	$\Delta\alpha\cos\delta$		$\Delta\delta$
	″		″
1)	−0.74	12)	+2.28
2)	+0.80	13)	+0.52
3)	+0.10	14)	−0.79
4)	+0.08	15)	−0.71
5)	−0.73	16)	+0.64
6)	−1.24	17)	+0.84
7)	+2.65	18)	+0.22
8)	−0.50	19)	+0.21
9)	−0.57	20)	−0.43
10)	−3.75	21)	+2.26
11)	+3.79	22)	−2.67

It remains to investigate within what limits $\partial\nu$ may vary and not produce results in contradiction with the observations. Substituting in the formula above given for the sum of the squares of the residuals values of $\partial\nu$ from 0 to −80″ there results

$\partial\nu$	[pvv]	$\partial\nu$	[pvv]
″ 0	″ 1741	″ −50	″ 1266
−10	1515	−60	1366
−20	1355	−70	1531
−30	1261	−80	1761
−40	1231		

None of these values of $\partial\nu$ between −20″ and −60″ would either materially alter the order of signs of the residuals when substituted in the equations of condition, nor do they introduce so great a variation in [pvv] as to be irreconcilable with the observations. Between the limits −30″ and −50″, it is evident that any value might be used at random, with almost equally good results. Some slight evidence, however, is at hand in the residuals afforded by the Nice observation of Aug. 21, which take form

$$+11''.03 + 0.198\,\partial\nu \quad \text{and} \quad -2.46 + 0.011\,\partial\nu$$

respectively. Assuming $\partial\nu$ at −40″ these residuals become of the same order numerically, *i.e.* $+3''.11$ and $-2''.90$.

Regarded as functions of $\partial\nu$ the definitive elements, referred to the mean equinox and ecliptic of 1894.0, are

$$\begin{aligned}
T &= 1894\ \text{April}\ 13.406\,912 + 0.000\,395\,\partial\nu \\
\omega &= 324^\circ\ 12'\ 22''.52 \quad +1.2046\,\partial\nu \\
\Omega &= 206\ 23\ 53.04 \quad -0.5347\,\partial\nu \\
i &= 86\ 59\ 18.19 \quad +0.8478\,\partial\nu \\
q &= 0.9830931 + 0.000001339\,\partial\nu \\
e &= 0.9911206 + 0.000002837\,\partial\nu
\end{aligned}$$

For $\partial\nu = 40''$ the period of revolution is 1143 years.

The orbit sustains peculiar relations to that of *Jupiter.* During the entire time of visibility, and for two or three years previous, the planet was near the orbit plane. From a cosmological standpoint the comet's previous history is of considerable interest. The computation of the perturbations by the major planets previous to discovery is now in progress, and the results will be presented later.

Syracuse University, Syracuse, N.Y., 1901 April 5.

A NEW DETERMINATION OF THE SOLAR MOTION,

By J. G. PORTER.

Nearly a decade ago, in *A.J.* 276, I published a determination of the solar motion based on my catalogue of 1340 proper-motion stars. This solution differed from all previous ones in the high declination of the apex resulting from stars with small motion; although Professor Boss had previously (*A.J.* 213) obtained from 253 stars with motion less than 0″.4 per year a value nearly as great. I have never been entirely satisfied with my former work, because, in the first place, no systematic corrections had been introduced in deriving the proper motions; and secondly, the method of Airy is subject to criticism as involving a hypothesis of doubtful validity.

Having in the mean time recomputed the motions of the stars used in the former work, and collected many more, I have thought it worth while to repeat the determination of the solar motion. Only the results will here be given. It is hoped to present a more detailed account of the investigation, together with some other studies based on proper motions, in a forthcoming number of the publications of the Cincinnati Observatory.

The data used in this investigation were the proper motions contained in the Publications of the Cincinnati Observatory, Numbers 13 and 14, the fundamental stars in the *Berl. Astr. Jahrbuch* with motion of 0″.08 and greater (revised values *A.N.* 3508-9), and a list of southern stars compiled for this especial purpose. This material includes

certainly most of the stars now known to have motion of 0″.15 per year and upwards, together with a considerable number with less motion.

The method adopted for deducing the position of the apex from this material is that proposed by Professor KAPTEYN in his paper, "The Determination of the Apex of the Solar Motion," in the proceedings of the Royal Academy of Sciences at Amsterdam, Feb. 21, 1900. The apex, according to this method, must be so chosen that the sum of the proper motions resolved in the direction of the anti-apex shall be a maximum. This condition is, of course, satisfied when the sum of the motions resolved perpendicular to this direction is zero. In the application of the method the surface of the sky was divided into sections about 15° square, and the mean of the positions and motions for the stars of the first and second groups falling in these several divisions were taken. Stars of the third and fourth groups were also united to some extent, but independently. No stars were included which fell within 30° of the assumed apex or anti-apex. The position of the apex in the first approximation was placed at $18^h\ 30^m$, +35°, in accordance with Professor NEWCOMB's conclusion (*A.J.* 457). The correction to the declination for the first group, however, coming out +18°.5, I judged it necessary to make another approximation. This was done for both the first and second groups, using for the apex $18^h\ 35^m$, +45°. The whole work was gone over, thus affording a check on the accuracy of the computations. In order to preserve the proper limit of distance from the new apex some few stars had to be rejected, and others added, but the close agreement of the results shows that this had no appreciable effect, and indeed, that the second approximation was hardly necessary.

The following are the final results.

Group	Yearly Motion	No. of Stars	*A*	*D*
	″		h m	°
I	Less than 0.2	1037	18 45	+54.1
II	0″.2 to 0.5	1063	18 37	+39.6
III	0″.5 to 1.25	235	18 25	+34.4
IV	Greater than 1.25	56	18 13	+43.5

The results from Group IV may be neglected, since for these stars the *motus peculiaris* in general far exceeds the parallactic motion, and their number is not great enough to eliminate its effect. The agreement of the declination of the apex from this group with the mean of the other values must be regarded as merely accidental. The results from the first three groups show a wide range, particularly in declination. That this difference is real, and is not due to any peculiarity of the method employed, can hardly be doubted. Indeed, the close approach of the results to those of my former determination by AIRY's method is rather remarkable. The question then arises which position represents most nearly the true apex of the solar motion.

Professor BOSS's conclusion (*A.J.* 213) that "for stars having proper motion greater than 10″ per century, the true criterion for estimating their average distance is very nearly independent of the magnitude, and is almost wholly some function of the apparent proper motion," is to some degree substantiated by the fact that the mean magnitudes of my first and second groups are very nearly the same; yet in this connection it should be remarked that while the second group is, for the northern sky, tolerably complete, the first group is not; and the stars which will in future be added to this group will nearly all be of magnitudes below the seventh. It may therefore be considered certain that eventually the average magnitude for this group will fall below that of the second group. The stars of the third and fourth groups, as was to be expected, show a distinct increase in brightness. There can be no doubt, however, from the researches of STUMPE and others, that for the stars here in question the amount of motion is a surer criterion than magnitude of their average distance from our system. As the mean motion of the second group is a little more than twice that of the first, we may assume that the average distance is about twice as great. They are, therefore, much more widely scattered in space, and hence are less likely to be affected by any common drift.

The fact that the apex from the third group is still further south lends plausibility to the conjecture that it is some local drift which causes the displacement of the apex. I am inclined to think that the low position of the apex given by the BRADLEY stars may be due in like manner to their comparative nearness, the great majority of these stars being brighter than the seventh magnitude.

It has long been recognized as true, and recent investigations of my own have confirmed the fact, that considerable numbers of stars in different parts of the sky are affected by common drift distinctly separate from that due to parallactic motion. The point I wish to emphasize, then, is this, that the more widely scattered in space are the stars used for determining the solar motion, the more likely are we to eliminate the effects of such drift. In this view of the case we should certainly give greater weight to the position of the apex as fixed by the first group. Excluding motions of over half a second, and giving double weight to Group I, we shall have as the result of the present investigation:

$$A = 18^h\ 42^m \quad , \quad D = +49°.3 \quad , \quad 2100 \text{ stars}$$

Cincinnati Observatory, 1901 *April* 10.

CH. 8093. *R INDI*,

R.A. $22^h\ 28^m\ 53^s$, Decl. $-67°\ 48'.3$ (1900),

BY ALEXANDER W. ROBERTS.

This star was discovered to be variable by GOULD, and is so noted both in the Zone Catalogue and in the General Catalogue.

A very careful examination of the star's light has been made at Lovedale during the past two years, and from these observations the following elements of variation are obtained:

Period,	209 days
Max. epoch,	1900 June 23
Min. epoch,	March 20 (?estimated only)
Limits,	$8^M.0 - 12^M.5$ (?)

The lower limit, $12^M.5$, is estimated from the form of the light-curve, as the faintest magnitude I can observe in the $3\frac{1}{2}$-inch Ross telescope is $11^M.2$.

The light-curve is regular and continuous, and the ascending phase is apparently slightly more rapid than the descending phase.

Lovedale, 1901 *February* 28.

LATITUDE OBSERVATIONS MADE AT THE IMPERIAL ASTRONOMICAL OBSERVATORY, AT KASAN,

BY M. A. GRATCHOF, OBSERVER OF THE OBSERVATORY.

[Communicated by Prof. DUBIAGO, Director.]

The following corrections to the results in *A.J.* 495 are due to February, 1901.

		φ	Pairs
Read:	1901 February 1	55° 47′ 22″.94	4
	9	23.02	16
	11	23.12	16
	13	23.20	14
	14	22.88	11
Read:	1901 February 18	55 47 23.38	8
	21	22.98	7
	26	23.08	8
	27	23.08	16

MONTHLY MEANS.

	φ	Pairs
1901 February 16	55° 47′ 23″.083	100

Kasan, 1901 *March* 24.

NEW COMET, *a* 1901.

From notices in the daily newspapers it appears that a very bright comet was discovered in the eastern sky on the morning of April 24, at the Cape of Good Hope.

It was also seen at Sydney, New South Wales, and at various other places in Australia.

On April 27 it was seen twenty minutes before sunrise, at the Yerkes Observatory, Williams Bay, Wisconsin, and followed until half an hour after sunrise.

It was fifteen degrees north of the sun, and was plainly visible to the naked eye. The nucleus appeared to be faint and diffuse, and there were two tails prominently visible.

Its motion is in a northeasterly direction, or away from the sun.

The present is perhaps as pertinent an occasion as any for a statement that requires to be made in the interest of astronomical truth. In consequence of changes agreed upon between Dr. KREUTZ and the Director of the Harvard College Observatory, in the method of addressing astronomical cable dispatches, the *Astronomical Journal* has for a year or two past been cut off from all such information. This abrogation of the arrangements originally made by Dr. KRUEGER with Mr. RITCHIE and the Editor of this Journal, which renders it necessary for the *Journal* to depend entirely on extraneous sources, does not appear to be in the interests of astronomical science. It cannot be supposed that Dr. KREUTZ would have assented to the change could he have been aware that it would lead to such a result, or be used for such a purpose. — ED.

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NOTES ON VARIABLE STARS, — No. 35,

BY HENRY M. PARKHURST.

DM. −6°6071. — *Aquarii*. While the two PETERS'S ecliptic charts confirm the DM. estimate, the CHACORNAC chart represents the star as 11^M. My photometric observations, upon final revision make the average error about $0^M.1$, and show that the star is not even fluctuating.

Missing DM. Star. +9°34 is taken from the list on the last page of the Third Catalogue. The first three observations below were taken from my star-maps, which included eight faint stars in the neighborhood, but omitted the missing star. From the observations below, the star appears to be nearly uniform at $11^M.4$.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors			Remarks
				1900							d	
7260	*Z Aquilae*	Max.	5297	Oct. 1	17	−30	5	9.1	–	–	–	
7261	*R Delphini*	Max.	5274	Sept. 11	15	−18	9	7.11	0.51	0.61	12	
7404	*R Microscopii*	Max.	5313	Oct. 20	13	− 2	9	8.17	0.20	0.21	6	Compared with *Ceres*
7435	*Y Aquarii*	Max.	5224	July 23	–	–	3	8.3	–	–	–	380 ? days
7448	*W Aquarii*	Max.	5223	July 22	11	+107	2		–	–	–	
7590	*Z Capricorni*	Max.	5308	Oct. 15	5	−178	9	9.28	0.81	1.23	32	−28, period *A.J.* 372
[7657]	— *Pegasi*	Max.	5308	Oct. 15	3	−15	9	9.31	3.28	3.16	57	Confirms period 204 days
7659	*T Capricorni*	Max.	5296	Oct. 3	61	− 3	7	9.10	–	–	–	
–	DM. −6°6071	–	–	–	–	–	–	9.32	–	–	–	See note above
8369	*W Pegasi*	Min.	5336	Nov. 12	4	−25	8	12.4	1.6	2.0	80	Assuming period 338 days
8512	*R Aquarii*	Max.	5366	Dec. 12	81	0	9	7.06	1.55	1.13	37	Apparently a coincidence
8622	*W Ceti*	Max.	5411	Jan. 26	–	–	6	–	–	–	–	Max. still in the twilight
–	DM. +9°34	–	–	–	–	–	–	11.4	–	–	–	See note above
103	*T Andromedae*	Max.	5411	Jan. 29	12	+15	5	8.0	–	–	–	
114	*S Ceti*	Max.	5405	Jan. 20	32	− 6	9	7.99	1.10	1.31	31	Rejecting obs. of Feb. 10
466	*U Piscium*	Max.	5333	Nov. 9	44	+11	6	–	–	–	–	
513	*R Piscium*	Max.	4962	Nov. 3	35	–	E	–	–	–	–	Obsns. interrupted 1899
"	"	Max.	5340	Nov. 16	36	+35	9	8.57	2.10	1.85	49	Light-curve much changed
715	*S Arietis*	Max.	5386	Jan. 1	36		E	–	–	–	–	
782	*R Arietis*	Max.	5365	Dec. 11	67	+21	9	7.67	1.33	1.08	32	
845	*R Ceti*	Max.	5381.7	Dec. 30	74	− 1	9	8.19	1.26	1.55	18	
893	*U Ceti*	Max.	5417	Feb. 1	25	–	E	–	–	–	–	
906	*R Trianguli*	Max.	5387	Jan. 2	14	+ 4	9	5.30	0.84	0.39	13	
976	*T Arietis*	Min.	5451	Mar. 7	33	–	E	–	–	–		[20 days
1113	*U Arietis*	Max.	5371	Dec. 17	–	–	6	–	–	–	–	Average variation of interval
1166	*X Ceti*	Max.	5385	Dec. 31	8	− 5	6	–	–	–	–	Period *A.J.* 138
[1217]	*Nova Persei*	Max.	5439.6	Feb. 23	–		–	0.0	–		–	> *Capella*, Mt. Hamilton
1222	*R Persei*	Max.	4890	Aug. 23	66	–	E	–	–		–	1899
"	"	Max.	5312	Oct. 22	68	–	E		–	–		1900
1577	*R Tauri*	Min.	5097	Mar. 18	13		E	–	–	–	–	1900
"	"	Min.	5422	Feb. 6	14	–	E	–		–	–	
1582	*S Tauri*	Max.	5475	Mar. 31	10		E		–	–	–	Earlier than elements

Individual Observations.

Including Observations by Arthur C. Perry

7260 *Z Aquilae.*

(Cont. from 482. Comp. Stars 393)

Julian	Calendar	Mag.
	1900	
5249.6	Aug. 17	11.8
5260.6	28	10.0
5276.5	Sept. 13	9.89$_2$
5290.5	27	9.08$_2$
5308.5	Oct. 15	9.43$_2$
5312.5	19	9.49$_2$
5331.5	Nov. 7	10.3]

7261 *R Delphini.*

(Continued from 482.)

Julian	Calendar	Mag.
	1900	
5257.6	Aug. 25	7.51$_2$
5258.6	26	7.06$_2$
5262.5	30	7.59$_2$
5263.5	31	7.82$_2$
5267.6	Sept. 4	7.53$_2$
5273.5	10	7.16$_2$
5274.5	11	7.06$_4$
5276.5	13	7.31$_2$
5282.5	19	7.71$_2$
5294.5	Oct. 1	8.26$_2$
5308.5	15	10.11$_2$
5317.5	24	9.29$_2$
5331.5	Nov. 7	10.6

7404 *R Microscopii.*

(Continued from 425.)

Julian	Calendar	Mag.
	1900	
5304.5	Oct. 11	8.5
5308.6	15	9.05$_2$
5309.5	16	8.70$_2$
5310.6	17	8.69$_2$
5312.5	19	8.17$_2$
5313.5	20	8.00$_2$
5314.5	21	8.17$_2$
5315.5	22	8.27$_2$
5317.5	24	9.19$_2$
5326.5	Nov. 2	10.08

7435 *Y Aquarii.*

(Continued from 482.)

Julian	Calendar	Mag.
	1900	
5205.6	July 4	9.1
5217.6	16	8.3
5223.6	22	8.35$_2$
5231.6	30	8.51$_2$
5235.6	Aug. 3	8.44$_2$
5248.6	16	8.94$_2$

7448 *W Aquarii.*

(Continued from 482.)

Julian	Calendar	Mag.
	1900	
5205.6	July 4	11.2
5217.6	16	11.2
5223.6	22	11.08$_2$
5231.6	30	11.71$_2$

7590 *Z Capricorni.*

(Continued from 372.)

Julian	Calendar	Mag.
	1900	
5262.6	Aug. 30	12.2
5275.5	Sept. 12	11.25$_2$
5284.5	21	10.39$_2$

7590 *Z Capric.*—Cont.

Julian	Calendar	Mag.
	1900	
5294.5	Oct. 1	9.65$_2$
5310.5	17	9.13$_2$
5313.5	20	9.02$_2$
5317.5	24	9.50$_2$
5326.5	Nov. 2	9.94$_2$
5334.5	10	10.16$_2$

[7657] — *Pegasi.*

(Continued from 482.)

Julian	Calendar	Mag.
	1900	
5228.6	July 27	11]
5249.6	Aug. 17	10.1
5260.6	28	9.76$_2$
5279.5	Sept. 16	9.51$_2$
5290.5	27	9.67
5312.5	Oct. 19	8.97$_2$
5326.5	Nov. 2	9.67$_2$
5334.5	10	9.41$_2$
5347.5	23	9.53$_2$
5368.5	Dec. 11	11.18$_2$

7659 *T Capricorni.*

(Continued from 432.)

Julian	Calendar	Mag.
	1900	
5262.6	Aug. 30	11.5
5279.5	Sept. 16	10.40$_2$
5285.5	22	9.17$_2$
5294.5	Oct. 1	8.65$_2$
5310.5	17	9.44$_2$
5313.5	20	9.41$_2$
5317.5	24	9.51$_2$
5326.5	Nov. 2	10.19$_2$

DM. −6°6071.

Julian	Calendar	Mag.
	1897	
4189.6	Sept. 21	9.0
4195.6	27	9.6
4196.5	28	9.20
4201.5	Oct. 3	9.43$_2$
4210.6	12	9.49
4213.6	15	9.47$_2$
4257.5	Nov. 28	9.37$_2$
4283.5	Dec. 24	9.4
	1898	
4546.5	Sept. 13	9.40$_2$
4576.5	Oct. 13	9.14$_2$
4605.5	Nov. 11	9.41$_2$
4638.5	Dec. 11	9.38$_2$
	1900	
5249.5	Aug. 17	9.19$_2$
5252.6	20	9.11$_2$
5263.6	31	9.11$_2$

8369 *W Pegasi.*

(Continued from 487.)

Julian	Calendar	Mag.
	1900	
5268.5	Sept. 5	9.54$_2$
5275.5	12	9.10$_2$
5284.5	21	10.24$_2$
5314.5	Oct. 21	11.50$_2$
5334.5	Nov. 10	12.36$_2$
5340.5	16	11.35:
5345.5	21	11.70$_2$
5378.5	Dec. 24	10.98$_2$
5380.5	26	11.84$_2$

8512 *R Aquarii.*

(Continued from 487.)

Julian	Calendar	Mag.
	1900	
5283.5	Sept. 20	10.5
5304.6	Oct. 11	9.66$_2$
5314.5	21	10.12$_2$
5322.5	29	9.13$_2$
5331.5	Nov. 7	7.75$_2$
5338.5	14	7.54$_2$
5347.5	23	7.53$_2$
5362.5	Dec. 8	7.01$_2$
5370.5	16	6.69$_2$
5378.5	24	7.54$_2$
5380.5	26	7.80$_2$
	1901	
5404.5	Jan. 19	7.86$_2$

8622 *W Ceti.*

(Continued from 487.)

Julian	Calendar	Mag.
	1900	
5338.5	Nov. 14	9.50$_2$
5347.5	23	9.55$_2$
5362.5	Dec. 8	8.66$_2$
5373.5	19	8.66$_2$
5381.5	27	8.20$_2$
	1901	
5387.5	Jan. 2	8.33
5388.5	3	8.09$_2$
5390.5	5	7.91$_2$
5392.5	7	7.97$_2$
5404.5	19	8.16$_2$
5411.5	26	6.73
5412.5	27	7.95$_2$
5421.5	Feb. 5	7.41$_2$

DM. +9°31.

Julian	Calendar	Mag.
	1862	
1425	Oct. 11	11]
1432	18	11]
1434	20	11]
	1896	
3895	Dec. 1	11.5
3897.6	3	11.83$_2$
	1897	
3930.5	Jan. 5	11.45
3975.5	Feb. 19	11.5
4138.7 to		
	1898	
4331.5	Feb. 10	11+
	6 dates	
4579.5	Oct. 16	11.47$_2$
4609.5	Nov. 15	11.9
	1900	
5339.5	Nov. 15	11]
5364.5	Dec. 10	11.3

103 *T Andromedae.*

(Cont. from 487. Comp. Stars 346)

Julian	Calendar	Mag.
	1900	
5314.5	Oct. 21	12]
5337.5	Nov. 13	11.5
5338.5	14	11.13$_2$
5345.5	21	11.60$_2$
5376.5	Dec. 22	9.09$_2$
5380.5	26	9.22$_2$
	1901	
5405.5	Jan. 20	8.05$_2$

103 *T Androm.*—Cont.

Julian	Calendar	Mag.
	1901	
5414.5	Jan. 29	8.3
5418.5	Feb. 2	8.06$_2$
5433.5	17	8.48$_2$

111 *S Ceti.*

(Continued from 314.)

Julian	Calendar	Mag.
	1900	
5314.5	Oct. 21	12]
5337.5	Nov. 13	10.1
5338.5	14	9.79$_2$
5345.5	21	9.58$_2$
5363.5	Dec. 9	9.23$_2$
5376.5	22	9.21$_2$
	1901	
5405.5	Jan. 20	7.77$_4$
5414.5	29	7.89$_2$
5418.5	Feb. 2	8.70$_2$
5426.5	10	6.51$_2$?
5428.5	12	8.52
5429.5	13	8.00

466 *U Piscium.*

(Continued from 468.)

Julian	Calendar	Mag.
	1899	
4927.6	Sept. 29	12]
	1900	
5314.5	Oct. 21	11.34$_2$
5337.5	Nov. 13	10.00$_2$
5346.5	22	11.16$_2$
5366.5	Dec. 12	10.98$_2$

513 *R Piscium.*

(Cont. from 468. Comp. Stars 468)

Julian	Calendar	Mag.
	1899	
4927.6	Sept. 29	11.1
4940.6	Oct. 12	10.27$_2$
4948.5	20	8.87$_2$
4949.6	21	9.24$_2$
	1900	
5313.6	20	8.67$_2$
5314.6	21	8.60$_2$
5315.6	22	9.12$_2$
5337.5	Nov. 13	8.40$_2$
5346.5	22	8.37$_2$
5362.5	Dec. 8	8.91$_2$
5373.5	19	9.34$_2$
5381.5	27	10.12$_2$

715 *S Arietis.*

(Continued from 438.)

Julian	Calendar	Mag.
	1900	
5314.5	Oct. 21	12]
5337.5	Nov. 13	12]
5374.5	Dec. 20	10.0
5376.5	22	10.41$_2$

782 *R Arietis.*

(Continued from 468.)

Julian	Calendar	Mag.
	1899	
4927.6	Sept. 29	9.1
4940.6	Oct. 12	9.39$_2$
	1900	
5285.6	Sept. 22	10.58$_2$
5317.5	Oct. 21	8.32$_2$
5334.5	Nov. 10	7.88$_2$
5346.5	22	8.08$_2$

782 *R Arietis*—Cont.

Julian	Calendar	Mag.
	1900	
5356.5	Dec. 2	8.13$_2$
5363.5	9	7.25$_2$
5370.5	16	7.75$_2$
5378.5	24	7.72$_2$
	1901	
5388.5	Jan. 3	9.02$_2$
5390.5	5	7.95$_2$

845 *R Ceti.*

(Continued from 438.)

Julian	Calendar	Mag.
	1900	
5339.5	Nov. 15	11.1
5376.5	Dec. 22	8.5
5378.5	24	8.39$_2$
5381.5	27	8.59$_2$
	1901	
5387.5	Jan. 2	8.49$_2$
5388.5	3	8.46$_2$
5390.5	5	8.56$_2$
5407.5	22	9.17$_2$

893 *U Ceti.*

(Cont. from 438. Comp. Stars 346)

Julian	Calendar	Mag.
	1900	
5347.5	Nov. 23	10.7
5373.5	Dec. 19	8.67$_2$
5381.5	27	7.83$_2$
	1901	
5414.5	Jan. 29	8.3
5421.5	Feb. 5	7.90$_2$
5431.5	15	7.59$_2$

906 *R Trianguli.*

(Continued from 487.)

Julian	Calendar	Mag.
	1900	
5260.6	Aug. 28	11.7
5312.6	Oct. 19	8.62$_2$
5317.6	24	8.32$_2$
5334.5	Nov. 10	7.70$_2$
5346.5	22	7.86$_2$
5356.5	Dec. 2	5.32$_2$
5362.5	8	5.83$_2$
5363.5	9	5.94$_2$
5372.5	18	6.30$_2$
	1901	
5386.5	Jan. 1	4.82$_2$
5387.5	2	5.43$_2$
5388.5	3	5.49$_2$
5390.5	5	4.73$_2$
5392.5	7	6.94$_2$
5394.5	9	5.92$_2$

976 *T Arietis.*

(Cont. from 468. Comp. Stars 403)

Julian	Calendar	Mag.
	1900	
5339.5	Nov. 15	9.2
5378.5	Dec. 24	8.74$_2$
	1901	
5424.5	Feb. 8	9.52$_2$
5434.5	18	9.10$_2$

1113 *U Arietis.*

(Cont. from 487 Comp. Stars 314)

Julian	Calendar	Mag.
	1900	
5340.5	Nov. 16	7.4
5345.5	21	7.74_2
5356.5	Dec. 2	7.61_2
5373.5	19	7.34_2
5383.5	29	7.43_2
	1901	
5423.5	Feb. 7	8.60_2

1166 *X Ceti.*

(Cont. from 468. Comp. Stars 468)

Julian	Calendar	Mag.
	1900	
5347.5	Nov. 23	9.8
5372.5	Dec. 18	9.26_2
5378.5	Dec. 24	9.03_2
5383.5	29	8.63_2
	1901	
5422.5	Feb. 6	10.57_2

[1217] *Nova Persei.*

Julian	Calendar	Mag.
	1901	
5440.5	Feb. 24	1.0
5441.3	25	1.2
5442.5	26	1.85_2
5443.5	27	$>2.47_2$
5443.5	27	3.04_2
5444.5	Feb. 28	3.52_2
5446.5	Mar. 2	2.21_2
5446.5	2	2.38_2
5450.5	6	3.18_2
5451.5	7	3.25_2
5456.5	12	3.38_4
5460.5	16	2.7
5462.5	18	2.23_4
5463.5	19	4.95_4
5465.5	21	4.80_2
5466.5	22	4.55_2
5467.5	Mar. 23	3.97_3
5473.5	29	4.70_3

1222 *R Persei.*

(Continued from 487.)

Julian	Calendar	Mag.
	1899	
4929.6	Oct. 1	8.2
4931.5	3	8.50_2
4940.6	12	8.88_2
	1900	
5347.5	Nov. 23	9.05_4
5364.5	Dec. 10	9.37_2
5371.5	20	9.47_2

1577 *R Tauri.*

(Continued from 468.)

Julian	Calendar	Mag.
	1900	
5056.6	Feb. 5	11.2r
5347.5	Nov. 23 to	
	1901	
5471.5	Mar. 27	12]
	4 dates	

2582 *S Tauri.*

(Continued from 468.)

Julian	Calendar	Mag.
	1900–1901	
5347.5	Nov. 23	12]
5376.5	Dec. 22	12]
5437.5	Feb. 21	10.21_2
5471.5	Mar. 29	11.1

Comparison-Stars, 1893–1900.

8369 *W Pegasi.*

Star	DM.		Mag.	n
I	+25°4917		7.88	10
1*I*	+25°4905		8.04	5
R	+25°4922		8.72	18
T	+25°4911		9.18	3
U	+24°4742		9.17	2
W	+25°4926		9.18	12
1*W*	+24°4762		9.24	13
Y	+25°4916		9.93	7
1*Y*	+25°4918		10.25	6
2*Y*	+25°4920		10.46	7
Z	+25°4919		10.51	4
1*Z*	+25°4923		9.70	2
a	6*s*2*p*	*Z*	10.57	2
b	3*f*	2*Y*	10.79	5
c	7*n*	2*Y*	10.75	6
d	2*n*2*f*	1*Z*	10.22	2

906 *R Trianguli.*

Star	DM.		Mag.	n
A	+34°469		4.96	19
C	+33°461		6.26	36
1*C*	+33°454		5.28	30
D	+34°462		6.41	2
1*E*	+34°471		6.14	9
1*F*	+33°458		7.02	21
G	+33°463		6.93	24
1*G*	+35°490		7.54	14
I	+35°498		7.68	12
J	+33°465		7.57	14
K	+33°462		7.92	11
N	+33°466		7.92	7
1*N*	+34°441		8.83	24
R	+34°451		8.92	14
1*R*	+35°491		8.87	12
S	+34°474		8.97	18
1*S*	+35°501		8.95	10
2*S*	+35°480		9.24	16
T	+33°464		8.67	8
1*T*	+35°492		8.90	12
W	+35°516		9.06	20
1*W*	+33°468		9.41	28
X	+35°493		9.71	14
Z	+33°467		9.58	29
1*Z*	+33°471		9.75	31
a	6*s*4*f*	*G*	9.73	1
b	9*f*	1*R*	11.26	18
c	3*n*2*p*	1*W*	10.53	11
d	3*s*6*p*	*Z*	10.46	6
e	3*n*2*f*	*Z*	10.66	24
g	3*n*	*U*	11.39	10
h	3*s*	*U*	11.06	22

[1217] *Nova Persei.*

Star	DM.	Mag.	n
A'	+45°1077	0.27	12
B^2	+49°917	1.66	19
$1B^2$	+44°1328	2.43	6
C^2	+41°395	2.77	9
F^2	+39°895	3.10	9
H^2	+47°876	2.76	6
K^2	+42°815	3.99	8
L^2	+44°631	3.95	7
$1N^2$	+47°843	1.45	7
O^2	+47°857	1.29	5
P^2	+49°857	4.24	2
R^2	+42°750	4.57	9
S	+43°674	5.62	6
1*A*	+49°899	5.01	3
2*A*	+49°902	4.78	3
D	+43°730	6.19	2

Nova Persei. My observations were confined to the daylight and bright twilight, and were ended in March, by the interference of the new building on the north. The magnitudes are derived from those given in the final table, obtained from the observations, based upon the Harvard Revised Photometry, and not connected with my asteroid basis. On Feb. 24 the star's position was not known until evening, when my photometric apparatus was not applicable to such bright stars. On Feb. 25, the comparison was made by daylight. On Feb. 26 I did not find it by daylight, but observed it very early in the twilight, the star running off the scale, at the magnitude stated in the second series, from the approaching darkness of the sky. The waves on March 2 and 18, may have resulted from the circumscribed conditions in which observations were made, with apparatus approximately and experimentally adapted to rapidly varying conditions, confining my choice of comparison-stars sometimes to bright and at other times to fainter stars, subject to different conditions of atmosphere and of twilight, brightness rapidly changing through each series of observations. In my series of observations of *Nova Aurigae* there were similar waves which were corroborated by other observers. In some cases the changes in brightness seemed conspicuous from visual examination, and additional observations were made for the express purpose of corroboration, although my time was limited throughout by its early disappearance behind the building referred to. For the same reason I could not verify the magnitudes of the comparison-stars further than as it occurred during the observations.

PHOTOGRAPHIC OBSERVATIONS OF MINOR PLANETS,

MADE AT THE LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,

By H. K. PALMER.

The following observations were made from plates taken with the Crossley Reflector, with the assistance of Mr. C. G. DALL. Several exposures were made on each plate, the plate being moved a little in declination between successive exposures. In the reduction the several exposures were treated as so many comparisons in a micrometer observation. The observations have been corrected for refraction, scale value and orientation, though the last two quantities might be changed somewhat by the use of definitive star-places. As precession and aberration simply affect the orientation, the observations have been reduced to 1901.0 directly.

The first table gives the positions of the asteroids as determined from each plate, while the second table gives the same positions as determined from each comparison-star.

The observations have been compared with ephemerides kindly furnished by Mr. CODDINGTON. This comparison is given in the columns O−C of the first table.

Positions Determined from Plates.

Plate	1900	Mt. H. M. T.	α	$\log p\Delta$	δ	$\log p\Delta$	O—C α	O—C δ	Exposures No.	Exposures Length
				(115) 1896 *CO.*						
1	M[illegible] 14	11 24 20	12 23 11.04	n9.260	+7 50 31.3	0.611	−0 13.61	+1 28.1	1	50s
2	14	11 30 20	12 23 10.79	n9.250	+7 50 33.1	0.611	−0 13.65	+1 28.1	1	50s
				(439) *Ohio.*						
3	[illegible] 8	10 50 52	12 13 18.11	n8.968	−8 42 10.2	0.802	+0 25.97	−0 7.3	3	5m
4	9	11 11 55	12 38.30	n8.613	32 39.7	0.801	+0 26.00	−0 7.1	3	5m
5	9	11 31 15	12 12 37.59	7.799	−8 32 31.2	0.801	+0 25.92	−0 7.6	3	5m

Plate 5 is a very poor one, the star images being very much elongated in right-ascension.

Positions Determined from Individual Stars.

Plate	$\Delta\alpha$	$\Delta\delta$	α	v	δ	v	*
1	+0 53.78	+ 3 11.1	12 23 11.02	−0.02	+7 50 32.0	+0.7	1
	+0 51.31	+ 8 15.6	11.01	0.00	32.2	+0.9	2
	+0 19.55	− 5 11.6	10.99	−0.05	30.7	−0.6	3
	+0 18.33	− 5 20.0	11.07	+0.03	30.7	−0.6	4
	+0 3.99	+12 2.5	11.07	+0.03	30.7	−0.6	5
2	+0 53.51	+ 3 12.7	12 23 10.78	−0.01	+7 50 33.6	+0.5	1
	+0 51.58	+ 8 17.0	.78	−0.01	33.6	+0.5	2
	+0 19.32	− 5 9.5	.76	−0.03	32.8	−0.3	3
	+0 48.08	− 5 17.9	.82	+0.03	32.8	−0.3	4
	+0 3.71	+12 1.6	.82	+0.03	32.8	−0.3	5
3	+1 19.90	− 1 16.7	12 13 18.08	−0.03	−8 42 10.3	−0.1	6
	+0 26.31	− 1 34.4	.21	+0.10	.3	−0.1	7
	+0 25.00	− 5 57.1	.09	−0.02	.0	+0.2	8
	−0 34.09	− 1 26.2	.09	−0.02	.2	0.0	9
4	+0 40.07	+ 7 14.0	12 12 38.25	0.05	−8 32 39.6	+0.1	6
	−0 13.18	+ 1 56.2	.39	+0.09	.7	0.0	7
	−0 14.80	+ 3 33.3	.29	−0.01	.6	+0.1	8
	−1 13.92	+ 8 1.3	.25	−0.05	.7	0.0	9
5	+0 39.36	+ 7 52.7	12 11 37.54	−0.05	−8 32 30.9	+0.3	6
	−0 14.17	+ 5 4.5	.70	+0.11	31.1	−0.2	7
	−0 15.51	+ 3 11.3	.58	−0.01	31.6	−0.4	8
	−1 14.63	+ 8 13.0	12 11 37.54	0.05	−8 32 31.0	+0.2	9

Mean Places of Comparison-Stars.

*	α 1901.0	δ 1901.0	Authority
1	12 22 17.24	+7 47 20.9	A. G. Leipzig 6123
2	22 19.50	+7 41 16.6	" " 6124
3	22 21.14	+7 55 12.3	" " 6125
4	22 22.74	+7 55 50.7	" " 6126
5	23 7.08	+7 38 28.2	" " 6134
6	11 58.18	8 40 23.6	½ (W₁ 12 668 + Paris 15704 + Rad., 3323)
7	12 51.87	8 37 35.9	Paris 15729
8	12 53.09	−8 36 12.9	Paris 15730
9	12 13 52.17	8 40 14.0	¼ (W₁ 12 704 + Paris 15757 + Schj. 1617 + Rad., 3331)

POSITION OF THE EQUINOX AND THE VALUES OF OTHER ELEMENTS DERIVED FROM RECENT GREENWICH AND WASHINGTON OBSERVATIONS OF THE SUN.

By SIMON NEWCOMB.

In the recently issued *Publications* of the U.S. Naval Observatory, Second Series, Vol. I, are found the results of meridian observations of the Sun from 1894 to 1899. As no other material for correcting the position of the equinox, except the annual results of the Greenwich observations, is likely to be soon available, it seems of interest to determine the correction from the observations in question. The monthly means of the corrections given by these observations to the Sun's position, as found in the *American Ephemeris*, are tabulated as follows:

Mean Monthly Corrections to the Ephemeris of the Sun from Hansen's Tables, Derived from Washington Observations, 1894 to 1899.

[The units are $0^{s}.001$ in Right-Ascension, and $0''.01$ in Declination. The subscript figures are number of observations.]

Corrections to Right-Ascension.

	1894	1895	1896	1897	1898	1899	Mean
January	...	...	-76_{9}	-57_{16}	-82_{12}	-65_{6}	$-0^{s}.069_{43}$
February	...	-70_{1}	..	-102_{6}	-39_{15}	-41_{8}	-0.051_{30}
March	...	-100_{2}	..	-61_{13}	-97_{12}	-47_{7}	-0.073_{34}
April	...	-154_{10}	..	-15_{19}	-52_{13}	-13_{10}	-0.045_{61}
May	...	-52_{4}	..	$+94_{16}$	-9_{16}	$+18_{17}$	$+0.028_{53}$
June	...	0_{2}	0_{2}	-4_{14}	$+9_{19}$	$+8_{10}$	$+0.001_{47}$
July	...	$+56_{5}$	$+23_{1}$	$+32_{16}$	$+53_{14}$	$+9_{15}$	$+0.033_{59}$
August	...	-29_{1}	-13_{18}	$+11_{17}$	$+11_{17}$	-21_{8}	-0.004_{59}
September	...	0_{4}	$+74_{10}$	-12_{20}	$+13_{21}$	0_{16}	$+0.011_{71}$
October	...	-78_{10}	-19_{12}	$+9_{11}$	$+6_{17}$	-47_{15}	-0.032_{74}
November	-111_{10}	-111_{5}	$+1_{11}$	-9_{24}	$+28_{16}$	-92_{12}	-0.050_{61}
December	-136_{9}	-154_{5}	-69_{14}	-31_{15}	$+29_{13}$	-82_{18}	-0.070_{65}

Corrections to Declination.

	1894	1895	1896	1897	1898	1899	Mean
January	..	-210_{1}	-80_{6}	$+2_{16}$	$+12_{12}$	$+96_{7}$	$-0''.02_{45}$
February	..	..	..	-132_{6}	-65_{15}	-41_{8}	-0.70_{29}
March	..	-135_{2}	..	$+55_{11}$	-92_{12}	-56_{8}	-0.37_{33}
April	..	-25_{10}	..	$+45_{13}$	-32_{13}	$+1_{18}$	$+0.03_{60}$
May	..	-70_{4}	..	$+99_{16}$	$+67_{16}$	$+82_{15}$	$+0.71_{51}$
June	..	-20_{2}	-10_{2}	$+98_{13}$	$+78_{19}$	-1_{10}	$+0.59_{46}$
July	..	-1_{5}	$+72_{12}$	$+108_{16}$	$+19_{14}$	$+15_{12}$	$+0.58_{59}$
August	..	-74_{9}	$+11_{17}$	$+41_{16}$	$+68_{17}$	$+49_{8}$	$+0.26_{67}$
September	..	-3_{4}	-31_{9}	0_{20}	-36_{19}	$+20_{15}$	-0.10_{67}
October	..	-22_{19}	$+55_{12}$	$+3_{11}$	$+9_{17}$	$+65_{15}$	$+0.19_{74}$
November	$+20_{11}$	-8_{5}	$+1_{9}$	$+45_{13}$	$+26_{16}$	$+111_{12}$	$+0.19_{55}$
December	$+51_{9}$	$+108_{5}$	$+28_{13}$	$+55_{15}$	-20_{13}	$+118_{19}$	$+0.56_{71}$

The observations up to June, 1899, were made with the 9-inch transit circle; the subsequent ones with the new 6-inch. Owing to this circumstance, and the gaps in the series, arising from the instrument being under repair, it does not seem worth while to treat the years separately. I have therefore taken the mean result for each month, which is found in the last column.

Very striking is the systematic discordances between the corrections in right-ascension, which show a well-marked annual period. This arises mainly from the errors of HANSEN's elements, especially of the longitude of the perigee. I have therefore computed the reduction from HANSEN's tables of the Sun to my new tables with the view of deriving the results from a comparison with the latter. This reduction was found by a comparison of the Sun's positions in the *American Ephemeris* for 1898, 1899

and 1900, with those in the *British Nautical Almanac*, whereby the reduction is found through the intermediary of the ephemeris from LEVERRIER's tables.

From the nature of the case the changes in the instrument need not give rise to question in the case of the right-ascensions. The reduction of the corrections to the new tables is as follows:

	$\Delta\alpha$ Ob.—H.	N.—H.	Ob.—N.		$\Delta\alpha$ Ob.—H.	N.—H.	Ob.—N.
	s	s	s		s	s	s
Jan.	—.069	—.006	—.063	July	+.033	+.098	—.065
Feb.	—.054	+.006	—.060	Aug.	—.004	+.083	—.087
Mar.	—.073	+.012	—.085	Sept.	+.011	+.056	—.045
Apr.	—.045	+.032	—.077	Oct.	—.032	+.016	—.048
May	+.028	+.070	—.042	Nov.	—.050	—.020	—.030
June	+.004	+.101	—.097	Dec.	—.070	—.026	—.044

Mean for the year: ΔR.A. = $-0^s.062$.

This mean correction shows that, assuming the tabular longitude of the Sun, which is based on many thousand observations from 1750 to 1892, to be correct, the right-ascensions of the stars used in the reductions, which are those of the *American Ephemeris*, are too small by $0^s.062$.

But we have also to inquire what correction to the absolute mean longitude of the Sun is given by the observations in declination. To get a good result requires that the observations shall be homogeneous, because the result depends on the comparison of observations made at opposite seasons. The observations during the last half of 1899 were made with a different instrument, and the large positive residuals for the last three months give rise to the suspicion of some systematic difference between the two instruments. I have therefore taken another set of means in which these observations are omitted. The mean monthly corrections then become as shown in the following table.

	Δ Decl. Ob.—H.	N.—H.	O.—N.	Wt.	$\cos\alpha$	$\sin\alpha$
	"	"	"			
Jan.	—0.02	+0.03	—0.05	3	+0.42	—0.91
Feb.	—0.70	+0.13	—0.83	2	+0.82	—0.57
Mar.	—0.37	+0.13	—0.50	2	+1.00	—0.09
Apr.	+0.03	+0.17	—0.14	4	+0.91	+0.42
May	+0.71	+0.25	+0.46	3	+0.57	+0.82
June	+0.74	+0.07	+0.67	3	+0.09	+1.00
July	+0.69	—0.26	+0.95	3	—0.42	+0.91
Aug.	+0.22	—0.32	+0.54	4	—0.82	+0.57
Sept.	—0.19	—0.29	+0.10	3	—1.00	+0.09
Oct.	+0.07	—0.06	+0.13	4	—0.91	—0.42
Nov.	+0.25	+0.10	+0.15	3	—0.57	—0.82
Dec.	+0.35	+0.02	+0.33	3	—0.09	—1.00

Putting $\Delta\lambda$ the correction to the Sun's absolute right-ascension:

x = $\sin\epsilon\,\Delta\lambda$;
$\Delta\epsilon$ = correction to obliquity;
z = common error of all the observed declinations;

each monthly mean correction gives rise to the equation of condition

$$x\cos\alpha + \Delta\epsilon\sin\alpha + z = \Delta\delta$$

The normal equations formed from these conditional equations are

$$\begin{aligned} 18.70x &+ 0.87\,\Delta\epsilon - 2.64z = -6.05 \\ 0.87 &+ 18.10 \quad + 1.23 = +6.37 \\ -2.64 &+ 1.23 \quad + 37.00 = +7.29 \end{aligned}$$

The solution of these equations gives

$$\begin{aligned} x &= -0''.32 \\ \Delta\epsilon &= +0.35 \\ z &= +0.16 \end{aligned}$$

Had the Pulkowa refractions been used the correction to the obliquity would be

$$\Delta\epsilon = +0''.35 - 0''.12 = +0''.23$$

a result which affords an index to the probable systematic error of the declinations.

From the value of x we have

Corr. to ☉'s absolute R.A. = $-0''.80$ = $-0^s.053$.

The annual discussions of the Greenwich observations, since 1892, when the corrections are reduced to the system N_1, give the following values of the corrections ΔE to the system of right-ascensions N_1:

	s
1893	+0.041
94	—0.007
95	—0.023
96	—0.011
97	+0.013
98	+0.032
99	—0.009
Mean	+0.005

We thus have the following corrections to the right-ascensions of the system in question.

From Washington observations, 1894 to 1899, assuming the Sun's tabular mean right-ascension to be correct,

$$\Delta E = +0^s.062$$

From the same, correcting the Sun's tabular right-ascension by the observations in declination

$$\Delta E = +0^s.009$$

From Greenwich observations, 1893–99,

$$\Delta E = +0^s.005$$

We conclude that, so far as the evidence goes, the right-ascensions in question are too small by an amount of which the most probable value may be set at $+0^s.02$ or $+0^s.03$.

COMET a 1901.

[From RITCHIE'S *Science Observer, Special Circular No. 129.*]

A message received on April 26, via Harvard College Observatory, announced the discovery of a very bright comet by HOLK of Queenstown, on April 23, giving also a position secured at the Cape of Good Hope, on April 24. An independent discovery message was also received from Arequipa, without date, but presumably May 2, with the rough position, 11ʰ 35ᵐ, R.A. 3ʰ 30ᵐ, Decl. South 1°. On May 8, two other Cape positions were received, together with the description, "Circular nebulosity greater than 1′ in diameter, and brighter than 3d magnitude, with well-defined nucleus, and a tail longer than 2°." On May 10 an orbit was received, which was computed by Dr. KREUTZ of Kiel, from the three Cape positions. The various data are here given:

POSITIONS.

Greenw. M.T.	R.A.	Decl.	Observers
1901	h m s	° ′ ″	
April 24.712	1 30 4.	+3 27 0	Cape
3.2145	3 10 32.1	−0 33 49	Cape
4.2187	3 51 29.2	−0 18 27	Cape

ELEMENTS.

T = 1901 April 24.22 Gr. M.T.

$$\left.\begin{aligned} \omega &= 202^\circ\ 50' \\ \Omega &= 109\ 57 \\ i &= 131\ 26 \end{aligned}\right\}\ \text{Mean Eq. 1901.0}$$

q = 0.2446

EPHEMERIS.

Gr. Midnight	R.A.	Decl.	Light
1901	h m s	° ′	
May 12	5 18 16	+2 43	0.11
16	5 41 4	4 8	
20	6 1 12	5 21	
24	6 20 24	+6 23	0.03

Light April 24 = 1.

For detailed ephemeris see p. 144.

NOTE ON THE INTERPOLATION OF LOGARITHMS.

BY HERMAN S. DAVIS.

The writer recently had need to make more than ten thousand interpolations between numbers whose logarithms were tabulated for equidistant arguments. As greater accuracy was desired than could be secured from a linear interpolation between the tabulated logarithms, and greater speed than by the conversion of the logarithms into numbers, and their reversion to logarithms after interpolation, the following table was devised:

FOR LINEAR INTERPOLATION OF NUMBERS WHOSE LOGARITHMS ARE GIVEN.

M	ARGUMENTS M AND DIFFERENCE BETWEEN THE LOGARITHMS.							TABULAR QUANTITY IS Q.		
	.01000	.02000	.03000	.04000	.05000	.06000	.07000	.08000	.09000	.10000
.02	0 0	0 1	0 2	0 4	0 6	0 8	1 2	1 5	2 0	2 5
.04	0 0	0 2	0 4	0 7	1 1	1 7	2 3	3 0	3 8	4 7
.07	0 1	0 3	0 7	1 2	1 9	2 8	3 8	5 4	6 4	8 0
.10	**0 1**	**0 4**	**1 0**	**1 7**	**2 7**	**3 9**	**5 3**	**7 0**	**8 9**	**11 0**
.15	0 1	0 6	1 3	2 4	3 8	5 5	7 5	9 8	12 5	15 5
.20	0 2	0 7	1 7	3 0	4 7	6 8	9 3	12 2	15 5	19 3
.25	**0 2**	**0 9**	**2 0**	**3 5**	**5 5**	**7 9**	**10 9**	**11 2**	**18 1**	**22 1**
.30	0 2	1 0	2 2	3 9	6 1	8 9	12 1	15 8	20 1	24 9
.40	0 3	1 1	2 5	4 4	7 0	10 0	13 7	17 9	22 7	28 0
.50	**0 3**	**1 2**	**2 6**	**1 6**	**7 2**	**10 4**	**11 1**	**18 1**	**23 3**	**28 7**
.60	0 3	1 1	2 5	4 4	6 9	9 8	13 1	17 4	22 0	27 2
.70	0 2	1 0	2 2	3 8	6 0	8 5	11 6	15 1	19 0	23 4
.75	**0 2**	**0 9**	**1 9**	**3 4**	**5 3**	**7 6**	**10 3**	**13 4**	**16 9**	**20 7**
.80	0 2	0 7	1 6	2 9	4 5	6 4	8 7	11 4	14 3	17 6
.85	0 1	0 6	1 3	2 3	3 6	5 1	6 9	9 0	11 3	13 9
.90	**0 1**	**0 4**	**0 9**	**1 6**	**2 5**	**3 6**	**1 9**	**6 3**	**7 9**	**9 7**
.93	0 1	0 3	0 7	1 2	1 8	2 6	3 5	4 6	5 7	7 0
.96	0 0	0 2	0 1	0 7	1 1	1 5	2 1	2 7	3 4	4 4
.98	0 0	0 1	0 2	0 4	0 5	0 8	1 1	1 4	1 7	2 1

This is to be used in accordance with the formula:

$$\text{Log}\,\{a_1 + M(a_2 - a_1)\} = \text{Log}\,a_1 + \{M(\log a_2 - \log a_1) + Q\}$$

where $Q = f\{M, (\log a_2 - \log a_1)\}$ *and is practically independent of* $\log a_2$ *and* $\log a_1$ *separately*. Q is always positive, but if $(\log a_2 - \log a_1)$ is negative, Q must be taken from the table with the argument $1-M$ instead of M. Q may be computed for any values of the arguments of which it is a function, but is here tabulated for use with logarithms of five decimals or less, and is expressed in units of the fifth decimal, for differences of logarithms up to 0.10000.

For illustration:

		Case I	Case II
Given, to the argument	n, $\log a_1 =$	9.74983	9.83983
and to the argument	$n+10$, $\log a_2 =$	9.83983	9.74983

Find $\log a$ for $n + 2.50$ in each case.

	$(\log a_2 - \log a_1)$ is	+0.09000	−0.09000
M = fraction of total interval =		0.25 and	$1 - M = 0.75$
	Q =	.00181	.00169
Hence by formula,	$\log a$ =	9.77414 and	9.81902

respectively.

International Latitude Observatory, Gaithersburg, Maryland, U.S.A.

EPHEMERIS OF COMET α 1901.

(Computed from KREUTZ's Elements, p. 143.)

EQUATORIAL COORDINATES.

$$x = r[9.85091] \sin(174^\circ\ 5.3 + v)$$
$$y = r[9.93810] \sin(298\ 52.4 + v)$$
$$z = r[9.93599] \sin(228\ 31.4 + v)$$

EPHEMERIS FOR GREENWICH MIDNIGHT.

1901	α	δ	log Δ	Brightness
	h m s	° ′		
May 20.5	6 4 12	+5 21	0.130	0.053
21.5	8 34	37		
22.5	12 45	5 53		
23.5	16 41	6 8		
24.5	20 24	23	0.170	0.035
25.5	24 0	36		
26.5	27 26	6 49		
27.5	30 42	7 1		
28.5	33 49	13	0.206	0.024
29.5	36 50	24		
30.5	39 45	35		
31.5	6 42 32	+7 45		
June 1.5	6 45 14	+7 55	0.239	0.018
2.5	47 51	8 5		
3.5	50 24	14		
4.5	52 51	23		
5.5	55 14	31	0.269	0.017
6.5	57 30	39		
7.5	6 59 43	47		
8.5	7 1 55	8 84		
9.5	7 4 4	+9 1	0.291	0.017

Brightness April 24 = 1.000.

An observation at the Lick Observatory by Mr. AITKEN on May 15, namely,

1901 May 15.6668 G.M.T., $\alpha = 5^h 38^m 25^s.8$, $\delta = +3^\circ 52' 12''$

gives the approximate correction to the above ephemeris, for that date, (O−C).

$$\Delta\alpha = +47^s, \quad \Delta\delta = -1'.$$

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MERIDIAN-OBSERVATIONS AT ALBANY IN 1897–8 AND THEIR RELATION TO SYSTEMS OF STANDARD STARS,

By LEWIS BOSS.

Previous to the removal of the Dudley Observatory to its present site in 1893 some progress had been made in the observation of stars in the region $-20°$ to $-40°$ of declination. The work was actively resumed at the new observatory in 1896. The places of about 8000 of the brighter stars, mostly within the limits $-22°$ to $-37°$ of declination, have been observed, and the reductions for these (and for about 3000 other stars) are in a forward state. The observations were designed to be differential. But it was foreseen that a difficulty would arise in applying this method in a region so far south. There were very few standard stars within that zone whose positions are known to a degree of accuracy comparable with those further north. Furthermore, even if a new discussion of standard star-positions in that region were undertaken, it was evident that some doubt might arise whether the system so derived might not be essentially discontinuous somewhere about declination $-25°$. It became desirable to devise some method by which clock-corrections and equator-points derived from Northern Standards might harmonize with those derived from standard stars within the zone under consideration. It was desirable that this should be accomplished without doing great violence to the direct testimony of the Albany Meridian-Circle. Accordingly, in parts of 1897 and 1898, observations were made upon a small list of the principal standard stars in conjunction with the systematic observation of nearly 500 stars within the limits of the zone in question. These 500 stars are to serve in their turn as the standards for a strictly differential process of reduction. In this work it was also thought that useful hints might be gained for the more strictly fundamental observations to be undertaken later.

Right-Ascension. Fundamentally, the polar deviation of the instrument in R.A. was based on successive transits of *Polaris*. There were but 16 of these, and the apparent probable error of the resulting R.A. is $\pm 0^s.18$. The right-ascensions of δ *Urs. Min.* and of a few other circumpolars were derived only from nights on which the polar deviation, n, could be ascertained from *Polaris*. For determination of clock-correction 66 stars were selected, of which the northernmost is α *Lyrae*, and the southernmost, α *Columbae*. The adopted right-ascensions of these stars were derived essentially in the following manner: The mean was taken between the right-ascensions derived from the Catalogue of AUWERS (*A.N.* 3508–9) and NEWCOMB's latest Catalogue for 1900, N_2. The R.A. of AUWERS first received the correction $+0^s.029$ to reduce it to the equinox of NEWCOMB, N_1. After approximations these means received systematic corrections indicated in the following table.

δ	Corr.	δ	Corr.
°	s	°	s
+40	−0.062	0	+0.008
+30	−0.047	−10	+0.015
+20	−0.028	−20	+0.037
+10	−0.000	−30	+0.060

These corrections applied, it made practically no difference in what declination the time-stars were found; all, from north to south, gave systematically the same clock-corrections.

Following are right-ascensions of polar stars determined by observation with the Albany instrument and compared with the standard catalogues of NEWCOMB and AUWERS respectively. All depend upon the observed right-ascension of *Polaris*. The final column shows how many times each R.A. was eventually used in the determination of n.

	R.A. 1900	No. Obs.	Corrections to N_2	Corrections to A	Same × cos δ N_2	Same × cos δ A	Times used
	h m s						
Polaris	1 22 32.92	16	−0.27	−0.50	−0.006	−0.011	108
51 *Cephei*	6 53 43.93	13	−0.06	−0.38	−0.003	−0.019	16
1 (*H*) *Drac.*	9 22 51.29	10	+0.04	+0.15	+0.001	+0.021	17
9 (*H*) *Drac.*	10 26 36.30	5	−0.01	+0.06	0.000	+0.014	7
ϵ *Urs. Min.*	16 56 12.11	5	−0.11	−0.06	−0.015	−0.008	6
δ *Urs. Min.*	18 4 32.47	32	−0.25	−0.25	−0.015	−0.015	59
76 (*F*) *Drac.*	20 19 50.73	4	+0.16	+0.20	+0.021	+0.027	5

Correction of N_2 $-0^s.0071 \sec\delta$ | Correction of A $-0^s.0084 \sec\delta$

The instrument proved to be exceptionally steady, the values of n 1^h to 12^h apart showing, as a rule, differences entirely comparable with those which might have been anticipated from the probable errors of determination. The values of n exhibit a well marked annual period. The collimations were determined by reversal over a basin of mercury, and were assumed to depend upon the temperature,— the coefficient found here being exactly that found at the old observatory. The collimation was determined a few times by reversal on *Polaris* and on the collimators. The systematic differences were small, but not sufficiently well determined to admit of adoption. The instrument has not yet been well investigated for errors arising from possible irregularities in the pivots. Levellings at the old observatory indicated that such effects are small.

The computations, as yet, have been carried only so far as was necessary to ascertain the most probable results for right-ascension when the observations are treated to be semi-fundamental as to $\Delta\alpha_\delta$. The difference, West - East, affords some test as to the probability that the instrument describes a true meridian.

Table of Differences, W.-E.

δ	Wt.	W.-E.	δ	Wt.	W.-E.
°		s	°		s
+68	5	+0.08	− 5	10	−0.009
51	10	+0.10	16	37	−0.008
41	13	−0.02	24	58	−0.022
30	40	−0.022	29	61	−0.022
21	32	+0.013	35	13	−0.019
12	58	+0.006	−40	50	−0.013
+ 6	29	+0.013			

As will be observed, the weights north of +30° are very small; the unit of weight is approximately that of one observation. There appears to be nothing in these numbers to throw serious doubt upon the values of right-ascensions derived from the mean of the East and West positions, especially since errors of collimation, as well as some others, tend to elimination in such means.

From observations north of −21° the casual probable error of a single R.A. is found to be ±0^s.027 sec δ; and from those south of −21° it is ±0^s.031 sec δ. From various considerations, including comparison with standard right-ascensions of known weight, the probable systematic error for one position seems to be about ±0^s.010. Tables of weights have been constructed accordingly. When the observations shall have been treated in a purely differential manner it is apparent that the casual probable errors will be much smaller.

Comparing, now, the deduced right-ascensions with the Catalogues of Newcomb (N_2) and Auwers, we have:

Comparison of Albany with Newcomb and Auwers.

δ	Wt.	Alb.—N_2	Wt.	Alb.—A
°		s		s
+87	90	−0.14	90	−0.15
67	10	−0.09	10	−0.03
53	21	−0.051	21	−0.033
42	22	−0.059	22	−0.042
30	51	−0.033	44	−0.031
21	35	−0.022	32	−0.005
12	67	−0.003	64	+0.026
+ 6	40	+0.006	40	+0.040
− 5	56	+0.007	50	+0.039
16	35	+0.025	27	+0.049
24	20	+0.049	67	+0.084
29	21	+0.055	62	+0.094
35	. .	. .	42	+0.079
−39	. .	. .	40	+0.102

To the numbers in the column, "Alb.—A," may be added −0^s.029 to make them comparable with those of the column "Alb.—N_2."

Comparison has also been made with my Catalogue of Southern Standard Stars, B_2, with the following results:

δ	Wt.	Alb—B_2
°		s
−24	67	+0.070
−29	62	+0.074
−35	42	+0.056
−39	44	+0.070

When these numbers are compared with others of recent epochs obtained through the use of reversible instruments they seem to require a revision of my right-ascensions of southern stars. It is also probable that the differences, "Alb.—N_2," between declinations, +20° to +60, though they may be largely due to systematic errors affecting the Albany observations, are not to be entirely explained in that way. In that region Newcomb finds a negative correction to the $\Delta\alpha_\delta$ of Auwers, and it appears probable that this correction should have been still larger in the same direction. This matter will shortly receive more specific attention at this observatory.

Declinations. The circles of the Olcott Meridian Circle have been thoroughly investigated for flexure and errors of graduation (*A.J.* 382–3). The flexure of the telescope has been measured repeatedly with care by the method of opposing collimators. These collimators are very small in relation to the instrument. In their use up to this time there is reason to suspect that there may be outstanding errors due to unsymmetrical illumination of the threads at the foci, though there is no proof that such errors exist. The zenith-distance observations throughout the year were connected together by observations of the nadir corrected for Chandler's variation of the latitude. In 1899 the zenith-distance micrometer gave out completely (from the disintegration of the threads of the female screw), and a new micrometer of different construction has been applied.

It is probable that serious deterioration of this micrometer occurred as early as November, 1897. This would impair the determination of latitude, but would have no marked effect on the observed declinations which are, in a broad sense, differential. The problem proposed was this: Given the system of declinations, B_1 and B_2, what value of the refraction constant will produce the best representation of these predicted declinations by the observed declinations from the Albany circle, the instrumental corrections having been determined as for a fundamental research. The refractions were not to be determined from circumpolar observations, but by means of assumed places of the stars. It is not necessary for the present purpose to enter into an explanation of the computing devices by which the final declinations were reached. It is sufficient to state that the declinations, as they finally stand, depend for the constant equator-point upon the adopted standard declinations, B_1 and B_2, that the Pulkowa refractions multiplied by 1.00074 were applied to the zenith-distances, and that, otherwise, the instrumental corrections were determined as for a fundamental research.

A test of the performance of the instrument can be derived by comparing declinations obtained when the clamp was West with those from clamp East; and this for each of the circles A and B. When the zenith-distances are formed directly from the nadir-observations, corrected for the refractions described in the preceding paragraph, and converted into declinations by the employment of latitude, $+42° 39' 12''.61$, the comparison of the results from the two positions of each circle lead to the following statement, condensed from the original.

W.-E. in Decl.

z	Circle A	Circle B
°	″	″
317	+0.01	−0.38
8	−0.12	−0.51
38	−0.07	−0.21
70	−0.03	−0.24

The constant difference for Circle A is $-0''.07$, and for Circle B is $-0''.29$. The nature of the individual results from which the foregoing were derived lends no support to the hypothesis that there is a marked variation, W.-E., in the different declinations. A term of correction due to flexure of the telescope and of the form, $b \cos z$, might be admitted, but it is unlikely that the coefficient b could be so large as $0''.1$. When the latitudes for each day are collected and combined in weighted means we have for the seconds of latitude,

Circle	Position	φ
		″
A	E	12.66
A	W	12.63
B	E	12.69
B	W	12.29

Considering all the available testimony it appears unlikely that the source of the discrepancy in the value of φ from B. W., is to be found in any defect of the adopted instrumental corrections. Indeed, not much weight should be attached to the discrepancy itself since if the series of determinations which make up B. W. be divided into two sections, we have

	Wt.	φ
		″
Before Nov. 1, '97	28	12.55
After Nov. 1, '97	16	11.85

It is probable that there was some defect in the determination of nadir-correction after Nov. 1. Whatever the source of the error it does not seriously affect the observations treated as differential.

The equator-points for the final declinations were determined from the predicted positions of the standard stars. When the quantities, W.-E., are formed in convenient groups we have

DECLINATION, W.-E.

δ	Wt.	A	Wt.	B
°		″		″
+90	8	+0.11	8	+0.48
63	5	−0.46	7	−0.24
37	4	+0.21	4	+0.04
28	9	−0.03	11	+0.20
16	15	+0.16	16	−0.21
+ 8	15	−0.12	14	+0.19
− 6	11	−0.16	11	−0.01
15	9	−0.04	7	−0.29
24	45	−0.02	45	−0.13
29	26	+0.04	26	+0.04
34	25	+0.03	25	−0.09
−39	5	−0.30	4	−0.48

The rather anomalous result at the pole for Circle B depends upon five stars only, and may be due in large part to an unlucky concurrence of local errors of graduation. From consideration of the numbers in general it can be assumed that there is no term of importance depending on uncorrected error of telescope-flexure of the form $b \cos z$. There is also small evidence of outstanding errors of graduation, or of flexure of the circles.

Next, I compare the definitive observed declinations with B_1 and B_2 over a range of declination from $+90°$ to $-42°$. For convenience the results are combined in groups. Column I exhibits the results from the direct comparison, the adopted refraction being: (Pulkowa) × (1.00074). The adopted flexure of the telescope was $+1''.16 \sin z$. If $+1''.46 \sin z$ had been adopted instead of that; if the refractions had been, (Pulkowa) × (0.99946), diminished instead of increased; and if the adopted value of the latitude had been increased by $+0''.04$; then we should have had the numbers in column II. Both columns are in the sense of corrections of the standard declinations.

Declinations Compared with Standard. (*Obs'd* – *B*).

δ	Wt.	I	II	δ	Wt.	I	II
°		"	"	°		"	"
+90	1	–0.30	–0.11	–5	5	+0.01	–0.09
77	1	–0.16	+0.01	15	3	+0.17	+0.07
60	1	–0.10	0.00	24	5	+0.11	+0.04
48	2	+0.23	+0.29	27	4	+0.02	–0.03
38	2	+0.03	+0.01	32	3	–0.01	–0.02
27	5	+0.11	+0.09	35	1	–0.11	–0.06
16	6	–0.01	–0.07	37	1	–0.30	–0.15
+8	7	+0.08	0.00	–40	0.5	–0.06	+0.25

The casual probable error of a single observation for stars north of –20° is ±0".31; for –25° it is ±0".41, and at –40° it is ±1".33. The constant, or systematic error, appears to be very exactly ±0".12 for a single position. The casual error will be made smaller when the observations are treated as strictly differential. The probable error of the unit of weight in the foregoing table is ±0".10.

The supposition that the adopted telescope-flexure could be in error so much as 0".3, though not unreasonable, is rather unlikely. All things considered it is clear that the "Declinations of Fixed Stars" and its Southern extension, *A.J.* 448–450, is consistent with the declinations observed at Albany. This evidence is of some value; for if the systems of the predicted and observed declinations respectively were, either of them, affected by serious irregularities, they could not be so well reconciled through an arc of 130° by a simple and probable assumption regarding the refraction. Evidence is accumulating that the standard declinations in question, B_1, represent, with good approximation, the whole course of observation throughout the nineteenth century.

PHOTOMETRIC OBSERVATIONS OF *EROS*,

BY HENRY M. PARKHURST.

The remarkable circumstances attending the late opposition of *Eros* made it desirable that special attention should be paid to observing it photometrically, determining its constants of brightness and their law, reducing its constants to the asteroid basis (*A.J.* 414), and obtaining with its aid standard groups of comparison-stars along its path of visibility. My photometric observations, 382 double extinctions, continued from September 13 to March 22, the first and the last available date. In the early observations the identification consisted in the absence of the observed star in the photographs of the region, kindly furnished me by Prof. Pickering, confirmed by its absence on subsequent evenings. In the later observations the identification included the change of position of the observed star during the observations of the evening. On two dates, although *Eros* was fully identified, it was evident that the result was affected by cloudiness, and those two observations were rejected. *Eros* was compared with a large number of standards, including four other asteroids.

The main objects sought were the precise values of the constants determining the brightness. In the Annals of the Harvard Observatory (Vol. XVIII: 69; and Vol. XXIX: 87), I have given tables tending to show that the quantity p, the factor of the diminution of brightness depending upon the phase angle, is a constant within the limits of the change of phase angle in the asteroids which had then been observed. In the recent opposition the change of phase angle has extended from 28°, the maximum before known, to 58°; far exceeding previous experience.

In my plan of operations I did not overlook the possibility of error from rotation and consequent changes of brightness; but took special pains to guard against the possibility of error in my determinations from this cause. Owing to the interference of an adjoining building on the north, and owing to the planet passing the zenith, where my dome did not permit its observation, it was usually impossible to make more than one series of observations in an evening, or to vary or to accurately predict the hours of observation. The changes of the sky during each series of observations, were necessarily obtained from *Eros* itself, using the mean brightness of *Eros*, and trusting to the balancing out of the errors from rotation. As an additional safeguard *Eros* was given due weight in determining the sky value, any change occurring in *Eros* itself extending over the times of the neighboring observations.

Even after the publication of the discovery of the rapid rotation and of the large variation in brightness, I could not investigate the question or vary my plan without serious interruption of my main work. Fortunately these questions do not need corroboration from me; it is sufficient to say that thorough examination of my observations has confirmed the variation in brightness, and the great changes in its extent, as well as the remarkable accuracy of the first determination of its period, and has demonstrated that notwithstanding these facts, no appreciable change is required in the reduction of the observations according to my original plan. Indeed, the substitution of a regular known cause for the irregularities which I had necessarily attributed to unknown variations of the sky, has given me greater confidence in the practical reliability of my observations and the accuracy of my final reduction.

The constants obtained from my final revision after the

conclusion of the observations, were: $G° = 9^M.78$, reducing both distances to unity; and $p = .037$. This quantity was computed separately for three principal divisions of the phase angle, the discrepancy being so small that it could be attributed to errors of observation or to inaccuracy of the comparison stars; for an average inaccuracy of less than $-0^M.03$ would be sufficient to produce the inaccuracy observed. The correction i, for defect of illuminated surface, depends upon the square of the phase angle; and therefore, if it is merged with p, as may be safely done with other asteroids, it would produce a small variation of p; but before the observations I had applied that correction separately. My conclusion is that p is a constant, even with the phase angle extended to 58°.

In order to test the accuracy of the different caps used in my observations, I reduced the observations with special reference to the accuracy of the scale for different degrees of brightness, comparing the observed magnitude with that computed from the differences in the distances. The errors were too small to be further noticed than by saying that I made the minute corrections required in the constants given above, carried one decimal further than above stated.

The explanation of the changes of brightness is not obvious. A spheroidal form seems a sufficient explanation of the observations. Whatever the explanation, it seems important to guard against the danger of error in the use of such asteroids as photometric standards; which may be done in two ways; first, by multiplying the observations of the asteroid used as a standard; and second, if the period of rotation can be ascertained, by making the observations in pairs at an interval equal to one-half the interval between the maxima, so that the errors from rotation may counterbalance each other. Recent observations of *Vesta* demonstrate that there is at present no appreciable fluctuation in its brightness. There is no sufficient reason for suspecting other asteroids of such changes. Certainty can only be reached with regard to any particular asteroid by alternate frequent comparison with standard stars, and by repeating this process after a sufficient lapse of time for the direction of the axis of rotation with regard to the earth to appreciably change.

OBSERVATIONS OF *NOVA PERSEI*, 1901.

By PAUL S. YENDELL.

I observed ANDERSON's new star in *Perseus* on twelve dates, from 1901 Feb. 24 to Mar. 19, inclusive. I found no opportunity for further observations from the latter date until early in May, when the star was too low in the northwest for useful work.

In reducing the observations the scale of the Harvard Photometry has been used, that being the only one available that included all the comparison-stars, from *Rigel*, $0^M.18$ to *l Persei*, $5^M.40$.

Dorchester, Mass., 1901 *May* 18.

The observed magnitudes are as follows:

1901	Mag.		1901	Mag.
Feb. 24	−0.08	full yellow	Mar. 7	2.75
25	1.10	" "	12	3.38
26	1.12		16	3.76
27	2.13		17	3.71 full orange
28	1.72		19	5.10
Mar. 2	2.23	[orange		
6	3.38	strong full		

Feb. 24 Mar. 6, moonlight

OBSERVATIONS OF COMET *a* 1901 AT THE CAPE OF GOOD HOPE.

[Communicated by Prof. KREUTZ.]

Greenw. M.T.	App. α	App. N.P.D.
	° ′ ″	° ′ ″
May 5.2178:	61 29 24	90 1 20
6.2124	64 48 43.5:	89 11 27
7.2096	67 36 3	89 19 15
11.2016	76 54 40.5	87 46 15
12.2347	78 54 31.5	87 22 21
13.2080	80 39 40.5	87 0 20

The (:) numbers are obscure, but scarcely to be interpreted otherwise.

ELEMENTS BY INNES.

Computed from May 3, 5 and 7.

T = 1901 April 24.24 Greenw. M.T.

$$\left.\begin{aligned} \omega &= 202° 58' \\ \Omega &= 110\ 10 \\ i &= 130\ 14 \end{aligned}\right\} 1901.0$$

q = 0.2425

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $1\frac{3}{4}$-INCH REFRACTOR,

By A. W. QUIMBY.

1900	Time	New Grs.	Total Grs	Total Spots	Fac. Grs.*	Def.
Jan. 1	9	–	1	1	1	fair
2	8		1	1	1	fair
3	11	–		–	1	good
4	8			–	–	poor
5	10					fair
6	3			–		fair
7	9	–	–		–	fair
8	9	–	–	–	–	fair
9	11	–		–	–	poor
10	2	–	–	–	–	poor
12	10	2	2	10	1	fair
13	9	–	2	22	1	fair
14	8	–	2	8	–	poor
15	10	–	2	5	–	poor
16	2	–	2	14	–	poor
17	10	–	1	7	1	fair
18	3	–	1	1	–	poor
21	9	–	–	–	–	fair
22	9	–	–	–	–	fair
23	9	–	1	2	1	fair
24	9		1	1	–	poor
26	9	1	2	2	1	poor
27	10	–	2	13	1	fair
28	9	–	2	10	2	fair
29	9	–	1	2	1	poor
30	3	–	1	10	–	fair
Feb. 1	2	–	1	7	–	fair
2	8	–	1	4	–	poor
3	10	–	1	1	–	fair
4	3	2	3	24	2	fair
5	8	–	3	14	2	fair
6	3		3	31	2	fair
7	11	–	2	24	1	poor
9	3	–	2	20	–	fair
10	9	–	1	1	–	fair
11	2	–	–	–	–	poor
13	2	–	–	–	–	poor
14	9	–	–	–	2	fair
15	8	–	–	–	1	poor
18	8	–	–	–	–	fair
19	8	–		–	–	poor
20	8		–	–	–	poor
21	2	–	–	–	–	fair
22	8	–	–			poor
23	3	–	–	–	1	fair
24	1	–	–	–	–	fair
25	8		–	–	–	fair
26	8		–	–	–	fair
27	8		–	–	–	fair
28	8		–	–	–	fair
Mar. 1	8		–	–		poor
2	8		–	–	1	fair
3	8	1	1	4	1	fair
4	8		1	10		fair
5	1		1	1	–	poor
6	5		1	10	–	poor
Mar. 7	4	–	1	8	–	poor
8	11	–	1	4	–	poor
9	11		1	7	1	fair
10	11	–	1	5	1	fair
11	9	–	1	2	1	fair
12	8	–		–	–	fair
13	8	–	–	–	–	fair
14	8	–	–	–	1	fair
16	8	–	–	–	–	fair
17	8	–	–	–	1	fair
18	9	–	–	–		fair
19	8	–	–	–	–	fair
20	8	–	–	–		fair
21	8	–	–	–	–	fair
22	8	–	–	–	1	fair
23	9	–	–	–	1	fair
24	8	–	–	–	1	fair
25	8	–	–	–	–	fair
27	8	1	1	3	1	fair
28	8	–	1	4	–	fair
29	8	1	2	4	–	fair
31	8	–	2	4	1	fair
Apr. 1	8	–	1	5	2	fair
2	7	–	1	3	–	poor
3	8	–	1	12	–	fair
4	9	–	1	10	–	fair
5	8	–	1	3	1	poor
6	9	–	1	2	1	poor
7	10	1	1	1	1	fair
8	9	–	1	2	1	fair
9	8	–	1	1	1	fair
10	8	–	1	1	–	poor
11	8	–	1	1	1	fair
13	6	–	1	3	–	fair
14	5	–	1	2	–	fair
15	9	–	–	–	–	fair
16	8	–	–	–		fair
17	5	–	–	–	1	fair
18	10	–	–	–		poor
19	4	1	1	1	1	good
20	7		1	1	1	poor
21	10	–	1	1	–	poor
22	1	–	1	1	–	poor
23	4	1	2	1	–	fair
24	8	–	2	11	2	good
25	8	1	3	22	2	fair
26	7		3	11	2	poor
27	8	1	4	17	2	poor
28	5	1	4	28	2	fair
29	8	–	2	70	2	good
30	8	–	2	15	2	fair
May 1	8	–	2	13	1	good
2	8	1	3	21	–	fair
3	12	–	3	22	–	fair
4	8	–	3	34	–	good
5	8	–	3	23	1	good
May 6	8	–	2	12	2	fair
7	8	–	1	2	2	fair
8	8	–	–	–	1	fair
9	6	–	–	–	–	poor
10	8	–	–	–	1	fair
11	8	–	–	–	1	fair
12	4	–	–	–	1	fair
13	8	–	–	–	–	fair
14	8	1	1	4	–	fair
15	8	–	1	3	–	fair
16	8	–	–	–	1	fair
17	12	–	–	–	–	poor
18	8	–	–	–	1	fair
20	11	–	–	–	–	poor
21	9	1	1	2	1	fair
22	10	–	1	7	1	fair
23	8	–	1	11	1	fair
24*	7	–	1	4	–	poor
25*	6	–	1	5	–	poor
26*	3	1	2	12	–	fair
27*	7	–	2	13	–	good
28*	9	–	2	12	–	good
29*	7	–	1	3	–	poor
30*	5	–	2	2	–	poor
31*	6	–	1	4	–	poor
June 1*	6	–	1	3	1	fair
2	7	–	1	5	1	fair
3	8	–	1	5	1	fair
4	7	–	–	–	2	fair
5	8	1	1	1	2	fair
6	8	–	–	–	–	fair
7	7	–	–	–	1	fair
8	8	–	–	–	–	fair
9	8	–	–	–	–	fair
10	8	–	–	–	–	fair
11	9	–	–	–	–	fair
12	8	–	–	–	–	fair
13*	6	–	–	–	–	poor
14*	10	–	–	–	–	poor
15*	7	1	1	3	–	poor
16	8	–	1	3	–	poor
17	6	–	1	13	–	poor
18	8	1	2	14	1	poor
19	8	–	2	36	1	good
20	8	–	2	22	1	poor
21	8	–	2	53	1	v. good
22	7	–	2	26	1	fair
23	8	–	2	8	2	fair
24	5	–	2	4	1	poor
25	7	–	1	1	–	poor
26*	7	–	–	–	–	poor
27*	7	–	–	–	–	poor
28*	6	–	–	–	–	poor
29	7	1	1	26	1	fair
30	8	–	1	8	1	poor

* $2\frac{1}{4}$-inch refractor; 36-inch focal length.

SYSTEMATIC CORRECTION OF RIGHT-ASCENSIONS OF SOUTHERN STANDARD STARS,

By LEWIS BOSS.

Since the publication of my paper on the positions of 179 Southern Standard Stars (*A.J.* 448–450), designated here for convenience, B_2, several star-catalogues have been published. Among these, two offer the testimony of reversible instruments, and at the same time contain many right-ascensions of Southern stars. These are the catalogues, Washington 1875 and Mt. Hamilton 1895. The comparatively low latitudes of these observatories encourage the hypothesis that the observations in R.A. may be entitled to fair weight even down to $-40°$ of declination. Therefore, when I found that these two catalogues, as well as the Albany observations, united in indicating a similar and decided correction for the right-ascensions of B_2, $-20°$ to $-40°$, it seemed desirable that the latter should be revised.

In the Southern hemisphere for the modern epoch we have been dependent very largely upon the Cape and Melbourne instruments which are used as non-reversible. Other things being equal, a reversible instrument, used as such in the determination of fundamental R.A., ought to produce more than twice the weight of result that can be got from a non-reversible instrument. For in the former case we not only get, in many respects, the effect of two instruments, but systematic errors in determination of collimation as well as those arising from defective illumination and other causes tend to eliminate upon reversal. GOULD's Cordoba observations were made with a reversible instrument; but the methods employed, necessarily governed by the requirements incident to an enormous mass of observations, leave something to be desired. I have been led to infer that, in the reduction of Professor UPDEGRAF's R.A.-observations (Cordoba 1889), predicted right-ascensions of the circumpolar stars were employed in the determination of n, *viz.*, GOULD's right-ascensions (*Cord. Obs.*, Vol. V, Int.). Since these contain observations as late as 1884, they might be supposed to be measurably precise in 1889. But Dr. AUWERS's investigations appear to point by inference to a very large periodic error in the adopted values of n in the Cordoba work of 1889 (*A.N.* 3413, p. 75). The term of correction found by Dr. AUWERS is, approximately,

$$+0^s.06 \; tg\,\delta \; \sin(210° + \alpha);$$

and it is very difficult to suppose that so large an inequality as this can have its origin solely in a defect of the predicted right-ascensions. It is doubtful whether this series is entitled to much weight in the establishment of a fundamental system, though it agrees fairly well with the conclusions of this paper. The Albany positions embrace only one year of observations when much more than half the observations was devoted to stars not in the Standard list. The Washington right-ascensions do not appear to be entitled to the highest confidence in respect to their probable freedom from systematic peculiarities; since the instrument, after a few years of use, appears to have been regarded as unfit for the determination of fundamental right-ascension, and its employment in that direction was abandoned. The Mt. Hamilton Catalogue contains a relatively small number of right-ascensions of Southern stars. In spite of these defects in the testimony of the reversible instruments, it has not seemed advisable to assign to the non-reversible instrument at the Cape, with four series of observations, a total weight of more than four, as against three for each of the four reversible instruments, represented in each instance by a single series of observations. In difference of observers and methods there is an undoubted increase of weight with increase in the number of series of observations with a given instrument; but, with especial reference to the problem in hand, the determination of $\Delta\alpha_\delta$, it is practically certain that there will remain with each instrument peculiarities which cannot be eradicated by repetition of use.

We first correct each series of right-ascensions to agree with NEWCOMB's equinox, N_1, and free them, so far as possible, from periodic terms of correction of the form, $\Delta\alpha_\alpha$. There still remain discrepancies, $\Delta\alpha_\delta$, varying with the declination, which exhibit the discordances found in the various attempts to describe a true meridian from a fixed point slightly north of the equator to the South pole. The following table shows these discordances in the sense of corrections to B_2. (See *A.J.* 448–450.)

CORRECTIONS, $\Delta\alpha_\delta$, FOR B_2, INDICATED BY OBSERVATION.

Series	Wt.	−20°	−25°	−30°	−35°	−40°
		s	s	s	s	s
Cape 30	1	−0.009	−0.013	−0.018	−0.022	−0.027
S. H. 32	1	0.000	+0.010	+0.032	+0.050	+0.050
Cape 33	2	+0.004	+0.001	−0.005	−0.007	−0.009
Cape 37	2	−0.018	−0.021	−0.024	−0.027	−0.030
So. 51	1	+0.021	+0.026	+0.030	+0.030	+0.030
Cape 59	1	−0.007	−0.009	−0.016	−0.024	−0.032
Melb. 62	1	+0.012	+0.015	+0.016	+0.017	+0.021
Melb. 68	1	+0.012	+0.012	+0.012	+0.012	+0.012
Cord. 76	3	+0.001	+0.010	+0.023	+0.037	+0.040
Melb. 76	1	−0.012	−0.016	−0.016	−0.016	−0.016
Cape 76	1	−0.012	−0.014	−0.009	−0.012	−0.003
Wn 79	3	+0.024	+0.030	+0.036	+0.043	+0.050
Cape 81	1	−0.001	−0.001	−0.002	−0.002	−0.002
Cord. 89	0	−0.009	+0.002	+0.052	+0.092	+0.096
Cape 89	1	+0.003	+0.002	0.000	−0.004	−0.008
Mt. H. 95	3	+0.041	+0.041	+0.041	+0.041	+0.041
Alb. 98	3	+0.068	+0.068	+0.068	+0.068	+0.068

From the foregoing numbers the following corrections to the preliminary system for 1875, and centennial variations

of those corrections, result. The last two columns exhibit corrections to the system for 1850 and 1900 respectively.

Systematic Corrections in R.A. for B_2. (179 Southern Standard Stars).

δ	1875	C.V.	1850	1900
°	s	s	s	s
−20	+0.0186	+0.075	−0.015	+0.037
25	+0.0202	+0.078	−0.015	+0.040
30	+0.0233	+0.082	−0.014	+0.044
35	+0.0257	+0.084	−0.013	+0.047
−40	+0.0259	+0.089	−0.013	+0.048

From the mode of derivation of the original corrections the correction of the system for −10° may be assumed zero. From −40° to −80° of declination I should adopt numbers interpolated from those at −40°[$+0^s.0259+0.089(T-1875)$] to zero at −80°. The corresponding corrections to the system of Auwers (*A.N.* 3431–32) are roughly these:

Corrections for the R.A. of Auwers (*A.N.* 3431–32).

δ	1850	1875	1900	C.V.
°	s	s	s	s
−20	−0.053	+0.012	+0.047	+0.143
25	−0.056	+0.014	+0.051	+0.157
30	−0.058	+0.021	+0.064	+0.174
35	−0.057	+0.026	+0.072	+0.184
−40	−0.057	+0.030	+0.077	+0.194

If Dr. Auwers had adopted Newcomb's equinox, N_1, the centennial variations of the corrections attributed to his system would have been smaller by about $0^s.085$, and, therefore about the same as those attributed to B_2.

That the revision pointed out in the foregoing is in the right direction does not seem to admit of much doubt; nor do I think that the corrections are exaggerated in amount. Of all the series from reversible instruments only those of St. Helena and Cordoba '89 exhibit outstanding values of $\Delta\alpha_\delta$ which are large enough to call for special comment. Cordoba '89 received no weight in this discussion for reasons stated. The St. Helena right-ascensions lay no claim to independence; they are confessedly, and almost of necessity, based on contemporary observations at other observatories. At the same time it is impossible to feel a high degree of confidence in the precision of the corrections actually found in this revision, as compared with the standard of accuracy attainable in the Northern hemisphere. The projected work with the new reversible instrument at the Cape of Good Hope, therefore, assumes a degree of importance which could not possibly attach to a similar undertaking in this hemisphere.

Cape '50. Having had occasion to make particular use of the Cape Catalogue for 1850, I have been led, recently, to revise the systematic corrections due to that catalogue, in order to reduce it to the system of B_2. For this purpose I possessed much more data than had been accessible to me for constructing the table of correction printed in *A.J.* 459. Usually in this catalogue there are not more than two observations of each star (often only one), even of the principal stars. This makes the derivation of corrections for it a process of unusual difficulty, though it may be hoped that the present result offers a fair approximation.

The revision in declination offers no material amendment to the results of my former paper.

The results for R.A. follow, and exhibit the corrections required to reduce the right-ascensions of Cape '52 to the system of B_2 as revised in the present communication.

R.A. Corrections, B_2 − Cape 52.

δ	Corr.	δ	Corr.
°	s	°	s
0	+0.016	−45	−0.079
−5	+0.016	50	−0.079
10	+0.015	55	−0.047
15	+0.010	60	−0.040
20	+0.005	65	−0.038
25	0.000	70	−0.036
30	−0.005	75	−0.02
35	−0.025	80	0.00
−40	−0.063	−85	0.00

The correction, $\Delta\alpha_\alpha$, adopted in *A.J.* 449, was deduced almost wholly from right-ascensions between declination parallels, +15° to −35°. Since the catalogue-positions refer mostly to stars south of −20°, it seems better to deduce $\Delta\alpha_\alpha$ from them. Proceeding in this manner we have:

$$\Delta\alpha_\alpha = -0^s.017 \sin\alpha - 0^s.010 \cos\alpha$$

These corrections are not very greatly different from those of my former paper; but they are still in need of revision. For this purpose a greater number of quasi-standard stars of comparison will be required. It is to be expected that eventually this increase of stars available for comparison in deducing the systematic error of Cape '50 will not necessitate special computations, but will naturally result from investigations in progress here in the interest of more general problems.

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A STUDY OF THE LIMITING MAGNITUDES OF THE CAPE PHOTOGRAPHIC DURCHMUSTERUNG,

By SIMON NEWCOMB.

Among the many important uses to which the Cape Photographic Durchmusterung may be put we must assign a high place to the determination of the law of distribution of the fixed stars. In the completeness with which it covers its field we have, as yet, nothing in the northern hemisphere equal to this work. It can hardly be supposed that any stellar object photographically brighter than $9^M.5$ comprised within the region from $-15°$ to the south pole, has failed to be included in it. It is also remarkable for the clearness and fullness with which the data on which it is founded are presented. These features render it a very attractive subject of study, and at the same time facilitate such corrections as farther research and comparison may show to be necessary. I have recently been studying its system of magnitudes, and have thus been led to conclusions which may be worthy of attention in questions growing out of its statistical data.

It has, I believe, been generally assumed that when the cloudless sky is photographed with a given instrument and a given time of exposure, stars down to some definite magnitude will be found on the plates. In accordance with this hypothesis the faintest stars photographed in the catalogue branch of the international chart of the heavens are assumed to be of the 11th magnitude.

Gill and Kapteyn, in the work under consideration, have rejected this standard as unreliable on the ground that, even under a cloudless sky, the variations in the steadiness and photographic transparency of the atmosphere are so wide that the size or intensity of a stellar image on a plate affords no sufficient basis for inferring the magnitude of the star which produced it. They have therefore reduced the magnitudes of the stars on each plate separately so as to reproduce the known visual magnitudes of stars found on the plate.

One result of this process will be obvious on an examination of the limiting magnitudes in different regions. These vary from $9^M.6$ on some plates to 11^M or even higher on others. The range of the limit of magnitude is therefore at least $1^M.5$.

The recent publication of the third part of Thome's *Cordoba Durchmusterung* suggests a comparison of the limiting magnitudes of this work with the other. It may be remarked that Thome's work extends to about $10^M.4$ of its scale, although beyond $9^M.9$ no decimals are given. It gives the impression of being quite uniform and complete up to the limiting magnitude, except, possibly, in the case of some clusters or dense agglomerations of stars. A very slight comparison served to raise the question whether the limiting magnitudes on Gill's plates were really as diverse as given in the catalogue. This question may be decided both by an independent comparison of the number of stars on different plates in different regions of the sky, as found in the C.P.D., and by a comparison with the Cordoba work. The numbers shown in the following tables were obtained in the following way:

From Table IV of the C.P.D. I made a selection of those plates not too near the galaxy in which the limiting magnitudes were either near the lower or higher extreme. The general average limit being $10^M.3$, plates were taken of which the limits are below $9^M.9$, or above $10^M.5$. The number of stars within the region covered by each plate, which was generally about $5°$ square, or $4° \times 5°$, was then counted. There was sometimes a slight indetermination as to the limiting right-ascension on the plate. But the uncertainty thus introduced in the count is too small to affect the result. A similar count was then made of the stars within the same region according to Thome. The results of these counts are shown in the following table in which is given:

The limiting right-ascensions and declinations of the plate, from which the plate itself may be identified in Gill's list of his Table IV;

The area in square degrees covered by the plates, or included in the count;

The number of stars within the region, according to the two authorities;

The mean density per square degree of the stars in each region, as thus found, called the richness of the region;

The limiting magnitude according to the C.P.D.;

The approximate galactic latitude of the center of the region covered.

As the plates chosen are not all very near the galactic pole it is desirable to make a reduction for the increasing density of the stars toward the galaxy. I find that, judging by SEELIGER's counts of the ARGELANDER-SCHOENFELD work outside the regions of the galaxy, the star-density can be approximately reduced to one standard by being multiplied by the square root of the sine of the galactic latitude.

Multiplying the richness by this factor we have the reduced richness of the sky given for each authority.

The last column, R_0, gives the reduced richness from the C.P.D., obtained in the following way. In the region of the 10th magnitude the number of stars of each magnitude is about three times that of the magnitude next below. That is to say, the number of stars may be approximately represented by the formula $N = 3^{M} h$, h being a constant depending on the area and the general mean richness. From this expression we derive the number of stars per square degree up to the 10th magnitude, or R_0, by the formula

$$R_0 = 3^{10-M} R$$

R being the reduced richness of the C.P.D. in the column preceding. The richness thus found is given to round units only.

COUNTS OF STARS FROM THE C.P.D. AND FROM THOME.

Extent in R.A.	Area Sq. Deg.	No. of Stars C.P.D.	No. of Stars THOME	Richness C.P.D.	Richness THOME	Lim. Mag. C.P.D.	Gal. Lat.	Reduced richness C.P.D.	Reduced richness THOME	R_0
Belt −26°.5 Decl.; 5° wide.										
2h 25m to 3h 32m	74.92	768	2478	10.3	33.1	9M.7	59°	9.5	30.5	13
21 10 to 21 56	51.44	613	1972	11.9	38.3	11.0	48	10.2	33.0	3
23 3 to 23 25	24.60	168	844	6.8	34.3	9.6	71	6.6	33.3	10
Belt −31°.5 Decl.; 5° wide.										
0 58 to 1 22	25.57	276	764	10.8	29.9	11.0	82	10.7	29.8	4
2 35 to 2 59	25.57	245	793	9.6	31.0	10.6	62	9.0	29.1	5
4 33 to 5 22	52.21	1250	2367	23.9	45.3	11.0	34	17.9	33.8	6
6 35 to 7 21	49.01	1202	4544	24.5	92.7	9.7	10	10.2	38.7	14
20 31 to 20 57	24.50	515	1288	21.0	52.6	10.8	40	16.8	42.3	7
22 58 to 23 46	51.13	560	1667	11.0	32.6	11.4	72	10.7	31.8	2
Belt −36° Decl.; 4° wide.										
0 14 to 0 38	19.41	171	608	8.8	31.3	9.8	81	8.7	31.1	11
1 4 to 1 29	20.22	150	636	7.4	31.5	9.8	79	7.3	31.2	9
3 35 to 4 50	60.67	796	2365	13.1	39.0	10.8	45	11.0	32.8	4
21 17 to 22 6	39.64	564	1768	14.2	44.6	9.9	51	12.5	39.3	14

It needs little argument to show that the great differences in the numbers R_0 must be entirely fictitious. Even if we suppose great differences in the reduced richness of the sky, the limits reached by the plates would be independent of the richness. But, in the present case, it is clear that the reduced number of stars per square degree imprinted on the plates is much more uniform than the number to the 10th magnitude as given in or inferred from the catalogue.

To see the same result from a little different point of view, suppose that we eliminate from the C.P.D. all stars of 10M.0 or higher. We shall then have left a nearly complete catalogue up to 9M.9, deficient only in that, on a few of the regions, the cataloguing stops at two or three-tenths below 9M.9. It is quite clear that, if the magnitudes given are correct, those regions of the sky in which 9M.7 9M.9 are wanting should be, in the general average, the poorest. But the contrary is actually the case in the catalogue. Those regions for which the magnitudes are 9M.8 or less appear as systematically the poorest.

Yet another way of considering the same subject is this. If the differences of the limiting magnitudes of the C.P.D. are real, then, when we compare this catalogue with that of Cordoba as to number of stars in different regions, the ratio C.P.D. ÷ Cordoba should be greatest when the limiting magnitudes are the most extreme. This only presupposes that the estimates of THOME are independent of the condition of the atmosphere at the time when the plates were taken at the Cape. Now, if we form the mean ratio of richness, THOME ÷ C.P.D., giving weights proportional to the number of plates, for the higher limits and the lower limits of magnitude respectively, as in the table given, we find

Mean limiting magnitude,	9M.7:	Thome ÷ C.P.D. = 3.3
" " "	10.8:	Thome ÷ C.P.D. = 2.8

The difference in these ratios would indicate a mean dif-

ference of limiting magnitude of only $0^M.22$ between the two sets. So far, therefore, as we can judge from these data, the mean limiting magnitude of the C.P.D. in the extreme seven regions investigated extends only about $0^M.22$ farther than in the six regions in which the mean limiting magnitude is $9^M.7$.

Our conclusion may be stated thus:

The magnitudes of the C.P.D. would have been nearer the truth without any correction for reduction to a visual standard, on the assumption that the limiting magnitude is the same on all the plates.

The only objection to the generality of this conclusion which I can see is that it is based on a few extreme cases. May it not be that, in other cases the magnitudes are improved by the corrections? I reply to this that it is only by extreme cases that the question can be tested. When, as in most cases, the corrections are too small to be subjected to a rigorous test, they become at the same time uncertain and unimportant. If the legitimate corrections are, as a general rule, less than two-tenths of a magnitude, they cease to be ascertainable with precision, and it would be better not to apply them at all.

The question which now presents itself is, granting that the limiting magnitude is nearly the same on all the plates, what is its mean value? In the absence of photometric determinations extending in sufficient number to nearly the extreme magnitude we can only find a mean limit relatively to the Cordoba system. The comparison can most readily be made by comparing numbers of stars. I find that, away from the galaxy, about one-third of THOME's stars are marked 10^M. From this, by the formula already given, it may be inferred that the stars thus marked range through $0^M.36$. Supposing them to start from $9^M.95$, this gives $10^M.31$ as the limit.

Away from the galaxy, the ratio number of stars, THOME $\div$ C.P.D. is about $2^M.7$. This corresponds to a difference of magnitude of $0^M.9$. Hence the general limit of the C.P.D. may be set at $9^M.4$ of the Cordoba system. There can, I conceive, be little doubt that the actual limit on the photometric scale is higher than this; perhaps $10^M.0$. If so, away from the galaxy THOME's DM. extends nearly or quite to $10^M.9$; and this seems quite likely.

In the denser galactic regions it seems from the counts given by THOME in his third volume that the limit is not very different for the two catalogues in the general means, though there are striking individual discrepancies. Thus, in the zone $-42°$ the C.P.D. seems to show a singular sparseness of stars through the galactic region 16^h to 18^h, though the limiting magnitude given in the catalogue is never below $9^M.8$, and generally exceeds $10^M.0$. This difference in the ratio of richness of the two catalogues for the galactic and non-galactic regions may arise in great part, as KAPTEYN has shown, from the general blueness of the galactic stars.

It would seem, on the whole, that the limit of magnitude of the Cape Photographic Durchmusterung is much nearer uniform than the limits given, and that it may be regarded as a fairly complete representation of the southern sky to a photographic magnitude of probably $10^M.0$.

In the same connection this comparison seems to show the great importance of the completion of the Cordoba work, which is now two-thirds done.

ON THE ERUPTIVE ENERGY OF THE STARS.

By J. WOODBRIDGE DAVIS.

The fact that the molecules of the different gases are equal in thermal capacity, but unequal in mass, must profoundly affect the behavior of great condensing bodies of gas. Suppose that a molecule of nitrogen and a molecule of hydrogen, starting at the same temperature, fall in company through a certain space, at the end of which their motion is arrested in such manner that the heat produced is wholly absorbed by the molecules themselves. This would be the case if two bodies of gas, each composed of a number of nitrogen molecules and the same number of hydrogen molecules, should come into collision in consequence of their mutual gravitational attraction. The nitrogen molecule generates fourteen times the quantity of heat generated by its lighter companion, and, being of the same thermal capacity, its increment of temperature would also be fourteen times as great, if it might retain its own heat; but, since the molecules are in juxtaposition, they necessarily arrive at the same temperature, and, therefore, share equally the heat generated by the pair. In this way the hydrogen acquires, mainly at the expense of the nitrogen, seven and a half times the heat due to its own fall under the gravitational attraction of the two gases. By the principle of gaseous diffusion, each gas is now acted upon by the other only in consequence of the gravitational attraction and the frictional resistance of the latter.* Hence, as the

* "If we apply the ordinary language about fluids to a single gas of the mixture, we may distinguish the forces which act on an element of volume as follows:

"1st. Any external force, such as gravity or electricity.

"2d. The difference of the pressure *of the particular gas* on opposite sides of the element of volume. [The pressure due to other gases is to be considered of no account.]

"3d. The resistance arising from the percolation of the gas through the other gases which are moving with different velocity."—Prof. J. C. MAXWELL, article "Diffusion." *Encyc. Brit.*, Vol. VII, 1877, p. 216.

result of a very slight fall of the two portions of the nebula, the hydrogen may become possessed of an energy sufficient to expand it to infinity against the gravitational attraction of the two gases. Held in check, however, by the frictional contact with the mass of the nitrogen, which is bereft of the power to lift itself so far as it fell, this energy expends itself slowly in forcing the lighter gas up through the heavier, and creating an outrushing hydrogen atmosphere.

To treat the problem more generally, let δ', δ'', be the relative densities of the two gases, δ' pertaining to the lighter gas; let the number of molecules of the heavier gas be y times the number of molecules of the lighter gas in each separated part of the nebula; and let H be the heat generated in the collision. Then, the portion of heat generated by the fall of the lighter gas is

$$\frac{\delta'}{y\,\delta''+\delta'}\,H \tag{1}$$

and the fraction of heat received by the lighter gas is

$$\frac{1}{y+1}\,H \tag{2}$$

The ratio of the heat received to the heat generated by the lighter gas is

$$\frac{y\,\delta''+\delta'}{y\,\delta'+\delta'} \tag{3}$$

which increases from 1 to $\delta'' \div \delta'$, as y increases from zero to infinity; that is, the smaller the proportion of the lighter gas, the more violently is it expelled. The quantity of heat acquired by the lighter gas at the expense of the heavier is

$$\left\{\frac{1}{y+1}-\frac{\delta'}{y\,\delta''+\delta'}\right\}H \tag{4}$$

which is a maximum when

$$y = \sqrt{\frac{\delta'}{\delta''}}\,, \quad \text{or when} \quad \frac{y\,\delta''}{\delta'} = \sqrt{\frac{\delta''}{\delta'}}$$

that is, the greatest mechanical effect in expelling the lighter gas occurs when the masses of the gases are proportional to the square roots of their relative densities.

Before dealing with celestial bodies of gas, it will be well to recall the behavior of mixed gases in a tall column at the earth's surface, as discussed by Professor MAXWELL in the article already referred to. A uniform mixture quickly arrives at a state of equilibrium as a mixture, in which the density increases downwardly. But the constituent gases are not in independent equilibrium. Suppose each to occupy alone a similar tube; then, when each has attained equilibrium under the same circumstances as to temperature and intensity of gravity, the rate of increase of density downward increases with the relative density of the gas. Each gas in the mixture has the same tendency to assume independent equilibrium as if it were alone, but its tendency is powerfully resisted by friction with the other gases. Hence, the process of attaining independent equilibrium, consisting of a downward movement of the heavier gases and an upward movement of the lighter, is exceedingly slow, and the result is a mixture richer in the heavier gases below and richer in the lighter gases above.

Consider now a non-rotating globe composed of any number of gases, the mixture in equilibrium, the constituent gases at any stage of their slow progress toward independent equilibrium. The temperature, and the acceleration of gravity, at each point, are necessarily the same for all the gases. Let

R be the radius of the globe;

r, g, the radius of any concentric spherical stratum of infinitesimal thickness, and the acceleration of gravity at that stratum;

p, ρ, τ, the pressure per unit surface, density, and absolute temperature of the fluid at that stratum;

m, h, the mass, and quantity of heat, from center to that stratum;

H, the quantity of heat in the globe;

W, the energy of the fall of all the molecules of the globe from an infinite distance to their present positions, no molecule passing any other;

δ, the relative density of any perfect gas or mixture of perfect gases in comparison with hydrogen;

$C_V \div \delta$, the specific heat of any perfect gas or mixture of perfect gases at constant volume, C_V being the value for hydrogen;

$C_p \div \delta$, the specific heat of any perfect gas or mixture of perfect gases at constant pressure, C_p being the value for hydrogen;

$C_H \div \delta$, the difference of the specific heats, $(C_p - C_V) \div \delta$, C_H being the value for hydrogen;

γ, the ratio of the specific heats, $C_p \div C_V$, approximately, 1.4.*

We have.
$$\frac{p}{\rho\tau} = C_H \div \delta\,\dagger \tag{5}$$

$$dp = -\rho\,dr\,g \tag{6}$$

$$dm = 4\pi\,r^2 dr\,\rho \tag{7}$$

With ρ', p', ρ'', p'', etc., to denote the densities and pressures of the several gases at any stratum, and δ', δ'', etc., to denote their respective relative densities, while s denotes the distance from the center of the globe to any point in space, we obtain,

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* PEABODY, 1.405; PRESTON, 1.408; WOOD, 1.406.

† PRESTON, *Theory of Heat*, 1894, p. 250; and other treatises on heat.

$$H = \int_0^R 4\pi r^2 dr\,\tau C_v \left\{ \frac{\rho'}{\delta'} + \frac{\rho''}{\delta''} + \text{etc.} \right\} = \text{, by (5)},$$

$$\int_0^R 4\pi r^2 dr \frac{C_v}{C_H}(p'+p''+\text{etc.}) = \frac{1}{\gamma-1}\int_0^R 4\pi r^2 dr\, p = \text{,}$$

by formula for integration by parts,

$$0 - \frac{1}{\gamma-1}\int_0^R \tfrac{4}{3}\pi r^3 dp = \text{, by (6)},$$

$$\frac{1}{\gamma-1}\int_0^R \tfrac{4}{3}\pi r^3 dr\,\rho g = \text{, by (7)}, \frac{1}{3(\gamma-1)}\int_0^R rg\,dm =$$

$$(8) \qquad \frac{1}{3(\gamma-1)}\int_0^R \left\{ dm\, g\, r^2 \int_r^\infty \frac{ds}{s^2} \right\} = \frac{W}{3(\gamma-1)}\text{.}^*$$

If the constituent gases are in independent equilibrium, then equations (5), (6), (7), (8), apply, also, to each constituent gas, p, ρ, δ, dm, H, W, pertaining then to that gas only, while r, R, τ, and g are identical for all the gases. Accordingly, the condition existing in the globe when each component gas is in independent equilibrium, is

$$(9) \qquad H = \frac{1}{3(\gamma-1)}\int_0^R rg\,dm = \frac{W}{3(\gamma-1)}\text{,}$$

$$H' = \frac{1}{3(\gamma-1)}\int_0^R rg\,dm' = \frac{W'}{3(\gamma-1)}\text{, etc.}$$

When the constituent gases are not in independent equilibrium, equations (5), (7), are true, and equation (6) is not true, for each gas; the first five members of equation (8), as expressions for the quantity of heat possessed by each gas, are true, as are the remaining four members, as expressions for the quantity of energy represented by the contraction of each gas, in company with the other gases, from infinite dimensions, divided by $3(\gamma-1)$; but equivalence between the first five members and the remaining four members does not exist except for the mixture.

To simplify the problem, suppose the globe to be composed of two gases whose relative densities are δ', δ'', δ' pertaining to the lighter gas. At any spherical stratum let y be the ratio of the number of molecules of the heavier gas to the number of molecules of the lighter gas. y is a variable with respect to r except in the case of uniform mixture. The total heat in each spherical stratum,

$$(10) \qquad dh = \frac{4\pi}{\gamma-1} r^2 dr\, p$$

is divided between the two gases so that each molecule of either gas possesses an equal share; therefore,

$$(11)\ H' = \int_0^R \frac{1}{y+1}dh, \quad H'' = \int_0^R \frac{y}{y+1}dh, \ H'+H'' = H$$

* This result was reached by A. RITTER in the case of a celestial globe composed of a single gas; see *Wiedemann's Annalen*, Vol. 8, 1879, p. 162; also, *Astrophysical Journal*, Vol. 8, 1898, p. 300, eq. (611).

The energies of the contraction of the two gases in company from infinite dimensions are severally

$$W' = \int_0^R rg\,dm', \quad W'' = \int_0^R rg\,dm'', \quad W'+W'' = W \quad (12)$$

$$\text{where} \qquad dm' = \frac{\delta'}{y\delta''+\delta'}dm, \quad dm'' = \frac{y\delta''}{y\delta''+\delta'}dm \qquad (13)$$

The superheating of the lighter gas in consequence of the fall from infinity causes it to rise through the heavier gas, resisted by friction; the deficit of heat in the heavier gas causes the latter to sink through the lighter, resisted by friction; the mixture is in equilibrium; hence, the effect is to drive out the lighter gas only. The energy required to lift every molecule of the lighter gas from its present position to the surface of the heavier, and thence on to infinity, while the molecules of the heavier gas remain fixed, is greater than W', and is composed of the following three parts: the energy required to lift every molecule to infinity against the attraction of the lighter gas, that required to lift every molecule to the surface of the heavier gas against the attraction of the heavier gas, and that required to lift every molecule thence on to infinity against the attraction of the heavier gas; that is, to lift to infinity a spherical layer dm', whose radius is r, requires the energy

$$rg'\,dm' + dm'\int_r^R g''\,dr + R\,g''_R\,dm' \qquad (14)$$

where g' is the part of the acceleration of gravity, g, at any point in the globe due to the lighter gas, and g'' is the part due to the heavier gas, and g''_R is the part of the acceleration of gravity at the surface of the globe due to the heavier gas. We have

$$g = E\frac{m}{r^2}, \quad g' = E\frac{m'}{r^2}, \quad g'' = E\frac{m''}{r^2}, \quad g''_R = E\frac{M''}{R^2} \quad (15)$$

where E is a constant. By means of (15) eliminate g', g'', g''_R, from (14):

$$E\,dm'\left\{\frac{m'}{r} + \int_r^R m''\frac{dr}{r^2} + \frac{M''}{R}\right\} \qquad (16)$$

But (17)

$$\int_r^R m''\frac{dr}{r^2} = \left[-\frac{m''}{r}\right]_r^R + \int_r^R \frac{dm''}{r} = -\frac{M''}{R} + \frac{m''}{r} + \int_r^R \frac{dm''}{r}$$

therefore, the energy required to lift all the molecules of the lighter gas out of the heavier and on to infinity is

$$U' = \int_0^R E\,dm'\left\{\frac{m}{r} + \int_r^R \frac{dm''}{r}\right\} \qquad (18)$$

$$= \int_0^R rg\,dm' + E\int_0^R\left\{dm'\int_r^R \frac{dm''}{r}\right\}$$

When the mixture is uniform, y is constant, and the heat possessed by the lighter gas is shown in equation (11) reduced to expression (2). The quantities of heat required by the lighter gas and by the whole globe, in order to insure equilibrium in each for a uniform mixture, are, by (9) and (13), as their respective masses: hence the lighter gas requires the fraction (1) of the total heat, and the excess of heat possessed by the lighter gas is the fraction shown in (4). We therefore arrive at the conclusions before stated, that, the smaller the proportion of the lighter gas, the more violently is it expelled, and that the greatest mechanical effect in expelling the lighter gas occurs when the masses of the gases are proportional to the square roots of their relative densities.

To compare the heat possessed by the lighter gas, (11), with the heat required to lift it out of the heavier gas and on to infinity, (18), we must know the law of the distribution of mass in the globe. For illustration, let it be

(19) $$\rho = \rho_0 \left\{ 1 - \frac{r}{R} \right\}^2 = \rho_0 (1-x)^2$$

where $r = Rx$, $dr = Rdx$. Then, from (7), (15), (6),

(20) $$m = 4\pi\rho_0 \int_0^r (1-x)^2 r^2 dr = 4\pi R^3 \rho_0 \int_0^x (1-x)^2 x^2 dx = 4\pi R^3 \rho_0 \left\{ \frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right\}$$

(21) $$g = E\, 4\pi R\rho_0 \left\{ \frac{x}{3} - \frac{x^2}{2} + \frac{x^3}{5} \right\}$$

$$p = E\, 4\pi R^2 \rho_0^2 \left\{ \frac{13}{900} - \frac{x^2}{6} + \frac{7x^3}{18} - \frac{23x^4}{60} + \frac{9x^5}{50} - \frac{x^6}{30} \right\}$$

in which the constant of integration of (6) is determined by the fact that $p = 0$, when $r = R$, or $x = 1$. For uniform mixture, the integrations indicated in (11), (18), can now be performed, and the results compared in terms of δ', δ'', y, as in expression (3). To obtain a numerical ratio, let the lighter gas be hydrogen, and let the relative density of the heavier gas be 40, which is probably much less than the relative density of a uniform mixture of the sun's gases without the hydrogen; and let the masses be as 4 is to 86; then,

$$\delta' = 1, \ \delta'' = 40, \text{ and } M' : M'' :: 4 : 86 :: \delta' : y\delta'', \text{ or } y = \frac{43}{80}$$

In this case the hydrogen has nearly two-thirds of the number of molecules in the globe, or occupies nearly two-thirds of its volume. We then find H', H'', W', W'', U', are respectively, 0.00860, 0.00463, 0.00070, 0.01517, 0.00137, multiplied by the common factor $E\pi^2 R^5 \rho_0^2$. The ratio of the heat possessed by the hydrogen to the heat required to hold it in equilibrium, is

$$H' \div \frac{W'}{3(\gamma-1)} = 14.74 \qquad (22)$$

The tendency of the heavier gas to sink is shown in the formula

$$H'' \div \frac{W''}{3(\gamma-1)} = 0.37 \qquad (23)$$

The equilibrium of the mixture appears as follows:

$$\left\{ H' + H'' \right\} \div \frac{W' + W''}{3(\gamma-1)} = 1 \qquad (24)$$

The ratio of the heat stored in the hydrogen to the energy necessary to lift it out of the heavier gas and on to infinity, is

$$H' \div U' = 6.28 \qquad (25)$$

With explosive violence the hydrogen would expand to infinite dimensions, were it not depressed by the friction incident to a tremendous depth of densely packed opposing molecules. Slowly, but powerfully, it is driven upward through and out of the heavier gas to form a continually rising atmosphere. As each spherical layer of this atmosphere is pushed outward from the attracting globe, it is constantly relieved of pressure, and, therefore, constantly expands; hence, the velocity of a particle of the rising atmosphere continually increases. As the particle traverses a certain constant distance, say a tenth of the radius of the globe, the ratio of the relief of pressure to the existing pressure, is exceedingly great near the globe and exceedingly small in the outer region removed a few radii from the globe; consequently the space-derivative of velocity diminishes rapidly outwardly, so that between the distance of a few radii from the globe and infinity the velocity is nearly uniform.

That the gases of the sun are in uniform mixture is perhaps the prevailing opinion among astronomers.* The theory advocated by Faye, Lane, and Ritter, to account for the upbringing of heat from the central parts to the surface by means of vertical currents, depends upon the principle of convective equilibrium. An essential condition for convective equilibrium is uniformity of mixture of the constituent gases, and the vertical currents incident to the theory act to maintain the uniformity.

If, however, the component gases were in the state of independent equilibrium indicated in equations (9), there would be no tendency to expel any of the gases. A discussion of this case, which would be too long for the present

* Young's *Sun*, 1898, p. 332.

paper, shows that the aggregations of the heavier gases near the center and of the lighter gases near the surface, incident to independent equilibrium, in a globe of the sun's mass and dimensions, would be so great that vertical currents would be powerfully resisted by gravity; the surface layer would contain only the lightest gas; except in so far as it might be modified by conduction and radiation of heat from the interior, the temperature of the surface of the globe would be absolute zero, and the temperature through a considerable depth from the surface would be near to absolute zero: the globe would be stagnant so far as radial currents are concerned.

To illustrate the effect of a compacting of the heavier gases toward the center, that is, of an approach to independent equilibrium of the gases, in lessening the excess of heat in the lighter gases, let us assume the problem already considered with the modification that the two gases are so distributed in the globe that the relative density, δ, of the mixture is no longer a constant, but varies from center to surface according to the equation

$$(26) \qquad \delta = 30 - 27x\,^*$$

Now

$$(27) \qquad \delta = \frac{y\delta'' + \delta'}{y+1} \quad . \quad \therefore y = \frac{\delta - \delta'}{\delta'' - \delta} = \frac{29 - 27x}{10 + 27x}$$

At the center there are nearly three times as many molecules of the heavier gas as of the lighter, and the relative density of the mixture is 30; at the surface there are more than eighteen molecules of hydrogen to one of the heavier gas, and the relative density is 3. Substitute for y in (11), (12), (13), (18), its value in terms of x, and perform the indicated operations to find that H', H'', W', W'', U', are equal each in its order to the common factor $E\pi^2 R^5 \rho_0^{\,2}$ multiplied by the coefficients, 0.00724, 0.00599, 0.00081, 0.01506, 0.00124. Consequently, the ratio (22) becomes 10.73; the ratio (23) takes the value 0.48; the ratio (24) remains equal to unity; and the ratio (25) is reduced to 5.84.

If the gases are maintained in uniform mixture by vertical currents, the tendency is to expel the hydrogen entirely, and the ratios (22), (25) increase with the diminution of the lighter gas. If the mixture is not stirred up, the gases seek a state of independent equilibrium in which a small portion of the hydrogen remains. In the passage to this state the hydrogen is for a long time under the treméndous stress indicated in ratios (22), (25), which according to the preceding paragraph decreases slowly while the gases are adjusting their equilibrium; hence the greater part of the hydrogen is driven out of the globe. The gases in the tall column at the earth's surface, already referred to, reach independent equilibrium without loss of a portion of the lighter gases only when the top of the column is sealed. In a tube of indefinite length open at the top, the uniform mixture has an upper free surface at a certain section of the tube; the process of attaining independent equilibrium of the gases involves the expulsion of a considerable portion of the lighter gases upward across this section.

In a globe composed of a great number of gases, each gas is urged upward by all the heavier gases and downward by all the lighter: therefore a series of the lighter gases is driven into the atmosphere. Whether the mixture is uniform, or the gases are in a state approaching independent equilibrium, every gas is present in some proportion at the surface of the globe.

The proposition that the sun generates an outrushing atmosphere is limited by the condition that such atmosphere shall be so thin as not to appreciably interfere with the motions of planets and comets. We are already aware of the existence of a body of matter surrounding the sun in the corona and zodiacal light that satisfies this condition; there is nothing in the theory to indicate that the atmosphere is denser than this body of matter. That the atmosphere is much rarer than cometic vapors is shown in the forms of comets' trains. When a rare medium in motion encounters a much denser and smaller body at rest, the former cannot immediately impart to the latter its own velocity. After a short interval the velocity of the denser body is so small that the accelerative power of the rare medium, which depends upon the difference of velocities, is but little less than it was at first. Consequently, when the difference of densities is very great, the motion of the denser body for a considerable distance is almost as if its velocity were uniformly accelerated. "The form of the comet's tail, on the supposition that it is composed of matter driven away from the sun with a uniformly accelerated velocity, has been several times investigated, and found to represent the observed form of the tail so nearly as to leave little doubt of its correctness." *

* The condition (26), with $\delta' = 1$, $\delta'' = 40$, combined with (27), (13), (7), (19), leads to the proportion, $M' : M'' :: 4 : 86$. This problem was first worked out from the condition (26), and the resulting ratio of masses, 4 : 86, was then used in the previous problem of uniform mixture, in order that the two problems might differ only in the distribution of the gases.

* NEWCOMB and HOLDEN'S *Astronomy*, 3d ed., 1887, p. 277.

523 W. 173 Street, New York.

OBSERVATIONS OF MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1900–1901 Greenw. M.T.	*	No. Comp.	Planet — * Δα	Planet — * Δδ	Planet's Apparent α	Planet's Apparent δ	log pΔ for α	log pΔ for δ	Obs.
				(270) *Anahita.*					
			m s	′ ″	h m s	° ′ ″			
Oct. 1 15 11 18	1	8 , 11	+0 20.18	+3 12.2	0 12 10.23	+ 9 19 22.7	n9.291	0.682	W
3 11 20 53	2	10 , 6	−1 41.01	−5 23.7	0 40 21.44	+ 9 5 16.9	n9.424	0.693	W
11 15 50 18	3	7 , 8	−0 10.50	+4 36.1	0 33 8.06	+ 8 8 21.6	n8.514	0.687	W
15 13 43 4	4	6 , 7	−0 29.01	+4 20.8	0 29 55.65	+ 7 40 37.3	n9.367	0.703	W
20 14 4 16	5	5 , 7	+0 20.93	−4 41.9	0 26 16.77	+ 7 6 33.1	n9.201	0.703	W
				(322) *Phæo.*					
Oct. 16 16 9 1	6	4	+1 48.54	+0 16.7	0 3 53.35	+11 16 19.8	n8.997	0.611	W
17 13 53 24	7	7 , 4	+1 4.76	+1 28.9	0 3 27.03	+11 8 49.3	n9.198	0.619	W
19 13 17 28	8	6 , 6	−0 7.28	+8 54.0	0 2 34.88	+13 51 23.6	n9.316	0.631	W
20 12 38 32	8	8 , 8	−0 30.95	+0 17.2	0 2 11.21	+13 42 16.8	n9.424	0.645	W
22 14 57 2	9	8 , 8	−0 18.50	−6 23.8	0 1 25.10	+13 24 18.4	7.935	0.620	W
				(17) *Thetis.*					
Oct. 17 15 28 57	10	10 , 6	−0 49.17	+7 29.3	1 51 11.45	+ 1 54 35.2	n9.242	0.752	W
Nov. 2 16 35 48	11	8 , 8	+0 5.84	+4 17.8	1 36 52.35	+ 0 40 1.3	8.991	0.762	W
				(16) *Psyche.*					
Nov. 23 15 33 3	12	10 , 8	+0 16.10	+1 10.2	4 18 35.27	+16 25 5.7	n9.248	0.590	F
28 15 27 38	13	8 , 12	−0 14.63	−2 41.8	4 14 2.03	+16 11 22.7	n9.150	0.587	F
30 15 8 22	14	8 , 9	+0 11.54	−3 11.2	4 12 14.55	+16 10 23.7	n9.203	0.590	F
				(78) *Diana.*					
Feb. 7 14 16 47	16	10 , 6	+2 5.60	−1 35.0	10 14 35.56	+13 34 25.0	n9.530	0.668	W
8 15 56 10	16	10 , 8	+1 3.02	−2 8.1	10 13 32.99	+13 33 51.9	n9.352	0.638	W
22 16 54 22	17	4 , 6	+0 32.95	+3 42.7	9 58 54.57	+13 24 56.3	8.272	0.619	W

Mean Places for 1900.0 and 1901.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	0 11 45.16	+4.59	+ 9 15 42.5	+28.0	Bruns & Peter, Leipzig II, A.G.C. 258
2	0 42 0.85	+4.60	+ 9 10 42.4	+28.2	" " " II, " 259
3	0 33 13.95	+4.61	+ 8 3 16.6	+28.9	" " " II, " 198
4	0 30 20.06	+4.60	+ 7 35 47.2	+29.3	" " " II, " 181
5	0 25 50.56	+4.59	+ 7 10 49.0	+29.0	" " " II, " 151
6	0 2 0.23	+4.58	+11 16 2.1	+31.0	" " " I, " 6
7	0 2 17.67	+4.58	+11 6 19.1	+31.0	" " " I, " 7
8	0 2 37.59	+4.57	+13 11 58.6	+31.0	" " " I, " 9
9	0 1 39.05	+4.55	+13 30 11.1	+31.1	" " " I, " 4
10	1 51 55.99	+4.63	+ 1 16 40.9	+25.0	Boss, Albany, A.G. Catal. 551
11	1 36 41.83	+4.68	+ 0 35 18.4	+25.1	Kortazzi, Nicolajew, A.G. Catal. 335
12	4 17 43.70	+5.47	+16 23 45.9	+ 9.6	Auwers, Berlin A.G. Catal. 1156
13	4 14 11.12	+5.54	+16 16 54.1	+10.1	" " " " 1138
14	4 11 57.47	+5.54	+16 13 24.6	+10.3	Micrometer comparison with (15)
15	4 11 45.01	. . .	+16 20 36.2	. . .	Auwers, Berlin A.G. Catal. 1124
16	10 12 27.38	+2.58	+13 36 17.0	−17.0	Bruns & Peter, Leipzig I, 3983
17	9 58 19.10	+2.52	+13 21 31.5	−17.9	" " " I, 3934

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TENTATIVE RESEARCHES UPON PRECESSION AND SOLAR MOTION.

By LEWIS BOSS.

The attempt to find the point in the sky toward which the sun's motion is directed is difficult, not only because of the hypothetical nature of the assumptions which must be made, but also because of the fact that the parallactic motion of the stars is entangled in a most perplexing way with other effects and motions. Some of these arise from the imperfection of observations, others from the unknown residuum of precessional motion, and still others from the motion which is peculiar to each star.

Eventually we may be able to apply something like a satisfactory test to determine whether the direction-element of the motion peculiar to each star is principally a matter of chance. That there is community of motion in groups of stars we know already. There is a notable group of this kind in *Ursa Major*, and another of numerous membership in *Taurus*. In various parts of the sky there are a great many such instances where two or more stars, separated by several minutes of arc, appear to have a common motion. Admitting the possibility that these star-streams are more prevalent than is commonly supposed, it may still be found that the hypothesis of chance motion of the stars is substantially correct, when the star-stream instead of the single star is taken as the unit.

It is no part of my present purpose to set forth the cumulative evidence in favor of the proposition as to the random character of the *motus peculiaris*. There appears to be no occasion for very serious doubt as to its efficiency to serve as a good working hypothesis until after we shall have cleared the determination of solar and precessional motion from other elements of uncertainty.

Under any hypothesis as to the character of stellar motions we may consider our origin of coordinates, to which the motions are referred, to be (so to speak) the moving resultant of all the motions considered. This is analogous to what we do in all physical problems dealing with motion. To be really useful, however, our hypothesis must mean that, if we divide the visible universe into volumes, each containing a very large number of stars, the resultant of all the motions for each unit of volume must be the same in velocity and direction as that of the entire volume made up of these units. This is the hypothesis which has been adopted, and various methods of computation have been devised for its practical application. These, precisely like the theory of probable error, are all necessarily founded on the antecedent requirement that the number of motions considered must be very large. The formulas are only rigorous when the actual apparent motions of the stars are parallactic only, due simply to the motion of the sun among the stars. Were it possible, then, we ought to prepare for the determination by computing values of the resultant apparent motions of the stars that might be considered, in each instance, fair approximations to the parallactic motions. To expect success otherwise would be much like attempting to determine probable error of the mean when the individual determinations are affected with large mistakes. The means for accomplishing this preliminary approximation have been pointed out and illustrated by more than one computer.

If in any area of sky we consider a large number of stars, all at distances from the sun not greatly differing from each other, it is obvious that the mean of all the motions exhibited by the separate stars will tend to be the measure of the parallactic effect for the mean distance of the stars in that part of space; provided that the resultant of the peculiar motions tends toward zero, according to the law of probabilities, and under the consequent assumption that the peculiar motion has no preference for any particular direction. The crucial questions are, then, what limits of distance can be included in a single combination, and how shall we arrive at a practical criterion for establishing such limits? For the larger proper motions there is no great difficulty, since the proper motion itself is a fairly good measure of relative mean distance. For seven-tenths of all the stars, it may be assumed that the thwart component is at least seven-tenths of the entire *motus peculiaris*. For only one-tenth of all the stars should the motion at right-angles to the line of sight be less than $0.44\mu_p$. And although the compounding of the peculiar motions with those of a parallactic nature complicates the problem as to an exact criterion of distance, the conditions

for its applicability become scarcely less favorable. Thus, when we have established two limits of proper motion in a special determination of the position of the solar apex, I think we may feel confident that the greater part of the stars considered are actually within the limits of distance indicated by the reciprocals of the limiting means of proper motion, and that the greater part of the remainder are not far outside those limits. But when we pass below the limit of about 10″ for the centennial motions, owing to the part played by error of observation, the application of this criterion becomes increasingly difficult, until for motions of 2″ or 3″, it practically fails. Yet it is precisely upon stars of this class that we must eventually rely for our most trustworthy determinations of the precessional and solar motions, because it is only in this class that we find a number of stars sufficiently great to afford reasonable ground of hope that the *motus peculiaris* can be virtually eliminated.

Fortunately, the brightness of stars, considered in large aggregations, also affords a rude criterion of distance, undoubtedly inferior to that of proper motion but still not without value. This conclusion is consistent not only with inherent probabilities, but also with direct evidence. Considering the stars in a general way the mean proper motion diminishes as the brightness decreases. Another form of proof may be drawn from the later conclusions of this paper.

In dealing with the comparison of STONE's Cape Catalogue with the Cape Catalogue for 1850, we may assume that the mean magnitude of the stars compared is not far from $6^{M}.5$; and, in spite of rather numerous exceptions, that the lower limit of magnitude is $7^{M}.0$. The principle of selection adopted tends to exclude all centennial proper motions greater than 25″; and, although, owing to errors of observation, many stars of somewhat larger proper motion have been included, it is probable that an even larger number of proper motions between 10″ and 25″ have been excluded; so that we may assume as a good approximation that 25″ is the upper limit of the centennial motions employed. Now if a determination of solar motion be founded upon stars of this average motion we shall find the solar parallactic motion, M, to be about $25'' \times 0.64 = 16''$. This may be termed the secular parallax of those stars, the average proper motion of which is 25″, and it is substantially what has been nominally found by actual computation. In the same way (as will appear later) I find from all of the stars contained in the comparison cited: $M = 4.07$. We have then for the lower limit of distance ($M = 10''$ being the unit), 0.62, and for the mean distance of all stars considered, 2.46. Assuming for the stars equal distribution in volume, we then have for the upper limit of distance: $\{2(2.46^3 - 0.62^3)\}^{\frac{1}{3}} = 3.0$. We are thus led to the remarkable conclusion that only about one-third of these stars lie outside of the limits of distance, 2.0 and 3.0; and that less than 1 per centum are outside the limits, 1.0 and 3.0. In all strictness this is probably not the case, but it is evident that something approximating these conditions must prevail.

In a more refined discussion, employing instead of two catalogues all that are available, together with accurate observations at the present date, it would be easy to fix the lower limit of distance with all sufficient definiteness at stars of centennial motion, 15″, distance, 1.0. If, then, we include only stars of the seventh magnitude, or brighter, we may be reasonably confident that for the great majority of the stars the relative distances will be included between 1.0 and 3.0; and, under such circumstances the theoretical objections against the very convenient method proposed and illustrated by AIRY would practically disappear.

The inference here to be drawn that the average centennial proper motion of the most distant seventh-magnitude stars, considered in masses, is about 4″, is no more surprising than the confirmatory inference which can be drawn from my paper upon the solar motion in relation to faint stars of small proper motion (*A.J.* 195–6). In that paper I find for stars from the seventh to ninth magnitudes, mean magnitude about $8^{M}.5$, $M = 2''.42$, corresponding to a proper motion of about 4″. These numbers are substantially confirmed by RISTENPART (*Veröffentlichen, Karlsruhe*, IV), from whose researches, corrected to conform to the systems, N_1 and B_1, I find, $M = 3''.3$, and the consequent centennial proper motion about 5″ for stars of mean magnitude about $8^{M}.5$. These facts appear to indicate that nearly all the stars which we ordinarily observe with meridian-instruments are comparatively near. The conviction impressed upon me through prolonged reflection upon the facts may be more definitely presented in the following hypotheses. 1°. The sun appears to be situated within a star-cluster of tolerably definite outlines. 2°. The dimensions of this may be stated by estimating the annual parallax of stars brighter than the seventh magnitude to be in very few instances less than 0″.003, and of the ninth magnitude, less than 0″.002.

At the risk of repetition I remark, again, that my studies upon the problem of determination of precessional and solar motion have led me to the conclusion that the best point of attack is offered, by the stars brighter than $7^{M}.0$, after excluding stars having centennial motion greater than 10″, and perhaps also stars of magnitudes above the fourth. With the problem so limited it seems to me that the theoretical difficulties would largely disappear, and we should have some hope of applying a test of value as to the validity of our fundamental hypothesis.

The choice of material suggested in the preceding paragraph aggravates a practical difficulty in the computation. Let us suppose that the individual determinations of proper motion have been arranged in groups of fifty or more in each. The means of these motions will represent observed parallactic motion, approximately. But combined with this will be the residual error of observation. In a general

discussion embracing the available number of star-catalogues we may safely rely upon the practical destruction, in the general means, of errors of observation that are purely casual. The case is different with the systematic errors, and, as I have already shown, these may be large enough to belong to the order of the quantities relied upon to furnish the observed parallactic motions. It is precisely this point which has been most ignored in the past, though it is a point of capital importance.

The following computation is not offered as a good practical illustration of the foregoing remarks in all particulars, but because it does seem to illustrate the relative importance of certain elements in the computation with unexpected force. Some years ago when the Cape Catalogue for 1850 was published, my attention was attracted to the tables on pages x and xiv of the Introduction. It was apparent that a large part of the quantities there exhibited was due to the parallactic effect, but since there then existed no standard catalogue of Southern Stars adequate to afford the means for determining the systematic corrections due to Cape '50 and Cape '80, respectively, there was no means of knowing to what extent the quantities in GILL's tables of comparison of these two catalogues were due to systematic discordance, and to what extent they were due to parallactic effects. The publication of my paper on 179 Southern Standard Stars (*A.J.* 448–450) offered an opportunity for the attempt to separate these two effects. Accordingly I corrected the means in GILL's tables for the difference of systematic corrections due to Cape '80 minus Cape '50, these corrections being taken from the tables in *A.J.* 449. Solving then by AIRY's method I obtained:

$$A = 242^\circ.9;\quad D = +43.1;\quad M = 3''.68.$$

The indicated correction for STRUVE's precession was $-0''.0214$. In Dr. GILL's tables the differences, C. '80 − C. '50, are combined in groups of four hours in R.A., and this greatly impairs the use of the means for a discussion like this. Again, these tables, designed mainly to exhibit systematic differences, included the stars with known proper motion, the proper motion having been applied to the differences for those stars.

It occurred to me, therefore, that it would be interesting to make the differences, C. '80 − C. '50, printed in the catalogue, the basis of a new discussion, revising those in which the proper motion had been applied, so that those stars should be treated exactly as GILL has treated the other stars. The individual differences were then collected into means covering generally two hours in right-ascension and 18° in declination. South of −54° the differences were first treated in sub-zones, and then combined. The improved material appeared to demand closer attention to the systematic corrections applicable to Cape '50. Meanwhile, through Albany observations and others, I had become aware that the system, B_2, of the 179 Southern Standard Stars required revision in right-ascension. The results of these several operations described in *A.J.* 499 were applied in the new investigation.

The essential results of the condensation of GILL's comparison for individual stars into mean centennial motions for the respective areas is exhibited in a succeeding statement. In order to convert the corrected difference of catalogues into centennial motions it became necessary to ascertain the difference of epochs for the several zones. The mean epoch of Cape '50 in R.A. is 1852.4, and in declination it is 1851.0. Then we have for difference of epochs:

Zone		Δ Epoch for α	Δ Epoch for δ
0° to -18°	0^h to 12^h	27.0	28.0
. . .	12 to 20	26.4	27.5
. . .	20 to 24	25.5	26.6
−18 to −36	. . .	26.3	27.4
−36 to −54	. . .	25.0	26.0
−54 to −72	. . .	24.2	25.3
−72 to −85	. . .	21.0	22.0

In the following statement the first two columns contain the approximate coordinates of the respective means. For stars in the group −72° to −85°, these are the means of the quantities actually used in α and δ equations. The third column contains the corrected $\Delta\alpha\cos\delta$ resulting from the combination of all the individual values of Cape '80 minus Cape '50 for the trapezoidal area in question, and the next column contains the corresponding value of $\Delta\delta$. The number of stars in each area appears in the fifth column, and this is always somewhat greater than the actual number of separate values of either $\Delta\alpha\cos\delta$, or of $\Delta\delta$, owing to the many incomplete observations in Cape '50. The sixth and eighth columns contain the values of the adopted mean centennial motions of the stars in the several areas, derived from the numbers in the third and fourth columns by means of the table of difference of epochs. The weights adopted in the final solution are contained in the seventh and ninth columns. They were computed in the following manner. The differences from the mean of each separate value of $\Delta\alpha\cos\delta$ and $\Delta\delta$ in a given area were treated like errors of observation. Weight unity was assigned to each difference, Cape '80 − Cape '50, except when the α, or δ, of Cape '50 depends on only one observation, when the weight, 0.6, was employed. In this manner the probable error of a single $\Delta\alpha\cos\delta$ is found to be $\pm 0^s.070 = \pm 1''.05$; of $\Delta\delta$, $\pm 0''.71$. The probable error of a centennial motion from each comparison of the catalogues would then be, for the mean difference of epochs, about $\pm 4''.0$ in R.A., and $\pm 2''.6$ in declination. It was then assumed that, from the effects of uncorrected systematic error, star-drift, etc., each mean of the table is also further subject to a constant probable error of about one-fifth the probable error of a single determination — about $0''.8$ for $\Delta\alpha\cos\delta$ reduced to centennial motion, and $0''.5$ for $\Delta\delta$. The unit of weight in $\Delta\alpha\cos\delta$ corresponds to about nine determinations, and its predicted probable error is approxi-

mately ±1″.7. The unit of weight in $I\delta$ corresponds to about five separate determinations, and its predicted probable error is, approximately, ±1″.4.

DATA FOR CONSTRUCTION OF THE CONDITIONAL EQUATIONS.

R.A.	South Decl.	Corr'd. $\Delta\alpha\cos\delta$	Corr'd. $\Delta\delta$	**	Centennial Motion $\Delta\alpha\cos\delta$	Wt.	$\Delta\delta$	Wt.
°		s	″		″		″	
2	8	+0.044	−0.56	43	+2.4	3	−2.1	4
29	7	+0.053	−0.15	37	+3.0	2	−0.5	4
62	7	+0.052	+0.06	39	+2.9	2	+0.2	4
86	8	+0.003	−0.44	68	+0.2	3	−1.6	5
120	9	−0.041	−1.04	35	−2.3	2	−3.6	4
148	7	−0.068	−0.98	56	−3.8	2	−3.5	5
175	8	−0.015	−0.68	18	−0.8	2	−2.4	2
209	11	−0.105	−0.89	13	−6.0	1	−3.2	2
240	10	−0.060	−0.41	10	−3.4	1	−1.5	1
269	10	−0.095	−1.27	10	−5.4	1	−4.7	1
301	10	−0.010	−0.63	14	−0.6	1	−2.3	2
332	10	+0.063	−0.46	24	+3.7	2	−1.7	4
359	28	−0.039	−0.68	44	−2.2	2	−2.5	5
30	30	+0.030	−0.76	47	+1.7	2	−2.8	5
61	29	−0.014	−0.55	58	−0.8	3	−2.0	5
91	30	−0.058	−0.38	94	−3.3	3	−1.4	6
123	29	−0.075	−0.51	100	−4.3	3	−1.9	6
150	29	−0.067	−0.22	49	−3.8	2	−0.8	5
179	30	−0.074	−0.94	47	−4.2	2	−3.4	5
214	27	−0.077	−1.17	53	−4.4	3	−4.3	5
242	28	−0.048	−0.83	199	−2.7	3	−3.0	6
270	29	−0.030	−0.98	223	−1.7	3	−3.6	6
300	27	−0.014	−1.22	125	−0.8	3	−4.5	6
328	28	+0.010	−1.03	61	+0.6	3	−3.8	5
0	45	+0.070	−0.61	70	+4.2	3	−2.4	6
31	44	+0.034	−0.25	81	+2.0	3	−1.0	3
60	43	+0.031	+0.02	77	+1.9	3	+0.1	6
92	44	−0.022	+0.12	120	−1.3	3	+0.5	6
120	45	−0.004	−0.26	192	−0.2	3	−1.0	6
147	46	−0.012	−0.19	87	−0.7	3	−0.7	6
180	46	−0.109	−0.57	74	−6.5	3	−2.2	6
210	46	−0.114	−1.06	70	−6.8	3	−4.1	6
240	45	−0.072	−0.71	104	−4.3	3	−2.7	6
271	44	−0.011	−1.19	89	−0.7	3	−4.6	6
302	45	+0.038	−1.20	49	+2.3	3	−4.6	5
332	46	+0.072	−0.98	64	+4.3	3	−3.8	6
359	62	+0.041	−0.46	69	+2.5	3	−1.8	6
31	64	+0.031	+0.17	70	+1.9	3	+0.7	5
59	62	−0.021	+0.21	42	−1.3	2	+0.8	5
90	62	−0.008	−0.27	59	−0.5	3	−1.1	5
121	61	0.000	−0.24	73	0.0	3	−1.0	6
150	61	−0.006	−0.22	130	−0.4	3	−0.9	6
180	62	−0.064	−0.34	85	−4.0	3	−1.3	6
209	61	−0.035	−0.61	48	−2.2	2	−2.4	5
241	61	−0.020	−0.85	102	−1.2	3	−3.4	6
268	63	−0.019	−0.73	62	−1.2	3	−2.9	6
301	63	+0.018	−0.95	65	+1.1	3	−3.8	6
329	62	+0.062	−0.80	70	+3.8	3	−3.2	6
358	77	+0.015	−0.05	22	+1.1	2	−0.2	4
40	78	−0.019	+0.22	22	−1.4	2	+1.0	3
80	79	+0.014	−0.10	15	+1.0	1	−0.4	2
122	77	−0.004	+0.80	14	−0.3	1	+3.6	2
154	77	−0.066	+0.14	29	−1.7	1	+0.6	4
187	80	−0.047	−0.16	13	−3.4	1	−0.7	2
225	79	−0.005	−0.53	14	−0.4	1	−2.4	2
292	78	+0.068	−0.52	19	+1.6	1	−2.4	4
326	77	+0.109	−0.28	20	+7.8	0.5	−1.3	4

The conditional equations were constructed from well known formulas adopted from the method first practically illustrated by AIRY.

$$a\,I\psi + bx + cy + dz = I\alpha\cos\delta$$
$$a'\,I\psi + b'x + c'y + d'z = I\delta$$

In which

$I\psi$ = Correction of STRUVE's luni-solar precession.
$x = M\cos D\cos A$.
$y = M\cos D\sin A$.
$z = M\sin D$.
ω = Obliquity of the ecliptic.
$a = \cos\omega\cos\delta + \sin\omega\sin\delta\sin\alpha$.
$b = \sin\alpha$.
$c = -\cos\alpha$.
$d = 0$.
$a' = \sin\omega\cos\alpha$.
$b' = \sin\delta\cos\alpha$.
$c' = \sin\delta\sin\alpha$.
$d' = -\cos\delta$.

The resulting normal equations are these:

For R.A.

$$\begin{array}{rrrcr}
+64.7\,I\psi & -13.6\,x & -\ \ 2.3\,y & & = -\ \ 68.3'' \\
-13.6 & +67.4 & +\ \ 1.1 & & = +\ \ 10.3 \\
-\ \ 2.3 & +\ \ 1.1 & +67.1 & & = -209.7
\end{array}$$

For declination.

$$\begin{array}{rrrrr}
+21.6\,I\psi & -35.1\,x & +\ \ 1.8\,y & -\ \ \ 1.0\,z & = +\ \ \ 1.2'' \\
-35.1 & +67.1 & -\ \ 4.4 & +\ \ \ 0.2 & = +\ \ \ 2.9 \\
+\ \ 1.8 & -\ \ 4.4 & +62.6 & +\ \ \ 0.5 & = -153.3 \\
-\ \ 1.0 & +\ \ 0.2 & +\ \ 0.5 & +139.0 & = +406.1
\end{array}$$

The equations from R.A. were solved in combination with the fourth equation from declination; the declination equations were also solved alone, but since $I\psi$ in them is practically indeterminate as to x, its value from the R.A.-equations is substituted; the two sets of equations were added and this gives the adopted solution, as it appears in the third column of results in the following table.

	R.A.	Decl.	Both	P.E.
	″	″	″	″
$I\psi$	−1.18	. . .	−0.93	±0.20
x	−0.03	−0.82	−0.31	±0.16
y	−3.16	−2.44	−2.82	±0.15
z	+2.92	+2.92	+2.92	±0.15
M	4.30	3.90	4.07	
	°	°	°	°
A	269.3	251.4	263.7	±3.6
D	+42.7	+48.5	+45.9	±2.8

The probable error of the unit of weight as it results from substitution in the conditional equations is ±1″.68, corresponding almost exactly to the predicted probable error of the weights in right-ascension.

An objection which may be fairly urged against the foregoing investigation, aside from its uncritical character in general, is that the centennial motions are based upon only two star-catalogues, neither of them ideal as to accuracy. A still more serious objection is the small mean difference of epoch which is only about a quarter of a century. A third drawback is that the discussion relates to only one hemisphere.

It will be found, however, that accuracy in the sense of mere casual error is a matter of minor importance; provided the number of stars employed is very great: for, in order to extinguish the *motus peculiaris* there already exists the necessity of employing the greatest possible number of stars. Under the circumstances, however, I was not prepared to find so high a degree of apparent accuracy in the solution, because of the short interval between the two catalogues. Any defect in applying systematic corrections appears magnified in a four-fold degree in the mean centennial motions; and these are the errors most to be feared. Moreover, the determination of the systematic errors of these two catalogues offers unusual difficulties as I have explained in a previous article as to Cape '50. For Cape '80 I have adopted a mean value of $\Delta\delta_a$, though it doubtless differs materially for the separate zones of which that catalogue is made up.

In order to get rid of the disadvantages associated with a discussion of this kind, which relates to one hemisphere only, it occurred to me that the present investigation might be combined with that by L. STRUVE (*Best. der Const. der Præc.*, St. Petersburg, 1887). Dr. STRUVE's discussion is based upon a comparison between the Pulkowa Catalogue for 1855 with Auwers–Bradley for 1755. No systematic corrections to the catalogues were applied. Whatever opinion one may entertain as to the value of any particular system of standard stars, one must admit that if such a system represents fairly well the entire course of observations during the present century it is much more likely to be free from systematic error than is any partial system made up by the combination of two catalogues only. The equinox of NEWCOMB, N_1, though published many years ago, corresponds very well with the best individual determinations of the equinox throughout the century; as has been shown during the past few years, and notably in NEWCOMB's researches of which the general results are presented in his "Astronomical Constants," pp. 88 and 96. The correction of N_1 is there given as

$$-0^s.005 \;\; +0^s.023\left(\frac{T-1850}{100}\right)$$

resulting from sun-observations. By combining with this, observations of *Mercury* and *Venus*, NEWCOMB makes the correction of N_1

$$+0^s.032 \;\; -0^s.020\left(\frac{T-1850}{100}\right)$$

It may well be doubted whether meridian-observations of objects like these, varying in apparent form with varying illumination, are well suited to the determination of the position of the equinox, though their testimony may not be wholly without value. If we adopt N_1, then all of STRUVE's centennial motions in R.A. require the correction due to Pulk. '55 — A.-B. 1755, which may be taken as

$$+0.001 \;\; +0^s.079 \;=\; +0^s.080$$

As to the writer's system of declination, B_1, several computers have accumulated much evidence during the past ten years to show that it approximates very closely to the weighted testimony of observation throughout the present century. The adoption of the system of declinations, B_1, would lead to the following corrections of the means of $\Delta\delta$ for STRUVE's zones.

Zone	Mean δ	Corr'n	Zone	Mean δ	Corr'n
	°	"		°	"
A	− 7.5	−1.66	E	+52	−0.28
B	+ 7.5	−1.66	F	+67	−0.49
C	+22.5	−1.67	G	+80	0.00
D	+37.5	−1.48			

The discussion of terms of correction of the form $\Delta\alpha_a$ and $\Delta\alpha_\delta$, as well as of $\Delta\delta_a$, would carry this discussion further than my opportunity permits at present, and might be regarded as scarcely worth while unless it can be founded on a thoroughly trustworthy basis. The large constant correction in R.A. to which I have called attention, as well as the still more important correction in declination, varying according to declination, might be supposed to cover the most important part of the correction required by the comparison, P. '55 — A.-B. 1755. I have, accordingly, corrected by a summary process the absolute terms of two of STRUVE's normal equations. Each of the absolute terms in STRUVE's conditional equations in R.A. (p. 12 ff.) requires a correction of $+1''.2 \cos\delta$. Each of the absolute terms in the equations for declination (p. 15 ff.) requires the correction for the corresponding zone as shown in the foregoing. For the present purpose we may assume without serious error that the effect of these constant corrections will practically disappear in the products with periodic terms. We may then form the products $(\cos\omega \cos\delta)\times p_1$ and $\cos\delta \times p_1$, for each zone, p_1, being STRUVE's adopted weights, and then add the sum of the first set of products to the first of the normal equations in R.A. upon page 19, and the sum of the second set of products to the last equation in declination on p. 20. We get for the value of

$$\Sigma[(\cos\omega\cos\delta)(\Delta\alpha\cos\delta)p_1]\,, \quad +101''.56\,;$$
and for $\Sigma[(\Delta\delta_\delta \cos\delta)p_1]\,,\quad +156''.21.$

After transformation of his Δn, STRUVE's equations then become:

For R.A.

$$\begin{array}{rrrcr} +83.19\,\Delta\psi & +17.06\,x & -\;9.07\,y & = & -\;92''.28 \\ +17.06 & +65.25 & +\;3.12 & = & -\;47.49 \\ -\;9.07 & +\;3.12 & +64.87 & = & -253.86 \end{array}$$

For declination.

$$\begin{array}{rrrrcr} +10.03\,\Delta\psi & +\;9.20\,x & -\;0.66\,y & -\;\;3.06\,z & = & -\;30''.33 \\ +\;9.20 & +17.61 & -\;1.40 & -\;\;2.93 & = & -\;23.66 \\ -\;0.66 & -\;1.40 & +16.52 & -\;\;3.69 & = & -\;50.74 \\ -\;3.06 & -\;2.93 & -\;3.69 & +101.99 & = & +384.46 \end{array}$$

The solution of these equations leads to the following values of the unknowns.

	R.A.	Decl.	Combined	P.E.
	"	"	"	"
$\Delta\psi$	−1.53	−2.38	−1.54	±0.17
x	−0.13	+0.30	−0.20	±0.18
y	−4.12	−2.36	−3.76	±0.18
z	+3.17	+3.52	+3.48	±0.16

M	5.39	4.24	5.12
	°	°	°
A	268.2	277.2	267.0
D	+40.1	+56.0	+42.8

The probable error of the unit of weight in R.A. is $\pm 1''.69$, and in declination $\pm 1''.38$, results almost identical with those found in the discussion of Cape '80 — Cape '50. The appended probable errors of the combined solution assume a mean probable error of the unit of $\pm 1''.60$. It thus appears that the combination, Pulk. — Bradley, founded on 2509 stars, extending over the area, $-15°$ to $+90°$ of declination, is of very nearly the same weight as the combination, Cape '80 — Cape '50, founded on 3587 stars, extending over the Southern hemisphere. In certain respects centennial motions derived from a 26-year period ought to have a weight of only one-fifteenth that which belongs to those deduced from a period of 100 years. But there are two other factors of great importance. In the first place, the weight of mean centennial motions will be in some measure dependent upon the number of stars comprised in those means. In the second place the precision of the means will depend in a very large measure upon the precision of the systematic corrections applied. I see no escape from the conclusion that this near equality of weight between the results for the two hemispheres is to be accounted for mainly on the hypothesis, that the much greater weight due to the greater interval is nearly counterbalanced by the greater weight due to the systematic corrections applied in the case of the shorter interval — this too in spite of the fact that the Cape Catalogue for 1850, in comparison with most star-catalogues of the last sixty years, offers unusual difficulties as to the ascertainment of the true systematic corrections applicable to it.

It is very plain that there are two elements in the influence which systematic error exerts in the solar-motion-problem. One element of error is inherent in the standard star-positions by means of which the systematic errors of individual star-catalogues are ascertained. Assuming the system to be virtually free from errors, the second difficulty arises in ascertaining the systematic error of the individual catalogues. The process must necessarily be limited to the number of stars in the standard catalogue common to those in the catalogue to be compared; and the accuracy of the result will further depend, not only upon the number of comparisons, but also upon the character of the catalogue compared, as to whether its errors are capable of expression in simple formulas, or curves, or whether they are of a complicated nature. If the systematic corrections for a modern star-catalogue can be ascertained and applied with four times the precision attainable for an older catalogue then it matters little if, in the former case, the interval be only one-fourth as great.

Obviously, however, it is desirable that the discussion should not be limited to two catalogues, since it may be supposed that the errors in the application of systematic corrections will tend toward complete compensation in proportion as the number of catalogues employed is increased, leaving finally only the systematic errors of the standard catalogue.

It is scarcely necessary here to call attention to the alteration of L. Struve's result here effected almost wholly by the application of systematic corrections. We have for the quantities which decidedly differ:

	Struve	Boss
	″	″
$h\psi$	-2.84	-1.54
M	4.36	5.12
	°	°
D	$+27.3$	$+42.8$

I now combine the investigation of Struve, corrected for systematic error, with that of the present communication from C. '80 — C. '50. The two sets of normal equations are of practically the same weight; they can therefore be combined simply by addition. But they do not relate to stars at the same distance. The secular parallax of Struve's stars (mean magnitude $6^{M}.0$) is $5''.12$, as found in the foregoing; and of the Southern stars (mean magnitude $6^{M}.5\pm$) it is $4''.07$. In order to reduce to the same standard the absolute terms of the normal equations for Cape '80 — Cape '50 must be multiplied by $5''.12 \div 4''.07 = 1.26$. In doing this the unit of weight has been altered so that both members of the equations must be multiplied by 0.631. We then multiply the left-hand members by 0.631, and the right-hand members by 0.794. This method has the effect of altering the value of $h\psi$ from Southern stars, but is sufficiently rigorous as to the other quantities. Proceeding in this manner we have as the combined normal equations, founded on 6096 stars from the north to the south pole, the following:

Equations from R.A.'s

$$\begin{array}{rrrcr}
+124.00\,h\psi & +\ \ 8.19\,x & -\ 10.51\,y & = & -146''.51 \\
+\ \ 8.19 & +107.75 & +\ \ 3.81 & = & -\ 39.31 \\
-\ 10.51 & +\ \ 3.81 & +107.24 & = & -419.86
\end{array}$$

Equations from declinations.

$$\begin{array}{rrrrcr}
+\ 23.66\,h\psi & -\ 12.94\,x & +\ \ 0.51\,y & -\ \ 3.72\,z & = & -\ 29''.38 \\
-\ 12.94 & +\ 59.93 & -\ \ 4.16 & -\ \ 2.82 & = & -\ 21.36 \\
+\ \ 0.51 & -\ \ 4.16 & +\ 56.05 & -\ \ 3.39 & = & -172.25 \\
-\ \ 3.72 & -\ \ 2.82 & -\ \ 3.39 & +192.72 & = & +706.83
\end{array}$$

The indetermination between $h\psi$ and x in the equations is now practically removed. Using the fourth declination-equation with the right-ascensions, we have the following results from the solutions.

	R.A.	Decl.	Combined	P.E.
	″	″	″	″
$h\psi$	-1.52	-0.91	-1.36	± 0.14
x	-0.10	-0.59	-0.35	± 0.13
y	-4.06	-2.89	-3.63	± 0.13
z	$+3.57$	$+3.59$	$+3.57$	± 0.12
M	5.40	4.65	5.11	
			°	°
A	268.6	258.4	264.6	$\pm$ 3.0
D	$+41.3$	$+50.5$	$+44.4$	$\pm$ 1.9

We may estimate from the method of combination that the value of $\Delta\psi$ is too large by about 0″.09, and consequently that the most probable correction of the luni-solar precession for 1830 from this computation is

$$-0''.0127 = \frac{\Delta\psi}{100}$$

This is very nearly the direct mean of the two solutions.

The agreement of the solutions in R.A. and declination separately treated must be regarded as satisfactory. The differences are certainly no larger than might have been anticipated alone from uncertainties in the application of systematic correction. I find myself strengthened in the conviction that the only road toward improvement in our knowledge both of the precessional and solar motions lies in attention to the systematic correction and in the multiplication of sources from which to draw the material of computation — the difference of epochs of the star-positions compared being actually of no more concern than the character for systematic accuracy of the positions themselves.

The present discussion seems to add very materially to the weight of opinion that the northern declination of the solar apex cannot be much less than 40°. It is usually estimated to be somewhere about +30°, or less, apparently because a succession of partial investigations, having Bradley's star-positions as the initial point, have so indicated. Evidently the best and most comprehensive one of these virtually supersedes all the others founded on less material of exactly the same kind.

As to the determination of precession, it is of interest to note that the right-ascensions in each of the star-catalogues employed are derived from transits by "eye-and-ear." Since the weight of $\Delta\psi$ comes largely from $\Delta\alpha\cos\delta$, this unity of method may be regarded as a distinct advantage, in the absence of any attempt to investigate errors of transit depending on the brightness of the star. The value of $\Delta\psi$ obtained from the declination-equations seems to be entitled to some consideration, since it is measurably free from any assumption as to A and D. We have for the probable error of $\Delta\psi$ in the separate solutions, $\pm 0''.15$ and $\pm 0''.37$, respectively, in R.A. and declination; so that a difference of 0″.58 in the two solutions does not appear to raise any very serious question as to the assumptions which underlie the computation, though it is certain that decided improvements in the treatment of systematic corrections are still possible. On the whole, this investigation appears to encourage the hope that a fairly satisfactory solution of the problem of precessional and solar motion is attainable; that the theoretical difficulties are not necessarily of a serious nature; and that the obstacles of a practical kind are by no means insurmountable.

Of determinations of solar motion which are analogous to the present in their dependence upon the testimony of stars having small proper motion, there are two. The mean magnitude of the stars employed in these is about 8ᵐ.6. One of these investigations is that of the writer (*A.J.* 195-6); and the other is due to Dr. F. Ristenpart (*Veröffentlichen, Karlsruhe*, Th. IV). I have corrected Dr. Ristenpart's investigation to conform with the system of declinations, B_1. We then have:

	M	A	D
A.J. 195-6	″ 2.4	264	+54
Karlsruhe	3.3	291	+44

Since each of these determinations is based upon a narrow zone of stars, there may be rather serious objections to their validity. But it is a somewhat singular fact that nearly all these limited and tentative researches upon the solar motion cluster about a certain mean value of the co-ordinates of the apex not far from $A = 275°$: $D = +45°$. Usually the differences are not greater than are easily attributable to the probable error of the computation from a purely numerical point of view. For instance, my attention was attracted to Table XI in the Introduction to the Harvard A.G. Zone. It seemed to me that these tables exhibited not only the systematic differences between the Harvard A.G. Zone and the catalogues with which it was compared, but that they were certain, from the method of derivation, to include the parallactic motion of stars of small proper motion, usually regarded as negligeable. Accordingly the systematic differences, H.—Pi., H.—Tay., and H.—R.C. '15, indicated by N_1 and B_1, were computed and subtracted from the quantities in Rogers's Table XI. The residuals were assumed to show the effect of solar parallactic motion. The choice of the three comparisons in question was for the purpose of limiting the investigation to the brighter stars. Proceeding then by the usual method, I obtained

$$M = 2''.4 \quad : \quad A = 284° \quad ; \quad D = +33°$$

Thus an important part of the quantities in Table XI of the Harvard Zone-Catalogue is simply due to the parallactic motion of the stars.

In the appendix of his Berlin A.G. Zone-Catalogue Dr. Auwers has published comparisons between that and star-catalogues of former years. In order to make a computation, analogous to that for the Cape catalogues and the Harvard Zone, I selected Dr. Auwers's comparisons of his zone with the catalogues of Piazzi and Taylor. Differences, $\Delta\alpha$ and $\Delta\delta$, which seemed to indicate a proper motion of 0″.1 were excluded. The values of $\Delta\alpha$ and $\Delta\delta$ were then made approximately conformable with the systems, N_1 in R.A., and B_1 in declination. The solution for solar motion gives

$$M = 3''.8 \quad ; \quad A = 274° \quad ; \quad D = +21°$$

For both the Harvard and the Berlin Zones it was assumed that they require the systematic correction due to the *Fundamental-Catalog*. There may be a very sensible error in this assumption.

Incidentally I desire to call attention again to the im-

portance of parallactic drift among the stars, the motions of which, individually, are usually regarded as negligeable. Attention was called to this point more than ten years ago in *A.J.* 196; but there appears to be necessity for repetition. The average amount of this parallactic effect may be approximately expressed in the following formulas of correction.

For α: $+0^s.10\,\tau\ \sin(\alpha+85^\circ)\sec\delta$
For δ: $+1''.6\,\tau\ \sin(\alpha+5^\circ)\sin\delta - 1''.9\,\tau\ \cos\delta$

τ is the fraction of a century. Obviously it will not do to assign to star-positions, reduced to the present date from the older zones without the application of proper motion, weights nearly equal to those assigned to good modern observations. Furthermore, no very high degree of refinement need be attempted in the discussion of definitive orbits of comets and minor planets, if any considerable proportion of the stars is reduced to the present epoch without application of proper motion from epochs even no more distant than those of the A.G. Zones.

It may be of interest to compare the deductions to be drawn in this paper as to the coordinates of solar motion with some of those derived from stars having comparatively large proper motion. The following table exhibits the results taken from the original sources.

Determination of the Solar Apex.

Computer	Method	Stars	M	A	D
			"	°	°
Bischof	Argel.	180	(34)	286	+18
Stumpe	Airy	551	14	287	+42
		340	30	280	+40
		105	61	288	+32
		58	206	285	+30
Boss	Airy	279	13	283	+44
Porter I	Airy	576	16	282	+51
		533	30	281	+40
		142	55	285	+34
		70	166	277	+35
Porter II	Kapteyn	1037	(10)	281	+54
		1063	(20)	279	+40
		235	(50)	276	+34
		56	(200)	273	+41

Bischof (*Inaug. Dish.*, Bonn, 1884) finds from the Airy method: $A = 290^\circ.8$; $D = +43^\circ.5$.

Stumpe (*A.N.* 2999 3000) proceeded under the misapprehension that the declination system, B_1, is identical with Auwers Bradley for 1755. Estimating the correction due to this oversight I should substitute for his numbers,

$$D\ +45^\circ,\ +42^\circ,\ +33^\circ,\ \text{and}\ +31$$

respectively; and I should conclude as his most probable coordinates of the apex from all the stars.

$$A = 285^\circ\quad;\quad D = +42$$

My own contribution (*A.J.* 213) has to do with a zone of stars only 4° in width, and is open to theoretical objections on that account. Its only interest arises from its close accordance with more extensive investigations less open to theoretical objections.

Porter's first investigation (*A.J.* 276) is based upon proper motions deduced from star-catalogues in general, but without the application of systematic corrections. The largest errors so produced ought to appear in the groups containing the smaller proper motions. I consider the most probable general result from this investigation to be

$$A = 281^\circ\quad;\quad D = +45^\circ$$

In his second investigation (*A.J.* 197) Professor Porter applied systematic corrections to the star-catalogues from which he derived his proper motions, according to what system does not appear. He also applied the new method of Kapteyn. Nevertheless his results are almost identical with those from his first investigation. Where the proper motions are deduced from all available star-catalogues the systematic errors tend to compensate each other to a certain extent; though, for the fainter stars, the main reliance for the older epochs will be the observations of Piazzi, and the zone-observations of Lalande and Bessel, all of which are subject to very large systematic errors. I conclude from Porter's second investigation,

$$A = 279^\circ\quad;\quad D = +46^\circ$$

This result differs from that which Professor Porter recommends. He assigns double weight to the group which has the smallest proper motions. If I were to make a difference in the weights it would be in favor of the second group, which actually contains more stars. For that group, also, any defect in the application of systematic corrections would be less harmful. If, as appears probable, Professor Porter has applied the corrections necessary to reduce his stars to the system of Dr. Auwers, the substitution of B_1 in place of Auwers, would increase the values of D, especially for the first two groups, carrying the first to $+56^\circ$, perhaps. Professor Porter has apparently taken no account of a hypothetical correction of the adopted coefficient of precession, and this may have influenced his result in a slight degree, though, of course, the larger the proper motions dealt with, the less proportional effect would be due to the neglect of precessional correction.

From the consensus of all the investigations the results for solar motion may be presented in the following form.

Stars	A	D
	°	°
Small motions, $8^M.5$	279	+49
Small motions, $6^M.0$	265	+44
Large motions	280	+45

From these I should adopt as the most probable coordinates of the solar apex,

$$A = 275^\circ\quad;\quad D = +45$$

This is a point about 7° in a northwesterly direction from α *Lyrae*.

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THE EFFECTS OF SECULAR COOLING AND METEORIC DUST ON THE LENGTH OF THE TERRESTRIAL DAY.

BY R. S. WOODWARD.

1. *Historical note.* In the fifth volume of the *Mécanique Céleste*, LAPLACE has devoted a chapter to an investigation of the effect of secular cooling of the earth on the length of the sidereal day.* He reaches the important conclusion that the day has not changed in length appreciably from this cause during the past two thousand years. This conclusion has been quoted often in popular and semi-popular works, but it does not appear that any one has sought to do more than to verify the formulas to which LAPLACE arrives after having adopted assumptions which seem quite inappropriate and approximations which are quite unnecessary.†

Stated briefly, the problem which LAPLACE set out to solve is this: Assuming (1) that the entire mass of the earth was initially of a uniform excess in temperature above that of surrounding space; (2) that it cools by a process of conduction to and radiation from the surface such that the diffusivity and emissivity are each constant; (3) that the cubical contraction of the mass is constant; what then will be the effect on the length of the day of secular cooling and consequent contraction of the earth's mass?

Analytically LAPLACE's solution of this problem is correct and complete. His method of treatment is similar to that which he applied with such great success to the problem of the figure of the earth; and the striking analogies of the analysis in the two problems led him to remark justly — "*J'ose esperer que les géomètres verront avec quelque intérêt cette nouvelle application de l'analyse par laquelle j'ai déterminé la figure des corps célestes et la loi de la pesanteur à leur surface.*"

The fault to be found with LAPLACE's treatment of the problem is not, therefore, with his generalities, but rather with the details of his application to the actual case of the earth. When he comes to this application he makes the unnecessary, and, as it appears to me, inappropriate additional assumption that the earth is in the last stages of cooling. This simplifies the calculation very much, but it ignores the interesting questions of the effects which take place during the earlier and intermediate stages of cooling, and of the total effect that may result during the entire process of cooling. LAPLACE's application is vitiated also by the undue importance he attaches to the rôle of the atmosphere and oceans in the process of secular cooling of the earth. In common with his eminent contemporaries, FOURIER and POISSON, LAPLACE seems to have entertained the view that the conduction of heat from the interior of the earth to its surface is controlled by the atmosphere and oceans. Hence it appeared essential to him to take strict account of FOURIER's "surface condition," although his observational data for that condition are not only meagre but probably untrustworthy.

2. *Object of present investigation.* Assuming the same conditions as those of LAPLACE, stated above, the following paper extends his researches, and shows how to determine the effect on the length of the day of the cubical contraction of the earth during any portion of, or during the entire history of, the process of secular cooling. It appears from the work below that LAPLACE's conclusion as to the imperceptible effect of such cooling during the past two thousand years will apply equally well to any such limited period of time in the history of cooling quite irrespective of the "surface condition." On the other hand, it is shown that the effect which may accrue in the entire history of cooling is very noteworthy, amounting, possibly, in the case of the earth, to as much as six per cent. in the length of the day. A final section of the paper is devoted to the allied question of the lengthening of the day due to accumulations of meteoric dust.

* *De la chaleur de la terre et de la diminution de la durée du jour par son refroidissement.* Chapitre IV, Livre XI, *Mécanique Céleste.*

† Some features of the mathematical work of LAPLACE's investigation are reviewed by TODHUNTER, *History of the Mathematical Theories of Attraction and the Figure of the Earth*, Vol. II, pp. 346–348. LAPLACE's conclusion as to the minute effect of secular cooling on the length of the day in historic times appears to the casual reader to be strengthened by a brief collateral investigation of PLANA in a Note "Sur la densité moyenne de l'écorce superficielle de la terre," *Astronomische Nachrichten*, No. 828, 1852.

3. *Theory of cooling sphere.* For the mathematical theory of a sphere cooling under the conditions here assumed I may refer to two papers published by me in the *Annals of Mathematics*, 1887.* It will be essential here, therefore, to give only so much of that theory as is pertinent to the present application.

Let r denote the radius, and u denote the excess in temperature above that of external space, at any time t after the initial epoch, of any stratum, $4\pi r^2 dr$, of the sphere. Let the diffusivity of the mass of the sphere be denoted by a^2. This is the quotient of the conductivity of the mass by its thermal capacity, and, as explained above, is assumed to be the same for all parts of the mass of the sphere and for all temperatures. Then these quantities are connected by the partial differential equation

$$(1) \qquad \frac{\partial(ru)}{\partial t} = a^2 \frac{\partial^2(ru)}{\partial r^2}$$

and the solution of the problem of the law of cooling consists in finding the value of (ru) from this equation subject to the following conditions.

Calling u_0 the initial uniform excess in temperature of the sphere above that of surrounding space,

$$(2) \qquad ru = ru_0 \quad \text{for} \quad t = 0$$

and in order that u may not be infinite for $r = 0$,

$$(3) \qquad ru = 0 \quad \text{for} \quad r = 0$$

In addition to these two relations which the integral of (1) must satisfy there is a third relation which defines the emission of heat at the surface of the sphere. Let K denote the conductivity, and H the emissivity of the mass of the sphere. Then, at all points of its surface, the rate at which heat is conducted thereto must be equal to the rate at which heat is carried away therefrom, or, in accordance with Fourier's theory,

$$-K\frac{\partial u}{\partial r} = Hu$$

This is Fourier's "surface condition," to which he and his contemporaries attached quite undue importance in the case of the earth. Calling r the radius of the surface of the sphere, and writing for brevity $h = H/K$, this condition takes the form

$$(4) \qquad \frac{\partial(ru)}{\partial r} + (rh-1)\,u = 0 \quad \text{for} \quad r = 0$$

Now it is readily found, as first shown by Fourier,† that (1), (3) and (4), are satisfied by

$$ru = \sum_{n=1}^{n=\infty} C_n e^{-a^2 t \lambda_n^2} \sin \lambda_n r \qquad (5)$$

if the successive values of λ_n are determined from the equation

$$r_0\lambda_n + (r_0 h - 1)\tan r_0\lambda_n = 0 \qquad (6)$$

and the condition (2) is also satisfied by (5) if the values of C_n are found from

$$ru_0 = \sum_{n=1}^{n=\infty} C_n \sin \lambda_n r \qquad (7)$$

which is what (5) becomes when $t = 0$.

Thus far everything is easy. But when it is essential to apply (5) one must encounter the difficulty presented by equation (6), which is transcendental in $r_0\lambda_n$. It is this difficulty, perhaps, which led Laplace to limit (5) to its first term in his application to the earth. Poisson carried the elaboration of (5) a step further without, however, attaining a much more tractable formula.* It was at this point that the problem contained in (5), (6), (7) was taken up in the second of my papers referred to above. The modification introduced is specially applicable to the conditions of the present investigation, and may therefore be briefly outlined here.

It is seen from equation (4) or (6) that rh must be a number, or that h must be the reciprocal of a length. Put $x = 1/rh$ and $\theta_n = r_0\lambda_n$. Then (6) may be written

$$(1-x)\tan\theta_n = -x\theta_n \qquad (8)$$

Now, since for the earth r_0 as expressed in ordinary units is large, and since h in the same units is not very small, x must be a small number. From certain observations made at the Paris Observatory, Poisson concluded that for the earth in that vicinity, h is a little greater than 1, using the metre as unit of length.† This would make x about 1/6300000.

Thus it appears advantageous to express the solution of (5) in two parts, one independent of, and the other dependent on the small quantity x; or, in other words, to develop (5) as a function of x by Maclaurin's series. Herein consists the modification carried out in detail in the papers referred to. If the solution is symbolized by S, it is seen at once to take the form

$$ru = S = S_0 + \left(\frac{\partial S}{\partial x}\right)_0 x + \left(\frac{\partial^2 S}{\partial x^2}\right)_0 \frac{x^2}{2} + \ldots \qquad (9)$$

* On the free cooling of a homogeneous sphere; and on the conditioned cooling and cubical contraction of a homogeneous sphere. *Annals of Mathematics*, Vol. III, pp. 75–88, 129–144.

† *Théorie Analytique de la Chaleur*, Paris, 1822.

* *Théorie Mathématique de la Chaleur*, Paris, 1835. Poisson's solution is given also in Riemann's *Partielle Differentialgleichungen und deren Anwendungen auf Physikalische Fragen*. Hattendorff's edition, 1882.

† *Théorie Mathématique de la Chaleur*, p. 502. The value given for h is 1.05719. It is plain, however, that this cannot apply with a precision indicated by so many significant figures to the entire surface of the earth. But it seems to have escaped the attention of Poisson that h might be a thousand times as great as the above value without sensibly affecting the secular cooling of the earth.

wherein the suffix zero signifies that the value of the quantity to which it is attached is to be taken for $x = 0$.

This method of treating the problem simplifies the expression of (5) very much, especially for the circumstances presented by the earth. For when $x = 0$, equation (8) shows that $\theta_n = n\pi$, whence S_0, or the part of (5) independent of x, is

$$(10) \qquad S_0 = \frac{2r_0u_0}{\pi}\sum_{n=1}^{n=\infty}\frac{(-1)^{n+1}}{n}e^{-a^2t(n\pi/r_0)^2}\sin n\pi\frac{r}{r_0}$$

This represents the complete solution for what I have called the case of free cooling; while the solution for the case of conditioned cooling is given by (9) when $x < 1$.

A method of deriving the differential coefficients $\frac{\partial S}{\partial x}$, etc., which appear in (9), is explained in the second of the papers cited; and a numerical example for the case of the earth is there worked out on the supposition that

$$x = 1/1000$$

From this example it appears that even for so large a value of x, which is probably a thousand times as great as it ought to be for the actual circumstances presented by the earth, the error which arises from the neglect of the terms of (9) in x is less than one per cent. My conclusion is, therefore, that for the earth the solution (10) is quite as precise as is required by the data of the problem. Geologically this conclusion is very important, for it means that the internal heat of the earth escapes as if the earth had neither atmosphere nor oceans. Or, to state the fact in another way, it may be said that, since the atmosphere and oceans are capable of dissipating a thousand to a million times as much heat as is conducted to the surface of the earth per unit time, they oppose no sensible obstacle to the secular cooling.

For the special application which is the primary object of this paper it is convenient to have (u_0-u) rather than ru as given by (10). By making $t = 0$ and $t = t$ in (10), and taking the difference of the results there is found

(11)
$$r(u_0-u) = \frac{2r_0u_0}{\pi}\sum_{n=1}^{n=\infty}\frac{(-1)^{n+1}}{n}\left\{1-e^{-a^2t(n\pi/r_0)^2}\right\}\sin n\pi\frac{r}{r_0}$$

1. *Effect of contraction on speed of rotation of earth.* As the earth contracts its moment of inertia, I, say, with respect to the axis of rotation diminishes; and, in accordance with the law of conservation of moment of momentum, the angular velocity ω, say, must simultaneously increase in such a way as to render the product $I\omega$ constant. Or, if τ denote the time of rotation of the earth at any time t when the moment of inertia is I,

$$I\frac{2\pi}{\tau} = C, \text{ a constant}$$

If the corresponding values of I and τ for $t = 0$ be designated by I_0 and τ_0,

$$I_0\frac{2\pi}{\tau_0} = C$$

and by a combination of this with the previous equation there results

$$\frac{\tau_0-\tau}{\tau_0} = \frac{\delta\tau}{\tau_0} = \frac{I_0-I}{I_0} = \frac{\delta I}{I_0} \qquad (12)$$

It is now required to express the variation of the moment of inertia of the earth, I, in terms of the time t after the initial epoch. At any such time let ρ be the density of a stratum $4\pi r^2dr$ at a distance r from the center, ρ being a function of r to be assigned later. Then, denoting latitude and longitude by φ and λ respectively, and the radius of the earth's surface by r_0, the moment of inertia

$$I = \int_0^{r_0}\rho r^4dr\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos^3\varphi d\varphi\int_0^{2\pi}d\lambda = \frac{8}{3}\pi\int_0^{r_0}\rho r^4dr \qquad (13)$$

By means of this result equation (12) becomes

$$\frac{\delta\tau}{\tau_0} = \frac{\delta\int_0^{r_0}\rho r^4dr}{\int_0^{r_0}\rho r^4dr} \qquad (14)$$

wherein it is to be observed that ρ and r_0 which appear in the integrations of the second member are to have values corresponding to $t = 0$.

To get the variation of the integral in the numerator of the second member of (14), observe that the mass of any stratum remains constant, or that

$$4\pi\rho r^2dr = \text{constant} \qquad (15)$$

Thus,

$$\delta\int\rho r^4dr = \delta\int(\rho r^2dr)r^2 = 2\int\rho r^3dr\,\delta r \qquad (16)$$

The value of δr which appears in this equation is the diminution of the radius of any stratum due to the cubical contraction of the mass below that stratum. If δu denote the fall in temperature of any stratum up to the time t, and ϵ denote the cubical contraction per degree of temperature,

$$4\pi r^2\delta r = 4\pi\epsilon\int_0^r\delta u\, r^2dr$$

whence

$$\delta r = \frac{\epsilon}{r^2}\int_0^r\delta u\, r^2dr \qquad (17)$$

Laplace's method of getting this result is less direct, but very instructive, and the substance of it is worth reproducing here. It is equivalent to this: From equation (15)

$$\delta(\rho r^2dr) = r^2dr\,\delta\rho + \rho\delta(r^2dr) = 0$$

whence
$$\delta(r^2dr) = -r^2dr\frac{\delta\rho}{\rho}$$

But to terms of the order $(\delta r)^2$

$$\delta(r^2 dr) = d\,(r^2 \delta r)$$

and hence

$$r^2 \delta r = -\int_0 r^2 dr \frac{\delta\rho}{\rho}$$

Now if v is the volume of any mass of density ρ, $v\rho =$ constant, and

$$\frac{\delta\rho}{\rho} = -\frac{\delta v}{v} = -\frac{\epsilon\, v\, \delta u}{v} = -\epsilon\,\delta u$$

Substitute this value in the above equation and the value given by (17) results.

Making use of equations (16) and (17), (11) becomes

$$(18) \qquad \frac{\delta\tau}{\tau_0} = \frac{2\epsilon \int_0^{r_0} \rho r\, dr \int_0^{r} \delta u\, r^2 dr}{\int_0^{r_0} \rho r^4 dr} = \frac{2\epsilon \int_0^{r_0} \delta u\, r^2 dr \int_r^{r_0} \rho r\, dr}{\int_0^{r_0} \rho r^4 dr}$$

LAPLACE gives the first only of these two equivalent forms. In deriving his formula he does not assign the limits of the integrations. In his subsequent application, however, he says "On a ensuite, en intégrant depuis r nul jusq' à $r = a$" ($a = r_0$). But instead of doing so he inverts the order of integration, and proceeds as required by the last member of (18).

5. *Aggregate effect of secular cooling on length of day.* Before proceeding to introduce the value of δu given by equation (11), and without assigning any law for the distribution of density ρ, it is possible to draw from (18) a remarkably simple expression for the shortening of the day due to the entire dissipation of the earth's heat. Thus, when $t = \infty$ in (11), $\delta u = u_0$, the initial uniform excess of the temperature of the earth's mass above that of surrounding space. Placing this value of δu in (18) its second member gives at once

$$(19) \qquad \frac{\delta\tau}{\tau_0} = \frac{2}{3} u_0 \epsilon$$

It is surprising that this result, which is obvious when the limits in the second member of (18) are assigned, should have escaped the attention of LAPLACE. It is still more surprising, perhaps, that this result should be independent of the law of distribution of density in the sphere.

A result so simple ought to be confirmed by an equally simple demonstration. Here is one which does not depend on equation (18). Let r'_0 denote the radius of the sphere after contraction is complete, or after it has fallen in temperature by u_0 degrees. Then

$$(a) \qquad r'_0 = r_0 \left(1 - \frac{1}{3} u_0 \epsilon\right)$$

Recurring to equation (13) it is seen that

$$I_0 = \frac{2}{3} \int_0^{r_0} r^2 (4\pi\rho r^2 dr) = \frac{2}{3} \int_0^{r_0} r^2 dM$$

if M denote the mass of the sphere. Hence if f be a suitable factor

$$I_0 = \frac{2}{3} f^2 r_0^2 M \qquad (b)$$

where fr_0 is the radius of gyration of the sphere. Similarly, after contraction, the moment of inertia is

$$I = \frac{2}{3} f^2 r_0'^2 M \qquad (c)$$

Inserting (b) and (c) in (12) and observing (a), there results

$$\frac{\delta\tau}{\tau_0} = 1 - \left(1 - \frac{1}{3} u_0 \epsilon\right)^2 = \frac{2}{3} u_0 \epsilon$$

to terms of the order neglected, which is the same as (19).

To get an idea of the magnitude of the change specified by (19) in the case of the earth, it may be assumed that u_0 cannot have been greater than 3000° C.; and that a proper value of ϵ is 3×10^{-5}, which is about that of iron. With these values (19) gives

$$\frac{\delta\tau}{\tau_0} = \frac{2}{3} \times 3000 \times 3 \times 10^{-5} = \frac{6}{100}$$

From this it follows that in the entire history of secular cooling of the earth the day may be shortened from this cause by something like six per cent. of its initial length.

6. *General expression for the effect of secular cooling on length of day.* It is evident from the preceding investigation that, quite contrary to the view entertained by LAPLACE, and after him by PLANA, the law of distribution of density in the earth plays a very unimportant rôle during the later stages of cooling in modifying the length of the day. It will be of interest therefore to determine what may happen under the assumed conditions during the earlier and during the intermediate stages of cooling. This requires the introduction of δu from equation (11) in (18), and the assignment of the density ρ of any stratum as a function of the radius of that stratum.

In his calculation LAPLACE adopts a law according to which the density increases in arithmetical progression from the surface to the center of the earth. It is not a matter of great importance what law of density is adopted; but in order that the problem may be worked out in accordance with the most plausible hypothesis applicable to the earth, I have made use of LAPLACE's favorite law of density, namely:

$$\rho = \rho_0 \frac{r_0 \sin ar}{r \sin ar_0} \qquad (20)$$

wherein ρ_0 is the density at the earth's surface, r and r_0

have the same meanings assigned above, and a is a constant.*

Before applying this more probable law of density, it will be instructive to set down the results given by equation (18) for the simplest case, namely, that of constant density. Thus, using both methods of integration specified by (18), it is found from the first that

$$(21)\qquad \frac{\delta\tau}{\tau_0} = \frac{60\,n_0\epsilon}{\pi^4}\sum_{n=1}^{n=\infty}\frac{1}{n^4}\left\{1-e^{-a^2t(n\pi/r_0)^2}\right\}$$

and from the second that

$$(22)\qquad \frac{\delta\tau}{\tau_0} = \frac{20\,n_0\epsilon}{3\pi^2}\sum_{n=1}^{n=\infty}\left(\frac{1}{n^2}-\frac{6}{\pi^2n^4}\right)\left\{1-e^{-a^2t(n\pi/r_0)^2}\right\}$$

The equivalence of these two results is easily established if not at once evident. It may suffice here to show that they each reduce, as they should, to (19) when $t = \infty$. This is seen at a glance, for in (21)

$$\sum\frac{1}{n^4} = \frac{\pi^4}{90}$$

and in (22)

$$\sum\left(\frac{1}{n^2}-\frac{6}{\pi^2n^4}\right) = \frac{\pi^2}{6}-\frac{\pi^2}{15} = \frac{\pi^2}{10}$$

Making use now of (11) and (20) in (18), and writing for brevity

$$(23)\qquad \gamma = ar_0 \quad\text{and}\quad \beta = \frac{\gamma}{\pi}$$

there results for the change in the length of the day at any time t after the initial epoch the following formula:

$$(24)\qquad \frac{\delta\tau}{\tau_0} = \frac{8\,n_0\epsilon\gamma^4\displaystyle\sum_{n=1}^{n=\infty}\left\{\frac{1}{(n^2-\beta^2)^2}+\frac{\frac{1}{2}\gamma\cot\gamma}{n^4-\beta^2n^2}\right\}\left\{1-e^{-a^2t(n\pi/r_0)^2}\right\}}{3\pi^4\left\{\gamma^2-2+(2\gamma-\frac{1}{3}\gamma^3)\cot\gamma\right\}}$$

In this formula, which is rather repulsive by reason of its complexity, dependence on distribution of density appears only through the constant γ. Assuming the ratio of the surface density to the mean density of the earth to be 2, the transcendental equation of the foot-note above gives $\gamma = 2.4606$, whence, through the second of (23), the numerical values of the coefficients of the factors involving the time in the series of (24) may be found.

To get a clear idea of the nature of this series as applied to the earth it will suffice to set down a few of its terms. Thus, writing for brevity

$$Q = e^{-a^2t(\pi/r_0)^2} \qquad (25)$$

equation (24) takes the following form:

$$\frac{\delta\tau}{\tau_0} = n_0\epsilon\left\{\begin{array}{l}+0.67522(1-Q)\\-0.00607(1-Q^4)\\-0.00144(1-Q^9)\\-0.00047(1-Q^{16})\\-0.00020(1-Q^{25})\\-\ldots\end{array}\right\} \qquad (26)$$

all terms after the first being negative.

The expression corresponding to (26) and derived from (21) for the case of uniform density, differs from (26) only in the values of the numerical coefficients. The first five of the latter, computed from $60/(n\pi)^4$ for $n = 1, 2, \ldots 5$, respectively, are

+0.61596	+0.00241
+0.03850	+0.00099
+0.00760	

A check on the sums of these coefficients and those of (26) is supplied by the fact stated in (19) that each should converge towards the limit 2/3. The sum of the first five coefficients of (26) is +0.66704, while the corresponding sum of the first five coefficients for the expression (21) is +0.66546. In either case the convergence of the series is rapid, that of (24) for the case of the earth being somewhat more rapid than that of (21).

7. *Time rate of change of length of day.* Evidently the rate at which the day changes in the earlier stages of cooling ought to be greater for a homogeneous planet than for one whose density increases from surface to center. This fact is shown as follows. Designating the coefficients, of which a few numerical values are given above, by the letter e with suffixes, the time differential of (21) or (24) gives, by the aid of (25),

$$\frac{\partial(\delta\tau)}{\tau_0\partial t} = n_0\epsilon\,(e_1Q+4e_2Q^4+9e_3Q^9+\ldots)\left(\frac{a\pi}{r_0}\right)^2 \qquad (27)$$

This shows that in any case the rate of change is greatest in the earlier stages of cooling, for Q has its maximum value, namely unity, when $t = 0$. On the other hand, in the later stages of cooling, alone considered by LAPLACE, this rate of change approaches zero, since $Q = 0$ for $t = \infty$. The rate of change is always greater for the case of homogeneity involved in (21) than for the case of central condensation specified by (20) and (24); for in the latter case $e_2, e_3, \ldots$ are all negative in (27).

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* The Laplacian distribution of density ρ, stress p, and potential V, in any spherical mass may be compactly specified by the three following equations:

$$\frac{\partial^2(rV)}{\partial r^2}+4\pi kr\rho = 0$$

$$dp = \rho dV, \quad\text{and}\quad dp = c\rho\,d\rho$$

The first of these is POISSON'S equation, k being the gravitation constant. The second is the hydrostatic law; and the third, in which c is a constant, is LAPLACE'S celebrated hypothesis so plausibly set forth in Chapitre I, Livre XI, of the *Mécanique Céleste*. The constant a which appears in (20) is given by $a^2c = 4\pi k$. The constant $ar_0 = \gamma$ is derived from the transcendental equation

$$\gamma^2 = 3\,\frac{\rho_0}{\rho_m}\,(1-\gamma\cot\gamma)$$

wherein ρ_0 is the surface density and ρ_m is the mean density of the earth.

For equivalent specifications of the same distribution, see Chap. II, Vol. V, *Mécanique Céleste*, or Arts. 824–829 of KELVIN and TAIT'S *Natural Philosophy*.

An inspection of (27) shows that the rate of change of the day in the earliest stages of cooling, or for $t = 0$ and $Q = 1$, would be about one and two-thirds times as fast for the case of homogeneity as for that defined by equation (20), the numerical coefficients of

$$u_0\epsilon\left(\frac{a\pi}{r_0}\right)^2$$

being in the two cases approximately 1.01 and 0.602 respectively.*

But the rate at which the day changes under the assumed circumstances is always small by reason of the smallness of the factor $(a\pi/r_0)^2$. Adopting for the earth Lord KELVIN's value of a^2, namely, 400, with the British foot as unit of length and the year as the unit of time,† the value of that factor is, in round numbers, of the same order of precision as the values assigned to u_0 and ϵ above, 9×10^{-12}. Taking the case of homogeneity, for which the rate of change in the day is greatest, the maximum value of the series in (27) is $10/\pi^2$. Thus it may be inferred from (27) that in the case of the earth $\partial(\delta\tau)/\tau_0\partial t$ has not been greater per year than

$$u_0\epsilon \times \frac{10}{\pi^2} \times 9 \times 10^{-12}$$

or, using the same values of u_0 and ϵ as above, and assuming τ_0 to have been 100000 seconds, that the change $\delta\tau$ in a year has not been greater than 82×10^{-9} seconds; or, for any period of two thousand years in the history of cooling, $\delta\tau$ has not been greater than 0.00016 seconds. LAPLACE was quite right therefore in the conclusion that the day has not changed appreciably from the cause in question in the past twenty centuries.

8. *Change in length of day during any interval after initial epoch.* For practical application of equation (26) to the case of the earth in its earlier stages of cooling—during the first ten million years, say—it will suffice to write

$$1 - Q = \left(\frac{a\pi}{r_0}\right)^2 n^2 t$$

neglecting the higher powers in the expansion of the exponential quantity defined by (25). Thus, for the earlier stages in question, (26) is very nearly

$$\frac{\delta\tau}{\tau_0} = \frac{3}{5}u_0\epsilon\left(\frac{a\pi}{r_0}\right)^2 t$$

or, with the values of the constants used above,

$$\frac{\delta\tau}{\tau_0} = 486\,t \times 10^{-15}$$

t being expressed in years. Supposing τ_0 to have been 100000 seconds, an overestimate doubtless,

$$\delta\tau = 486\,t \times 10^{-10} \tag{28}$$

From this it appears safe to conclude that the length of the day will not change, or has not changed, as the case may be, by so much as a half second in the first ten million years after the initial epoch.

By means of (26) the change during any time t after the initial epoch is readily computed. Although this change goes on exceedingly slowly, it will be practically all accomplished in less than a million million years. In fact, I have shown by a computation given in the second of the papers on a cooling sphere cited above, that the earth will accomplish about 95 per cent. of its cubical contraction in less than 300,000,000,000 years.*

9. *Effect of meteoric dust on length of day.* It will be of interest in connection with the preceding investigation to estimate the retarding effect on the speed of rotation of the earth of its accumulations of meteoric dust. The amount of such accumulations per year, say, is exceedingly small in comparison with the mass of the earth; but since they go on continuously, apparently, it can be only a question of time when their effect will be appreciable. It is especially desirable to know whether this effect is of about the same order as, or of a lower order than, that due to secular cooling of the earth.

It will be assumed that the distribution of meteors falling to the earth is uniform with respect to its surface, and that they neither increase nor decrease the angular velocity of the earth by impact upon it. In this case, then, as well as in that already considered, equation (12) applies. Here, however, it is seen by a glance at equation (13) that

$$\delta I = \frac{\partial I}{\partial t}\,t, \text{ and } \frac{\partial I}{\partial t} = \frac{8}{3}\pi\int_r^{r+\Delta r}\rho r^4\,dr$$

wherein Δr is the thickness of a shell of dust accumulated in any time t on a sphere of radius r, and ρ is the in-

* The exact values of these numbers are given by $10/\pi^2$ and by

$$\frac{2\gamma^4}{3\pi^2\{\gamma^2-2+(2\gamma-\frac{1}{3}\gamma^3)\cot\gamma\}} = 0.60248 \text{ for } \gamma = 2.4606$$

respectively. The first of these expressions comes from the summation in (21) after multiplying it by n^2 and making $t = 0$. The second expression comes from the corresponding summation in (24). The sum that has to be evaluated in the latter case is

$$\sum\left\{\frac{n^2}{(n^2-\beta^2)^2}+\frac{\frac{1}{2}\gamma\cot\gamma}{n^2-\beta^2}\right\}$$

Observing that $\beta = \gamma/\pi$, it may be shown that this sum is $\pi^2/4$, whence the expression above.

The mathematical reader's attention may be called here to a remarkable theorem in pure analysis which results from (24) when $t = \infty$, since (24) must then reduce to (19).

† KELVIN and TAIT's *Natural Philosophy*, Part II, Appendix D.

* A very simple and closely approximate formula for the cubical contraction of a large sphere under the assumed conditions, for all but the latest stages of cooling, is

$$8\pi r_0^3 u_0\epsilon\left(\frac{a}{r_0}\sqrt{\frac{t}{\pi}}-\frac{a^2t}{2r_0^2}\right)$$

the notation being the same as that used in the text. If t be made equal to $r_0^2/(4a^2)$ in this (which for the earth makes t about 273×10^9 years) the above-mentioned result follows.

crease in density per unit time at the surface of this sphere. Assuming ρ to be constant,

$$\frac{\partial I}{\partial t} = \frac{8}{3}\pi\rho\, r^4 \Delta r$$

to terms of the first order in Δr.

Now let N denote the number of meteors which fall into the earth's atmosphere per year, and let μ denote the average mass of such bodies. Then

$$4\pi\rho r^2 \Delta r = N\mu$$

and substituting the value of $\rho\Delta r$ from this in the preceding equation, there results

$$\delta I = \frac{\partial I}{\partial t}\, t = \frac{2}{3}\, N\mu\, r^2 t$$

The moment of inertia of the earth is known to be, nearly enough for the present purposes, one-third of the product of its mass by the square of its radius. That is, calling the mass M and the radius r, one may write for I_0 in (12)

$$I_0 = \frac{1}{3}\, Mr^2$$

Hence equation (12) becomes for the present purposes

$$\frac{\delta\tau}{\tau_0} = 2N\frac{\mu}{M}\, t \tag{29}$$

t being expressed in years from any epoch.

To apply this formula I shall adopt the estimate for N given by the late Professor H. A. Newton in his article on meteors in the ninth edition of the *Encyclopædia Britannica*, namely, that not less than twenty millions of meteors fall to the earth daily, or that

$$N = 20 \times 10^6 \times 365.25$$

The value of M is known to be about 6×10^{24} kilogrammes. Thus, in round numbers

$$\frac{\delta\tau}{\tau_0} = \frac{5}{2}\, \mu t \times 10^{-15}$$

or, assuming $\tau_0 = 100000$ seconds and $\mu = 0.001$ kilogramme, both of which are probably overestimates,

$$\delta\tau = \frac{5}{2}\, t \times 10^{-13} \tag{30}$$

This shows that at the rate of accumulation assumed it would take a million million years to produce a change in the length of the day as great as a quarter of a second. Even if the average value of μ were one kilogramme, it would require a thousand million years to produce that change.

A comparison of (28) with (30) shows that the shortening of the day from secular cooling goes on much more rapidly than the lengthening of day from meteoric dust: the ratio of the two changes being in round numbers 200000. In other words, it seems reasonable to suppose that the total effect from secular cooling will accrue before the effect from meteoric dust will begin to be appreciable.

OBSERVATIONS OF THE NEW STAR IN *PERSEUS*.

By ZACCHEUS DANIEL.

Cloudy weather interfered a great deal with the observation of Anderson's new star in *Perseus*. However, I was able to observe it on twenty-three dates before its disappearance from the evening sky. The observations were made by Argelander's method, and each depends on a careful comparison with one star brighter and another fainter than the variable. An effort was made to make the values in grades of the two intervals represent their relative sizes. All observations before April 2 were made with the naked eye, those between that date and May 6 were made with a 1.25-inch finder, having a field of 3° 45′, and a power of 10, while the last observation was made with a four-inch refractor. All the observations before March 7 were made in moonlight, and all those in May were made in twilight. On account of the low altitude of the star the observations made after April 16 are not so good as the others.

The magnitudes of the comparison-stars used in reducing these observations were taken from the Harvard Photometric *Durchmusterung* (H.C.O. Annals, Vol. 45).

At the first observation the star had a strong yellow color which later changed to a decided orange. With the telescope it always showed a strong flash of red. The last seen of the star with the naked eye was on April 27.

The following are the Greenwich Mean Times of observation and the observed magnitudes:

Julian Day	1901	Mag.	Seeing
2415441.60	Feb. 25	0.92	fair, moon
443.52	27	1.65	fair, clouds
444.58	28	1.65	fair
450.60	Mar. 6	2.92	poor, haze
451.56	7	2.92	fair, haze
456.52	12	2.96	fair
460.51	16	3.60	good
461.52	17	3.45	good
462.53	18	3.72	good
465.54	21	4.33	fair
466.62	22	4.55	fair
474.56	30	4.33	fair, moon
476.55	Apr. 1	4.42	fair, moon
486.60	11	5.40	fair
491.58	16	5.77	fair
501.56	26	5.77	fair, moon
502.58	27	4.55	fair, moon
503.57	28	5.35	poor, haze
504.56	29	5.95	poor, smoke
506.56	May 1	5.30	poor, clouds
509.57	4	5.77	poor, clouds
510.56	5	5.95	fair
2415511.56	6	6.16	poor, clouds

Bucknell University, Lewisburg, Penn., 1901 *June* 21.

OBSERVATIONS OF COMET a 1901.*

By R. G. AITKEN.

1901 Mt. Hamilton M.T.	✶	No. Comp.	☍ — ✶ $\Delta\alpha$	☍ — ✶ $\Delta\delta$	☍'s apparent α	☍'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
			s	′ ″	h m s	° ′ ″		
May 15 7 53 34	1	8 , 8	4.22	−5 15.5	5 38 25.50	+3 52 7.7	9.672	0.721
16 7 48 15	2	10 , 8	+13.64	+1 33.2	5 41 8.47	+1 12 12.4	9.671	0.723

Mean Places for 1901.0 of Comparison-Stars.

✶	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	5 38 28.92	+0.80	+3 57 31.8	−8.6	Boss, Albany A.G. Catal. 1867
2	5 43 54.01	+0.82	+1 10 18.0	−8.8	Boss, Albany A.G. Catal. 1908

Both observations were made with the 12-inch telescope of this Observatory, and Δa was measured directly with the micrometer. No illumination was used, the wires standing out black against the bright twilight sky.

The comet was about as bright as an $8\frac{1}{2}$-magnitude star. It had a well-defined nucleus, almost stellar, surrounded by a circular nebulosity nearly 1′ in diameter. No tail was seen.

On May 16, two stars (Albany A.G. Catal., Nos 1908 and 1911), rated as $8^{M}.2$ and $8^{M}.7$ respectively, were in the field of the telescope with the comet, whose brightness could therefore be estimated very accurately. It was certainly fainter than the $8^{M}.2$ star, but seemed rather brighter than $8^{M}.7$. Lack of comparison-stars and cloudy skies prevented further observations until May 21. By that time the comet had become fainter than a ninth-magnitude star, and could not be seen well enough for accurate measures. In spite of repeated careful search, it has not been seen since.

It may be well to add that the comet was looked for both in the morning and evening skies from April 26 until the Arequipa observation was received. Thereafter the evening sky was searched without success until the evening of May 14, when an approximate position for the comet was obtained from the circles of the 12-inch telescope.

Lick Observatory, University of California, 1901 *June* 13.

**From Supplement to No. 501.*

ON THE OBSERVATORY AT QUITO.

The government of the Republic of Ecuador has courteously placed the Observatory of Quito at the disposal of the French Commission charged with the remeasurement of the Peruvian arc. M. GONNESSIAT, Astronomer of the Observatory of Lyons, has been intrusted with the direction of the establishment. The new Director has taken possession of his post, and is actively engaged in organizing astronomical, meteorological and magnetic services. He will be deeply grateful to astronomers and meteorologists if they will now address their publications to the Observatory of Quito, which in return will send its own publications immediately upon their appearance. F. GONNESSIAT.

Observatorio de Quito, Ecuador.

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LATITUDE, AND THE VARIATIONS OF LATITUDE, DETERMINED WITH THE PRIME VERTICAL TRANSIT AT THE U.S. NAVAL OBSERVATORY,

By Assistant Astronomer GEORGE A. HILL.

[Published by permission of the Superintendent.]

In *A.J.* 404 I published the results of observations, up to 1896 October 15, made with our Prime Vertical Transit for the purpose of determining the latitude, and the variation of latitude at the Naval Observatory.

The declinations from which those latitudes were derived depended upon the star-places as given in Professor Boss's catalogue. In the present paper will be found a new discussion of those observations, as well as those made since that date, and all based upon the star-places from Professor Newcomb's New Catalogue.

Upon several occasions writers in astronomical and scientific journals have suggested, and some have made the statement, that the Prime Vertical Transit at the Naval Observatory is not an instrument of a very high order of construction, and is not capable of giving first-class work. Those statements and suggestions, so far as I have been able to glean from their reading, have been confined to the expression of an opinion, without being substantiated by an investigation of its results to indicate what defects it might have. I have constantly observed with the instrument for the past eight years, and after an experience extending over that period I feel somewhat competent to express an opinion.

Within the past decade there have been but three instruments mounted in the prime vertical that have published observations having in view the determination of the variation of latitude, and in that connection I desire to submit for comparison a table containing the observed and computed variation at Kiel, Pulkowa and Washington, from observations made by the prime vertical instruments at each of those observatories.

Kiel.				Pulkowa.				Washington.			
Date	Obs'd	Comp'd	Diff.	Date	Obs'd	Comp'd	Diff.	Date	Obs'd	Comp'd	Diff.
1892	″	″	″	1893	″	″	″	1894	″	″	″
Aug. 18	+0.27	+0.07	+0.20	June 10	−0.35	−0.12	−0.23	Apr. 10	+0.03	0.00	+0.03
Sept. 3	+0.38	+0.11	+0.27	July 22	−0.25	−0.08	−0.17	May 11	−0.17	−0.05	−0.12
Sept. 16	+0.18	+0.14	+0.04	Aug. 19	−0.36	−0.06	−0.30	June 17	0.00	−0.09	+0.09
Oct. 17	+0.36	+0.16	+0.20	Sept. 19	−0.23	−0.03	−0.20	July 14	+0.03	−0.09	+0.12
Nov. 21	+0.50	+0.17	+0.33	Oct. 16	−0.31	+0.01	−0.32	Sept. 17	−0.17	−0.05	−0.12
Dec. 15	+0.21	+0.15	+0.06	Nov. 15	−0.41	+0.05	−0.49	Oct. 15	+0.01	0.00	+0.01
1893 Jan. 13	+0.49	+0.10	+0.39	Dec. 22	−0.36	+0.08	−0.44	Nov. 19	+0.07	+0.06	+0.01
Feb. 17	−0.14	+0.01	−0.15	1894 Feb. 16	−0.31	+0.09	−0.40	Dec. 16	+0.15	+0.08	+0.07
Mar. 24	+0.10	−0.06	+0.16	Mar. 18	−0.13	+0.08	−0.21	1895 Jan. 14	−0.01	+0.08	−0.09
Apr. 12	+0.11	−0.11	+0.22	Apr. 15	−0.12	+0.06	−0.18	Feb. 19	−0.10	+0.04	−0.14
May 10	−0.36	−0.16	−0.20	May 16	−0.31	+0.04	−0.35	Mar. 18	−0.12	+0.01	−0.13
June 27	−0.49	−0.19	−0.30	June 22	−0.23	+0.02	−0.25	Apr. 15	+0.02	−0.03	+0.05
July 30	−0.30	−0.14	−0.16	July 18	−0.07	+0.01	−0.08	May 23	−0.14	−0.09	−0.05
Aug. 14	−0.41	−0.09	−0.32	Aug. 7	−0.20	0.00	−0.20	June 16	−0.09	−0.13	+0.04
Sept. 15	−0.21	−0.01	−0.20	Sept. 21	−0.24	−0.03	−0.21	Aug. 24	−0.19	−0.07	−0.12
Nov. 4	+0.02	+0.10	−0.08	Oct. 22	−0.22	−0.04	−0.18	Sept. 15	−0.02	−0.08	+0.06
Dec. 7	−0.18	+0.13	−0.31	Nov. 11	−0.13	−0.03	−0.10	Oct. 16	−0.10	−0.08	−0.02
1894 Jan. 2	−0.07	+0.12	−0.19	Dec. 10	−0.19	−0.03	−0.16	Nov. 14	+0.36	−0.07	+0.43
				1895 Feb. 13	+0.03	−0.05	+0.08	Dec. 15	+0.31	−0.03	+0.34
				Mar. 21	+0.09	−0.05	+0.14	1896 Jan. 16	+0.26	+0.03	+0.23
				Apr. 24	+0.06	−0.02	+0.08	Feb. 15	+0.12	+0.08	+0.04
				May 11	+0.01	+0.01	+0.03	Mar. 13	+0.10	+0.12	−0.02

These data have been taken, for Kiel, from the *Astronomische Nachrichten*, Band 140, pp 105 106, and for Pulkowa and Washington, which are contemporaneous series, from Dr. Albrecht's Report on the Variation of Latitude for 1898.

The first column of the table contains the mean date of the latitude for that month; the second contains the observed variation of the latitude; the third gives the computed variation as determined by Dr. Albrecht, and the last column contains the difference between the observed and the computed.

As I have previously remarked, the Washington instrument has been the subject of some criticism, coupled with the statement that first-class work could not be secured with it. An examination of the last column in the series of observations made with the Kiel instrument will be very significant when those differences are compared together. I invite attention also to the Pulkowa differences, and to the fact that a glance down the last column in that series indicates large systematic errors in the observed latitude determined with the largest of the three instruments. It extends from June 1893 up to the middle of 1895.

The Pulkowa prime vertical is the instrument with which Struve determined the constant of aberration that has been used in the English, American, and German ephemerides for the past forty years. Professor Kostinsky who observed these latitudes is an able astronomer. Has the Pulkowa prime vertical also degenerated into an ordinary instrument after these fifty years, when its results are compared with those made with zenith-telescopes?

I give this comparison of a continued series made with these instruments, and I make no comment upon them, leaving to the unbiased astronomer these data that he may judge for himself how much justice there has been in the criticisms upon the Washington instrument.

The method of observing a star on the prime vertical is so simple, and the instrumental errors may be eliminated in so complete a manner that, as a preface to the observations to be published in this paper, I desire to give the result of my investigation upon the question of how completely they have been eliminated.

To consider each step of that investigation, I wish to remark that the prime vertical method of observing a star, either for latitude, or for its declination, depends upon the transit-time from the east to the west vertical, as shown by the face of a clock, one-half of the elapsed time measures one side of a spherical triangle, the other two sides being 90° minus the latitude, and 90° minus the declination.

The hour-angle thus determined can be influenced to give an erroneous value by being involved in the following conditions:

First, the rate of the clock. If that is appreciable, it will increase or decrease the length of the hour-angle, depending upon its sign. Second, a change of atmospheric conditions during the interval after the star has crossed the east vertical, and continued until it has passed the west side, sufficient to produce an appreciable differential refraction. Third, a wrong level-determination of the axis of the instrument. Fourth, a change of azimuth during the elapsed time between the star's crossing the east and the west verticals. Fifth, the probability that the pivots are not true cylinders, or are conical in shape. Their unequal size, if such obtained, would not enter the observation as an error because it would be eliminated by the method of securing the position of the star. Seventh, the verticality of the transit-threads.

The first condition mentioned, the clock rate, has been eliminated from all my observations.

It can be demonstrated that, for a star having as large a zenith-distance as 1° 43′ in the meridian, when observed on the prime vertical, the instrument may be moved as much as 1.7 seconds of time before an error as large as 0″.01 will occur in the declination, from that cause.

For stars nearer the zenith that deviation of time of course increases, amounting to something like five seconds for *α Lyrae*.

I assume it is understood that the most complete method of observing with the prime vertical is used, that is, a reversal of the instrument on both the east and the west verticals. Four level-readings are always made before and after each set of transits, or sixteen for each observation.

As an example of the stability of the instrument I give here the level corrections to the eight stars that were observed between $17^h\ 45^m$ and $0^h\ 46^m$ on October 19, 1900, or over a period of 7 hours. The temperature was 56°.3 when I commenced, and it was 46°.0 when I finished. The figures in the first column are the right-ascension of the several stars.

Sidereal Hour h m	Level Correction ″
18 33	−0.49
20 56	− .64
21 39	− .63
21 49	− .53
22 12	− .61
22 38	− .78
23 16	− .94
23 55	−0.81

With our instrument, during the interval that a star, whose meridian zenith-distance is 1° 43′ south, is passing from the first thread before reversal, to the same thread after the instrument is reversed, it has decreased its zenith-distance by 22′, and on the west side, during the time it is being observed, it will increase it by a like amount.

In that is the elegance of the prime vertical method of observing. Each transit over the several threads is made at the same zenith-distance on both sides of the meridian for each of the threads.

Now suppose a marked change in the temperature and barometer happened after the star was observed on the east side, what element of differential refraction would it produce, and would it be worth while to consider it as an error in the observation needing to be removed? This can be best illustrated by an example. We will assume when the observer secured the transit of a star, east, the one with the largest zenith-distance of any I am observing, namely, 15° 44′ in the prime vertical, that the temperature of the air was 65°, the barometer reading 29.90 inches, and its attached thermometer indicated also 65°. The star I have taken for the example is *θ Aurigae*, at 37° 11′ north declination. The elapsed time between verticals for it is $2^h\,40^m$. We will also assume that on the west vertical the meteorological instruments read 50°, 29.60 in., and 50°. By using these data, we shall find that under this abnormal change of the elements its effect upon the hour-angle would be to change it by $0^s.003$, or in other words the hour-angle would be $0^s.003$ less on the west side than it had been on the east. This quantity is too small to consider when making the reduction of the observation, and its effect upon the latitude can be assumed as zero.

Now for a moment consider the question of lateral refraction. I understand by that term the effect produced by the sun shining on the upright shutters of the transit-house, as has obtained in the old and new prime vertical observing rooms here in Washington, sufficient to change, laterally, the position of the star as seen through the column of heated air. In the first place it has never been demonstrated that lateral refraction great enough in magnitude to bias a transit of a star across the prime vertical exists other than in theory. Granting that it does, the object-glass of the instrument is not in that stratum of air that may be heated by the sun shining on the shutters, but it is a stratum from 10 to 12 feet below it. It does not make the least difference what changes are happening in the successive layers through which the light from the star is seen, so far as any refractive effect is concerned. The only shell of air that can produce refractive influences on the ray of light is that one which is directly over the object-glass, and the last through which the ray passed.

In dealing with lateral refraction all that conspires in it to influence a prime vertical observation is its effect upon the hour-angle, which it would, if potent enough, carry toward the north, making it shorter.

Take the simple formula $\tan\delta = \tan\varphi\cos h$, in which δ, φ and h are the declination, latitude, and hour-angle. Differentiate it, and we have $d\,\delta = -\tfrac{1}{2}\sin 2\delta\tan t\,dt$.

Assume the error dt, or lateral refraction as $0''.2$, and its maximum effect upon *α Lyrae* (the only star I am able to observe with our instrument at mid-day), would be $0''.01$. If lateral refraction is a function of the cosine of the sun's azimuth, as has been suggested by Prof. NEWCOMB, in *A.J.* 263, page 182, then at any other time than the coincidence of the star's crossing the prime vertical, and the sun the meridian, it would be less than $0''.01$.

The third condition relates to the error of level of the axis, and the investigation that has been given it. As previously stated, four separate readings are made before and after each set of transits.

To examine thoroughly the probability of any systematic error arising from level-determinations, I have frequently made observations by reflection over mercury. In this connection it may be well to add that I do not know of any other observer, constantly using the prime vertical, who is testing his level-results by that method.

One who observes a star direct and also over the mercury has the best test that can be employed to check level-corrections. Theory explains that, for whatever level is secured, in its application to the reduction of an observed latitude, to free it from inclination of the axis when the transits were obtained, its sign will be opposite in a reflected to what it is in a direct observation. Therefore, observations made on following days and by the two methods would make any systematic error of level plainly evident.

The following table contains individual latitudes, given in two columns, of reflected and direct observations. In each instance the former corresponds to a date within one to three days of the latter. It will be noted that a systematic difference does not exist between these: they were made in the day-time as well as at night, and they are entirely controlled by the seeing that obtained when they were made:

Direct Observations	Reflected Observations
38° 55′ 14″.33	38° 55′ 14″.50
14.96	14.42
14.76	14.50
14.80	14.39
14.67	15.03
14.47	14.47
14.62	14.81
14.24	14.94
14.65	14.60
13.90	14.41
14.56	14.83
14.63	14.36
14.43	14.85
14.07	14.29
38 55 14.54	38 55 14.60

The fourth condition is the probability of a change of azimuth taking place while the star is passing from the east to the west vertical. Let δ express the declination, h the hour-angle of the star, and φ the latitude, then,

$$\cos h = \tan\delta\cot\varphi \qquad (1)$$

In the event of a change of azimuth, its effect would be to add algebraically an interval of time to h. If we differentiate equation (1), with respect to δ and h, we have

(2) $$\Delta\delta = -\Delta_0 \cos^2\delta \tan\varphi \sin h$$

Now let L represent the angle which the meridian of the instrument makes with the true meridian, and $a = L \sin\varphi$, in which a will express the azimuth of the rotation axis, put $\Delta h = \frac{1}{2}\Delta L$, and eliminate h from equation (2), we have

(3) $$\Delta\delta = \frac{1}{2}\Delta L \cdot \frac{\cos\delta\sqrt{[\sin(\varphi+\delta)\sin(\varphi-\delta)]}}{\cos\varphi}$$

(4) $$= \Delta a \frac{\cos\delta\sqrt{[\sin(\varphi+\delta)\sin(\varphi-\delta)]}}{\sin 2\varphi}$$

For *α Lyrae* the following table, computed by formula (4), represents the effect of a change of azimuth, by tenths of a second of arc, upon the observed declination.

a	$\Delta\delta$	a	$\Delta\delta$
″	″	″	″
0.1	0.005	0.6	0.030
.2	10	.7	35
.3	15	.8	40
.4	20	.9	45
0.5	0.025	1.0	0.050

The elapsed time for the several stars I am observing between the east and the west vertical, varies from one hour to two hours and forty minutes. In that period the changes of temperature are usually small and I do not think them sufficient to disturb the azimuth as much as 0″.3, which would produce an error of 0″.01 in the declination. However, to have control of the azimuth, a mark would remove all doubt upon that point.

The collimation-error of the instrument need not be considered, for the process of observing eliminates that.

In the field of the eyepiece are seven threads, in two groups, each side of the central thread, and far enough from that to permit the instrument to be reversed after the star has passed the last thread, and before it would again cross it. In other words, the motion of the star is so slow that the instrument can be reversed after the object has made a transit over either of these groups and again reset. This is the Struve method of observing, and it removes the error of collimation from each set of transits.

The fifth condition is one that I have given a thorough examination. The polished surface of each pivot is an inch wide, but the central portion that rests in the Y's is about two-tenths of an inch. I made the following experiments to ascertain if either pivot had a conical form, or was not properly centered in the cube, and if so, how much error was likely to arise from a departure from a true cylinder for each pivot.

The instrument resting its position for observing, I placed the level on the pivots as far toward the south as possible. I then read the level, moved the frame a slight distance toward the north, and then made another reading. I continued these readings until the level had been moved as far as possible toward the north. It was then moved back by these several steps to the south end of the pivot. The level was reversed, and two sets of readings forward and back, were made, reversed back to its first position, and the same process gone through with.

I made a scale, and divided the length of the pivot over which these experiments could be made, into seven parts, and by using that I was confident the level was placed in practically the same position for each of these readings. The following table contains the results, for the two positions of the clamp, and also the direction in which the object-glass was facing.

This experiment is a delicate one, and it may be expected to have variations in the readings. It will be noticed that the central portion of each pivot has become slightly worn from the use of the instrument during the past eight years, but these level-results do not indicate that the pivots to the prime vertical are conical in form. If they were so, then these readings, as the frame is moved along on the pivot, would have an increasing or decreasing value, depending upon which end of the pivot was the larger. In the fifth experiment a gradual decrease of the level is somewhat apparent.

The part of the pivot that rests in the Y, and also the opposite side upon which the level frame is placed, when observing with the instrument, is that portion in these readings half-way on either side of the central result in each set.

I do not deem these investigations conclusive, and they are to be continued. In the table the plus sign shows that the south end of the axis was high.

Cl. / O.G. North East	South East	South East	North East	North West	South West
″	″	″	″	″	″
−1.77	+1.23	+1.45	−1.00	−1.37	+2.41
−1.03	+ .60	+1.22	− .74	−1.24	+2.08
− .18	+ .01	+ .98	− .56	−1.06	+1.39
− .38	− .43	+ .57	− .84	− .51	+ .86
− .41	− .31	+ .09	−1.28	− .16	+ .80
− .92	− .34	− .06	−1.61	− .11	+ .80
−1.06	+ .14	+ .53	−1.72	+ .24	+1.38

The verticality of the transit threads is from time to time tested by the aid of a small collimator temporarily mounted in the observing room.

In observing with our prime vertical I have always recorded the transits on the chronograph. I have found that more convenient, and in observing a star that is constantly changing its zenith-distance, it is more conducive to accuracy, in my judgement, to have but one thought to occupy the mind when observing instead of attempting to keep the beat of the clock in the mind, a pencil and the observing book in the hand with which to record, and at the same time either depress or raise the eye-end of the transit to keep the star in the center of the field. But above all, the observer should refrain from constantly changing the focus of the eye, by transferring the vision from a slightly illumi-

nated field to the face of a clock made visible by a lamp. There is more vicious personal equation in transits, when an observer changes the position of his eye from a large metal face made bright by a light to a field crossed with small dark lines faintly illuminated, than there is in all other forms of personal equation combined.

Observing in the prime vertical and in the meridian are not at all similar. In the meridian a transit is across threads that are at right-angles to the motion of the star, and the motion, except in stars well to the north, is quite rapid. On the prime vertical the motion of the star over the threads is at an acute angle with respect to their position. Moreover, the star has not that rapid motion as in meridian-transits, and appears to slide along the thread, producing practically a bisection. The cause is the resultant of two motions of the star, one downward or upward, depending upon which vertical it is being observed, and the other across the field of the eyepiece.

The equatorial interval of the transit-threads in our prime vertical, is three seconds, and for *α Lyrae*, due to its position, the interval is about thirty-one seconds.

The smallest interval between threads in any of the stars I am observing is eleven seconds, and to consider that a form of personal equation can exist when observations are recorded on the chronograph, and made on the prime vertical, with intervals of motion from four to ten times as large as would obtain for an equatorial motion, does not seem probable. In the near future I hope to have the data to confirm or disprove that theory.

The determinations of latitude that follow are based upon the star-positions as printed in Prof. NEWCOMB's New Fundamental Catalogue, and are a rediscussion of all observations to conform to that catalogue. The table includes observations up to the end of 1900, and there are 1522 separate observations. All have been reduced from mean to apparent place by the use of the Besselian star numbers, as given in the *American Ephemeris* for log A and log B. Log C and log D, which depend upon the aberration, I have computed myself, using as the constant 20″.50.

The following is a list of stars used in obtaining these latitudes, their places for 1900.0, and also their proper motions.

Name of Star	R.A.	Mean Declination 1900.0	Proper Motion	Name of Star	R.A.	Mean Declination 1900.0	Proper Motion
	h m s	° ′ ″	″		h m s	° ′ ″	″
ρ Andromedae	0 15 51	+37 24 52.85	−0.037	*e Herculis*	17 14 13	+37 23 46.32	+0.059
μ Andromedae	0 51 12	37 54 25.13	+0.007	*ρ Herculis*	17 20 14	37 14 15.80	+0.005
ρ Persei	2 57 59	38 27 10.20	−0.115	*θ Herculis*	17 52 49	37 15 49.08	+0.004
θ Aurigae	5 52 12	37 12 20.41	−0.095	*α Lyrae*	18 33 33	38 41 25.71	+0.275
Groombridge 1450	8 26 25	38 21 34.10	−0.179	*θ Lyrae*	19 12 54	37 57 20.09	+0.006
38 *Lyncis*	9 12 37	37 13 32.91	−0.133	15 *Cygni*	19 40 40	37 6 45.98	+0.040
31 *Leo. min.*	10 22 6	37 13 10.79	−0.112	40 *Cygni*	20 23 52	38 6 42.52	−0.059
α Canum Venatic.	12 51 21	38 51 36.29	+0.048	61[1] *Cygni*	21 2 25	38 15 27.05	+3.244
γ Bootis	14 28 3	38 41 44.25	+0.142	*τ Cygni*	21 16 50	37 37 6.35	+0.433
295 *Bootis*	14 45 11	38 13 24.01	+0.113	72 *Cygni*	21 30 41	38 5 8.23	+0.100
μ Bootis	15 20 43	+37 43 10.00	+0.081	10 *Lacertae*	22 34 46	+38 31 46.94	−0.011

MONTHLY MEANS OF THE OBSERVED LATITUDE.

Epoch	No.	φ 38° 55′ +
1894		″
April 22	10	14.73
May 14	13	14.34
June 19	37	14.31
July 13	30	14.55
Sept. 17	16	14.54
Oct. 14	27	14.49
Nov. 17	36	14.56
Dec. 15	42	14.55
1895		
Jan. 8	10	14.39
Feb. 18	17	14.40
Mar. 18	24	14.43
Apr. 14	12	14.67
May 24	15	14.31
June 9	12	14.29
Aug. 23	9	14.50
Sept. 15	27	14.54
Oct. 16	37	14.55
Nov. 16	21	14.85
Dec. 16	27	14.83

Epoch	No.	φ 38° 55′ +
1896		″
Jan. 14	18	14.87
Feb. 15	18	14.66
Mar. 13	25	14.65
Apr. 16	16	14.69
May 17	19	14.57
June 15	17	14.76
July 15	7	14.80
Aug. 15	20	14.67
Sept. 13	12	14.58
Oct. 18	24	14.35
Nov. 19	10	14.51
Dec. 16	22	14.31
1897		
Jan. 16	17	14.47
Feb. 16	17	14.19
Mar. 20	21	14.33
Apr. 17	30	14.54
May 18	32	14.65
June 15	27	14.47
July 19	10	14.58

Epoch	No.	φ 38° 55′ +
1897		″
Aug. 19	20	14.41
Sept. 9	15	14.35
Oct. 11	12	14.12
Nov. 16	25	14.52
Dec. 30	18	14.56
1898		
Feb. 18	30	14.36
Mar. 13	23	14.20
Apr. 10	12	14.24
May 18	18	14.38
June 17	21	14.56
July 17	24	14.42
Aug. 14	13	14.57
Sept. 19	45	14.49
Oct. 18	24	14.50
Nov. 16	15	14.66
Dec. 13	23	14.56
1899		
Jan. 24	10	14.54
Feb. 15	8	14.54
Mar. 16	17	14.36

Epoch	No.	φ 38° 55′ +
1899		″
Apr. 18	39	14.24
May 19	26	14.30
June 15	38	14.30
July 17	21	14.24
Aug. 15	28	14.40
Sept. 15	32	14.48
Oct. 17	31	14.52
Dec. 2	28	14.82
1900		
Jan. 24	7	14.81
Feb. 17	7	14.58
Mar. 20	11	14.28
Apr. 9	14	14.32
June 17	6	14.75
July 15	19	14.56
Aug. 12	8	14.46
Sept. 12	15	14.52
Oct. 21	21	14.64
Dec. 7	14	14.58

The following are the yearly mean latitudes, taken from the table just preceding. The first column is the year, the second the number of observation upon which the mean is based, and the third is the latitude for the year.

Year	No. Obs.	Lat. of Instru.	Year	No. Obs.	Lat. of Instru.
		° ′ ″			° ′ ″
1894	211	+38 55 14.51	1898	248	+38 55 14.45
1895	211	.52	1899	281	.43
1896	205	.61	1900	122	.55
1897	244	+38 55 14.46		1522	+38 55 14.50

Reduction to center of clock room, —.52

Latitude of Naval Observatory, +38 55 13.98
±0″.03

Washington, D.C.

ON THE ORBIT OF ξ *BOOTIS*,

($\alpha = 14^h\ 46^m.8\ \ ;\ \ \delta = +19°\ 31′$),

BY GEORGE C. COMSTOCK.

In the work entitled "Researches on the Evolution of the Stellar System," Vol. I, Professor T. J. J. See has derived for the binary star ξ *Bootis*, an orbit based upon a fairly continuous series of observations extending from 1821 to 1895, supplemented by a few scattered observations by Sir W. Herschel made prior to the former date. The degree of precision attributed to this orbit by the author is represented by the following quotation from page 157 of the work cited: "It is possible that the period may be varied by so much as one year, and that the eccentricity is uncertain to the extent of about ±0.02; larger alterations in these quantities are not to be expected, and the values of the other elements are correspondingly well determined."

My attention has been especially called to this orbit and to its derivation by the large discordances between the ephemeris based upon it and the results of my own recent observations. These discordances, which in the present year amount to nearly 10° in position-angle and 0″.9 in distance, have accumulated during the past six years, since the orbit represents very well the observations prior to 1895. This rapid accumulation of errors points to something abnormal in the data upon which the orbit is based, and suspecting that the errors were contained in the early observations by Herschel, I have plotted upon Professor See's diagram representing the apparent orbit of the star, the following observations which comprise all the data subsequent to 1895 that are now accessible to me. These observations may be compared with Professor See's ephemeris, a part of which is here reproduced.

Date	θ	ρ	Obs.	Observer
	°	″		
1896.43	219.1	2.74	3	Greenwich observers
1896.49	222.7	2.80	3	Comstock
1897.33	219.2	2.82	1	Greenwich
1897.47	219.0	2.93	2	Brown
1897.50	216.2	2.90	2	Comstock
1898.54	212.6	2.78	3	Comstock
1899.21	208.7	2.72	3	Aitken
1899.40	208.1	2.70	2	Greenwich
1899.42	208.5	2.71	1	Brown
1899.43	208.1	2.99	3	See
1899.54	208.6	2.77	3	Comstock
1900.44	198.2	2.98	1	Greenwich
1901.46	196.1	2.72	5	Comstock

Professor See's Ephemeris.

	°	″
1896.50	221.2	2.65
1897.50	216.2	2.53
1898.50	210.1	2.40
1899.50	203.1	2.25
1900.50	195.7	2.06
1901.50	186.1	1.83

Through the plotted points corresponding to these observations and through the points of Professor See's diagram for all dates later than 1820 I have drawn the best ellipse that I can fit to them, and find that this curve, whose area is about 40 per cent. greater than that of Professor See's orbit, represents very well the recent observations, while between the years 1840 and 1895 it is practically indistinguishable from Professor See's curve. Between 1820 and 1840 the new ellipse fits the plotted points rather better than does the old one, the sums of the squares of the residual errors, in millimeters, being 289 as against 328 for Professor See's curve. During the past 80 years, therefore, within which the companion star has moved over an arc of rather more than 140°, there is no considerable period within which Professor See's ellipse represents the data better than does the larger one which I have substituted for it, while in some measure at the beginning of the period and in very marked degree at its close the new ellipse is to be preferred to the old one.

Of Professor See's data there remain to be considered only the two plotted points, to which he assigns the dates 1780 and 1804, which I have not used in drawing the apparent orbit. The observations corresponding to these points are given by Professor See and by Sir John Herschel (A Synopsis of all Sir William Herschel's Micrometrical Measurements, etc.), as follows:

Herschel			See		
Date	θ	ρ	Date	θ	ρ
	°	″		°	″
1780.662	. . .	$1\frac{1}{2}$D	1780.69	24.1	3.23
1781.266	. . .	3.23	1804.25	353.9	6 ±
1782.285	24.12	. .			
1804.252	353.90	>3.23			

In a paper read before the Royal Astronomical Society in 1833, Herschel, in commenting upon these observations

and certain others of the same star, remarks that it "seems impossible to conciliate them all with any supposable orbit," an opinion in which I think everyone must concur who examines the data. In the same paper he reproduces what is apparently a memorandum made by W. HERSCHEL in 1804, that the companions of ξ *Bootis* are "farther apart than those of π *Bootis*," and comments upon it as follows: "The distance of the latter star is 6″.889, and since a judgment of this kind, to be decisive, supposes some considerable difference between the objects compared it is probable that the interval between the two stars must have been 9″ or 10″ at the least." However, for his own use in computing an orbit of this star, he tabulates the distance as "$>7''$." My apparent orbit makes the distance between the stars corresponding to the position-angle 353°.9 to be 7″.3, and in view of the above quotations it seems to me to conform to the observed data quite as closely as does Professor SEE's assumed distance of 6″±.

I am unable to construct from Sir JOHN HERSCHEL's Synopsis the observation which Professor SEE gives for the date 1780.69, but assuming it to be as printed, I find corresponding to this position-angle in my apparent orbit a distance of 5″.0, while Professor SEE's orbit furnishes 2″.8. The discrepancy of 1″.8 furnished by the former orbit does not seem to me intolerable in view of the errors affecting many of HERSCHEL's observations, but whatever interpretation may be placed upon this particular observation, I prefer the concurrent testimony of the modern results, which are quite irreconciliable with any such distance as that assumed by Professor SEE for the epoch 1780.

Washburn Observatory, 1901 July.

The observations subsequent to 1820 are hardly sufficient to determine a satisfactory orbit, but as I have not found it feasible to draw through the plotted points of the diagram any ellipse differing greatly from my apparent orbit, I have derived from the latter, by the method of KLINKERFUES, the following elements which may serve as a preliminary determination. For the sake of comparison Professor SEE's elements are placed beside them.

	COMSTOCK	SEE
P	172 years	128.0 years
T	1908	1903.90
e	0.49	0.721
a	5″.10	5″.5578
☊	165°.	10°.5
i	35.	52.28
λ	8.	239.25
η	−2.095	−2.8125

A criterion for deciding between these elements will be furnished by the observations of the next few years. According to Professor SEE the motion in position-angle will be very rapid, and the distance will decrease notably, so that for the epoch 1904.50 we shall find the coordinates of the companion as given below, while from my apparent ellipse I find them to be, approximately, as shown in the second line of the table.

Epoch	θ	ρ	Computer
	°	″	
1904.50	125.5	1.03	See
1904.50	180.	2.6	Comstock

OBSERVATIONS OF COMET 1900 III (*GIACOBINI*),

MADE AT THE CHAMBERLIN OBSERVATORY, UNIVERSITY PARK, COLORADO,

BY CHARLES J. LING.

The following observations were made with the Bruce Micrometer on the twenty-inch equatorial, the magnifying power being two hundred. The right-ascension observations are chronographic. The declination bisections were made while the object was drifting through the field.

1901 Univ. Park M.T.	∗	No. Comp.	☄ − ∗ $\Delta\alpha$	☄ − ∗ $\Delta\delta$	☄'s apparent α	☄'s apparent δ	log $p\Delta$ for α	log $p\Delta$ for δ
	h m s		m s	′ ″	h m s	° ′ ″		
Jan. 10 8 39 4	1	20 , 6	−2 55.93	−12 15.4	0 36 56.51	−22 45 21.9	9.578	0.834
10 8 53 19	2	10 , 2	−2 23.75	− 7 57.9	0 36 58.78	−22 45 1.2	9.600	0.826
14 7 20 15	3	20 , 6	−4 10.06	− 1 39.4	0 57 21.44	−22 13 49.0	9.382	0.870
14 7 36 29	1	20 , 6	−3 36.94	− 7 15.0	0 57 25.13	−22 13 41.6	9.432	0.864
15 7 47 29	4	20 , 6	+1 26.08	+ 1 36.9	1 2 28.13	−22 4 19.7	9.459	0.860
16 7 55 1	5	20 , 6	+3 50.71	+ 8 33.5	1 7 26.41	−21 54 32.0	9.475	0.857
16 8 9 46	6	20 , 6	+4 11.47	+14 26.4	1 7 30.53	−21 54 19.1	9.508	0.850
18 8 6 22	7	20 , 8	−1 5.97	+ 5 32.8	1 17 13.54	−21 33 12.6	9.496	0.852
18 8 20 45	8	19 , 6	−2 38.03	+ 3 44.3	1 17 15.82	−21 33 5.4	9.526	0.845

Mean Places for 1901.0 of Comparison-Stars.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	s	
1	0 39 51.77	+0.67	−22 33 5.1	−1.4	Yarnall 371
2	0 39 21.87	+0.66	−22 36 58.9	−1.4	Argentine General Catalogue 665
3	1 1 30.45	+0.72	−22 12 4.4	−5.2	Weiss's Argelander 514
4	1 1 1.35	{ +0.72 { +0.70	−22 5 51.4	{ −5.2 { −5.2	" " 506
5	1 3 35.00	+0.70	−22 3 0.2	−5.3	" " 533
6	1 3 15.36	+0.70	−22 8 40.2	−5.3	" " 531
7	1 18 18.73	+0.78	−21 38 39.7	−5.7	" " 658
8	1 19 53.06	+0.79	−21 36 44.0	−5.7	" " 677

University Park, Colorado, 1901 *July* 12.

CORRECTIONS TO OBSERVATIONS OF COMET *b* 1900 II, IN *A.J.* 491–492,

By CHARLES J. LING.

On account of a systematic error in the reductions of the comparison-stars above 60° north declination, the positions of Comet *b*, 1900 II, in *A.J.* 491–492, pp. 95, 96, are incorrect after 1900 Aug. 6. The following are all the data there given that require emendation:

Correct Comet's Apparent Places.

1900	Comet's Apparent α	Comet's Apparent δ	log. *p* Δ for α	log. *p* Δ for δ
	h m s	° ′ ″		
Aug. 15	3 43 11.44	+71 29 53.5	*n*0.113	0.755
17	4 2 24.14	77 56 9.7	*n*0.240	0.710
20	4 51 58.09	82 12 12.2	*n*0.343	0.764
21	5 21 3.93	83 26 51.6	*n*0.520	0.592
22	5 59 11.17	84 28 41.7	*n*0.651	0.414
23	6 46 10.08	85 13 24.0	*n*0.668	0.595
25	8 41 18.83	85 50 50.1	9.261	0.855
Sept. 28	14 25 21.18	+69 18 56.3	0.010	0.763

Correct Comparison-Star Places.

*	α (1900.0)	Red. to app. place	δ (1900.0)	Red. to app. place
	h m s	s	° ′ ″	″
15	3 55 19.03	+ 6.59	+74 21 45.1	− 9.6
16	4 5 36.13	+ 7.79	77 49 43.4	11.0
17	4 47 48.88	+10.04	82 22 1.6	14.6
18	5 11 47.06	+11.15	83 46 34.9	16.3
19	5 53 51.50	+ 9.77	84 11 51.6	18.5
20	6 40 4.64	+ 8.02	85 19 46.6	20.4
21	8 47 3.56	− 2.22	85 57 9.5	21.2
22	14 27 26.14	− 2.58	+69 29 30.0	− 2.9

PROPER MOTION OF BONN A.G. CATALOGUE, NOS. 2435 AND 2485,

By CHARLES J. LING.

Star No. 2435 of the A.G. Catalogue (Bonn) has a proper motion of about $+0^s.053$ in R.A., and $-0''.31$ in Declination. This star was used as a comparison-star in *Eros*-observations on Oct. 2 and 3, 1900, and the positions of *Eros* did not agree with those obtained by comparisons with Bonn, Nos. 2423 and 2427, which were also used as comparison-stars on Oct. 2 and 3 respectively. This led to a micrometric comparison of No. 2435 with Nos. 2423 and 2427, the results of which show the above proper motion.

No. 2435 was also compared with No. 2485, and a study of all of the results show a proper motion for No. 2485 of $-0^s.012$ in R.A., and $-0''.03$ in Declination.

A study of R. H. Tucker's Meridian-Circle Observations in the Lick Observatory Bulletin, No. 1, confirms the fact of a large proper motion of No. 2435, though his observations seem to indicate slightly larger values. The proper motion of No. 2485 appears to be confirmed.

University Park, Colorado, 1901 *July* 5.

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ECLIPSE-CYCLES.

By JOHN N. STOCKWELL.

Having devoted considerable time during several years to the computation of ancient eclipses, the question of their periodicity has very naturally received some attention; and the results of my studies on the subject are deemed of sufficient importance to be preserved for the benefit of astronomers. The problem of eclipse-cycles is not new; but astronomers during the past three thousand years seem to have been content with the celebrated Chaldean eclipse-cycle of 18 years and 10 days, called the *Saros*, which has been known from the remotest antiquity. There is, however, another eclipse-cycle of somewhat longer period, but vastly more accurate, to which I wish to call the attention of astronomers.*

In order to investigate this cycle we may observe that according to observation, the line of conjunction of the sun and moon advances in longitude in each lunation through an arc of $29^\circ.105\,616\,7892$; and during the same interval of time the moon's node retrogrades in longitude through an arc of $1^\circ.564\,872\,5724$. If we denote the moon's longitude by nt, and the longitude of the moon's ascending node by Ω, and suppose them to be in the same straight line at a given instant, at the end of one lunation we shall have:

$$nt-\Omega = 30^\circ.670\,489\,3616 \quad (1)$$

This equation will give $6(nt-\Omega)=184.022\,936\,1696$ (2)

$$41(nt-\Omega)=177.490\,063\,8256 \quad (3)$$

The sum of (2) and (3) is $47(nt-\Omega)=1.512\,999\,9952$ (4)

The sum of (3) and (4) is $88(nt-\Omega)=179.003\,063\,8207$ (5)

The sum of (4) and (5) is $135(nt-\Omega)=180.516\,063\,8159$ (6)

The sum of (5) and (6) is $223(nt-\Omega)=359.519\,127\,6366$ (7)

The sum of (6) and (7) is $358(nt-\Omega)=180.035\,191\,4525$ (8)

Each of these equations represents an *eclipse-cycle*; but the first four are very imperfect and will receive no further attention.

The sun's ecliptic limits being 18° on each side of the node, it follows that there is an arc of 36° along the ecliptic through which the moon must pass in each lunation; and if the sun happened to be in the same arc an eclipse would take place. Suppose now that a conjunction took place at say 17° to the westward of the node, there would be a small eclipse; and since the line of conjunction advances with respect to the node 30°.67 in each lunation, it follows that another conjunction and eclipse would take place at a distance of 13°.67 to the eastward of the node. Moreover, in five lunations the line of conjunction advances 153°.35, and this added to 13°.67 gives 167°.02; or a fifth conjunction and eclipse would take place at a distance of 12°.98 to the westward of the other node. If we subtract this from 30°.67 we get 17°.69, as the distance to the eastward from the node at which a conjunction and small eclipse would take place. We thus see that eclipses of the sun may take place at intervals of one lunation and also at intervals of five lunations, although the usual interval is six lunations. We also see that there may be two eclipses within the ecliptic limits of each node, and consequently there may be four eclipses of the sun in the course of a year. The usual number, however, is only two, one at each node, at intervals of six lunations.

In the cycle of 88 lunations there are 88 conjunction points distributed along the ecliptic at distances of 4°.0909 from each other; and since the ecliptic limits at the two nodes amounts to 72°, or one-fifth of the circumference, it follows that there will be one-fifth as many eclipses as there are lunations in the cycle. In this cycle there will therefore be seventeen or eighteen eclipses; of which ten or eleven will be central somewhere on the earth. Each of these eclipses will begin its cycle as a small one near one pole of the earth. In 88 lunations it will recur at a considerably greater distance from the opposite pole; and after five or six recurrences it will become central near the poles and gradually approach the central latitudes of the earth during eleven or twelve recurrences. It will then recede from the equator according to the same law that determined its previous approach; and finally after about

*Since the above article was put into type I have learned that OPPOLZER had previously made use of the 29-year cycle in his tables of eclipses. — S.

250 years it will have passed beyond the ecliptic limits and remain there for more than a thousand years. The cycle of 88 lunations or 7.11 years, although convenient for a short period of time would require too frequent adjustment to be of much value in the discussion of the eclipses of ancient times. In general, eclipses would happen at new conjunction points every 14½ years.

The cycles of 135 lunations and of 223 lunations possess nearly the same degree of accuracy; but the former gives 27 eclipses, each of which will recur either as a partial or central eclipse, about 70 times, embracing some 761 years; while the latter gives about 45 eclipses, each of which recurs about 75 times, and includes a period of 1350 years. Neither of these cycles is convenient or useful in the calculation and discussion of ancient eclipses; although the cycle of 223 lunations has become justly celebrated as the only eclipse-cycle known; and was discovered, or at least made known to the Arabians by the ancient Chaldeans. In general in these two cycles eclipses would happen at new conjunction points at intervals of 28 and 30 years respectively.

We shall now consider the eclipse-cycle of 358 lunations. This cycle seems to possess every desirable attribute for the prediction and discussion of eclipses in all ages. Its period of about twenty-nine years (more exactly 28 years and 345 days), is not long enough to make it cumbersome and unwieldy; but its most valuable quality is the extreme slowness with which the most important eclipse-elements change in passing from cycle to cycle. We have already seen that if a conjunction takes place exactly at a node, after 358 lunations a conjunction will take place only 0°.0352 or 2′ 6″ beyond the other node; whereas, in the celebrated Chaldean cycle of 18 years, a new cycle would begin at a distance of 28′ 51″ from the node. This slow change of the elements is of the utmost importance in the theory of eclipses; and we shall here explain and illustrate it with all the detail its importance requires.

As there are 358 conjunction points in this cycle, it follows that they must be uniformly distributed along the ecliptic at intervals of 1°.005587; and about 72 of these conjunction points must be within the ecliptic limits of both nodes; and that number of eclipses will occur in each cycle. If then a conjunction point has come within the ecliptic limits a small partial eclipse will take place near one pole of the earth; and after 358 lunations a second conjunction and eclipse will take place only 2′ 6″ nearer the opposite node and a little larger eclipse would take place at the opposite pole. This process will be repeated about 200 times, embracing a period of 5800 years before the conjunction point approaches sufficiently near the node for the eclipse to become central. It will then continue to be central during about 625 cycles or 18100 years, during which time it will have passed to the equator and retreated again to the poles, when it will become a partial eclipse, growing smaller and smaller in each successive cycle, for a period of 5800 years, when the conjunction point, at which it took place, will have passed beyond the ecliptic limit, where it will remain during a period of about 118400 years. In general, a new conjunction point will come within the ecliptic limits on an average of about once in an interval of 113.54 years; but just as many would pass out as came in.

A conjunction point therefore remains within the ecliptic limits during a period of about 1025 cycles, which amounts to about 29600 years; and is many times longer than history or tradition reaches into the past. In order to give practical value to this cycle, it is therefore only necessary to prepare a table showing the *mean elements* of all the eclipses which occur during the brief period of 358 lunations or *twenty-nine* years. This I have done in Table I for eclipses of the sun, and in Table II for eclipses of the moon; which will now be explained.

It is a matter of no consequence as to what period of time these 358 lunations cover; but as all astronomical tables must have an *epoch*, I have adopted the time of *mean new moon* of May 28, A.D. 1900, as the *epoch* of the tables for *past eclipses;* and the time of *mean new moon* June 17, A.D. 1871, as the *epoch* for *future eclipses.* In days of the JULIAN PERIOD these dates are, May 28, 1900 $=2415168^d.1988$; and June 17, $1871=2404596^d.2475$, in Washington mean time of the JULIAN PERIOD.

In the preparation of Table I, I have supposed the sun's ecliptic limits to be 25° instead of 18°, on each side of the node; so that the table really includes about twenty-eight conjunction points which are at present outside of the ecliptic limits, but which will gradually approach and finally come within them. This extension of the table beyond present needs adds about 6000 years to the life of the table, and enables us to investigate with very little labor, eclipses that took place 35000 years ago; and also those that will occur during 35000 years to come. The table, therefore, in its present form really embraces a period of 70000 years; and by filling in omitted values its usefulness may easily be extended to all past and future time. The numbers given are independent of the secular variations of the elements of the moon's motion; for the secular variation of the node is the same as that of the mean longitude and consequently disappears in the term $nt_0 - \Omega_0$ of the table.

For past eclipses of the moon, the *epoch* of Table II, is the JULIAN day $2415182^d.9632$ corresponding to June 13, A.D. 1900; and for future eclipses it is the JULIAN day $2504611^d.0128$, corresponding to July 2, 1871. The lunar ecliptic limits being much less than those of the sun, a much less elaborate degree of expansion has been deemed sufficient.

TABLE I.

SOLAR ECLIPSE-CYCLE OF 358 LUNATIONS.

Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0$	ω_0	$nt_0 - \Omega_0$	Future Eclipses N
			°	°	°	
0	1900 May 28	2415168.1979	65.2755	350.1818	174.6450	358
5	1899 Dec. 31	5020.5449	279.7475	333.7383	21.2926	353
6	1899 Dec. 2	4991.0143	250.6419	330.4496	350.6221	352
11	1899 July 7	4843.3614	105.1137	314.0057	197.2697	347
12	1899 June 7	4813.8308	76.0081	310.7170	166.5992	346
17	1899 Jan. 11	4666.1779	290.4801	294.2735	13.2467	341
18	1898 Dec. 12	4636.6473	261.3745	290.9848	342.5762	340
23	1898 July 17	4488.9945	115.8463	274.5409	189.2238	335
24	1898 June 18	4459.4638	86.7407	271.2522	158.5533	334
29	1898 Jan. 21	4311.8109	301.2126	254.8084	5.2009	329
35	1897 July 28	4134.6273	126.5790	235.0762	181.1779	323
41	1897 Feb. 1	3957.4439	311.9452	215.3436	357.1550	317
46	1896 Sept. 6	3809.7909	166.4172	198.9001	203.8025	312
47	1896 Aug. 8	3780.2603	137.3116	195.6114	173.1320	311
52	1896 Mar. 13	3632.6074	351.7834	179.1676	19.7796	306
53	1896 Feb. 13	3603.0768	322.6778	175.8788	349.1091	305
58	1895 Sept. 18	3455.4239	177.1498	159.4353	195.7566	300
59	1895 Aug. 19	3425.8931	148.0440	156.1463	165.0862	299
64	1895 Mar. 25	3278.2404	2.5160	139.7028	11.7337	294
65	1895 Feb. 24	3248.7098	333.4104	136.4141	341.0632	293
70	1894 Sept. 29	3101.0569	187.8822	119.9703	187.7108	288
71	1894 Aug. 30	3071.5263	158.7766	116.6815	157.0403	287
76	1894 Apr. 4	2923.8733	13.2487	100.2380	3.6878	282
82	1893 Oct. 9	2746.6898	198.6149	80.5055	179.6649	276
88	1893 Apr. 15	2569.5062	23.9813	60.7733	355.6419	270
93	1892 Nov. 18	2421.8533	238.4531	44.3295	202.2895	265
94	1892 Oct. 20	2392.3227	209.3475	41.0408	171.6190	264
99	1892 May 25	2244.6698	63.8193	24.5969	18.2666	259
100	1892 Apr. 26	2215.1392	34.7137	21.3082	347.5951	258
105	1891 Nov. 30	2067.4862	249.1857	4.8647	194.2436	253
106	1891 Oct. 31	2037.9557	220.0801	1.5760	163.5731	252
111	1891 June 6	1890.3028	74.5519	345.1322	10.2207	247
112	1891 May 7	1860.7722	45.4463	341.8435	339.5502	246
117	1890 Dec. 11	1713.1192	259.9184	325.3999	186.1977	241
118	1890 Nov. 11	1683.5886	230.8128	322.1112	155.5272	240
123	1890 June 16	1535.9357	85.2846	305.6674	2.1748	235
129	1889 Dec. 21	1358.7522	270.6508	285.9349	178.1519	229
134	1889 July 27	1211.0992	125.1228	269.1914	24.7994	224
135	1889 June 27	1181.5686	96.0172	266.2027	354.1289	223
140	1889 Jan. 30	1033.9157	310.4890	249.7588	200.7765	218
141	1889 Jan. 1	1004.3851	281.3834	246.4701	170.1060	217
146	1888 Aug. 6	0856.7322	135.8555	230.0266	16.7535	212
147	1888 July 8	0827.2016	106.7499	226.7379	316.0830	211
152	1888 Feb. 11	0679.5487	321.2217	210.2941	192.7306	206
153	1888 Jan. 13	0650.0181	292.1161	207.0054	162.0601	205
158	1887 Aug. 18	0502.3651	146.5881	190.5619	8.7076	200
159	1887 July 19	0472.8346	117.4823	187.2728	338.0372	199
164	1887 Feb. 22	0325.1816	331.9543	170.8293	184.6847	194
170	1886 Aug. 28	2410147.9982	157.3206	151.0968	0.6618	188
176	1886 Mar. 4	2409970.8146	342.6870	131.3646	176.6388	182
Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0$	ω_0	$nt_0 - \Omega_0$	Future Eclipses N

TABLE I. SOLAR ECLIPSE-CYCLE OF 358 LUNATIONS. — *Continued.*

Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0$	ω_0	$nt_0 - \Omega_0$	Future Eclipses N
181	1885 Oct. 8	2409823.1617	197.1588	114.9208	23.2864	177
182	1885 Sept. 8	9793.6311	168.0532	111.6321	352.6159	176
187	1885 Apr. 13	9615.9781	22.5252	95.1886	199.2634	171
188	1885 Mar. 15	9616.4176	353.4197	91.8998	168.5929	170
193	1884 Oct. 18	9468.7916	207.8915	75.4560	15.2405	165
194	1884 Sept. 19	9439.2610	178.7859	72.1673	344.5700	164
199	1884 Apr. 24	9291.6112	33.2577	55.7235	191.2176	159
200	1884 Mar. 26	9262.0806	4.1521	52.4348	160.5171	158
205	1883 Oct. 30	9114.1276	218.6241	35.9912	7.1946	153
206	1883 Sept. 30	9084.8970	189.5185	32.7025	336.5241	152
211	1883 May 6	8937.2411	43.9903	16.2587	183.1717	147
217	1882 Nov. 10	8760.0605	229.3568	356.5265	359.1487	141
223	1882 May 16	8582.8770	54.7230	336.7940	175.1258	135
228	1881 Dec. 20	8435.2240	269.1950	320.3505	21.7733	130
229	1881 Nov. 20	8405.6935	240.0892	317.0614	351.1029	129
234	1881 June 26	8258.0405	94.5612	300.6180	197.7504	124
235	1881 May 26	8228.5099	65.4556	297.3292	167.0799	123
240	1880 Dec. 30	8080.8570	279.9274	280.8851	13.7275	118
241	1880 Dec. 1	8051.3221	250.8218	277.5967	343.0570	117
246	1880 July 6	7903.6734	105.2938	261.1532	189.7045	112
252	1880 Jan. 11	7726.4899	290.6600	241.4207	5.6816	106
253	1879 Dec. 12	7696.9593	261.5544	238.1320	335.0111	105
258	1879 July 18	7549.3063	116.0264	221.6885	181.6586	100
264	1879 Jan. 22	7372.1228	301.3926	201.9560	357.6357	94
269	1878 Aug. 27	7224.4700	155.8644	185.5121	204.2833	89
270	1878 July 28	7194.9391	126.7589	182.2234	173.6128	88
275	1878 Mar. 3	7047.2864	341.2309	165.7799	20.2603	83
276	1878 Feb. 1	7017.7558	312.1253	162.4912	349.5898	82
281	1877 Sept. 9	6870.1029	166.5970	146.0474	196.2371	77
282	1877 Aug. 8	6840.5723	137.4914	142.7587	165.5669	76
287	1877 Mar. 13	6692.9194	351.9635	126.3151	12.2144	71
288	1877 Feb. 12	6663.3888	322.8580	123.0264	341.5439	70
293	1876 Sept. 17	6515.7359	177.3298	106.5826	188.1915	65
294	1876 Aug. 19	6486.2053	148.2242	103.2939	157.5210	64
299	1876 Mar. 24	6338.5521	2.6960	86.8501	1.1687	59
305	1875 Sept. 29	6161.3688	108.0624	67.1178	180.1456	53
311	1875 Apr. 5	5984.1853	13.1286	47.3853	356.1227	47
316	1874 Nov. 8	5836.5323	227.9006	30.9418	202.7702	42
317	1874 Oct. 10	5807.0017	198.7950	27.6531	172.0997	41
322	1874 May 15	5659.3188	53.2669	11.2092	18.7473	36
323	1874 Apr. 15	5629.8182	24.1613	7.9205	348.0768	35
328	1873 Nov. 19	5482.1652	238.6338	351.4770	194.7243	30
329	1873 Oct. 20	5452.6347	209.5275	348.1880	164.0539	29
334	1873 May 25	5304.9817	63.9995	331.7445	10.7014	24
335	1873 Apr. 26	5275.4511	34.8939	328.4558	340.0309	23
340	1872 Nov. 29	5127.7982	249.3657	312.0120	186.6785	18
341	1872 Oct. 31	5098.2676	220.2601	308.7233	156.0080	17
346	1872 June 6	4950.6146	74.7321	292.2798	2.6555	12
352	1871 Dec. 11	4773.4311	260.0983	272.5469	178.6326	6
357	1871 July 16	4625.7781	114.5703	256.1034	25.2801	1
358	1871 June 17	2401596.2475	85.4647	252.8151	354.6096	0
Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0$	ω_0	$nt_0 - \Omega_0$	Future Eclipses N

TABLE II.

Lunar Eclipse-Cycle of 358 Lunations.

Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0 + 180°$	ω_0	$nt_0 - \Omega_0$	Future Eclipses N
0	1900 June 13	2415182.9632	259.8283	351.8262	9.9800	358
6	1899 Dec. 16	5005.7797	85.1946	332.0938	185.9572	352
12	1899 June 22	4828.5962	270.5609	312.3614	1.9343	346
18	1898 Dec. 27	4651.4126	95.9272	292.6290	177.9113	340
24	1898 July 3	4474.2291	281.2935	272.8966	353.8884	334
30	1898 Jan. 7	4297.0456	106.6598	253.1642	169.8655	328
36	1897 July 13	4119.8620	292.0261	233.4318	345.8425	322
41	1897 Feb. 16	3972.2091	146.4980	216.9881	192.4901	317
47	1896 Aug. 23	3795.0256	331.8643	197.2557	8.4671	311
53	1896 Feb. 27	3617.8421	157.2306	177.5233	184.4442	305
59	1895 Sept. 3	3440.6586	342.5969	157.7909	0.4213	299
65	1895 Mar. 10	3263.4750	167.9632	138.0585	176.3983	293
71	1894 Sept. 11	3086.2915	353.3295	118.3261	352.3754	287
77	1894 Mar. 21	2909.1080	178.6958	98.5937	168.3524	281
82	1893 Oct. 21	2761.1550	33.1677	82.1500	15.0000	276
88	1893 Apr. 30	2584.2715	218.5340	62.4176	190.9770	270
94	1892 Nov. 4	2407.0879	43.9003	42.6852	6.9541	261
100	1892 May 10	2229.9044	229.2666	22.9528	182.9312	258
106	1891 Nov. 15	2052.7209	54.6329	3.2204	358.9082	252
112	1891 May 22	1875.5374	239.9992	343.4880	174.8853	246
118	1890 Nov. 26	1698.3539	65.3655	323.7556	350.8623	240
124	1890 June 2	1521.1702	250.7318	304.0232	166.8390	234
129	1890 Jan. 5	1373.5173	105.2037	287.5795	13.4870	229
135	1889 July 12	1196.3338	290.5700	267.8471	189.4640	223
141	1889 Jan. 16	1019.1503	115.9363	248.1147	5.4411	217
147	1888 July 22	0841.9668	301.3026	228.3823	181.4181	211
153	1888 Jan. 27	0664.7833	126.6689	208.6499	357.3952	205
159	1887 Aug. 3	0487.5998	312.0352	188.9175	173.3723	199
165	1887 Feb. 7	0310.4162	137.4015	169.1851	349.3493	193
171	1886 Aug. 14	2410133.2326	322.7678	149.4527	165.3264	187
176	1886 Mar. 19	2409985.5797	177.2397	133.0090	11.9739	182
182	1885 Sept. 23	9808.3962	2.6060	113.2766	187.9510	176
188	1885 Mar. 30	9631.2127	187.9723	93.5442	3.9280	170
194	1884 Oct. 4	9451.0292	13.3386	73.8118	179.9051	164
200	1884 Apr. 9	9276.8457	198.7049	54.0794	355.8822	158
206	1883 Oct. 15	9099.6622	24.0712	34.3470	171.8592	152
212	1883 Apr. 21	8922.4786	209.4375	14.6146	347.8363	146
223	1882 May 31	8597.6421	249.2757	338.4385	10.4609	135
229	1881 Dec. 5	8420.4586	74.6420	318.7061	186.4380	129
235	1881 June 11	8243.2751	260.0083	298.9737	2.4150	123
241	1880 Dec. 16	8066.0915	85.3746	279.2413	178.3921	117
217	1880 June 21	7888.9080	270.7409	259.5089	354.3691	111
253	1879 Dec. 27	7711.7245	96.1072	239.7765	170.3462	105
270	1878 Aug. 12	7209.7045	321.3117	183.8680	8.9479	88
276	1878 Feb. 10	7032.5210	146.6780	164.1356	184.9249	82
282	1877 Aug. 23	6855.3374	332.0413	144.4032	0.9020	76
288	1877 Feb. 27	6678.1539	157.4106	124.6708	176.8790	70
294	1876 Sept. 2	6500.9704	342.7769	104.9384	352.8561	64
300	1876 Mar. 9	6323.7869	168.1432	85.2060	168.8332	58
311	1875 Apr. 19	5998.9504	207.9814	49.0299	191.4578	47
317	1874 Oct. 24	5821.7668	33.3477	29.2975	7.4348	11
323	1874 Apr. 30	5644.5833	218.7140	9.5651	183.4119	35
329	1873 Nov. 4	5467.3998	44.0803	349.8327	359.3890	29
335	1873 May 11	5290.2163	229.4466	330.1003	175.3660	23
341	1872 Nov. 15	5113.0328	51.8129	310.3679	351.3431	17
347	1872 May 21	4935.8492	240.1792	290.6355	167.3201	11
352	1871 Dec. 26	4788.1963	94.6511	274.1918	13.9677	6
358	1871 July 2	2404611.0128	280.0174	254.4594	189.9447	0
Past Eclipses N	Date	Day of Julian Period T_0	$nt_0 = n't_0 + 180°$	ω_0	$nt_0 - \Omega$	Future Eclipses N

In the tables N denotes the number of any conjunction point counted continuously from the beginning of the cycle; nt_0 and $n't_0$ denote the mean longitudes of the moon and sun respectively, while ω_0 denotes the longitude of the moon's perigee, and Ω_0 denotes the longitude of the moon's ascending node. All these longitudes are referred to the mean equinox of 1850.

If N exceeds 358, the conjunction which it denotes will not be found in the table: and we shall put

$$\frac{N}{358} = i + \frac{N - 358\,i}{358}$$

in which i denotes the number of complete cycles contained in N lunations; and if we put $N' = N - 358\,i$, N' will take the place of N in the table.

The changes in the tabular numbers for any number i of complete cycles are given by the equations

$$\Delta T_0 = 10571^{d}.95041505.i \qquad \Delta\omega_0 = 97^\circ.366702655.i$$
$$\Delta nt_0 = 339^\circ.810810534.i \qquad \Delta(nt_0 - \Omega_0) = 180^\circ.035191453.i$$

In these equations i is negative for past eclipses.

The computation of the values of these quantities may be very greatly facilitated by means of the following:

AUXILIARY TABLE.

i	$i\,.\,\Delta T_0$	$i\,.\,(\Delta nt_0 = \Delta n't_0)$	$i\,.\,\Delta\omega_0$	$i\,.\,\Delta(nt_0 - \Omega_0)$
	d	°	°	°
1	10571.95041505	339.810810534	97.366702655	180.035191453
2	21143.90083010	319.621621068	194.733405310	0.070382906
3	31715.85124515	299.432431602	292.100107965	180.105574359
4	42287.80166020	279.243242136	29.466810620	0.140765812
5	52859.75207525	259.054052670	126.833513275	180.175957265
6	63431.70249030	238.864863204	224.200215930	0.211148718
7	74003.65290535	218.675673738	321.566918585	180.246340171
8	84575.60332040	198.486484272	58.933621240	0.281531624
9	95147.55373545	178.297294806	156.300323895	180.316723077

which contains the first nine multiples of the coefficients of i.

If we add the values of ΔT_0, etc., to the quantities in the tables we shall have the numbers corresponding to any time whatever. We shall now illustrate the manner of using these tables by a few examples.

As a first example let us inquire if there was an eclipse of the sun on Oct. 20, B.C. 3784. This date corresponds to the 339611th day of the JULIAN PERIOD; and if we subtract it from the JULIAN date of the epoch for past eclipses, we shall find the interval between the two dates to be 2075557 days; and if this number be divided by the number of days in a lunation, which is $29^{d}.530588775$, the quotient will be $N = -70285$; the negative sign being taken because the given date precedes the epoch of the cycle. If we divide N by 358 we get $i = -196$, and a remainder $N' = 117$; which shows that the given date precedes the conjunction numbered 117 of the eclipse-cycle by exactly 196 cycles.

If we substitute this value of i in the preceding equations we shall find

$$\Delta T_0 = -2072102^{d}.28135 \qquad \Delta\omega_0 = -3^\circ.87372$$
$$\Delta nt_0 = -2^\circ.91887 \qquad \Delta(nt_0 - \Omega_0) = -6^\circ.89752$$

And if we add these numbers to the corresponding ones in the table we shall find, as the approximate elements of that conjunction.

$$T_0 = 339610^{d}.8379 \qquad \omega_0 = 321^\circ.5262$$
$$nt_0 = n't_0 = 256^\circ.9995 \quad \text{and} \quad nt_0 - \Omega_0 = 179^\circ.3002$$

These longitudes are all referred to the mean equinox of 1850; and would be the correct mean elements for that conjunction, were it not for the secular changes in the elements of the moon's motion which have taken place since that early date. We see, however, from the value of $nt_0 - \Omega_0$ that the conjunction took place very near to the moon's descending node, and consequently the sun was centrally eclipsed near the equatorial regions of the earth. We may here observe that if we add the preceding values of ΔT_0, etc., to all the numbers in the table we should obtain a complete cycle of eclipses for that ancient date; and if we change the signs of those quantities and add them, we should obtain a similar cycle of eclipses for a date 5700 years in the future.

The subject of the secular changes of the elements of the moon's motion does not come within the scope of this article, and we must therefore be content with the assumption of unchanging elements, which is nearly enough correct for our present purpose.

As a second example let us inquire if there was an eclipse of the sun Oct. 10, B.C. 2136, of which the JULIAN date is 941533 days. The interval between that date and the epoch of the cycle is 1473625 days, which gives

$$N = -49902, \quad i = -139 \text{ and } N' = 140$$

That conjunction therefore took place just 139 cycles before number 140 of the table, which was apparently outside the ecliptic limits; but if we compute the variation of the elements during 139 cycles we shall find

$$\Delta T_0 = -1469501^d.1077 \qquad \Delta\omega_0 = -213°.9717$$
$$\Delta nt_0 = -73°.7027 \qquad \Delta(nt_0 - \Omega_0) = -184°.8916$$

If we add these corrections to the corresponding numbers in the table we get for the elements of that conjunction

$$T_0 = 941532^d.8080 \qquad \omega_0 = 35°.7871$$
$$nt_0 = 236°.7863 \qquad nt_0 - \Omega_0 = 15°.8849$$

We thus see that the conjunction took place considerably within the ecliptic limits; and the effect of the secular variations since that early date is such as to bring the conjunction point 3°.8527 nearer the node; so that there was really a large eclipse, although it was nowhere central. This is the celebrated eclipse which was visible in China, and for which the astronomers Ho and Hi were beheaded because they failed to predict it. It is also further important as being the earliest eclipse of which we have authentic records.

As a third example, we shall inquire into the circumstances of an eclipse which happened on May 18, B.C. 603. We easily find $N = -30946$, $i = -86$, and $N' = 158$. The eclipse was therefore the 86th recurrence of number 158 of the table; and the mean elements at that time were

$$T_0 = 1501314^d.6294 \qquad \omega_0 = 97°.0255$$
$$nt_0 = 82°.8584 \qquad nt_0 - \Omega_0 = 5°.6812$$

This is the celebrated eclipse which was predicted by THALES.

As a fourth example, we shall investigate the elements of an eclipse which will take place Aug. 11, A.D. 1999. In this case the interval from the epoch of the table is 46806 days; and gives $N = 1585$, $i = 4$, and $N' = 153$; and from this it appears that the eclipse will be the fourth return of number 153, which took place October 30, 1883. The change of elements are readily found to be

$$\Delta T_0 = +42287^d.8017, \qquad \Delta\omega_0 = 29°.4668$$
$$\Delta nt_0 = 279°.2432 \qquad \Delta(nt_0 - \Omega_0) = +0°.1408$$

and the mean elements of that eclipse become

$$T_0 = 2451402^d.2293 \qquad \omega_0 = 65°.4580$$
$$nt_0 = 137°.8673 \qquad nt_0 - \Omega_0 = 7°.3354$$

As a fifth and last example, we shall investigate the elements of the moon's eclipse which was observed at Babylon March 19, B.C. 721. The interval between that date and the epoch of the table is 957028 days; and it gives

$$N = -32408, \quad i = -90, \quad \text{and} \quad N' = 188$$

Therefore that eclipse corresponds with number 188 of the cycle, which took place March 30, 1885; and was the *ninetieth* return of the Babylonian eclipse. That eclipse has become celebrated as being the earliest recorded lunar eclipse; and it has played a conspicuous part in its historical relation to the lunar theory. We readily find

$$\Delta T_0 = -951475^d.5374 \qquad \Delta\omega_0 = -123°.0032$$
$$\Delta nt_0 = -342°.9729 \qquad \Delta(nt_0 - \Omega_0) = -\ \ 3°.1672$$

and the resulting mean elements of that eclipse are

$$T_0 = 1458155^d.6743 \qquad \omega_0 = 330°.5410$$
$$nt_0 = 205°.0000 \qquad nt_0 - \Omega_0 = 0°.7608$$

The eclipse therefore took place very near to the node, and must have been total.

These few examples sufficiently illustrate the manner of using the tables, and show how easily an approximate knowledge of an eclipse occurring at any time in the past may be obtained by the expenditure of only a few minutes of labor. The elements thus obtained are the correct *mean elements* for the given date; but they must be corrected for the secular changes in the elements of the moon's motion which have taken place during the interval of time between the date of the eclipse and the year 1850, in order to obtain the *true elements*. Then by applying a few of the periodic inequalities to the mean places of the sun and moon, we easily obtain the coordinates of these bodies, and are thus enabled to determine the path of the moon's shadow across the earth.

Cleveland, 1901 *July* 15.

COMET *b* 1901,

(Return of ENCKE's periodical comet.)

ENCKE's comet has been observed by Dr. H. C. WILSON, at Carleton College Observatory, Northfield, Minnesota, in the following position:

1901 Aug. 5.8924 G.M.T., $\alpha = 6^h\,2^m\,2^s.8$, $\delta = +31°\,42'\,30''$

A sweeping ephemeris of this comet, computed by THOMBERG, has been communicated from Kiel by Dr. KREUTZ. Interpolated for intermediate dates it is as follows:

1901	α	δ	Brightness
	h m s	° ′	
Aug. 9.5	6 27 24	+31 26	1.00
11.5	6 41 32	31 8	
13.5	6 56 4	30 44	
15.5	7 10 58	30 13	
17.5	7 26 12	29 34	
19.5	7 41 45	28 46	
21.5	7 57 36	+27 49	2.00

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR,

By A. W. QUIMBY.

1901	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan. 1	8	–				fair
2	8				–	fair
3	8				–	poor
4	8			–	–	fair
5	9				–	fair
6	9			–		fair
7	8				–	poor
8	8	–	–	–	–	fair
9	8	–			–	poor
12	8	–		–	–	poor
13	9	–	–	–	–	fair
14	9	–	–		–	fair
15	9		–	–	–	fair
16	2		–	–	–	fair
17	11	–	–		–	v. poor
18	1	–		–	–	fair
19	9		–	–	–	fair
20	9	–	–	–	–	fair
21	10				–	poor
22	9	–	–	–	–	fair
23	11		–	–	–	poor
24	2	–	–	–	–	poor
25	3			–	–	fair
26	9	–	–	–	–	fair
27	2	–	–	–	–	fair
28	11	–	–	–		fair
29	3			–	–	fair
31	9			–	–	fair
Feb. 1	9	–	–	–		fair
2	9				–	fair
4	12	1	1	1	1	fair
5	8	–	1	1	–	fair
6	9	–	1	1	–	fair
7	9	–	1	1	–	fair
8	9		1	1	–	fair
10	9		–	–	–	fair
11	9	–		–	–	fair
12	9	–		–	–	fair
13	8	–	–		–	fair
14	8	–		–		fair
15	1		–		–	fair
16	8	–	–			fair
17	9	–				fair
18	8				–	fair
19	10	–	–	–	–	poor
20	8			–	–	fair
21	9					poor
22	10					fair
23	8				–	fair
24	9					fair
25	8			–		fair
26	8					fair
27	8		–			fair
28	9					fair
Mar. 1	9		–	–	1	fair
2	1	1	1	1	1	fair
3	8		1	1	1	fair
4	8	1	2	6		fair
5	8		2	5	–	fair
6	3	–	1	5	–	fair
7	8	–	1	1	–	fair
8	3	1	2	6	–	poor
11	12	–	–	–	–	poor
12	8	–	–	–		fair
13	8	–	–	–		fair
14	8	–	–	–	–	poor
15	3	–	–	–		fair
16	8	–	–	–	–	poor
17	9	–	–	–	–	fair
18	8	–	–	–	–	good
19	8	–	–	–	–	good
21*	9	–	–	–	–	poor
22*	7	–	–	–	–	fair
23*	8	–	–	–	–	good
27	5	–	–	–	–	fair
28	10	–	–	–	–	poor
29	8	–	–	–	–	fair
30	8	–	–	–	–	fair
31	8	–	–	–	–	fair
Apr. 1	8	–	–	–	–	fair
2	8	–	–	–	–	fair
3	1	–	–	–	–	poor
4	11	–	–	–	–	poor
5	8	–	–	–	–	fair
7	1	–	–	–	–	poor
8	7	–	–	–	–	poor
9	8	–	–	–	–	fair
10	8	–	–	–	–	fair
11	8	–	–	–	–	fair
12	8	–	–	–	–	fair
13	8	–	–	–	–	fair
14	1	–	–	–	–	poor
15	11	–	–	–	–	poor
16	4	–	–	–	–	fair
17	8	–	–	–	–	fair
21	8	–	–	–	–	fair
22	9	–	–	–	–	fair
23	10	–	–	–	–	poor
25	2	–	–	–	–	fair
26	7	–	–	–	–	fair
27	7	–	–	–	–	fair
28	8	–	–	–	–	fair
29	7	–	–	–		fair
30	7	–	–	–		fair
May 1	8	–		–	–	fair
2	1			–		fair
3	9					fair
4	7	–	–	–		fair
5	8	–	–	–	–	fair
6	7	–	–	–	–	fair
7	8	–	–	–	–	fair
8	8	–		–	–	fair
9	6	–	–	–	–	poor
11	8	–	–	–	–	fair
12	7	–	–	–	–	good
13	4	–	–		–	fair
14	7	–	–	–	–	good
15	8	–	–	–	–	fair
16	8	–	–	–	–	fair
17	8	–	–	–	–	fair
18	5	–	–	–	–	fair
20	6	1	1	10	1	poor
21	5	–	1	35	–	poor
22	0	–	1	35	–	poor
23	11	–	1	52	–	fair
24	7	–	1	25	–	fair
27	5	–	1	10	–	poor
28	7	–	1	7	–	poor
29	4	–	1	10	–	poor
30	7	–	1	13	1	fair
31	6	–	–	–	–	poor
June 1	7	–	–	–	–	fair
2	8	–	–	–	–	fair
3	8	–	–	–	–	fair
4	7	–	–	–	–	fair
5	7	–	–	–	–	fair
6	7	–	–	–	–	fair
7	6	–	–	–	–	poor
8	7	–	–	–	–	fair
9	8	–	–	–	–	fair
10	7	–	–	–	–	fair
11	7	–	–	–	–	fair
12	7	–	–	–	–	fair
13	7	–	–	–	–	fair
14	7	–	–	–	–	poor
15	11	–	–	–	–	poor
16	1	–	–	–	–	poor
17	8	1	1	5	–	fair
18	1		1	10	–	fair
19	8	–	1	10	–	fair
20	7	–	1	3	–	poor
21	3	–	1	1	–	poor
22	8	–	1	1	–	poor
23	8	–	1	1	–	poor
24	8	1	2	6	–	fair
25	8	–	2	4	1	poor
26	7	–	–	–	1	fair
27	7	–	–	–	–	fair
28	7	–	–	–	–	fair
29	8	–	–	–	–	poor
30	8	–	–	–	–	fair

* 2½-inch refractor

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DOUBLE-STAR MEASURES,

By W. A. COGSHALL.

[Communicated by the Director of the Lowell Observatory.]

The following are measures of stars from the "First Catalogue of Double and Multiple Stars Discovered at the Lowell Observatory" [*A.J.* 431–432], between 0^h and 18^h right-ascension.

Those observations were made by Mr. BOOTHROYD which are followed by "B"; the remainder were made by myself. When both observed the same star I have used both initials to distinguish the work of each. The methods of observation, etc., are the same as previously described in these columns.

Three stars in the latter part of the list, λ_1 202, λ_1 203 and λ_1 253 were wrongly identified in the original list, but are here retained in order to preserve the measures. The approximate places of two of them are given in this table.

λ_1 1. C. $8^M.5$; $11^M.3$.

$\alpha = 0^h\,0^m\,51^s.1$; $\delta = -30°\,53'\,3''.3$

t	θ_0	ρ_0
	°	″
1898.911	323.0	4.70
1899.586	324.1	4.74
.602	322.0	4.46
1899.366	323.0	4.63

λ_1 2.

$\alpha = 0^h\,2^m\,40^s.2$; $\delta = -23°\,3'\,50''.8$.

1899.914

I have not been able to find any companion. Have examined the star with all apertures and eye-pieces.

λ_1 3. 8^M ; $9^M.7$.

$\alpha = 0^h\,4^m\,46^s.1$; $\delta = -34°\,20'\,26''.9$

1899.647	191.0	1.14
.675	189.0	0.65
1899.661	190.0	0.90

λ_1 4. $8^M.8$; 11^M.

$\alpha = 0^h\,16^m\,36^s.1$; $\delta = -28°\,47'\,33''.9$.

1898.780	251.9	3.29
.862	254.0	3.06
.906	252.5	3.33
1898.849	252.8	3.23

λ_1 5. $8^M.7$; $11^M.5$.

$\alpha = 0^h\,20^m\,42^s.7$; $\delta = -35°\,22'\,56''.9$

1898.925	75.5	3.64
1899.600	74.8	3.78
.602	73.0	3.61
1899.376	74.4	3.68

λ_1 7. $8^M.3$; 12^M.

$\alpha = 0^h\,23^m\,42^s.6$; $\delta = -33°\,19'\,0''.7$

t	θ_0	ρ_0	
	°	″	
1898.862	198.5	9.05	B.
1899.630	200.2	8.99	
.647	200.0	9.28	
1900.039	198.2	9.16	
1899.772	199.5	9.14	C.

λ_1 8. $7^M.8$; $13^M.3$.

$\alpha = 0^h\,20^m\,32^s.9$; $\delta = -19°\,21'.6$

1898.922	116.4	5.99
1899.647	117.0	6.43
.668	117.6	6.11
1899.412	117.0	6.18

This star is C.DM. (−19°) 55 instead of O.A. 236 as originally published.

λ_1 9. $8^M.7$; $13^M.3$.

$\alpha = 0^h\,44^m\,23^s.8$; $\delta = -31°\,10'\,5''.8$

1898.925	245.9	3.59	B.
1899.600	245.5	3.57	
.630	243.9	3.61	
.647	244.8	3.62	
1899.626	244.7	3.60	

λ_1 10. $8^M.1$; $10^M.7$.

$\alpha = 0^h\,57^m\,32^s.3$; $\delta = -22°\,8'\,36''.1$

1898.758	321.0	4.68	
.845	325.4	–	
.862	322.5	4.05	
.914	327.1	4.25	
1898.844	324.0	4.33	B.

λ_1 11. 8^M ; $8^M.7$.

$\alpha = 1^h\,13^m\,56^s.6$; $\delta = -27°\,2'\,6''.2$

t	θ_0	ρ_0
	°	″
1898.763	314.4	2.16
.780	313.2	2.06
.791	314.6	2.05
1898.778	314.1	2.09

λ_1 12.

$\alpha = 1^h\,14^m\,42^s.6$; $\delta = -25°\,28'\,21''.6$

1899.033

I have examined this star with all powers, and with none of them can I see any trace of elongation. Seeing much better than for some time.

λ_1 13. 8^M ; $8^M.3$.

$\alpha = 1^h\,16^m\,59^s.2$; $\delta = -24°\,39'\,10''.4$

1899.600	326.2	0.32
.668	316.8	0.38
.695	312.8	0.36
1899.654	318.6	0.35

λ_1 14. $6^M.3$; $12^M.2$.

$\alpha = 1^h\,24^m\,48^s.2$; $\delta = -22°\,8'\,48''.5$

1898.862	248.9	21.78
.906	250.1	21.51
.911	250.3	21.76
1898.893	249.8	21.68

λ_1 15. $8^M.2$; $9^M.7$.

$\alpha = 1^h\,36^m\,25^s.1$; $\delta = -22°\,13'\,30''.2$.

1898.722	313.1	3.05	
.862	310.9	2.97	
.925	313.7	3.11	
1898.836	312.6	3.04	B.

λ_1 16. $8^M : 8^M.8$.

$\alpha = 2^h\ 1^m\ 20^s.3 \quad : \quad \delta = -22°\ 37'\ 49''.5$

t	θ	ρ
	°	″
1899.668	39.6	0.40
.689	36.0	0.51
.695	40.1	0.67
1899.684	38.6	0.53

λ_1 17. $7^M.8 : 11^M.3$.

$\alpha = 2^h\ 7^m\ 23^s.4 \quad : \quad \delta = -21°\ 18'\ 58''.6$

1898.862	357.5	8.36
.925	356.3	8.54
1899.022	358.4	8.50
1898.936	357.3	8.47

λ_1 18. $8^M : 8^M.8$.

$\alpha = 2^h\ 16^m\ 16^s.2 \quad : \quad \delta = -35°\ 34'\ 11''.6$

1899.046	55.8	1.11
.668	56.8	1.10
.695	61.4	1.01
1899.470	58.0	1.07

λ_1 19. $7^M.5 : 8^M.8$.

$\alpha = 2^h\ 36^m\ 2^s.3 \quad : \quad \delta = -24°\ 33'\ 54''.9$

1899.046	323.6	0.57	B.
1899.668	328.0	0.39	
.673	323.8	0.76	
1900.039	327.1	0.40	
1899.793	326.3	0.52	C.

λ_1 20. $4^M : 15^M$.

$\alpha = 2^h\ 46^m\ 30^s.1 \quad : \quad \delta = -21°\ 24'\ 55''.3$

1899.022	126.6	16.87	B.

Single distances.

λ_1 21. $8^M : 8^M$.

$\alpha = 2^h\ 47^m\ 14^s.2 \quad : \quad \delta = -21°\ 42'\ 6''.8$

1899.668	93.2	0.34
.673	92.0	0.36
.706	97.0	0.39
1899.682	94.1	0.36

λ_1 22. $8^M : 11^M.5$.

$\alpha = 3^h\ 8^m\ 26^s.1 \quad : \quad \delta = -30°\ 25'\ 25''.5$

1899.046	341.9	0.77	B.(1)
1899.706	340.5	1.19	
.717	340.1	1.08	
1899.712	340.3	1.13	C.

(1) Seeing very bad.

λ_1 23.

$\alpha = 3^h\ 13^m\ 56^s.7 \quad : \quad \delta = -22°\ 52'\ 36''.0$

No measure was made of this star. I examined it on three different occasions, and each time it had some appearance of duplicity, although the seeing was not good enough to get any accurate results.

λ_1 25. $7^M.3 : 12^M.7$.

$\alpha = 3^h\ 22^m\ 33^s.7 \quad : \quad \delta = -28°\ 54'\ 38''.2$

t	θ_0	ρ_0
	°	″
1898.985	15.9	9.89
1899.000	17.7	10.15
.046	12.0	10.17
1899.010	15.2	10.07

λ_1 26.

$\alpha = 3^h\ 28^m\ 7^s.7 \quad : \quad \delta = -24°\ 57'\ 20''.8$

1899.014 (1)
.692 (2)
.706 (3)

(1) Seeing very good; possibly elongated in 220±, but if so the star is very close.

(2) Definition the best I have ever seen; stars *very* steady; no trace of any elongation.

(3) Seeing very fair; nothing definite with 750 diams.

λ_1 27. $8^M : 9^M$.

$\alpha = 3^h\ 28^m\ 25^s.7 \quad : \quad \delta = -19°\ 35'\ 43''.4$

1898.985	349.2	0.56	B.
1899.706	351.7	0.59	
.709	351.9	0.61	
.744	348.7	0.61	
1899.720	350.8	0.60	C.

λ_1 28. $8^M : 12^M.5$.

$\alpha = 3^h\ 31^m\ 8^s.6 \quad : \quad \delta = -29°\ 25'\ 26''.0$

1898.985	101.6	9.85
1899.717	104.8	10.30
.744	105.6	9.92
1899.182	104.0	10.02

λ_1 33.

$\alpha = 3^h\ 45^m\ 36^s.9 \quad : \quad \delta = -22°\ 15'\ 15''.6$

I have never been able to find this star, nor anything like it near this place. Have searched several times.

λ_1 34. $7^M.5 : 13^M$.

$\alpha = 4^h\ 9^m\ 26^s.2 \quad : \quad \delta = -25°\ 47'\ 0''.1$

1899.014	54.7	19.20
.046	49.4	18.80
.709	56.4	19.00
1899.256	53.5	19.00

λ_1 35. $8^M.3 : 14^M$.

$\alpha = 4^h\ 15^m\ 27^s.2 \quad : \quad \delta = -31°\ 19'\ 5''.4$

1899.706	124.2	2.10
.750	126.5	2.56
.758	123.6	2.26
1899.738	124.8	2.31

λ_1 36. $7^M : 13^M$.

$\alpha = 4^h\ 16^m\ 25^s.0 \quad : \quad \delta = -19°\ 34'\ 22''.4$

1899.014	347.6	8.00	B.
1899.717	347.1	7.71	
.744	347.5	7.84	
1900.039	346.3	7.52	
1899.833	347.0	7.69	C.

λ_1 37. $7^M.7 : 12^M.2$.

$\alpha = 4^h\ 26^m\ 49^s.8 \quad : \quad \delta = -25°\ 11'\ 41''.8$

t	θ_0	ρ_0
	°	″
1899.706	38.1	11.62
.717	38.4	11.44
.733	37.3	11.38
1899.719	37.9	11.48

λ_1 38. $8^M : 12^M.5$.

$\alpha = 4^h\ 33^m\ 9^s.9 \quad : \quad \delta = -29°\ 29'\ 48''.5$

1899.706	238.3	6.10
.717	236.2	6.40
.744	235.3	6.47
1899.722	236.6	6.32

λ_1 39.

$\alpha = 4^h\ 43^m\ 22^s.7 \quad : \quad \delta = -20°\ 59'\ 20''.0$

Have not been able to find this star.

λ_1 41. $8^M : 13^M.2$.

$\alpha = 4^h\ 48^m\ 46^s.6 \quad : \quad \delta = -30°\ 49'\ 33''.0$

1899.014	118.9	9.24	B.
1899.750	123.1	9.06	
1900.088	124.7	8.98	
1899.919	123.9	9.02	C.

λ_1 44. $7^M : 11^M.3$.

$\alpha = 4^h\ 57^m\ 7^s.2 \quad : \quad \delta = -23°\ 51'\ 21''.4$

1899.717	332.1	1.50
.733	332.6	1.42
.750	334.8	1.52
1899.733	333.2	1.48

λ_1 46. $8^M.3 : 12^M.2$.

$\alpha = 5^h\ 3^m\ 4^s.2 \quad : \quad \delta = -22°\ 46'\ 53''.6$

1899.758	14.3	12.27
.788	14.3	10.12
.791	16.2	13.33
1899.779	14.9	11.91

λ_1 47.

$\alpha = 5^h\ 5^m\ 30^s.1 \quad : \quad \delta = -22°\ 36'\ 54''.1$

1899.744	57.0	2.80
.750	54.9	3.20
.758	55.2	3.06
1899.751	55.7	3.02

λ_1 48. $9^M.5 : 11^M$.

$\alpha = 5^h\ 8^m\ 51^s.3 \quad : \quad \delta = -28°\ 34'\ 37''.2$

1899.008	3.2	2.85
.799	3.5	2.34
1899.404	3.4	2.60

λ_1 49. $8^M.3 : 13^M.3$.

$\alpha = 5^h\ 13^m\ 17^s.5 \quad : \quad \delta = -18°\ 14'\ 19''.9$

1899.750	194.7	1.17
.807	197.0	1.55
.815	196.9	1.45
1899.801	196.2	1.39

λ_1 51. $7^{M}.2$: $7^{M}.7$.

$\alpha = 5^{h}\ 15^{m}\ 13^{s}.8$; $\delta = -41^\circ\ 8'\ 52''.7$

t	θ_0	ρ_0	
1899.758	–	0.67	
1900.030	96.5	0.51	
1899.894	96.5	0.59	

λ_1 53.

$\alpha = 5^{h}\ 23^{m}\ 20^{s}.1$; $\delta = -20^\circ\ 59'\ 52''.4$

I have tried to measure this object on two occasions. On the first night the seeing was poor, and I was inclined to think the star might be all right. On the second night the seeing was good, and I could see nothing to indicate duplicity.

λ_1 54. $8^{M}.3$; $13^{M}.2$.

$\alpha = 5^{h}\ 28^{m}\ 15^{s}.9$; $\delta = -27^\circ\ 43'\ 46''.3$

1899.008	266.9	13.54	B.
1899.791	268.4	13.52	
.793	269.2	12.86	
.807	270.1	13.43	
1899.797	269.2	13.27	C.

λ_1 56. $8^{M}.2$: $11^{M}.7$.

$\alpha = 5^{h}\ 46^{m}\ 43^{s}.0$; $\delta = -24^\circ\ 22'\ 2''.6$

1898.843	258.1	6.59
1899.008	252.4	6.46
.845	251.7	5.32
1899.232	254.1	6.12

λ_1 57. $6^{M}.5$; 14^{M}.

$\alpha = 5^{h}\ 51^{m}\ 38^{s}.6$; $\delta = -21^\circ\ 42'\ 4''.5$

1899.845	110.6	24.12
[.862(1)	96.2	34.14]
.895	110.0	23.57
1899.870	110.3	23.84

(1) Probably another star.

λ_1 58. 8^{M} : $11^{M}.7$.

$\alpha = 5^{h}\ 58^{m}\ 36^{s}.0$; $\delta = -21^\circ\ 48'\ 16''.6$

1899.793	214.1	1.75
.807	213.1	1.61
.810	213.1	1.56
1899.803	213.4	1.64

λ_1 60. $7^{M}.7$; $12^{M}.2$.

$\alpha = 6^{h}\ 2^{m}\ 38^{s}.9$; $\delta = -32^\circ\ 2'\ 3''.7$

1899.008	97.3	26.16	B.
1899.838	99.8	25.26	
.843	99.9	25.74	
1899.840	99.9	25.50	C.

λ_1 62. $7^{M}.7$: $7^{M}.7$.

$\alpha = 6^{h}\ 7^{m}\ 1^{s}.7$; $\delta = -22^\circ\ 48'\ 8''.2$

AB

1899.008	110.1	0.34
.906	113.0	0.35
1899.457	111.5	0.35

$\frac{AB}{2}\ C$

t	θ	ρ
1899.008	324.8	24.57
.906	325.4	24.40
1899.457	325.1	24.48

λ_1 63. $7^{M}.8$: $12^{M}.7$.

$\alpha = 6^{h}\ 7^{m}\ 32^{s}.1$; $\delta = -22^\circ\ 46'\ 15''.4$

1899.838	165.7	16.85
.845	164.9	16.89
.862	166.3	17.25
1899.848	165.6	17.00

λ_1 64. $8^{M}.2$: $13^{M}.3$.

$\alpha = 6^{h}\ 9^{m}\ 5^{s}.1$; $\delta = -25^\circ\ 46'\ 49''.0$

1898.835	71.0	8.08	B.
1899.793	66.7	7.57	
.807	69.3	7.63	
.810	69.6	7.45	
1899.803	68.5	7.55	C.

λ_1 65. $7^{M}.6$; 12^{M}.

$\alpha = 6^{h}\ 18^{m}\ 52^{s}.5$; $\delta = -33^\circ\ 49'\ 10''.6$

1900.000	107.1	17.02
.793	109.1	17.05
.807	109.5	16.89
.845	109.0	16.94
1900.611	108.7	16.97

λ_1 67. 9^{M} : 11^{M}.

$\alpha = 6^{h}\ 24^{m}\ 52^{s}.2$; $\delta = -23^\circ\ 31'\ 17''.2$.

1899.046	212.3	2.48

λ_1 68. $4^{M}.7$; $13^{M}.8$.

$\alpha = 6^{h}\ 27^{m}\ 41^{s}.4$; $\delta = -23^\circ\ 20'\ 48''.2$

AB

1899.000	142.9	25.69	B.
1900.845	142.6	24.27	
.848	144.3	24.37	
.862	142.9	23.53	
1900.852	143.3	24.06	C.

AC $C = 14.5$

1900.000	302.6	18.15	B.
1900.845	303.7	28.28	
.848	303.4	30.99	
.862	301.7	27.75	
1900.852	302.9	29.01	C.

λ_1 69. 6^{M} : 12^{M}.

$\alpha = 6^{h}\ 31^{m}\ 2^{s}.5$; $\delta = -42^\circ\ 1'\ 7''.1$

1899.906	8.3	10.82

λ_1 71. $6^{M}.5$: $13^{M}.5$.

$\alpha = 6^{h}\ 48^{m}\ 59^{s}.6$; $\delta = -26^\circ\ 49'\ 57''.3$

AB

1899.807	98.0	10.25
.843	98.5	10.22
1899.825	98.2	10.23

AC $C = 13.7$

t	θ	ρ
1899.807	248.5	19.91
.843	248.1	20.21
1899.825	248.3	20.05

λ 72. $6^{M}.8$: $13^{M}.2$.

$\alpha = 6^{h}\ 51^{m}\ 18^{s}.1$: $\delta = -21^\circ\ 54'\ 30''.7$

1898.997	36.3	13.51
1899.192	39.9	12.56
.793	41.0	13.05
1899.327	39.1	13.04

λ_1 74. $6^{M}.3$; $13^{M}.7$.

$\alpha = 6^{h}\ 56^{m}\ 4^{s}.4$; $\delta = -21^\circ\ 58'\ 45''.1$

1899.046	228.9	13.15	B.
1899.845	233.3	12.68	
.848	232.6	12.81	
.862	232.5	12.80	
1899.852	232.8	12.76	

Faint pair near λ_1 74. 10^{M} ; 11^{M}.

1899.046	345.8	2.34	B.
1899.845	349.8	1.25	
.848	352.2	1.50	
.862	349.7	1.40	
1899.852	350.6	1.38	C.

λ_1 75. $7^{M}.3$; $13^{M}.2$.

$\alpha = 7^{h}\ 13^{m}\ 41^{s}.2$; $\delta = -25^\circ\ 48'\ 20''.9$

1899.046	[1.9]	12.27
.197	6.9	12.38
.200	7.7	12.62
1899.148	7.3	12.42

λ_1 76. 6^{M} : $13^{M}.5$.

$\alpha = 7^{h}\ 16^{m}\ 51^{s}.4$; $\delta = -26^\circ\ 46'\ 35''.3$

1899.848	215.7	8.42
.906	217.1	8.56
1899.877	216.4	8.49

λ_1 78.

$\alpha = 7^{h}\ 18^{m}\ 51^{s}.3$; $\delta = -25^\circ\ 34'\ 31''.2$

AB

1899.914	291.4	1.67
1900.033	290.8	1.62
.052	288.1	1.58
1900.000	290.1	1.62

AC

1899.914	12.1	2.77
1900.033	7.9	2.16
.052	10.6	2.25
1900.000	10.2	2.49

AD

1899.914	31.0	6.90
1900.033	31.3	6.99
.052	29.1	6.93
1900.000	30.5	6.94

λ_1 79. 8^M ; $8^M.2$.

$\alpha = 7^h\,22^m\,15^s.7$; $\delta = -27°\,57'\,45''.2$

t	θ	ρ	
	°	"	
1899.200	282.3	0.47	
.214	296.1	0.60	
1899.207	289.2	0.53	B.
1900.033	300.2	0.51	C.

λ_1 81. $7^M.5$; $8^M.3$.

$\alpha = 7^h\,27^m\,35^s.8$; $\delta = -20°\,43'.0$

1898.997	38.6	8.92	
1899.022	38.6	8.86	
.046	34.3	8.96	
1899.022	37.2	8.91	B.

This star is S.DM. 20°1999, instead of O.A. 7012.

λ_1 82. $8^M.7$; $9^M.2$.

$\alpha = 7^h\,30^m\,15^s.8$; $\delta = -36°\,9'\,36''.3$

1899.197	319.9	7.20	
.211	349.3	7.15	
1899.204	349.6	7.17	B.

λ_1 83. $7^M.7$; $13^M.2$.

$\alpha = 7^h\,30^m\,59^s.3$; $\delta = -25°\,53'\,57''.6$

1898.997	201.6	9.07
1899.200	203.1	8.80
.211	203.2	8.94
1899.137	202.7	8.94

λ_1 84. $6^M.3$; $11^M.8$.

$\alpha = 7^h\,35^m\,49^s.7$; $\delta = -19°\,25'\,48''.5$

1898.845	285.8	8.26
.997	285.2	8.30
1899.197	286.7	8.32
1899.013	285.9	8.29

λ_1 85. 5^M ; $13^M.7$.

$\alpha = 7^h\,39^m\,30^s.3$; $\delta = -28°\,10'\,24''.1$

1899.848	34.2	26.08
.906	33.8	25.96
.914	34.5	26.22
1899.889	34.2	26.09

λ_1 86. $4^M.8$; $12^M.3$.

$\alpha = 7^h\,43^m\,56^s.0$; $\delta = -25°\,41'\,20''.1$

1899.906	199.8	26.30
1900.033	198.1	26.95
.052	198.1	26.95
1899.997	198.3	26.73

λ_1 87. $6^M.5$; 14^M.

$\alpha = 7^h\,45^m\,21^s.9$; $\delta = -19°\,57'\,5''.8$

1899.859	151.1	4.72

Very difficult.

λ_1 95. $6^M.7$; $14^M.5$.

$\alpha = 7^h\,58^m\,4^s.6$; $\delta = -20°\,2'.4$

t	θ	ρ
	°	"
1899.914	190.8	12.30
1900.014	191.2	12.39
1899.964	191.0	12.34

This star is S.DM. 19°2205, and not O.A. 8009.

λ_1 97. 8^M ; $13^M.5$.

$\alpha = 8^h\,10^m\,5^s.8$; $\delta = -37°\,25'\,18''.2$

1900.082	3.6	2.57

λ_1 98. 7^M ; 13^M

$\alpha = 8^h\,11^m\,9^s.4$; $\delta = -35°\,23'\,3''.9$

1898.931	65.4	5.54
1899.914	66.7	5.26

λ_1 101. $8^M.7$; 12^M.

$\alpha = 8^h\,35^m\,11^s.7$; $\delta = -28°\,1'\,31''.2$

1898.931	84.7	7.71	
1899.214	89.5	7.08	
1899.072	87.1	7.40	B.

λ_1 105. 7^M ; $13^M.5$.

$\alpha = 8^h\,41^m\,19^s.2$; $\delta = -26°\,46'\,39''.7$

A B

1899.914	115.3	20.95
.985	117.5	20.60
1900.014	118.2	20.99
1899.971	117.0	20.85

A C $C = 13.5$

1899.914	147.0	20.95
.985	147.6	20.49
1900.014	146.7	20.42
1899.971	147.1	20.62

A D $D = 13.5$

1899.914	248.9	22.01
.985	249.2	21.72
1900.014	248.6	. .
1899.971	248.9	23.36

λ_1 106. $6^M.5$; 12^M.

$\alpha = 8^h\,40^m\,15^s.6$; $\delta = -23°\,25'\,27''.7$

A B

1899.197	223.3	16.83	B.
1899.997	224.4	16.53	
1900.014	[196.2]	16.51	
.033	224.5	16.69	
1900.015	224.5	16.58	C.

This star is A.G.C. 11831, and not A.G.C. 11980.

B C $C = 12.7$

1899.197	332.2	4.17	B.
1899.997	330.1	2.30	
1900.014	325.8	2.27	
.033	333.2	3.27	
1900.015	329.7	2.61	C.

λ_1 107. 6^M ; $13^M.7$.

$\alpha = 8^h\,51^m\,13^s.9$; $\delta = -27°\,17'\,45''.1$

t	θ	ρ
	°	"
1899.197	269.1	24.18
.914(1)	[279.0]	23.37
1900.077	267.6	23.91
1899.729	268.3	23.82

(1) Very bad seeing.

λ_1 116. $8^M.5$; $11^M.7$.

$\alpha = 9^h\,41^m\,43^s.7$; $\delta = -28°\,6'\,37''.4$

1898.922	203.5	2.69
.997	202.1	2.24
1899.008	203.3	2.30
1898.976	203.0	2.41

λ_1 120. $8^M.2$; $13^M.7$.

$\alpha = 10^h\,37^m\,50^s.1$; $\delta = -33°\,7'\,57''.5$

1898.997	293.0	6.05
1899.008	293.2	6.26
.175	293.8	5.83
.244	294.5	5.90
1899.106	293.6	6.01

λ_1 121. 6^M ; $14^M.7$.

$\alpha = 10^h\,38^m\,4^s.1$; $\delta = -32°\,11'\,31''.5$

1898.997	220.1	18.08
1899.175	218.5	17.37
1899.086	219.3	17.73

λ_1 131. $4^M.8$; $11^M.5$.

$\alpha = 11^h\,18^m\,22^s.1$; $\delta = -35°\,36'\,58''.2$

1898.997	127.5	31.32	
1899.244	125.9	31.68	
.345	126.2	30.39	
1899.195	126.5	31.13	B.

λ_1 133. $5^M.5$; 13^M.

$\alpha = 11^h\,36^m\,44^s.3$; $\delta = -31°\,56'\,37''.7$

1899.244	345.3	26.74	C.
1899.337	246.0	26.33	B.

λ_1 135.

$\alpha = 11^h\,42^m\,16^s.4$; $\delta = -29°\,43'\,22''.0$

1899.342

I cannot see this star double.

λ_1 136. $9^M.5$; $12^M.7$.

$\alpha = 11^h\,43^m\,54^s.6$; $\delta = -35°\,22'\,40''.2$

1899.337	258.6	7.62
.345	262.2	7.21
1899.341	260.4	7.42

λ_1 142. $7^M.7$; $9^M.5$.

$\alpha = 11^h\,56^m\,41^s.1$; $\delta = -34°\,5'\,38''.5$

1899.189	242.0	0.75
1900.370	239.8	0.45
.386	238.3	0.46
1899.982	240.0	0.55

λ_1 143. $7^{M}.7$; $7^{M}.8$.

$\alpha = 11^h\,58^m\,32^s.5$; $\delta = -38°\,27'\,2''.4$

t	θ_0	ρ_0
	°	″
1900.389	221.2	0.45
.400	216.4	[0.75]
.408	217.2	0.52
1900.399	218.3	0.49

λ_1 144. 6^{M} ; $13^{M}.2$.

$\alpha = 12^h\,0^m\,48^s.4$; $\delta = -35°\,8'\,14''.0$

1899.189	175.4	25.82
1900.071	176.1	24.58
.079	175.6	24.21
.096	176.1	24.72
1900.082	175.9	24.50

λ_1 148. 7^{M} ; $13^{M}.5$.

$\alpha = 12^h\,8^m\,54^s.8$; $\delta = -35°\,59'\,59''.1$

1900.079	63.4	19.11
.096	63.7	18.98
.115	62.6	19.01
1900.063	63.2	19.03

λ_1 150. $6^{M}.3$; $13^{M}.5$.

$\alpha = 12^h\,18^m\,33^s.3$; $\delta = -29°\,46'\,50''.6$

1899.197	98.3	18.37
.345	101.4	17.89
1900.071	95.9	17.38
1899.538	98.5	17.88

λ_1 152. 8^{M} ; $11^{M}.3$.

$\alpha = 12^h\,20^m\,8^s.7$; $\delta = -30°\,34'\,34''.2$

1899.189	86.0	2.34
1900.079	84.0	2.02
.096	84.3	2.29
1899.788	84.8	2.22

λ_1 155. $9^{M}.2$; $12^{M}.2$.

$\alpha = 12^h\,24^m\,38^s.8$; $\delta = -34°\,14'\,13''.0$

1900.079	117.9	9.41
.370	118.0	9.39
.386	118.4	9.58
1900.278	118.1	9.47

λ_1 156. $8^{M}.7$; $9^{M}.7$.

$\alpha = 12^h\,30^m\,45^s.6$; $\delta = -35°\,16'\,23''.1$

1900.096	126.9	0.73
.389	121.8	0.51
.406	121.9	0.44
1900.297	123.5	0.56

λ_1 158.

$\alpha = 12^h\,33^m\,17^s.5$; $\delta = -38°\,50'\,32''.4$

Have not been able to find this star.

λ_1 160. $9^{M}.5$; $11^{M}.2$.

$\alpha = 12^h\,40^m\,58^s.9$; $\delta = -38°\,53'\,58''.8$

1900.386	183.0	1.77
.389	185.1	2.04
.416	185.9	1.59
1900.397	184.7	1.80

λ_1 161. 7^{M} ; $12^{M}.5$.

$\alpha = 12^h\,42^m\,40^s.6$; $\delta = -35°\,9'\,35''.2$

A B

t	θ_0	ρ_0
	°	″
1900.449	110.0	33.14

B C $C = 13.5$

1900.449	96.7	5.82

λ_1 162. $10^{M}.5$; $13^{M}.5$.

$\alpha = 12^h\,44^m\,0^s.6$; $\delta = -34°\,12'\,57''.4$

1900.079	228.3	5.19
.416	225.9	5.89
.425	230.0	6.32
1900.307	228.1	5.80

λ_1 164. 8^{M} ; $13^{M}.3$.

$\alpha = 12^h\,46^m\,18^s.7$; $\delta = -32°\,48'\,2''.2$

A B

1900.408	193.5	2.24
.425	193.7	2.33
.441	193.0	2.60
1900.425	193.4	2.39

A C $C = 13.5$

1900.408	119.5	42.04

λ_1 165. $6^{M}.5$; $13^{M}.2$.

$\alpha = 12^h\,47^m\,42^s.1$; $\delta = -38°\,50'\,47''.1$.

1900.386	235.3	19.86
.389	231.7	19.82
.416	231.6	20.43
1900.397	232.9	20.04

A B (COGSHALL) $B = 12.5$

1900.389	165.5	2.71
.416	163.8	2.75
1900.402	164.7	2.73

λ_1 167. $8^{M}.3$; 13^{M}.

$\alpha = 12^h\,55^m\,11^s.5$; $\delta = -35°\,36'\,42''.5$

1899.200	118.9	23.60
.430	118.4	23.41
.436	117.6	23.31
1899.355	118.3	23.44

λ_1 169. 7^{M} ; $12^{M}.3$.

$\alpha = 12^h\,58^m\,15^s.0$; $\delta = -33°\,43'\,46''.2$

1900.079	235.2	10.40
.096	235.0	10.98
.123	235.5	11.00
1900.099	235.2	10.79

λ_1 171. 8^{M} ; 8^{M}.

$\alpha = 13^h\,9^m\,19^s.0$; $\delta = -34°\,6'\,3''.8$

1899.433	295.5	0.50	B.
1900.449			

Found this star by SEE's field, and after trying 500, 750, 1000 diameters, I can see nothing to indicate any duplicity. Star is certainly under 0″.2, if double at all.

λ_1 172. $7^{M}.5$; $12^{M}.7$.

$\alpha = 13^h\,9^m\,40^s.4$; $\delta = -33°\,37'\,25''.5$

t	θ_0	ρ_0
	°	″
1900.389	350.3	4.31
.406	347.0	4.44
.408	345.6	4.05
1900.401	347.6	4.27

A C $C = 14$

1900.389	226.3

λ_1 174.

$\alpha = 13^h\,11^m\,21^s.1$; $\delta = -30°\,3'\,49''.9$

1900.449

I have examined this star carefully, and cannot believe it double, either as Dr. SEE observed it, or any other way.

λ_1 175. $8^{M}.2$; $13^{M}.2$.

$\alpha = 13^h\,12^m\,57^s.3$; $\delta = -31°\,34'\,39''.6$

1899.427	117.1	11.64
1900.079	116.9	12.09
.386	117.3	12.22
1899.964	117.1	11.98

A C $C = 13$

1899.227	117.1	40.51
1900.079	116.9	40.77
.386	117.4	40.91
1899.964	117.1	40.73

λ_1 178. 8^{M} ; $13^{M}.7$.

$\alpha = 13^h\,21^m\,38^s.9$; $\delta = -32°\,12'\,48''.3$

1899.200	227.5	14.34
.345	233.1	14.06
.430	232.9	14.39
1899.325	231.2	14.26

λ_1 181.

$\alpha = 13^h\,27^m\,57^s.6$; $\delta = -37°\,44'\,38''.1$

A B

1900.433 (1)

1900.438 (2)

(1) The *A B* of this system is not apparent. Possible elongation in 90°. Not more than 0.25 in any case.

(2) This star is not double. Seeing good, and no companion visible under $\frac{3}{4}$, $\frac{1}{2}$ or $\frac{3}{8}$-eye-pieces.

$\frac{A\,B}{2}$ *C*

$7^{M}.5$; 12^{M}.

1900.433	17.4	28.31

λ_1 184. 7^{M} ; 10^{M}.

$\alpha = 13^h\,32^m\,4^s.1$; $\delta = -34°\,33'\,14''.0$

1900.441	313.4	1.15
.449	315.2	1.50
1900.446	314.3	1.32

λ_1 185. $8^{M}.2$; $12^{M}.5$.

$\alpha = 13^h\,32^m\,21^s.1$; $\delta = -36°\,58'\,25''.1$

1899.189	14.6	17.49
.430	15.0	17.56
1899.309	14.8	17.52

λ_1 187. 9^{M} ; 13^{M}.

$\alpha = 13^{h}\,36^{m}\,37^{s}.4$; $\delta = -36°\,52'\,14''.9$

t	θ	ρ	
1899.427	38.9	4.26	
.436	37.8	5.32	
1899.431	38.3	4.79	B.

λ_1 188. $7^{M}.8$; $12^{M}.5$.

$\alpha = 13^{h}\,37^{m}\,7^{s}.1$; $\delta = -32°\,9'\,54''.0$

1899.200	276.5	18.00	
.345	274.4	18.25	
.427	276.8	17.94	
1899.324	275.9	18.06	

λ_1 189. 8^{M} ; 13^{M}.

$\alpha = 13^{h}\,46^{m}\,34^{s}.6$; $\delta = -30°\,17'\,24''.5$

1899.441	254.8	13.46	B.
1900.079	254.8	12.64	
.423	254.1	12.62	
.389	253.3	11.81	
1900.197	254.1	12.36	

λ_1 190. $7^{M}.8$; $10^{M}.5$.

$\alpha = 13^{h}\,48^{m}\,7^{s}.0$; $\delta = -29°\,47'\,25''.9$

1899.189	223.6	7.43	
.345	223.6	6.92	
.430	223.2	7.17	
1899.321	223.5	7.17	

AC $C = 13.5$

1899.189	142.8	31.94	
.345	143.4	29.66	
.430	142.9	31.25	
1899.321	143.0	30.95	

λ_1 191. $7^{M}.3$; $12^{M}.7$.

$\alpha = 13^{h}\,48^{m}\,49^{s}.4$; $\delta = -34°\,6'\,10''.3$

1900.123	155.1	19.32 (1)	
.192	155.7	19.68	
.406	156.4	19.41	
1900.240	155.7	19.47	

(1) Seeing bad.

λ_1 192. $7^{M}.8$; $11^{M}.8$.

$\alpha = 13^{h}\,50^{m}\,18^{s}.1$; $\delta = -38°\,10'\,21''.9$

1900.416	185.6	2.77	
.425	185.2	2.65	
.438	184.8	2.85	
1900.426	185.2	2.76	

λ_1 193. 8^{M} ; $13^{M}.7$.

$\alpha = 13^{h}\,51^{m}\,13^{s}.0$; $\delta = -27°\,10'\,13''.4$

1899.441	163.8	5.83	
1900.079	162.7	6.54	
.406	164.8	6.21	
1899.975	163.8	6.19	

λ_1 194. 8^{M} ; $8^{M}.5$

$\alpha = 13^{h}\,51^{m}\,27^{s}.5$; $\delta = -27°\,1'\,57''.6$

t	θ_0	ρ_0	
		$''$	
1899.200	321.1	0.62	B.
1900.433	310.0	0.55	
.436	312.0	0.62	
.438	311.5	0.55	
1900.436	311.2	0.57	C.

λ_1 195. $7^{M}.7$; $13^{M}.3$.

$\alpha = 13^{h}\,55^{m}\,39^{s}.1$; $\delta = -31°\,8'\,4''.2$

1899.345	237.7	22.00	
1900.416	237.6	21.89	
.425	239.2	22.29	
1900.062	238.2	22.06	

λ_1 199. $7^{M}.5$; $12^{M}.7$.

$\alpha = 14^{h}\,4^{m}\,35^{s}.4$; $\delta = -29°\,36'\,54''.2$

1899.438	227.0	8.11	B.
1900.411	225.9	8.32	
.416	227.0	8.36	
.436	225.0	8.20	
1900.421	226.0	8.29	C.

λ_1 200. 8^{M} ; 12^{M}.

$\alpha = 14^{h}\,5^{m}\,31^{s}.7$; $\delta = -29°\,18'\,45''.3$

1899.441	96.6	10.09	B.

λ_1 201. $7^{M}.5$; $11^{M}.5$.

$\alpha = 14^{h}\,8^{m}\,2^{s}.7$; $\delta = -31°\,34'\,39''.9$

1899.436	200.7	17.46	
1900.416	199.8	17.44	
.433	200.1	17.31	
1900.095	200.2	17.40	

λ_1 202. $8^{M}.5$; $8^{M}.7$.

$\alpha = 14^{h}\,10^{m}\,59^{s}.9$; $\delta = -29°\,30'\,25''.2$

AB

1899.427	109.5	0.90	
.433	107.7	0.84	
.438	110.6	0.84	
1899.433	109.3	0.86	

Wrongly identified by SEE. About 5^{m} west of this place.

$\frac{AB}{2}$ $C = 12.3$

1899.427	134.8	29.66	
.433	135.9	30.27	
.438	134.8	30.73	
1899.433	135.2	30.22	B.

λ_1 203. $8^{M}.2$; $12^{M}.0$.

$\alpha = 14^{h}\,13^{m}\,37^{s}.4$; $\delta = -29°\,11'\,47''.4$

1899.430	95.3	10.04	
1900.079	95.2	9.50	
.416	95.6	9.85	
1899.975	95.1	9.80	

Wrongly identified by SEE. Star is about 5^{m} west of this place.

λ_1 204. $4^{M}.5$; 14^{M}.

$\alpha = 14^{h}\,14^{m}\,28^{s}.6$; $\delta = -37°\,25'\,31''.0$

t	θ_0	ρ_0	
	°	$''$	
1900.458	89.8	33.99	

λ_1 205. $7^{M}.7$; $13^{M}.7$.

$\alpha = 14^{h}\,23^{m}\,26^{s}.8$; $\delta = -36°\,35'\,29''.7$

1900.416	63.9	11.82	
.425	67.3	12.10	
.433	68.8	11.00	
1900.425	66.7	11.64	

λ_1 206.

$\alpha = 14^{h}\,26^{m}\,2^{s}.6$; $\delta = -31°\,44'\,30''.7$

1899.433(1)
.438(2)
1900.438(3)

(1) The faint companion is very easy, but the large star has no appearance of being double, with this seeing.

(2) Nothing seen of close companion. Seeing fair.

(3) I can see the faint companion of this system, although sky is bright, with full moon. The $10^{M}.5$, comp. 190° 1″ is not there. Stars quite small, and at times quite steady, but I can see no such companion.

λ_1 208. $6^{M}.2$; $11^{M}.5$.

$\alpha = 14^{h}\,32^{m}\,19^{s}.3$; $\delta = -34°\,50'\,29''.3$

1899.189	19.9	23.15	
.200	20.0	23.19	
.427	20.6	23.23	
1899.272	20.2	23.19	B.

λ_1 210. $8^{M}.5$; $8^{M}.5$.

$\alpha = 14^{h}\,35^{m}\,57^{s}.9$; $\delta = -32°\,23'\,8''.3$

AB

1899.430	256.4	0.47	B.

$\frac{AB}{2}$ C

1899.430	346.3	25.38	B.

$\frac{AB}{2}$ D

1899.430	20.3	25.99	B.

λ_1 211. 8^{M} ; 13^{M}.

$\alpha = 14^{h}\,41^{m}\,19^{s}.9$; $\delta = -36°\,32'\,11''.1$

1899.189	180.1	13.70	
.200	180.1	13.47	
.433	178.9	13.99	
1899.274	179.7	13.72	

λ_1 212.

$\alpha = 14^{h}\,41^{m}\,51^{s}.1$; $\delta = -31°\,49'\,41''.4$

1899.441

I have not been able to see this star double

λ_1 214. $7^M.3$; $13^M.5$.

$\alpha = 14^h\,44^m\,47^s.9$; $\delta = -35°\,58'\,14''.8$

A B

t	θ_0	ρ_0	
	°	″	
1899.441	258.2	4.85	
1900.416	258.3	4.62	
.444	259.2	4.62	
1900.190	258.6	4.70	

A C $C = 13.2$

1899.441	58.2	38.66	
1900.416	58.7	38.86	
.444	57.4	37.82	
1900.100	58.1	38.45	

λ_1 215. $8^M.3$; $8^M.3$.

$\alpha = 14^h\,50^m\,21^s.4$; $\delta = -34°\,13'\,32''.8$

A B

1899.427	93.5	0.56	
.430	96.4	0.53	
.438	94.4	0.59	
1899.432	94.8	0.56	

$\frac{AB}{2}$ $C = 10.3$

1899.427	24.1	48.64	
.430	23.6	49.00	
.438	24.1	48.66	
1899.432	23.9	48.77	

λ_1 217. $5^M.5$; $13^M.7$.

$\alpha = 14^h\,56^m\,52^s.0$; $\delta = -32°\,14'\,55''.5$

1899.189	117.4	36.24	
.197	118.4	35.50	
.200	122.0	35.97	
1899.195	119.3	35.90	B.

λ_1 221. $7^M.5$; $13^M.0$.

$\alpha = 15^h\,9^m\,31^s.1$; $\delta = -36°\,37'\,32''.2$

1900.416	39.0	5.22	
.425	39.3	5.17	
.438	39.2	4.88	
1900.426	39.2	5.09	

λ_1 222. $8^M.8$; $11^M.7$.

$\alpha = 15^h\,9^m\,58^s.2$; $\delta = -30°\,21'\,26''.5$

1899.200	329.5	13.41	
.468(1)	329.8	14.34	
.473(2)	328.7	13.13	
1899.380	329.3	13.64	

(1) Seeing very bad.
(2) Thin clouds and moonlight.

λ_1 223.

$\alpha = 15^h\,10^m\,3^s.7$; $\delta = -30°\,12'\,23''.6$

I have not been able to find this star.

λ_1 224. $10^M.2$; $12^M.3$.

$\alpha = 15^h\,11^m\,57^s.5$; $\delta = -31°\,41'\,24''.4$

t	θ_0	ρ_0	
	°	″	
1899.427	343.0	3.39	B.
1900.416	346.2	1.81	
.425	339.5	2.04	
.444	340.9	2.23	
1900.428	342.2	2.03	

λ_1 225. $8^M.5$; $11^M.5$.

$\alpha = 15^h\,12^m\,9^s.3$; $\delta = -31°\,28'\,57''.3$

1899.197	111.8	12.49	
1900.449	111.4	12.76	
.479	110.7	13.08	
1900.464	111.0	12.92	

λ_1 226. 6^M ; $11^M.5$.

$\alpha = 15^h\,12^m\,33^s.6$; $\delta = -30°\,50'\,36''.4$

1899.200	70.3	21.25	B.
1900.479	71.8	21.29	C.

This companion may be variable, as Dr. See rated it $12^M.5$, and it is now on the very limit of vision with the 24-inch.

λ_1 227. $10^M.5$; 13^M.

$\alpha = 15^h\,12^m\,32^s.9$; $\delta = -30°\,47'\,37''.6$

1900.449	120.5	7.78	
.455	120.5	7.44	
.458	119.5	7.65	
1900.454	120.2	7.62	

λ_1 230. 9^M ; 10^M.

$\alpha = 15^h\,15^m\,56^s.9$; $\delta = -28°\,56'\,30''.1$

1899.200	148.2	3.47	
.468	150.0	3.45	
.482	148.2	3.60	
1899.383	148.8	3.41	

λ_1 231. $8^M.2$; 14^M.

$\alpha = 15^h\,16^m\,6^s.1$; $\delta = -36°\,40'\,7''.1$

1900.449	245.1	12.89	
.458	245.1	12.70	
.463	242.0	12.56	
1900.457	244.1	12.72	

λ_1 232. 9^M ; $12^M.7$.

$\alpha = 15^h\,16^m\,49^s.1$; $\delta = -28°\,37'\,57''.1$

1900.425	45.5	4.96	
.458	43.8	5.10	
1900.441	44.6	5.03	

λ_1 233. $7^M.5$; 14^M.

$\alpha = 15^h\,18^m\,2^s.5$; $\delta = -26°\,56'\,52''.2$

1900.455	223.8	13.79	
.458	226.0	14.35	
.476	224.0	13.87	
1900.463	224.6	14.00	

λ_1 235. 9^M ; $11^M.7$.

$\alpha = 15^h\,22^m\,32^s.0$; $\delta = -33°\,58'\,54''.1$

t	θ_0	ρ_0	
	°	″	
1899.427	258.5	5.34	
1900.416	257.1	5.25	
.425	258.5	5.34	
1900.420	257.8	5.30	

λ_1 236. 9^M ; $9^M.5$.

$\alpha = 15^h\,22^m\,49^s.3$; $\delta = -38°\,17'\,10''.0$

1900.482	244.3	0.79	

λ_1 237. $7^M.7$; $12^M.3$.

$\alpha = 15^h\,23^m\,5^s.1$; $\delta = -35°\,3'\,20''.4$

1899.425	289.9	9.23	
1900.425	287.1	9.78	
.438	289.4	9.43	
1900.431	288.2	9.60	

λ_1 238. 7^M ; $7^M.5$.

$\alpha = 15^h\,27^m\,13^s.8$; $\delta = -24°\,8'\,58''.0$

A B

1900.509	144.2	0.29	
.512	148.9	0.30	
1900.510	146.5	0.30	

A C $C = 6.2$

1900.509	119.1	9.12	
.512	119.7	9.00	
1900.510	119.4	9.06	

λ_1 241. $8^M.7$; $10^M.3$.

$\alpha = 15^h\,30^m\,54^s.6$; $\delta = -23°\,20'\,52''.7$

1900.441	29.2	0.83	
.458	24.8	0.67	
.479	23.1	0.61	
1900.460	25.7	0.70	

λ_1 242. $8^M.2$; $9^M.2$.

$\alpha = 15^h\,31^m\,39^s.6$; $\delta = -30°\,55'\,18''.9$

1900.449	6.1	0.61	
.482	4.7	0.54	
.493	. .	0.78	
1900.475	5.4	0.64	

λ_1 243. 8^M ; 10^M.

$\alpha = 15^h\,33^m\,44^s.4$; $\delta = -31°\,3'\,35''.5$

1900.441	29.7	0.85	
.458	28.1	0.87	
.479	29.5	0.87	
1900.460	29.1	0.86	

λ_1 244. $8^M.5$; $9^M.2$.

$\alpha = 15^h\,35^m\,30^s.1$; $\delta = -31°\,31'\,28''.4$

1899.197	31.3	0.87	
.482	37.0	0.71	
1899.340	34.1	0.79	B.

λ_1 245. $9^M.5$: $9^M.5$.

$\alpha = 15^h 36^m 31^s.8$: $\delta = -34° 22' 32''.7$

t	θ	ρ
1900.509	173.2	0.31

λ_1 246. $8^M.2$: 11^M.

$\alpha = 15^h 36^m 59^s.0$: $\delta = -27° 39' 21''.0$

1900.416	312.9	13.32
.449	311.1	13.76
.455	310.8	13.33
1900.440	311.6	13.47

λ_1 247. $7^M.3$: $13^M.2$.

$\alpha = 15^h 37^m 13^s.0$: $\delta = -39° 7' 31''.0$

1900.425	277.6	13.02
.438	277.5	12.43
.458	278.8	13.03
1900.440	278.0	12.83

λ_1 250. $8^M.3$: $9^M.2$.

$\alpha = 15^h 44^m 40^s.3$: $\delta = -34° 44' 35''.3$

1900.463	98.7	0.60
.479	96.5	0.59
.482	96.1	0.57
1900.475	97.1	0.59

λ_1 251. 3^M : $12^M.5$.

$\alpha = 15^h 50^m 42^s.6$: $\delta = -28° 55' 19''.1$

1899.175	98.0	38.89	
.427	95.0	37.49	
.449	97.9	38.62	
1899.350	97.0	38.33	B.

λ_1 253. 8^M : $12^M.3$.

$\alpha = 15^h 53^m 17^s.0$: $\delta = -35° 33' 26''.3$

1900.458	130.5	11.84
.463	129.2	11.99
.479	127.8	12.13
1900.467	129.2	11.99

This star is wrongly identified by Dr. SEE.

λ_1 255. $8^M.2$: $9^M.5$.

$\alpha = 15^h 54^m 8^s.0$: $\delta = -25° 58' 31''.1$

1899.436	343.3	15.06
.482	342.8	14.99
1899.459	343.0	15.02

AC $C = 14$

1899.436	22.0 ±	11.70
.482	20.7	12.52
1899.459	21.3	12.11

λ_1 256. 8^M : $11^M.7$.

$\alpha = 15^h 54^m 24^s.3$: $\delta = -35° 42' 35''.8$

1899.197	128.3	12.65	
.430	126.8	13.09	
1899.313	127.6	12.87	B.

λ_1 257. 8^M : $11^M.8$.

$\alpha = 15^h 55^m 2^s.2$: $\delta = -28° 3' 0''.9$

t	θ	ρ
1899.175	340.1	6.35
.197	341.9	6.21
.200	339.9	6.23
1899.191	340.6	6.26

λ_1 260. $8^M.3$: $9^M.3$.

$\alpha = 15^h 55^m 33^s.0$: $\delta = -28° 10' 36''.0$

1899.425	22.4	5.91
.427	23.3	6.05
.460	23.5	5.93
1899.440	23.1	5.96

λ_1 261. 8^M : $13^M.7$

$\alpha = 15^h 55^m 38^s.6$: $\delta = -28° 1' 26''.8$

1900.425	13.8	9.87
.433	12.8	11.14
.458	14.1	9.98
1900.439	13.6	10.33

λ_1 263. $7^M.2$: $13^M.8$.

$\alpha = 15^h 58^m 19^s.7$: $\delta = -38° 56' 26''.2$

1900.449	218.5	8.78
.463	217.4	8.68
.479	215.6	8.43
1900.461	217.2	8.63

λ_1 264. $7^M.7$: $9^M.7$.

$\alpha = 15^h 59^m 20^s.6$: $\delta = -32° 34' 55''.2$

1900.455	335.5	0.64
.458	332.7	0.57
.482	335.9	0.55
1900.465	334.7	0.59

AC $C = 11.7$

1900.455	8.3	9.53
.458	8.1	9.75
.482	8.7	9.83
1900.465	8.4	9.70

λ_1 265. $6^M.3$: $13^M.5$.

$\alpha = 16^h 1^m 52^s.6$: $\delta = -38° 49' 27''.7$

1900.438	299.0	15.41
.449	299.6	15.56
.463	298.4	15.69
1900.450	299.0	15.55

λ_1 268. $7^M.7$: $8^M.2$.

$\alpha = 16^h 7^m 59^s.9$: $\delta = -38° 52' 33''.7$

1899.175	168.1	2.03	B.
1900.138	166.6	1.75	
.179	164.1	1.74	
1900.158	165.3	1.75	C.

λ_1 270. $7^M.7$: $13^M.2$.

$\alpha = 16^h 10^m 37^s.1$: $\delta = -29° 29' 38''.7$

t	θ	ρ
1899.425	137.9	9.12
.427	140.7	8.88
1899.426	139.3	9.00

λ_1 276. $7^M.5$: $14^M.8$.

$\alpha = 16^h 19^m 17^s.5$: $\delta = -33° 20' 11''.8$

1900.416	154.3	14.55
.425	154.4	14.83
.455	154.5	15.05
1900.432	154.4	14.81

λ_1 278. $8^M.3$: $8^M.8$.

$\alpha = 16^h 19^m 39^s.8$: $\delta = -31° 0' 12''.5$

1900.438	323.5	0.80
.455	328.0	0.78
.479	324.8	0.69
1900.457	325.4	0.76

λ_1 280. $8^M.7$: $14^M.7$.

$\alpha = 16^h 28^m 54^s.3$: $\delta = -36° 54' 3''.4$

1900.416	245.5	7.81
.463	245.5	7.25
.476	245.5	6.73
1900.452	245.5	7.26

λ_1 281. $8^M.5$: $13^M.5$.

$\alpha = 16^h 30^m 25^s.1$: $\delta = -33° 50' 58''.4$

AB

1900.438	252.3	8.36
.476	252.1	8.36
.479	251.1	8.30
1900.464	251.8	8.34

AC $C = 14.7$

1900.438 ([1])		
.476	244.2	17.28
.479	243.5	16.70
1900.477	243.8	16.99

([1]) Moon too bright to see well enough.

λ_1 282. $8^M.3$: $12^M.7$.

$\alpha = 16^h 32^m 59^s.0$: $\delta = -34° 0' 25''.1$

1900.425	209.1	2.40
.438	207.2	2.65
.476	207.3	2.48
1900.446	207.9	2.51

λ_1 285. 9^M : 14^M.

$\alpha = 16^h 38^m 7^s.5$: $\delta = -27° 14' 51''.1$

1900.425	261.8	13.65
.455	259.9	13.63
.463	258.4	13.78
1900.448	260.0	13.69

λ_1 289. $6^M.7$: $14^M.7$.

$\alpha = 16^h\,44^m\,14^s.4$: $\delta = -31^\circ\,27'\,46''.0$

t	θ	ρ
	°	″
1900.458	99.3	3.23
.463	101.1	3.00
.476	99.3	3.01
1900.466	99.9	3.08

λ_1 290. $7^M.8$: 14^M.

$\alpha = 16^h\,45^m\,21^s.5$: $\delta = -35^\circ\,17'\,18''.1$

1900.438	75.6	9.28
.455	73.2	9.47
.463	72.7	9.39
1900.452	73.8	9.38

λ_1 291. 8^M : 13^M.

$\alpha = 16^h\,46^m\,7^s.8$; $\delta = -25^\circ\,25'\,51''.2$

1899.602	3.7	2.26

λ_1 305. $10^M.5$; 14^M.

$\alpha = 16^h\,48^m\,31^s.4$; $\delta = -39^\circ\,21'\,18''.3$

1900.493	328.7	6.97

λ_1 306. 9^M ; 13^M.

$\alpha = 16^h\,48^m\,32^s.7$; $\delta = -39^\circ\,19'\,16''.4$

1900.463	350.5	14.82

λ_1 307. 10^M ; 13^M.

$\alpha = 16^h\,48^m\,32^s.9$; $\delta = -39^\circ\,17'\,23''.2$

1900.463	95.1	7.41

λ_1 311. $7^M.2$: 15^M.

$\alpha = 16^h\,49^m\,52^s.5$; $\delta = -31^\circ\,8'\,43''.8$

1900.458	130.9	3.11
.479	129.8	2.72
.482	131.3	2.82
1900.473	130.7	2.88

λ_1 312. $9^M.2$: $13^M.7$.

$\alpha = 16^h\,49^m\,54^s.8$; $\delta = -31^\circ\,13'\,14''.2$

1900.476	349.7	8.96
.479	351.5	9.34
.482	352.3	9.11
1900.479	351.2	9.13

λ_1 313. $8^M.3$: $10^M.8$.

$\alpha = 16^h\,50^m\,10^s.9$; $\delta = -38^\circ\,16'\,24''.8$

1899.463	230.4	2.48
.476	232.1	2.50
.600	228.9	2.44
1899.513	230.5	2.47

λ_1 314. 7^M ; 14^M.

$\alpha = 16^h\,50^m\,29^s.7$; $\delta = -32^\circ\,10'\,36''.7$

A B

1900.458	22.1	15.06
.479	21.0	14.21
1900.469	21.5	14.63

B C $C = 15$

t	θ	ρ
	°	″
1900.458	328.8	5.77
.479	330.5	5.12
1900.469	329.6	5.44

λ_1 317. 8^M : $13^M.3$.

$\alpha = 16^h\,55^m\,39^s.2$: $\delta = -29^\circ\,31'\,31''.5$

1900.425	356.5	9.35
.438	354.7	9.00
.458	357.9	9.40
1900.440	356.4	9.25

λ_1 318. 8^M ; $8^M.5$.

$\alpha = 16^h\,59^m\,20^s.6$: $\delta = -38^\circ\,29'\,29''.5$

1899.600	243.3	0.45

λ_1 319. 8^M ; $12^M.5$.

$\alpha = 17^h\,2^m\,23^s.9$: $\delta = -26^\circ\,46'\,19''.2$

1899.427	210.3	7.46
.575	211.5	7.69
.594	211.0	7.45
1899.532	210.9	7.53

λ_1 325. $8^M.3$; 9^M.

$\alpha = 17^h\,15^m\,44^s.4$: $\delta = -30^\circ\,24'\,6''.0$

1899.430	233.3	3.57
.463	233.2	3.49
.473	234.5	3.45
1899.455	233.7	3.50

λ_1 326. $8^M.7$: $9^M.2$.

$\alpha = 17^h\,16^m\,5^s.4$: $\delta = -20^\circ\,38'\,31''.8$

1899.476	12.0	0.55
.575	11.8	0.56
.589	11.7	0.59
1899.547	11.8	0.57

λ_1 329.

Cannot find any such star near this place.

λ_1 330. $8^M.5$: $9^M.7$.

$\alpha = 17^h\,23^m\,47^s.5$: $\delta = -30^\circ\,10'\,40''.6$

1899.463	170.3	1.86
.509 [1]	171.8	1.65
.575	170.2	1.63
1899.546	170.8	1.71

[1] Stars not separated.

λ_1 332. 8^M : 12^M.

$\alpha = 17^h\,24^m\,18^s.5$: $\delta = -27^\circ\,7'\,26''.7$

A B

1899.427	189.1	7.47	

A C $C = 12.5$

1899.427	0.2	15.98	B.
1899.610	189.9	7.49	
.613	191.4	7.66	
1899.611	190.6	7.57	

A C

t	θ	ρ
	°	″
1899.610	359.2	15.98
.613	358.0	16.13
1899.611	358.6	16.06

λ_1 336. $9^M.2$: $12^M.5$.

$\alpha = 17^h\,35^m\,24^s.5$; $\delta = -18^\circ\,35'\,2''.0$

1899.592	97.6	3.34
.630	94.8	3.07
.638	96.4	3.34
1899.620	96.3	3.25

This is not O. A. 17158, but is S. DM. 18°4617.

λ_1 337. $8^M.3$; $9^M.3$.

$\alpha = 17^h\,39^m\,12^s.0$; $\delta = -28^\circ\,0'\,37''.6$

1899.471	10.6	9.63
473	10.8	9.83
.575	10.6	9.65
1899.506	10.7	9.70

λ_1 342. $6^M.7$; $7^M.3$.

$\alpha = 17^h\,46^m\,43^s.2$; $\delta = -34^\circ\,52'\,14''.1$

1899.600	283.9	0.46
.602	287.0	0.41
.605	287.3	0.48
1899.602	286.1	0.45

λ_1 343. $7^M.5$; $11^M.5$.

$\alpha = 17^h\,51^m\,12^s.4$; $\delta = -35^\circ\,59'\,38''.5$

A B

1899.463	16.2	3.40
.471	16.4	3.00
.473	15.5	3.31
.504	16.4	2.84
1899.478	16.1	3.14

A C $C = 13.1$.

1899.163	97.1	10.73
.471	97.8	10.85
.473	98.3	11.64
.504	98.0	11.04
1899.478	97.8	11.07

λ_1 344.

$\alpha = 17^h\,52^m\,4^s.2$: $\delta = -36^\circ\,50'\,59''.1$

Can find no such star near this place. Seeing very fine.

λ_1 346. $5^M.3$: $13^M.2$.

$\alpha = 17^h\,58^m\,37^s.8$: $\delta = -29^\circ\,35'\,3''.2$

A B

1899.610	234.5	32.87
.613	233.3	32.88
.649	234.0	32.47
1899.624	233.9	32.74

A C $C = 13.2$

1899.610	101.2	47.30
.613	100.7	47.99
.649	100.2	48.17
1899.624	100.7	47.92

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY OBSERVATORY,

By R. E. BRUCE and L. E. CROUCH, Students in Astronomy.

W M T 1900–1901		Groups N	Groups S	Spots in Groups N	Spots in Groups S	Totals Groups	Totals Spots	Def.
	d h							
Oct. 12	0	1	0	7	0	1	7	F
16	2	0	1	0	9	1	9	G
16	22	0	1	0	10	1	10	E
17	22	0	1	0	29	1	29	G
18	22	0	1	0	12	1	12	P
21	23	0	1	0	54	1	54	F
22	23	0	1	0	–	1	–	P
24	22	0	1	0	14	1	14	F
31	22	0	0	0	0	0	0	P
Nov. 1	22	0	0	0	0	0	0	F
5	3	0	0	0	0	0	0	F
5	22	0	0	0	0	0	0	G
7	23	0	0	0	0	0	0	G
8	22	0	0	0	0	0	0	P
9	22	0	0	0	0	0	0	P
11	22	0	0	0	0	0	0	P
12	22	0	1	0	1	1	1	E
13	22	0	1	0	1	1	1	E
16	1	0	1	0	1	1	1	P
16	23	0	1	0	1	1	1	F
27	22	0	0	0	0	0	0	P
30	20	0	0	0	0	0	0	F
Dec. 2	22	0	0	0	0	0	0	G
5	22	0	0	0	0	0	0	P
9	22	0	0	0	0	0	0	P
11	1	0	0	0	0	0	0	F
16	23	0	0	0	0	0	0	G
17	22	0	0	0	0	0	0	G
18	22	0	0	0	0	0	0	G
25	23	1	0	3	0	1	3	F
26	23	0	0	0	0	0	0	P
27	23	0	0	0	0	0	0	F
1901 Jan. 1	0	0	0	0	0	0	0	F
2	2	0	0	0	0	0	0	F
3	1	0	0	0	0	0	0	P
3	22	0	0	0	0	0	0	
4	22	0	0	0	0	0	0	
7	23	0	0	0	0	0	0	P
8	23	0	0	0	0	0	0	G
10	2	0	0	0	0	0	0	F
13	23	0	0	0	0	0	0	F
18	3	0	0	0	0	0	0	F
18	22	0	0	0	0	0	0	P
20	23	0	0	0	0	0	0	E
21	23	0	0	0	0	0	0	P
25	2	0	0	0	0	0	0	P
27	22	0	0	0	0	0	0	G
28	22	0	0	0	0	0	0	F
29	22	0	0	0	0	0	0	P
31	22	0	0	0	0	0	0	P
Feb. 1	21	0	0	0	0	0	0	F
7	3	1	0	1	0	1	1	P
8	1	1	0	1	0	1	1	F
10	21	0	0	0	0	0	0	F

W M T 1901		Groups N	Groups S	Spots in Groups N	Spots in Groups S	Totals Groups	Totals Spots	Def.
	d h							
Feb. 12	3	0	0	0	0	0	0	P
12	21	0	0	0	0	0	0	F
14	2	0	0	0	0	0	0	F
15	22	0	0	0	0	0	0	G
17	22	0	0	0	0	0	0	E
18	21	0	0	0	0	0	0	F
20	2	0	0	0	0	0	0	G
21	0	0	0	0	0	0	0	G
21	23	0	0	0	0	0	0	F
22	22	0	0	0	0	0	0	F
26	23	0	0	0	0	0	0	F
28	1	0	0	0	0	0	0	F
Mar. 1	2	1	1	12	2	2	14	F
5	2	1	0	–	0	1	–	P
5	22	1	0	2	0	1	2	F
6	23	1	0	1	0	1	1	P
8	0	2	0	16	0	2	16	G
12	2	0	0	0	0	0	0	P
13	1	0	0	0	0	0	0	F
14	23	0	0	0	0	0	0	P
17	23	0	0	0	0	0	0	P
19	0	0	0	0	0	0	0	P
21	23	0	0	0	0	0	0	G
22	22	0	0	0	0	0	0	E
24	0	0	0	0	0	0	0	F
26	22	0	0	0	0	0	0	G
27	22	0	0	0	0	0	0	P
28	23	0	0	0	0	0	0	G
29	22	0	0	0	0	0	0	E
Apr. 11	22	0	0	0	0	0	0	F
12	20	0	0	0	0	0	0	G
16	22	0	0	0	0	0	0	G
17	23	0	0	0	0	0	0	F
18	23	0	0	0	0	0	0	P
26	1	0	0	0	0	0	0	F
26	23	0	0	0	0	0	0	P
27	23	0	0	0	0	0	0	G
May 1	1	0	0	0	0	0	0	F
1	21	0	0	0	0	0	0	F
2	23	0	0	0	0	0	0	E
5	22	0	0	0	0	0	0	F
6	23	0	0	0	0	0	0	E
7	20	0	0	0	0	0	0	E
9	0	0	0	0	0	0	0	G
11	2	0	0	0	0	0	0	G
13	5	0	0	0	0	0	0	G
13	23	0	0	0	0	0	0	G
16	0	0	0	0	0	0	0	G
16	23	0	0	0	0	0	0	G
21	1	1	0	22	0	1	22	F
21	21	1	0	40	0	1	40	G
22	21	1	0	45	0	1	45	F
June 16	23	1	0	8	0	1	8	G
		11	12	158	134	26	292	

For explanations, see *A.J.* 466.

The number of different groups observed was 10, containing 138 different spots. Six groups, with 74 spots, were north of the equator, while 4 groups, with 64 spots, were south. No group was within 2° of the equator. Eight had a latitude less than 10°, while 2 were on the parallel of 10°. The spot observed in November had the same latitude, 6° S., and nearly the same longitude, 118°, as the mean of the October group. The group in June was probably a reappearance of the May group.

The longitudes of five northern and three southern groups were between 90° and 180°. One northern and one southern were between 270° and 360°.

The location of the current minimum can not be determined, as yet, but a comparison of these observations with those made during the preceding college year (*A.J.* 484) seems to indicate a decrease in solar activity. Doubtless conclusions based upon the number of sun-spots, alone, should not be considered final in locating maxima and minima, although they may be as reliable as those based upon any other *one* set of data. For the purpose of investigating the last maximum, Mr. CROUCH made use of all the observations of Rev. A. W. QUIMBY since January, 1890. Mr. QUIMBY kindly furnished any that had not been published. Four curves were constructed with intervals of 6, 60, 120 and 300 days respectively, from which was concluded a maximum at 1893.75. This result, in itself, may be of but little scientific value, though it is interesting to note that it agrees almost perfectly with the epoch given by NEWCOMB in the *Astrophysical Journal* for January, 1901. It should be added that Mr. CROUCH knew nothing of this epoch until his conclusion was reached.

ORBIT OF Σ3062.

By JOSHUA LARSON.

Most of the elements of this star hitherto published have been obtained by the graphical method. The latest and best elements are, evidently, those obtained by Dr. T. J. J. SEE (*A.N.* 3292). His elements represent the true places so nearly that a great improvement could not be expected. Nevertheless, my elements, which were obtained by the least-square method, do show a marked diminution of the residuals over those of Dr. SEE.

Great care was exercised in collecting the observations, and the original sources were consulted in every case as far as possible. The averages given by the different observers were verified, and every measure available was looked up at least twice in the original source.

A few of SCHIAPARELLI's measures, since about 1887, which are still unpublished, were taken on the authority of Dr. SEE (*A.N.* 3292), and some of MÄDLER's and STRUVE's, which I did not have access to, were kindly furnished by Prof. S. W. BURNHAM, who also verified my list of observations and added such measures as might have been overlooked.

Owing to their doubtful character, the measures of 1782, 1783 and 1823 were omitted altogether from the computation.

As provisional elements, I took the following by Dr. SEE (*A.N.* 3292):

$$\begin{aligned} T &= 1836.26 \\ e &= 0.450 \\ \Omega &= 47^\circ.15 \\ i &= 43^\circ.85 \\ \lambda &= 90^\circ.90 \\ \mu &= +3^\circ.4413 \\ a &= 1''.3712 \end{aligned}$$

Forming equations of condition from the observations of position-angle only, with suitable weights, I find the following

NORMAL EQUATIONS.

$$\begin{array}{rrrrrrrrr} +9.25x & -13.36y & -19.02z & -18.66s & +20.89t & -12.82u & -\overset{\circ}{6}.19 & = & 0 \\ & +131.76 & +62.66 & +257.91 & -305.66 & +109.93 & -3.19 & = & 0 \\ & & +117.55 & +101.45 & +194.54 & +67.86 & +90.17 & = & 0 \\ & & & +550.65 & -167.61 & +210.77 & -2.42 & = & 0 \\ & & & & -1260.38 & -270.73 & -4.57 & = & 0 \\ & & & & & +105.40 & +17.84 & = & 0 \end{array}$$

Solving these equations (and dividing $\delta\mu$ by 10, and δe by 57.296), we have

	Weight	Prob. Error
$x = \delta i = +3^\circ.63$	0.613	$\pm 1^\circ.547$
$y = \delta\lambda = +10^\circ.46$	0.386	$2^\circ.457$
$z = \delta e = -0.006$	4.189	0.226
$s = \delta\mu = -0^\circ.2898$	0.944	$1^\circ.005$
$t = \delta T = +0^y.63$	3.333	$0^y.285$
$u = \delta\Omega = -3^\circ.50$	0.784	$1^\circ.210$

Probable error of 1 observation of weight 1 $= \pm 0^\circ.94833$.

These values furnish the following

FINAL ELEMENTS (1900):

$P = 114.15$ (years)	$i = 47^\circ.48$
$T = 1836.89$	$\lambda = 101^\circ.36$
$e = 0.441$	$\mu = +3^\circ.1515$
$\Omega = 43^\circ.65$	$a = 1''.4919$

The sum of the squares of the residuals was reduced from 254.66 to 112.66.

The semi-major axis was determined from the distance-measures, which gave

$$\begin{aligned} \delta a &= +0''.1207 \\ \text{or} \quad a &= 1.4919 \end{aligned}$$

The above final elements furnish the following comparison with the observations:

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o-\theta_c$	$\rho_o-\rho_c$	n	Observer
1831.71	87.5	90.0	0.82	0.72	−2.5	+0.10	2	W. Struve
1833.71	108.6	109.3	0.56	0.61	−0.7	−0.05	3	W. Struve
1835.66	132.6	133.9	0.41	0.56	+1.3	−0.05	5	W. Struve
1836.65	147.6	147.1	0.47	0.57	−0.5	−0.10	8–5	Σ5; OΣ3–0
1837.78	157.9	161.2	0.49	0.60	−3.3	−0.11	3	W. Struve
1840.55	186.6	188.4	0.65	0.74	−1.8	−0.19	7–6	OΣ4; Da. 3–2
1841.72	193.2	196.7	0.89	0.80	−3.5	+0.09	9	Mädler 7; Da. 2
1842.80	207.3	203.3	0.87	0.87	+4.0	0.00	1	Mädler
1843.69	209.0	208.0	0.91	0.92	+1.0	−0.01	4	Mädler 3; Da. 1
1844.50	213.8	211.9	0.85	0.96	+1.9	−0.11	5	Mädler
1845.61	216.9	216.7	0.96	1.02	+0.2	−0.06	1	Mädler
1846.50	220.7	220.2	1.04	1.06	+0.5	−0.02	19	OΣ2; Mädler 17
1847.53	225.1	223.7	1.12	1.10	+1.2	+0.02	5	Mädler
1848.55	229.4	227.3	1.15	1.14	+2.1	+0.01	3	OΣ2; Da. 1
1849.19	232.5	230.3	1.09	1.16	+2.2	−0.07	3	O. Struve
1850.56	233.4	233.6	1.22	1.21	−0.2	+0.01	7–6	OΣ3; Mädler 3; Da. 1–0
1851.37	236.4	235.9	1.21	1.22	+0.5	−0.01	12	OΣ2; Mädler 8; Mädler 2
1852.48	238.5	237.9	1.21	1.24	+0.6	−0.03	15	Mädler 2; OΣ3; Mädler 10
1854.47	246.6	244.1	1.40	1.29	+2.5	+0.11	13–7	OΣ4; Da. 3; De. 6–0
1855.59	247.6	246.9	1.28	1.31	+0.7	−0.03	14–12	OΣ3; De. 8–6; Mo. 3
1856.65	248.8	249.5	1.32	1.32	−0.7	0.00	7	De. 3; Winn 1; OΣ2; Mädler 1
1857.56	252.0	251.7	1.30	1.34	+0.3	−0.04	10–9	OΣ3; Sec. 3; De. 4–3
1858.51	252.4	254.0	1.2	1.34	−1.6	−0.14	2–1	Dembowski
1859.16	255.3	255.5	1.46	1.35	−0.2	+0.11	3	O. Struve
1861.80	265.2	261.6	1.21	1.36	+3.6	−0.15	2	Mädler
1862.60	263.7	263.4	1.44	1.36	+0.3	+0.08	15–8	OΣ2; De. 11–4; Mädler 2
1863.73	265.9	266.1	1.42	1.37	−0.2	+0.05	11–10	Kn. 1; De. 9–8; Da. 1
1864.73	268.7	268.2	1.40	1.38	+0.5	+0.02	7–4	Dembowski
1865.71	271.0	270.4	1.32	1.38	+0.6	−0.06	11	De. 6–5; Kn. 3; Tal. 2–3
1866.65	272.5	273.0	1.39	1.38	−0.5	+0.01	14–18	OΣ2; Tal.3; Hv.3; De.0-4; Hl.5; Sec.1
1867.82	275.5	275.1	1.52	1.38	+0.4	+0.14	9	De. 7; New. 2
1868.82	277.2	277.3	1.48	1.39	−0.1	+0.11	6–7	De. 4; Tal. 0–1; OΣ2
1869.75	279.9	279.3	1.48	1.39	+0.6	+0.11	6	Dembowski
1870.48	281.2	280.9	1.52	1.39	+0.3	+0.14	15–16	OΣ2; Gl. 1; Tal. 5–6; De. 7
1871.62	284.0	283.3	1.42	1.39	+0.7	+0.03	8	Gl. 1; De. 7
1872.70	285.1	285.8	1.44	1.39	−0.7	+0.05	9	De. 6–5; Kn. 2; Ws. 1; Brw. 0–1
1873.76	287.7	288.1	1.47	1.39	−0.4	+0.08	12–10	De. 9–7; W.&H.&S. 1; Gl. 2
1874.80	290.2	288.6	1.40	1.39	+1.6	+0.01	9	De. 6; Sea. & Sm. 1; Gl. 2
1875.68	292.5	292.4	1.48	1.40	+0.1	+0.08	11	De. 6; Du. 5
1876.84	294.7	294.9	1.49	1.40	−0.2	+0.09	15–14	De. 5; OΣ1; Dk. 3; W.&S. 1; Pl.5–4
1877.68	296.4	296.7	1.47	1.40	−0.3	+0.07	8	De. 4; Dk. 4
1878.75	300.9	299.0	1.44	1.40	+1.9	+0.04	9	De. 4; Dk. 5
1879.70	301.8	301.1	1.45	1.41	+0.7	+0.04	13–12	Hl. 8; Dk. 2; Sea. & Sm. 3–2
1880.54	305.0	303.7	1.49	1.41	+1.3	+0.08	12	Sea & Sm. 2; β6; Dk. 4
1881.60	304.3	305.1	1.56	1.42	−1.0	+0.14	15–13	Jed. 3–2; β3; Sea. 3; Big. 1; Hl. 4
1882.58	306.9	306.0	1.40	1.42	+0.9	−0.02	15–13	Jed.7; Sea.&Sm.3–2; OΣ1; Dk.4–3
1883.68	310.4	309.5	1.60	1.42	+0.9	+0.18	14	Sea. 2; Ew. 9; Hl. 3
1884.48	311.7	311.2	1.26	1.43	+0.5	−0.17	2	Seabroke
1885.80	316.1	313.4	1.46	1.43	+2.7	+0.03	5	Hall
1886.56	314.8	314.7	1.44	1.44	+0.1	0.00	8–7	Sea. & Sm. 3–2; Hl. 5
1887.08	313.9	316.5	1.43	1.44	−2.6	−0.01	9–6	Sch. 6–3; Tar. 3
1888.66	319.3	319.7	1.41	1.45	−0.4	−0.04	11	Sch. 1; Hl. 4; Sch. 6
1889.79	322.0	322.0	1.44	1.46	0.0	−0.02	8	β3; Hl. 4; Highton 1
1890.64	324.2	323.7	1.41	1.47	+0.5	−0.05	9	Hig. 2; Big. 1; Hl. 5; Sch. 1
1891.53	325.0	325.5	1.44	1.47	−0.5	−0.03	7–6	Hig.1; See 1; Fuss 1–0; Col.2; Sch.2
1892.88	327.4	328.1	1.18	1.18	−0.7	0.00	9	Com.3; Hig. 1; Col. 2; Jo. 1; Sch. 2
1893.69	330.1	329.7	1.44	1.48	+0.4	−0.04	9	Glas. 2; Com. 2; H.& S. 3; Sch. 2
1894.61	331.2	331.4	1.72	1.49	−0.2	+0.23	6–5	Big. 3–2; Glas. 1; H.&S. 2
1895.49	332.0	331.1	1.52	1.50	−1.1	+0.02	30–25	D.&M.2; Big.7-6; Ho.3; Com.6; See 4;
1896.76	334.6	335.5	1.65	1.51	−0.9	+0.14	7	Hu.1; Morg.3 [Ml.2; Col.1; Dk.4-0
1897.64	336.6	337.1	1.54	1.51	−1.0	+0.03	16	Doolittle 5; Dk. 1; Aitken 3; Hu. 4
1898.51	337.3	338.7	1.61	1.52	−1.4	+0.09	1	Maw 3; Bryant 1
1900.74	344.8	342.7	1.39	1.54	+1.7	−0.15	5–4	Sola 1; Dk. 4–3

New Haven, Conn., 1901 *June*.

MICROMETRIC DETERMINATION OF THE DIAMETER OF *VENUS*,

By D. A. DREW.

In 1898, while engaged at the Lowell Observatory of Flagstaff, Arizona, I made the following measures of the diameter of *Venus* with the 24-inch refractor of that observatory. The measures were made primarily for the purpose of determining the diameter of this planet for my own personal satisfaction, since the diameters obtained by different persons using different methods disagree among themselves very considerably. As the work may possibly be of some interest to some one else I publish it here.

For all the work but two eye-pieces were used, giving respectively powers of 165 and 124; but the power of 124 was not often used. A diaphragm containing a circular aperture of a diameter of one-half a millimeter was always placed between the eye and the ocular, and for a number of the measures an amber-colored glass screen was used in addition to the diaphragm.

In order to diminish as much as possible the effects of refraction, the measures were made with the planet as near the meridian as circumstances would allow. As a result the measures were made in broad daylight, a circumstance that would tend to lessen in a considerable degree the effects of irradiation.

In the table the G.M.T. is given with sufficient accuracy to the nearest hundredth of a day. In the column $\theta-90°$ will be found the angle at which the micrometer was set while measuring, θ being taken from the *American Ephemeris*. The seeing is evidently marked on a scale of ten, and quite severely marked, too, everything being taken into consideration that would tend to make the measuring difficult or to vitiate the results. The columns d and $d\,(\Delta=1)$ contain respectively the measures, and these same measures reduced to unit distance and corrected for refraction. The measures made with the colored screen are indicated by c. The measures made with a power of 124 are indicated by a. The residuals are denoted by v, and k gives the ratio, as determined by measuring, of the illuminated portion of the apparent disk to the whole disk considered as the area of a circle. With the few exceptions that are noted, the diameter was obtained from three double measures.

G.M.T. 1898		$\theta-90$ °	Seeing	d ″	d $\Delta=1$ ″	v ″	k
Mar.	27.41	253.0	1	9.644	16.27	−0.61	0.965
	28.39	253.2	1–2	10.333*	17.41	+0.53	0.960
	31.42	253.1	1	10.407	17.48	+0.60	0.990
April	1.42	252.8	2–3	9.746	16.35	−0.53	0.995
	9.40	252.7	2–3	9.780	16.24	−0.64	0.982
	17.52	253.6	2–3	10.036	16.46	−0.42	0.971
	24.40	254.9	2–4	10.347	16.76	−0.12	0.986
	25.55	255.2	2–3	9.992	16.15	−0.73	1.000
May	22.48	265.1	4–6	10.835	16.36	−0.52	0.915

* Four double measures.

G.M.T. 1898		$\theta-90$ °	Seeing	d ″	d $\Delta=1$ ″	v ″	k
May	24.54	265.4	3–5	10.933	16.41	−0.47	0.915
	26.53	267.5	2–4	10.712	15.97	−0.91	0.903
	29.48	268.5	3–5	11.189	16.52	−0.36	0.895
June	3.43	271.5	2–4	11.609	16.84	−0.04	. .
	4.55	272.0	4–6	11.403	16.48	−0.40	. .
	7.33	273.5	1–3	11.490**	16.43	−0.45	0.861
	13.38	276.5	2–4	11.963	16.70	−0.18	0.861
	13.39	276.5	4–6	12.076 *c*	16.86	−0.02	0.853
	13.50	276.5	3–7	12.143 *c*	16.94	+0.06	0.846
	14.48	276.9	5–7	12.233	17.00	+0.12	0.838
	14.48	276.9	2–5	12.199 *c*	16.95	+0.07	0.843
	15.45	277.4	3–6	12.215	16.90	+0.02	0.839
	15.47	277.4	3–5	12.236 *c*	16.93	+0.05	0.840
	16.51	277.9	6–7	12.138	16.72	−0.16	0.835
	16.52	277.9	4–7	12.144 *c*	16.73	−0.15	0.836
	17.51	278.3	3–5	12.189	16.72	−0.16	0.826
	17.51	278.3	3–5	12.168 *c*	16.69	−0.19	0.829
	23.54	281.0	5–6	12.492	16.68	−0.20	0.828
	23.54	281.0	5–7	12.486 *c*	16.68	−0.20	0.828
	25.48	281.0	3–5	12.828	16.98	+0.10	0.827
	25.49	281.0	3–6	12.810 *c*	16.95	+0.07	0.826
	26.49	282.2	3–5	12.796	16.85	−0.03	0.823
	26.50	282.2	4–6	12.761 *c*	16.80	−0.08	0.823
	27.57	282.6	2–4	12.714	16.67	−0.21	0.813
	27.57	282.6	2–4	12.674 *c*	16.61	−0.27	0.815
	28.50	283.0	3–5	12.773	16.66	−0.22	0.814
	28.51	283.0	2–5	12.758 *c*	16.64	−0.24	0.817
	29.51	283.4	4–7	12.922	16.77	−0.11	0.799
	29.52	283.4	4–8	12.843 *c*	16.66	−0.22	0.808
	30.52	283.8	2–3	12.935	16.70	−0.18	0.803
	30.52	283.8	2–4	12.936 *c*	16.70	−0.18	0.799
July	7.55	286.3	4–6	13.611 *c*	16.95	+0.07	0.785
	7.56	286.3	3–5	13.596	16.93	+0.05	0.800
	18.44	289.5	2–3	14.247 *c*	16.67	−0.21	0.749
	18.47	289.5	2–4	14.275	16.70	−0.18	0.733
	19.44	289.7	1–3	14.850	17.27	+0.39	0.735
	19.44	289.7	1–4	14.601 *c*	16.97	+0.09	0.742
	21.50	290.2	3–4	14.706	16.89	+0.01	0.729
	21.51	290.2	3–5	14.674 *c*	16.85	−0.03	0.731
	23.49	290.7	3–5	14.785 *c*	16.77	−0.11	0.729
	23.49	290.7	3–4	14.775	16.76	−0.12	0.733
Aug.	10.49	293.2	3–5	17.016	17.04	+0.16	0.674
	10.49	293.2	3–4	17.030 *c*	17.05	+0.17	0.673
	26.33	293.4	2–4	20.062	17.65	+0.77	. .
	26.35	293.4	3–4	20.082† *c*	17.66	+0.78	. .
Sept.	1.50	293.1	1–2	20.454††	17.01	+0.13	0.591
	2.45	293.0	1–2	20.381	16.80	−0.08	. .
	2.45	293.0	2–3	20.382 *c*	16.80	−0.08	. .
	4.50	292.8	2	20.423	16.51	−0.37	. .
	4.50	292.8	2–3	20.410 *c*	16.49	−0.39	. .
	5.49	292.7	1	20.589	16.48	−0.40	0.559
	5.49	292.7	1–2	20.662 *c*	16.55	−0.33	0.567
	6.51	292.5	1–2	20.932	16.59	−0.29	0.551
	6.52	292.5	1–2	20.879 *c*	16.55	−0.33	0.554

** Two single measures.
† Five single measures.
†† Three single measures.

G.M.T. 1898	θ—90	Seeing	d	d Δ = 1	v	k
Sept. 8.44	292.3	1–2	20.996	16.33	−0.55	0.549
8.47	292.3	1	20.974 e	16.29	−0.59	0.550
10.47	292.1	1–2	22.099	16.83	−0.05	0.554
10.48	292.1	2	22.097 e	16.82	−0.06	0.540
11.51	291.9	1–2	22.213	16.75	−0.13	0.552
11.52	291.9	1–2	22.197 e	16.73	−0.15	0.552
12.44	291.8	1–2	22.617	16.88	0.00	0.546
12.45	291.8	1–2	22.634 e	16.89	+0.01	0.548
13.43	291.7	1–3	22.918	16.92	+0.04	0.537
13.44	291.7	1–2	22.869 e	16.88	0.00	0.539
14.52	291.5	2–3	23.212	16.95	+0.07	0.532
14.54	291.5	2–3	23.202 e	16.94	+0.06	0.531
17.44	291.1	1–2	24.225	17.13	+0.25	0.526
17.45	291.1	1–2	24.088 e	17.03	+0.15	0.529
18.44	290.9	1–3	24.364	17.03	+0.15	0.510
18.44	290.9	2–3	24.405 e	17.06	+0.18	0.511
20.47	290.5	3	25.335	17.31	+0.43	0.506
20.49	290.5	3	25.387 e	17.34	+0.46	0.503
21.52	290.4	1–3	25.274	17.07	+0.19	0.496
21.52	290.4	2	25.220 e	17.03	+0.15	0.498
22.48	290.2	1–3	25.627	17.11	+0.23	0.495
22.49	290.2	2	25.603 e	17.09	+0.21	0.496
23.47	290.0	1–2	25.580	16.88	0.00	0.494
23.47	290.0	1–2	25.561 e	16.87	−0.01	0.494
25.47	289.6	2	26.142	16.84	−0.04	0.495
25.47	289.6	2	26.166 e	16.86	−0.02	0.493
26.50	289.4	2	26.697	16.99	+0.11	0.481
26.51	289.4	2–3	26.708 e	17.00	+0.12	0.484
27.50	289.2	1–2	26.906	16.92	+0.04	0.479
27.51	289.2	2–3	26.821 e	16.87	−0.01	0.482
28.51	289.0	3	27.240	16.92	+0.04	0.473
28.52	289.0	3	27.280 e	16.95	+0.07	0.473
30.53	288.6	3	28.016	16.97	+0.09	0.449
30.54	288.6	2–3	28.053 e	16.99	+0.11	0.450
Oct. 4.49	287.7	2	30.698	17.66	+0.78	0.414
4.50	287.7	2	30.732 e	17.68	+0.80	0.413
5.54	287.5	2	30.830	17.49	+0.61	0.413
5.54	287.5	2	30.787 e	17.46	+0.58	0.412
9.53	286.7	2	31.736	17.03	+0.15	0.402
9.53	286.7	2	31.773 e	17.05	+0.17	0.402
14.44	285.7	1–2	33.970	16.99	+0.11	0.391
14.45	285.7	2	34.009 e	17.01	+0.13	0.384
14.51	285.7	2	35.002 a	17.49	+0.61	. .
14.52	285.7	2	34.746 ae	17.36	+0.48	. .
17.37	285.1	1	35.430	16.96	+0.08	0.354
17.38	285.1	2	35.460 e	16.97	+0.09	0.353
17.49	285.1	2	35.445 a	16.94	+0.06	0.355
17.50	285.1	2	35.446 ae	16.94	+0.06	0.355
21.48	284.5	2	36.885 a	16.57	−0.31	0.330
21.48	284.5	2	36.897 ae	16.59	−0.29	0.328
21.51	284.5	2	37.843	17.00	+0.12	0.323
21.52	284.5	2	37.792 e	16.99	+0.11	0.322
23.52	284.2	1–2	39.118	17.03	+0.15	0.309
23.52	284.2	1–2	39.050 e	17.00	+0.12	0.310
26.44	284.0	2	41.383	17.17	+0.29	0.285
26.45	284.0	2	41.345 e	17.15	+0.27	0.285
27.46	283.9	1–2	41.551	16.96	+0.08	0.276
27.47	283.9	1–2	41.572 e	16.96	+0.08	0.275
29.45	283.8	2	42.442	16.76	−0.12	0.248
29.45	283.8	2	42.589 e	16.82	0.06	0.247
Nov. 1.45	283.6	1–2	45.652 e	17.15	+0.27	0.218
Nov. 1.45	283.6	1–2	45.614	17.14	+0.26	0.218
1.45	283.7	1	46.152 e	16.61	−0.27	0.197
1.45	283.7	1	46.467	16.61	−0.27	0.211
5.44	283.7	2	49.726	17.48	+0.60	0.185
5.45	283.7	2	49.557 e	17.42	+0.54	0.195
6.42	283.8	2	48.892	16.92	+0.04	0.179
6.43	283.8	3	48.948 e	16.93	+0.05	0.179
12.44	284.8	1–2	51.167	17.04	+0.16	0.116
12.45	284.8	1–2	51.174 e	17.04	+0.16	0.116
14.48	285.4	1	55.703	17.01	+0.13	0.107
14.48	285.4	1	55.743 e	17.02	+0.14	0.107
15.36	285.7	1–2	56.406	16.99	+0.11	0.080
15.37	285.7	1	56.507 e	17.02	+0.14	0.080
22.41	289.0	2	60.552	16.77	−0.11	0.042
22.42	289.0	2	60.705	16.82	−0.06	0.041

An examination of the table shows that there is no practical difference between the measures made without the glass screen and those made with it. Our examination also reveals the fact that a number of the measures give very large residuals. These poor measures are probably the result of a number of causes, not the least of which, perhaps, was poor judgement in setting the wires.

If we now let d_0 represent the diameter obtained from the total 139 measures we obtain

$$d_0 = 16''.885 \quad \pm 0''.017$$

If we apply to these 139 measures Peirce's Criterion we find that the measure for May 26 is to be rejected. The remaining 138 measures give the diameter

$$d'_0 = 16''.891 \quad \pm 0''.017$$

We now reject all measures made with a power of 124, and separate those remaining into two groups, one containing 74 measures — one for each day of observation — made with a power of 165 and the diaphragm alone, the other containing 59 measures made on 58 different days with a power of 165 and the diaphragm and glass screen combined. The 74 measures of the first group yield a diameter

$$d_{1_0} = 16''.85 \quad \pm 0''.026$$

Applying Peirce's Criterion we find that no measures are to be rejected. The 59 measures of the second group yield a diameter

$$d_{2_0} = 16''.92 \quad \pm 0''.022$$

Applying the Criterion to this group we find that the measures for Aug. 26, Oct. 4 and Sept. 8 are to be rejected. The remaining 56 measures give a diameter

$$d'_{2_0} = 16''.90 \quad \pm 0''.016$$

These different determinations would seem to indicate that the diameter of the planet cannot be much different from 16''.9.

Examining again the residuals in the table we discover that they possess a peculiar characteristic, namely, the first residuals are nearly all negative, while the last are nearly

all positive. If we divide them into three groups we shall find that of the first forty-six 13 are positive and 33 negative; of the second forty-six 23 are positive, 20 negative and 3 zero; of the last forty-seven 38 are positive and 9 negative. The apparent diameter of the planet must, then, have undergone a peculiar variation from first to last.

In order to show up the nature of this variation we divide the 139 measures into seven groups of twenty measures each, except the last, which contains but nineteen. These seven groups yield the seven following diameters:

16″.63, 16″.75, 16″.91, 16″.82, 17″.08, 17″.03 and 16″.97.

We take also the 74 measures made with a power of 165, and the diaphragm alone, and divide them into seven groups, the first six containing ten measures each, and the last fourteen. These seven groups yield the following diameters:

16″.59, 16″.63, 16″.82, 16″.81, 17″.00, 17″.07 and 16″.99.

These groups of diameters show conclusively that the apparent diameter of the planet steadily increased up to a certain point, and then as steadily decreased. The variation is now clear enough, but how shall we go about discovering its causes? There lies the difficulty. It seems reasonable to assume that this variation is due to a combination of causes, such as the size, form and brilliancy of the planet's disk. We have good reason, I think, for assuming that irradiation plays no unimportant rôle in this matter, when we notice that the apparent diameter of the planet reached its maximum value in the latter part of October, and that the planet attained its greatest brilliancy on the twenty-sixth of the same month. It is a rather remarkable coincidence, to say the least.

Baraboo, Wis., 1901 *July* 13.

SECULAR PERTURBATIONS OF THE *EARTH* FROM THE ACTION OF *JUPITER*,

By ERIC DOOLITTLE.

The elements employed in this computation were from Dr. G. W. Hill's "*New Theory of Jupiter and Saturn*," pp. 19, 192, 554 and 558.

The Earth.	*Jupiter.*
$\pi = 100°\ 21'\ 39''.73$	$\pi' = 11°\ 54'\ 31''.67$
$i = 0\ \ 0\ \ 0.00$	$i' = 1\ \ 18\ \ 12.10$
$\Omega = -\ \ -\ \ -$	$\Omega' = 98\ \ 56\ \ 19.79$
$e = 0.01677114$	$e' = 0.04825511$
$n = 1295977''.416$	$n' = 109256''.62552$
$\log a = 0.0000000$	$\log a' = 0.7162374$
$m = \frac{1}{327000}$	$m' = \frac{1}{1047.879}$

Epoch 1850.0 G.M.T.

As in previous cases, the work has been duplicated from the beginning, and all known test equations have been applied.* The orbit of the *Earth* was divided into twelve parts with regard to the eccentric anomaly, but the agreement of the final sums used in obtaining the differential coefficients indicates that a division of the orbit into eight parts only would probably have been sufficient.

The constants of the orbit were found to have the values,

$I = 1°\ 18'\ 12''.10$	$\log k = p9.9998865$
$II = 1\ \ 25\ \ 19.94$	$\log k' = p9.9999998$
$II' = 272\ \ 58\ \ 11.88$	$\log c = p8.7995614$
$K = 88\ \ 27\ \ 5.264$	$c = +0.063032044$
$K' = 88\ \ 27\ \ 10.859$	

The equation, $\sin q \,.\, \frac{1}{2} A_1^{(0)} + \cos q \,.\, B_0^{(0)} = 0$, was found to give the residual $-0.000,000,000,0093$. When m' was left indefinite, the resulting values of the differential coefficients were the following:

		log coeff.
$\left[\frac{de}{dt}\right]_{00} =$	$-\ \ 85''.760340\ m'$	$n1.9332865$
$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} =$	$+7298.7450\ \ m'$	$p3.8632482$
$\left[\frac{dp}{dt}\right]_{00} =$	$-\ \ 26.316855\ m'$	$n1.4202340$
$\left[\frac{dq}{dt}\right]_{00} =$	$-\ 168.14734\ \ m'$	$n2.2256900$
$\left[\frac{dL}{dt}\right]_{00} =$	$-9631.7202\ \ m'$	$n3.9837038$

When finally the mass of *Jupiter* given above was adopted, ($m' = 1 \div 1047.879$), the following values were obtained for the secular variations:

$$\left[\frac{de}{dt}\right]_{00} = -\ 0''.081\,841\,849$$

$$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} = +\ 6.965\,2565$$

$$\left[\frac{dp}{dt}\right]_{00} = -\ 0.025\,114\,105$$

$$\left[\frac{dq}{dt}\right]_{00} = -\ 0.160\,464\,46$$

$$\left[\frac{dL}{dt}\right]_{00} = -\ 9.191\,6336$$

The values found by LEVERRIER are stated in the *Annales de l'Observatoire de Paris*, Tome II, p. 59, and Tome IV, pp. 11 and 12; those of NEWCOMB are given on pages 336 and 377 of the "*Secular Variations of the Four Inner Planets*," while the values of

$$\left[\frac{dp}{dt}\right]_{00} \text{ and } \left[\frac{dq}{dt}\right]_{00}$$

* In the duplication, addition and subtraction tables of logarithms were used. The following error occurs in ZECH's "*Tafeln der Additions-und Subtractions-Logarithmen*," second edition, p. 790: 560×.6 should be 336.0 instead of 326.0.

have been computed by Dr. HILL in the "*New Theory of Jupiter and Saturn*," pp. 511 and 512. If all these results are reduced to the above value of m', they will compare as follows:

	LEVERRIER	NEWCOMB	HILL	Method of GAUSS
	"	"		"
$\left[\frac{de}{dt}\right]_{00} =$	−0.08182	−0.08182	. . . . .	−0.0818118*
$e\left[\frac{d\pi}{dt}\right]_{00} =$	+0.11679	+0.11677	. . . . .	+0.1168153
$\left[\frac{dp}{dt}\right]_{00} =$	−0.02501	−0.02511	−0.0251149	−0.0251144
$\left[\frac{d\Omega}{dt}\right]_{00} =$	−0.16041	−0.16017	−0.1604628	−0.1604645
$\left[\frac{dL}{dt}\right]_{00} =$	−9.1916	. . . .	. . . . .	−9.191634

* The results of NEWCOMB were computed to one more figure than was published, in order to insure the accuracy of the final figure as here given. It is probable that this rather large difference arises from terms neglected in the series employed by LEVERRIER and NEWCOMB, since their results agree so exactly. The method of GAUSS gives the following values for $\left[\frac{de}{dt}\right]_{00}$ when the orbit of the *Earth* is divided into but *six* parts:

From the six even points of division, −0″.0818428
From the six odd points of division, −0″.0818409

The final value from twelve points of division is of course the mean of these two, and is theoretically correct to terms of the eleventh order in e and $\sin I$.

The Flower Observatory, 1901 *July* 23.

ANNOUNCEMENT AS TO THE PUBLICATION OF A NEW CATALOGUE OF VARIABLE STARS.

The Council of the *Astronomische Gesellschaft* has undertaken the preparation of a new Catalogue of Variable Stars and has delegated to the undersigned Committee the conduct of this work. The Committee request observers of variable stars who have considerable unprinted series of observations, which would be useful in the correction of elements, either to publish them soon or to communicate them to the member of the Committee in charge (Prof. G. MÜLLER, Potsdam Observatory).

The Committee also announces that it will from the present time undertake the definitive notation of newly discovered variables as soon as their light-fluctuations are certainly ascertained. A list will shortly be published of the names of variables found in recent years which have heretofore remained unnamed.

The Committee on Publication of a
Catalogue of Variable Stars:

DUNÉR, HARTWIG, MÜLLER, OUDEMANS.

CONTENTS.

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OBSERVATIONS OF *EROS*,

MADE AT THE CHAMBERLIN OBSERVATORY, UNIVERSITY PARK, COLORADO.

By CHARLES J. LING.

The following observations of *Eros* were made with the Bruce Micrometer on the twenty-inch equatorial. The magnifying power was two hundred. The right-ascension observations are chronographic, and the declination-bisections were made while the object was drifting through the field.

1900 Univ. Park M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
June 6 14 29 30	1	20 , 6	− 2 23.28	+10 28.4	0 4 52.72	+ 5 38 54.6	n9.626	0.726	+1.97	+11.0
6 14 47 29	2	10 , 6	− 2 49.04	+ 2 1.7	0 4 52.73	5 39 26.8	n9.611	0.722	1.97	11.0
24 14 24 11	3	25 , 9	+ 1 41.88	+ 26.9	0 36 12.06	10 59 37.7	n9.601	0.684	2.34	12.0
24 14 52 21	4	20 , 6	− 1 52.57	− 9 54.9	0 36 14.34	11 0 2.6	n9.565	0.673	2.41	11.9
25 14 20 56	5	20 , 6	− 19.30	+14 42.8	0 37 55.31	11 17 51.9	n9.602	0.683	2.45	12.1
25 14 44 10	6	20 , 6	− 3 54.20	− 7 48.3	0 37 56.53	11 18 8.2	n9.574	0.673	2.43	11.9
26 14 17 59	7	20 , 8	+ 55.84	− 5 3.5	0 39 37.43	11 35 58.9	n9.604	0.681	2.49	12.1
26 14 35 21	8	20 , 8	− 1 14.68	+ 2 22.0	0 39 38.80	11 36 12.3	n9.583	0.674	2.47	12.1
27 14 32 41	9	20 , 6	− 2 3.91	+17 12.5	0 41 22.11	11 54 21.3	n9.584	0.671	2.50	12.2
27 15 9 12	10	10 , 3	−10 46.15	+ 1 18.7	0 41 22.69	11 54 51.7	n9.529	0.654	2.45	11.9
29 14 48 7	11	20 , 8	− 58.20	− 9 7.0	0 44 46.62	12 31 10.8	n9.557	0.656	2.48	12.1
July 2 13 49 17	12	20 , 8	− 1 9.71	+ 46.3	0 49 47.13	13 25 34.9	n9.622	0.677	2.62	12.3
2 14 19 2	13	20 , 6	+ 3 47.54	− 6 43.0	0 49 48.70	13 26 1.1	n9.590	0.660	2.64	12.4
3 14 44 13	14	20 , 6	− 5 48.32	+ 1 40.4	0 51 21.49	13 44 43.7	n9.551	0.642	2.61	12.2
5 15 4 46	15	10 , 3	− 4 55.34	− 2 33.7	0 54 55.82	14 22 7.6	n9.508	0.623	2.67	12.2
6 14 27 4	16	20 , 6	− 2 18.67	− 1 28.7	0 56 33.83	14 40 11.6	n9.569	0.640	2.71	12.2
6 14 40 37	17	20 , 6	− 4 7.12	−10 46.1	0 56 34.76	14 40 22.7	n9.548	0.632	2.70	12.2
27 13 25 48	18	20 , 6	− 1 44.62	− 2 31.3	1 30 51.66	21 20 26.8	n9.603	0.582	3.33	12.7
27 13 40 3	19	20 , 6	− 2 38.11	− 4 9.2	1 30 52.56	21 20 39.7	n9.584	0.567	3.32	12.6
27 13 59 39	20	20 , 8	+ 27.02	+ 9 47.7	1 30 54.21	21 21 1.3	n9.554	0.548	3.33	12.8
29 15 5 24	21	20 , 6	+ 1 55.50	− 1 53.4	1 34 7.69	22 1 11.0	n9.382	0.473	3.39	12.8
29 15 20 14	22	20 , 6	− 1 37.95	+ 3 25.6	1 34 8.81	22 1 25.0	n9.332	0.462	3.38	12.6
30 13 49 10	23	20 , 6	+ 3 4.70	+ 7 19.6	1 35 37.18	22 19 56.3	n9.561	0.538	3.42	13.0
31 13 46 7	24	20 , 8	− 28.85	+10 3.6	1 37 10.86	22 39 48.9	n9.563	0.534	3.44	12.2
31 14 2 1	25	30 , 9	− 2 56.75	+ 6 16.4	1 37 11.75	22 39 54.6	n9.534	0.516	3.42	12.5
Aug. 2 13 51 51	26	20 , 8	+ 1 1.67	− 1 34.4	1 40 17.63	23 19 37.3	n9.547	0.512	3.49	12.6
2 14 2 46	27	20 , 6	− 3 3.30	− 9 40.8	1 40 18.33	23 19 45.4	n9.523	0.499	3.48	12.2
6 13 31 25	28	20 , 8	+ 1 9.23	− 7 30.5	1 46 24.24	24 39 36.5	n9.570	0.505	3.63	12.6
6 13 49 10	29	20 , 6	− 4 51.61	− 1 49.4	1 46 25.52	24 39 51.0	n9.538	0.483	3.61	12.2
20 10 59 0	30	20 , 6	+ 1 13.74	+ 14.3	2 6 37.91	29 26 35.9	n9.706	0.620	4.10	12.3
20 11 12 46	31	20 , 6	+ 1 35.66	− 9 45.7	2 6 38.79	29 26 48.9	n9.700	0.600	4.10	12.3
20 11 23 40	32	16 , 6	− 4 2.51	+ 6 24.3	2 6 39.35	29 27 0.3	n9.693	0.582	4.08	12.0
20 11 41 4	33	20 , 8	− 33.66	− 4 47.1	2 6 40.35	29 27 15.3	n9.680	0.554	4.10	12.2
21 12 25 41	34	20 , 6	+ 1 26.89	− 1 8.8	2 8 5.21	29 49 9.1	n9.633	0.468	4.14	12.3
21 12 40 27	35	20 , 6	+ 2 41.04	+10 28.1	2 8 5.92	29 49 19.5	n9.612	0.441	4.14	12.5
22 13 21 11	36	20 , 8	+ 51.96	− 4 29.7	2 9 29.60	+30 11 11.7	n9.535	0.353	+4.18	+12.3

1900 Univ. Park M.T.	⁎	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
			m s	″	h m s				s	″
Aug. 22 13 37 8	37	20 , 6	+ 2 5.89	+ 5 51.0	2 9 30.16	+30 11 24.8	n9.497	0.222	+4.18	+12.4
25 11 28 23	38	20 , 6	− 8 13.92	− 7 40.0	2 13 22.57	31 13 41.0	n9.728	0.534	4.21	11.4
22 10 13 25	39	20 , 6	+ 0 15.76	− 6 28.8	2 41 3.73	41 39 2.4	n9.709	0.249	5.42	11.8
22 10 59 50	40	20 , 6	+ 1 29.05	+10 25.2	2 41 4.15	41 39 19.8	n9.687	0.222	5.40	12.0
24 10 56 12	41	20 , 6	− 11.64	+ 5 53.2	2 42 3.72	42 21 59.7	n9.689	0.135	5.48	11.9
24 11 9 37	42	20 , 8	+ 37.46	− 1 28.5	2 42 4.15	42 25 11.7	n9.668	0.062	5.50	11.9
28 11 5 4	43	20 , 8	− 42.86	− 39.4	2 43 29.58	43 56 13.2	n9.662	9.913	5.69	11.9
28 11 21 30	44	20 , 6	+ 2 24.96	+ 5 6.5	2 43 29.69	43 56 28.6	n9.631	9.762	5.68	12.3
29 9 41 8	45	20 , 6	− 1 39.68	−11 40.7	2 43 42.85	44 17 25.0	n9.765	0.335	5.75	11.9
29 9 53 37	46	20 , 6	− 1 29.53	−21 11.7	2 43 42.92	44 17 39.1	n9.754	0.285	5.76	11.9
Oct. 2 11 19 59	47	20 , 8	− 55.99	− 8 21.3	2 44 5.62	45 26 15.6	n9.612	9.287	5.92	12.5
2 11 31 24	48	20 , 6	− 1.03	+ 52.8	2 44 5.32	45 26 27.2	n9.585	8.646	5.91	12.6
3 10 22 9	49	20 , 6	− 9.40	+ 7 17.0	2 44 6.10	45 47 30.8	n9.715	9.988	5.95	12.7
3 10 37 11	47	20 , 12	− 55.54	+13 5.4	2 44 6.09	45 47 45.4	n9.692	9.859	5.94	12.6
4 9 50 48	50	20 , 8	+ 1 2.47	+ 9 38.6	2 44 3.57	46 9 2.3	n9.752	0.152	5.99	13.0
4 10 3 41	51	20 , 8	− 1 2.30	−16 44.1	2 44 3.69	46 9 16.3	n9.737	0.073	6.02	12.7
6 10 35 8	52	20 , 8	+ 1 11.87	+ 5 0.6	2 43 46.14	46 53 20.6	n9.683	0.630	6.08	13.4
6 10 47 49	53	20 , 6	− 3 36.49	+ 8 10.7	2 43 46.16	46 53 31.5	n9.659	0.364	6.08	12.8
8 9 49 20	54	20 , 6	+ 1 28.31	− 2 22.5	2 43 11.88	47 35 24.2	n9.746	9.990	6.18	13.7
8 10 1 2	55	20 , 6	+ 1 48.35	−11 23.9	2 43 14.69	47 35 32.5	n9.728	9.876	6.19	13.7
9 9 30 22	56	20 , 6	− 3 3.92	− 4 37.1	2 42 53.13	47 56 7.1	n9.766	0.084	6.24	13.3
9 9 50 17	57	20 , 8	− 1 6.52	+ 3 39.7	2 42 53.00	47 56 27.7	n9.742	9.925	6.23	13.6
10 8 13 13	58	20 , 6	− 3 8.16	+ 2 24.8	2 42 29.26	48 15 46.6	n9.823	0.427	6.28	13.5
10 8 25 40	59	20 , 6	− 4 9.67	+ 6 9.8	2 42 28.88	48 15 59.7	n9.817	0.381	6.28	13.4
11 9 22 27	60	20 , 8	+ 1 14.96	− 2 10.0	2 41 57.51	48 37 18.2	n9.771	0.048	6.36	14.3
11 9 33 10	61	20 , 6	+ 1 36.80	− 3 4.7	2 41 57.09	48 37 29.7	n9.758	9.961	6.34	14.3
13 9 27 30	62	20 , 8	+ 1 4.78	+ 3 25.4	2 40 13.74	49 17 33.4	n9.760	9.888	6.45	14.8
13 9 39 42	63	20 , 6	− 4 4.51	+10 38.8	2 40 13.30	49 17 43.6	n9.743	9.739	6.45	14.2
16 9 15 51	64	20 , 8	− 35.46	+ 3 25.7	2 38 20.59	50 14 52.2	n9.765	9.791	6.60	15.6
16 9 27 51	65	20 , 8	− 1 15.79	+ 2 59.0	2 38 20.13	50 15 0.2	n9.749	9.596	6.60	15.5
17 9 59 58	66	20 , 6	− 1 20.46	+ 4 44.2	2 37 21.96	50 33 43.2	n9.687	9.437	6.65	15.4
17 10 16 56	67	20 , 6	− 3 53.65	− 4 20.6	2 37 21.27	50 33 54.8	n9.650	9.724	6.66	15.4
18 9 21 7	68	20 , 8	− 1 8.71	+ 2 47.9	2 36 22.25	50 51 2.0	n9.749	9.420	6.69	16.1
18 9 37 4	69	20 , 6	− 1 16.32	−10 15.4	2 36 21.49	50 51 12.8	n9.724	8.241	6.71	15.7
22 8 48 36	70	20 , 6	+ 2 10.53	− 6 44.8	2 31 33.02	51 56 31.9	n9.777	9.549	6.81	18.0
22 9 4 32	71	20 , 8	+ 1 5.65	+ 9 37.2	2 31 31.79	51 56 40.9	n9.754	8.756	6.79	17.8
23 8 29 16	72	20 , 6	− 3 20.50	−11 21.5	2 30 10.17	52 11 20.3	n9.796	9.764	6.88	17.6
23 8 47 37	73	20 , 6	+ 2 52.37	− 28.7	2 30 9.24	52 11 29.7	n9.773	9.378	6.83	18.4
24 9 29 19	74	20 , 6	− 2 20.91	+ 3 18.7	2 28 38.48	52 26 17.2	n9.693	n9.783	6.88	18.2
24 9 45 55	75	20 , 6	+ 3 27.22	+ 2 40.2	2 28 37.26	52 26 26.6	n9.655	n9.932	6.86	19.0
25 9 2 25	76	20 , 6	+ 1 8.67	+ 2 22.2	2 27 8.21	52 39 34.6	n9.737	n9.449	6.88	19.5
25 9 17 51	77	20 , 6	+ 3 0.96	− 4 9.3	2 27 7.22	52 39 46.3	n9.708	n9.733	6.89	19.3
26 8 42 4	78	20 , 6	− 1 56.48	− 5 57.6	2 25 33.51	52 52 16.5	n9.764	n8.632	6.97	19.0
26 9 6 40	79	20 , 6	+ 3 29.81	−13 28.4	2 25 31.66	52 52 28.1	n9.722	n9.676	6.95	19.8
27 8 24 18	80	20 , 6	+ 3 3.25	− 6 3.8	2 23 55.08	53 4 8.2	n9.784	9.210	6.97	20.2
27 8 38 11	81	20 , 6	+ 3 6.03	− 3 39.4	2 23 54.13	53 4 21.2	n9.763	n9.001	6.96	20.2
31 9 23 15	82	20 , 6	− 2 59.61	+ 1 10.7	2 16 37.75	53 44 59.0	n9.624	n0.102	7.07	21.5
31 9 35 57	83	20 , 6	− 3 1.96	+ 2 25.4	2 16 37.19	53 45 2.8	n9.585	n0.154	7.08	21.6
Nov. 2 8 15 7	84	20 , 6	− 1 36.27	− 3 56.9	2 12 50.78	53 59 28.9	n9.751	n9.624	7.08	22.7
2 8 27 24	85	20 , 6	− 2 1.51	− 10.1	2 12 50.00	53 59 33.7	n9.731	n9.802	7.07	22.6
3 8 37 13	86	20 , 6	+ 2 37.84	+ 1 26.1	2 10 49.85	54 5 41.2	n9.699	n9.956	7.03	23.7
3 8 58 19	87	20 , 6	+ 5 11.16	+ 35.4	2 10 47.87	54 5 47.2	n9.648	n0.085	6.99	24.2
5 8 47 6	88	20 , 6	− 2 11.77	− 8 48.5	2 6 15.73	54 11 58.4	n9.648	n0.095	7.08	24.0
5 8 59 15	89	20 , 6	− 3 11.31	−10 11.5	2 6 11.54	54 15 0.4	n9.612	n0.118	7.09	23.9
6 7 11 19	90	20 , 6	− 2 20.11	− 6 19.2	2 1 48.28	54 17 55.6	n9.767	n9.548	7.09	24.6
6 7 56 29	91	20 , 6	− 2 55.99	− 8 10.5	2 1 17.33	54 17 58.2	n9.718	n9.740	7.08	24.5
7 8 17 13	92	20 , 6	− 2 4.71	− 7 45.2	2 2 38.71	54 20 15.3	n9.614	n0.151	7.07	25.1
7 9 2 32	93	20 , 6	− 2 34.57	− 9 55.6	2 2 37.37	54 20 16.2	n9.563	n0.205	7.08	25.0
9 8 50 1	94	20 , 6	+ 1 39.41	+ 8 18.4	1 58 31.45	54 21 29.3	n9.565	n0.204	6.98	26.7
9 9 3 1	95	20 , 6	+ 3 5.89	+ 2 21.6	1 58 29.88	54 21 29.1	n9.516	n0.240	6.98	26.9
10 8 15 16	96	20 , 6	− 2 44.89	− 6 49.2	1 56 30.51	+54 20 33.8	n9.654	n0.090	+7.01	+26.6

1900–01 Univ. Pk. M. T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Nov. 10 8h 41m 7s	97	20 , 6	+ 3m 25.68s	− 4.8″	1h 56m 28.84s	+54° 20′ 32.2″	n9.576	n0.194	+6.97s	+27.5″
12 10 46 33	98	20 , 8	+ 37.31	+ 7 45.7	1 52 20.69	54 15 25.7	n9.109	n0.335	6.93	28.1
12 11 7 28	99	20 , 6	+ 3 58.64	+ 43.5	1 52 18.82	54 15 23.0	n8.765	n0.347	6.90	28.5
13 7 2 59	100	20 , 6	− 4 50.49	+14 48.3	1 50 36.94	54 11 45.6	n9.783	n9.152	6.95	27.9
13 7 19 44	101	20 , 6	− 5 9.86	+14 0.0	1 50 35.29	54 11 43.1	n9.738	n9.794	6.96	27.8
14 7 43 35	102	20 , 6	+ 3 36.41	−19 36.5	1 48 36.73	54 6 38.0	n9.672	n0.035	6.87	29.5
19 6 53 54	103	20 , 6	− 1 54.42	+ 2 43.5	1 39 47.56	53 26 38.8	n9.713	n9.824	6.68	30.9
19 7 16 30	104	10 , 3	− 3 13.75	+ 3 23.1	1 39 45.79	53 26 30.3	n9.662	n0.006	6.70	30.6
20 7 11 17	105	20 , 8	+ 39.79	−11 39.4	1 38 11.96	53 15 34.1	n9.659	n9.997	6.62	31.5
20 7 23 52	106	20 , 6	− 4 17.84	− 1 48.0	1 38 10.96	53 15 30.5	n9.626	n0.067	6.67	30.9
22 7 10 3	107	20 , 6	+ 2 40.36	+ 6 39.3	1 35 18.21	52 51 0.7	n9.630	n0.029	6.50	32.3
22 7 20 6	108	20 , 6	− 2 47.12	+ 9 12.7	1 35 18.01	52 50 52.0	n9.601	n0.079	6.57	31.7
29 6 37 38	109	20 , 6	− 2 42.40	+10 0.7	1 28 18.25	50 59 46.4	n9.606	n9.926	6.14	31.5
29 6 56 48	110	29 , 6	+ 5 19.55	−11 1.5	1 28 17.74	50 59 35.0	n9.550	n0.027	6.06	32.3
30 7 6 2	111	20 , 6	− 3 8.24	− 4 57.2	1 27 12.63	50 40 36.6	n9.491	n0.071	6.24	33.8
Dec. 1 7 28 8	112	20 , 8	+ 46.22	+ 2 3.8	1 27 15.11	50 21 12.7	n9.370	n0.135	6.14	34.3
1 7 49 16	113	20 , 6	+ 5 13.76	− 8 39.1	1 27 14.34	50 20 54.0	n9.234	n0.177	6.09	34.8
8 6 52 43	114	20 , 6	− 3 35.88	+ 3 58.1	1 27 10.38	47 52 49.1	n9.388	n9.974	5.96	34.6
8 7 18 8	115	20 , 6	− 4 31.22	− 2 9.2	1 27 10.86	47 52 32.3	n9.304	n0.022	5.99	34.4
10 6 59 57	116	19 , 6	+ 3 14.59	+ 2 52.9	1 28 8.23	47 6 31.8	n9.306	n9.969	5.83	35.2
10 7 19 18	117	20 , 6	+ 4 0.94	+ 7 53.4	1 28 8.61	43 6 15.0	n9.170	n0.076	5.82	35.3
11 7 19 39	118	20 , 8	− 1 17.60	+ 5 55.0	1 28 17.50	46 42 39.7	n9.136	n9.999	5.86	34.7
11 7 32 37	119	20 , 6	− 1 36.52	− 7 0.7	1 28 48.29	46 42 28.3	n8.999	n0.020	5.88	34.7
12 6 51 4	120	20 , 6	− 1 46.88	− 7 36.1	1 29 31.73	46 19 17.3	n9.314	n9.902	5.85	34.6
12 7 25 10	121	20 , 6	+ 5 20.75	−11 19.7	1 29 33.56	46 18 43.1	n9.046	n9.988	5.75	35.3
14 6 50 19	122	19 , 6	− 2 18.23	− 1 33.6	1 31 20.33	45 30 53.5	n9.276	n9.853	5.79	34.4
14 7 4 53	123	20 , 6	− 2 15.83	+ 7 30.2	1 31 20.66	45 30 39.1	n9.173	n0.899	5.80	34.1
15 6 8 0	124	20 , 6	+ 3 0.45	−12 34.0	1 32 22.06	45 7 1.5	n9.465	n9.524	5.70	34.7
15 6 17 20	125	20 , 6	− 3 34.23	+ 1 59.2	1 32 22.46	45 6 50.6	n9.429	n9.623	5.78	34.1
17 7 43 8	126	20 , 6	+ 2 37.38	− 2 21.1	1 34 48.19	44 16 43.9	n8.262	n9.865	5.66	34.4
17 7 58 54	127	20 , 6	− 2.17	− 34.5	1 34 48.80	44 16 29.7	n8.403	n9.863	5.69	34.4
18 6 56 45	128	20 , 6	+ 4 2.64	− 9 47.2	1 36 9.74	43 51 26.5	n9.137	n9.746	5.64	34.3
18 7 10 37	129	20 , 6	+ 2 43.67	− 2 1.8	1 36 10.34	43 51 10.6	n8.999	n9.785	5.65	34.2
31 5 59 13	130	20 , 6	− 3 10.34	+ 7 39.3	2 7 34.02	38 18 15.1	n9.333	9.690	5.67	29.8
31 6 12 55	131	20 , 5	− 3 11.33	+ 4 21.8	2 7 36.35	38 17 57.6	n9.269	9.613	5.68	29.8
1901 Jan. 2 6 7 54	132	20 , 6	+ 4 5.78	+ 3 37.2	2 6 37.90	35 27 14.2	n9.228	n9.906	1.89	12.5
10 10 35 15	133	20 , 6	+ 1 45.52	− 7 58.8	2 28 41.49	33 58 21.0	9.631	0.359	1.93	11.0
14 9 2 29	134	20 , 6	− 2 26.56	−10 28.1	2 41 57.89	32 16 55.9	9.414	0.193	1.95	9.2
16 9 4 29	135	20 , 6	− 2 7.90	− 8 13.7	2 48 27.39	31 25 18.5	9.422	0.231	1.95	8.8
16 9 14 39	136	20 , 6	− 2 50.02	− 7 30.9	2 48 28.61	31 24 47.5	9.454	0.252	1.96	8.7
17 8 58 42	137	20 , 8	− 19.46	− 7 35.8	2 51 44.06	30 59 11.3	9.402	0.237	1.95	8.5
17 9 12 45	138	20 , 6	+ 1 49.95	−11 27.7	2 51 46.12	30 59 28.2	9.263	0.263	1.94	8.7
18 9 17 33	139	20 , 8	+ 47.63	− 9 58.8	2 55 6.57	30 34 6.5	9.465	0.289	1.95	8.3
21 9 25 55	140	20 . 8	+ 1 0.48	+ 47.0	3 5 19.79	29 18 14.1	9.484	0.340	1.95	7.4
21 9 37 46	141	20 , 6	+ 1 15.75	+ 2 27.2	3 5 21.58	29 18 2.4	9.513	0.360	1.94	7.4
30 9 22 57	142	20 , 6	+ 1 19.98	+ 56.4	3 37 20.51	25 35 12.0	9.470	0.427	1.94	4.2
30 9 33 38	143	20 , 6	− 2 23.91	+ 2 18.1	3 37 22.02	25 35 1.7	9.485	0.434	1.97	4.0
Feb. 4 9 24 22	144	20 , 6	+ 2 59.46	− 19.6	3 55 43.97	23 34 10.3	9.471	0.470	1.95	2.6
4 9 37 53	145	20 , 6	+ 3 2.02	− 3 57.6	3 55 45.84	23 33 56.0	9.503	0.484	1.95	2.6
5 9 17 45	146	20 , 6	+ 4 11.97	− 1 31.[illegible]	3 59 25.46	23 10 23.3	9.452	0.471	1.93	2.3
5 9 29 7	147	20 , 6	− 3 4.76	+ 2 0.8	3 59 27.04	23 10 11.2	9.482	0.482	1.97	+ 1.9
12 9 31 17	148	20 , 8	− 16.39	− 8 19.9	4 25 31.71	20 25 51.5	9.491	0.534	1.94	− 0.5
12 9 48 10	149	20 , 6	+ 2 42.43	− 1 45.0	4 25 36.71	20 25 40.3	9.519	0.545	1.92	0.4
15 8 28 11	150	20 , 6	− 2 5.75	− 6 0.4	4 36 38.59	19 18 29.0	9.308	0.507	1.96	1.6
15 8 46 52	151	19 , 6	− 2 59.54	+ 8 4.7	4 36 41.22	+19 18 16.9	9.382	0.518	1.96	1.7
May 8 9 16 26	152	20 , 6	− 3 30.59	−13 10.5	9 17 2.93	− 3 12 44.0	9.514	0.767	1.88	19.1
June 13 9 11 56	153	20 , 6	− 2 52.18	+17 44.2	11 1 43.18	10 9 45.1	9.572	0.792	2.05	19.9
17 9 30 16	154	19 , 6	+ 1 11.13	+ 2 47.8	11 12 51.14	10 53 59.6	9.601	0.787	2.08	20.0
17 9 40 27	155	10 , 3	+ 2 26.16	+ 9 5.6	11 12 52.91	−10 54 5.0	9.613	0.784	+2.07	−20.4

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	0 7 14.03	+ 5 28 25.2	Leipsic II, A.G. 34
2	0 7 39.80	5 37 14.1	" " 36
3	0 34 27.75	10 58 58.8	Glasgow 174
4	0 38 4.50	11 9 15.6	Göttingen 213
5	0 38 12.16	11 2 57.0	Munich 392
6	0 41 48.30	11 25 44.6	Yarnall 389
7	0 38 39.10	11 40 50.3	Göttingen 217
8	0 40 51.01	11 33 38.2	" 232
9	0 43 23.55	11 36 56.6	" 249
10	0 52 7.39	11 53 21.1	Cincinnati (13) 124
11	0 45 42.24	12 40 5.7	Schjellerup 290
12	0 50 51.22	13 24 36.3	Cincinnati (13) 119
13	0 45 58.52	13 32 31.7	Munich 467
14	0 57 18.20	13 42 51.1	Cincinnati (13) 137
15	0 59 48.49	14 24 29.1	" (13) 143
16	0 58 49.79	14 44 28.1	Yarnall 561
17	1 0 39.18	14 50 56.6	Cincinnati (13) 145
18	1 32 32.95	21 22 45.4	Berlin (B) A.G. 499
19	1 33 27.35	21 24 36.3	" " " 501
20	1 30 23.96	21 11 0.8	" " " 481
21	1 32 8.78	22 2 51.6	" " " 494
22	1 35 43.38	21 57 46.8	" " " 508
23	1 32 29.06	22 12 23.7	" " " 497
24	1 37 36.27	22 29 33.1	" " " 515
25	1 40 5.08	22 33 25.7	" " " 522
26	1 39 12.47	23 20 59.1	" " " 519
27	1 43 18.15	23 29 14.2	" " " 537
28	1 45 11.38	24 16 54.4	" " " 550
29	1 51 13.52	24 41 28.2	" " " 580
30	2 5 20.07	29 26 9.3	Cambr. (Eng.) A.G. 1163
31	2 4 59.03	29 36 22.3	" " " 1161
32	2 10 37.78	29 20 24.0	" " " 1209
33	2 7 9.91	29 31 50.2	" " " 1176
34	2 6 34.18	29 50 5.6	" " " 1173
35	2 5 20.74	29 38 38.9	" " " 1164
36	2 8 30.46	30 15 29.1	" " " 1188
37	2 7 20.39	30 5 21.4	" " " 1178
38	2 21 32.25	31 21 9.6	Washington (2) 515
39	2 40 12.55	41 15 19.4	Bonn A.G. 2371
40	2 39 29.70	41 28 42.6	" 2352
41	2 42 12.88	42 18 54.6	" 2395
42	2 41 21.19	42 29 28.3	" 2383
43	2 44 6.75	43 56 40.7	" 2424
44	2 40 59.05	43 54 9.8	" 2376
45	2 45 16.78	44 28 53.8	" 2440
46	2 45 6.69	44 38 48.9	" 2439
47	2 44 55.69	45 34 27.1	Comp. with 48 and 49*
48	2 44 0.44	45 25 21.8	Bonn A.G. 2423
49	2 44 9.55	45 39 31.4	" 2427
50	2 42 55.44	45 59 10.7	" 2409
51	2 44 59.97	46 25 47.7	" 2436
52	2 42 28.19	46 48 6.6	" 2398
53	2 47 16.57	46 45 8.0	" 2463
54	2 44 40.36	47 37 33.0	" 2387
55	2 44 20.15	47 46 42.7	" 2381
56	2 45 50.81	48 0 30.9	" 2447
57	2 43 53.29	47 52 31.4	" 2421
58	2 45 31.44	48 13 8.3	" 2442
59	2 46 32.27	+48 9 36.5	" 2455
60	2 40 36.19	+48 39 13.9	Bonn A.G. 2367
61	2 40 13.95	48 40 20.1	" 2363
62	2 39 32.51	49 13 53.2	" 2349
63	2 44 41.36	49 6 50.6	" 2434
64	2 38 49.45	50 11 10.9	" 2342
65	2 39 29.32	50 11 45.7	" 2348
66	2 41 35.77	50 28 43.6	Camb. (Mass.) A.G. 1286
67	2 41 8.26	50 38 0.0	" " " 1282
68	2 37 24.27	50 47 58.0	" " " 1253
69	2 40 31.10	51 1 12.5	" " " 1272
70	2 29 15.68	52 2 58.7	" " " 1201
71	2 30 19.35	51 46 45.9	" " " 1207
72	2 33 23.79	52 22 24.2	" " " 1227
73	2 27 10.04	52 11 40.0	" " " 1189
74	2 30 52.54	52 22 40.3	" " " 1208
75	2 25 3.18	52 23 27.4	" " " 1177
76	2 22 52.66	52 36 52.9	" " " 1156
77	2 23 59.37	52 43 36.3	" " " 1168
78	2 27 23.02	52 57 55.1	" " " 1191
79	2 21 54.90	53 5 36.7	" " " 1150
80	2 20 44.86	53 9 51.8	" " " 1138
81	2 20 41.15	53 7 40.4	" " " 1137
82	2 19 30.32	53 40 26.8	" " " 1127
83	2 19 35.07	53 42 15.9	" " " 1130
84	2 14 19.97	54 3 3.1	" " " 1092
85	2 14 47.47	53 59 21.2	" " " 1095
86	2 8 4.98	54 3 51.4	" " " 1036
87	2 4 59.72	54 4 47.6	" " " 1012
88	2 8 50.42	54 23 22.9	" " " 1046
89	2 10 18.76	54 24 51.0	" " " 1056
90	2 7 1.60	54 23 50.2	" " " 1023
91	2 7 36.21	54 25 44.2	" " " 1030
92	2 4 36.38	54 27 35.4	" " " 1007
93	2 5 4.84	54 29 46.8	" " " 1013
94	1 56 45.06	54 12 44.2	" " " 950
95	1 55 17.01	54 18 37.6	" " " 936
96	1 59 8.36	54 26 56.4	" " " 970
97	1 52 56.19	54 20 9.5	" " " 921
98	1 51 36.15	54 7 11.9	" " " 913
99	1 48 13.28	54 11 11.0	" " " 880
100	1 55 20.48	53 56 29.4	" " " 937
101	1 55 38.19	54 0 15.3	" " " 941
102	1 44 53.45	54 25 45.0	" " " 849
103	1 44 35.30	53 23 24.4	" " " 819
104	1 42 52.84	53 22 36.6	" " " 830
105	1 37 25.55	53 26 42.0	" " " 789
106	1 42 22.13	53 16 47.6	" " " 825
107	1 32 31.38	52 43 49.1	" " " 735
108	1 37 58.56	52 41 8.6	" " " 794
109	1 30 54.51	50 49 14.2	" " " 718
110	1 22 52.13	51 10 4.2	" " " 659
111	1 30 44.63	50 45 0.0	" " " 717
112	1 26 22.78	50 18 34.6	" " " 686
113	1 21 51.49	50 28 58.3	" " " 646
114	1 30 10.30	47 48 16.4	Bonn A.G. 1350
115	1 31 36.09	47 54 7.1	" 1366
116	1 24 17.81	47 3 6.7	" 1277
117	1 24 1.85	46 59 16.3	" 1264
118	1 29 59.24	+46 36 10.0	" 1339

*	α	δ	Authority
	h m s	° ′ ″	
119	1 30 18.93	+46 48 54.3	Bonn A.G. 1344
120	1 31 12.76	46 26 18.8	" 1363
121	1 24 6.06	46 29 27.5	" 1266
122	1 33 32.77	45 31 52.7	" 1406
123	1 33 30.69	45 22 34.8	" 1405
124	1 29 15.91	45 19 0.8	" 1326
125	1 35 50.91	45 4 17.3	" 1438
126	1 32 5.15	44 18 30.6	" 1380
127	1 34 45.28	44 16 30.1	" 1421
128	1 32 1.46	44 0 55.9	" 1378
129	1 33 21.02	43 52 38.2	" 1403
130	2 10 38.69	38 10 6.6	Weisse's Bessel II, 3
131	2 10 42.00	38 12 52.5	" " II, 4
132	2 2 30.23	35 23 24.5	Washington 2d Cat. 458
133	2 26 54.04	34 6 18.8	Glasgow 2d Catal. 201
134	2 44 22.50	32 26 14.8	Weisse's Bessel II, 1012
135	2 50 33.34	31 33 23.4	" " II, 1151
136	2 51 16.67	31 32 9.7	Washington 2d Cat. 607
137	2 52 1.57	+31 7 8.6	Weisse's Bessel II, 1184
138	2 49 54.23	+31 10 47.2	W.B. II, 1138 and 1139
139	2 51 16.99	30 43 57.0	W.B. II, 1239 and 1240
140	3 4 17.36	29 17 19.7	Cambr. (Eng.) A.G. 1593
141	3 1 3.88	29 15 27.8	" " " 1590
142	3 34 58.59	25 34 11.4	" " " 1789
143	3 39 43.96	25 32 39.6	" " " 1834
144	3 52 42.56	23 34 27.3	Berlin (B) A.G. 1274
145	3 52 41.87	23 37 51.0	" " " 1273
146	3 55 11.56	23 11 54.7	" " " 1294
147	4 2 29.93	23 8 8.5	" " " 1336
148	4 25 49.16	20 34 11.9	" " " 1452
149	4 22 52.39	20 27 25.7	" " " 1443
150	4 38 12.38	19 21 31.0	Berlin (A) A.G. 1280
151	4 39 38.80	+19 10 13.9	" " " 1284
152	9 20 31.64	− 2 59 14.1	Munich Ann. I. 1130
153	11 4 33.31	− 9 51 11.0	" " 6332
154	11 11 38.23	−10 56 27.1	" " 6515
155	11 10 24.68	−11 2 50.5	" " 6491

*Comparison-Star, No. 47, is No. 2435 of the A.G. (Bonn) Catalogue. On account of discrepancies in the position of *Eros* where this star was used as comparison-star, micrometric comparisons were made with Nos. 48 and 49. The comparisons show a proper motion of No. 47 of $+0^s.053$ in R.A. and $-0''.31$ in Decl. See *A.J.* 503, p. 184.

ON THE CORDOBA DURCHMUSTERUNG AND SOME CONCLUSIONS DERIVED FROM IT.

BY SIMON NEWCOMB.

The three volumes of the Cordoba *Durchmusterung* which have thus far appeared cover the zone of the sky between the parallels of −22° and −52°. Within these limits are catalogued the positions and magnitudes of 489,662 stars, a larger number than is contained in the catalogues of ARGELANDER and SCHÖNFELD combined. If the work is extended to the pole on the same scale, which is to be hoped, the positions of some 250,000 stars between −52° and the south pole will be added. If the whole sky were covered on the same scale, the number of stars included would be about 2,400,000.

Apart from the great and permanent scientific value of this work, the interest which attaches to its prosecution at a point so distant from the world's centers of astronomical activity, and under difficulties little known to our public, naturally attracts attention to it. Through the courtesy of Professor PICKERING I have been enabled to devote part of a season of leisure at a mountain resort to a careful examination of the work, the results of which may not be entirely devoid of interest. I shall call especial attention to those features which seem to suggest an improved method in its prosecution, with the hope that it may be thus rendered yet more valuable to the astronomy of the future.

I.

I begin with a study of the system of magnitudes. The estimates of magnitude are given to tenths up to $9^M.9$, after which, with unimportant exceptions, we have only mag. 10, without fractions. In this the observers may have followed the system of the northern work, in which all stars at the limit are classed as of $9^M.5$.

The first feature to which I shall invite attention is the relative number of estimates of each individual magnitude. To show this I have made a count of several strips around or near the south galactic pole (R.A. 0^h 10^m; Decl. −27°). Five zones were selected, scattered through the three volumes, and in each the first 800 magnitudes from 0^h of R.A. were classified as to their value. The result is shown in the following table:

TABLE I.

Number of Stars of Each Magnitude in Certain Strips of the Cordoba Durchmusterung.

Mag.	−23°	−32°	−40°	−42°	−50°	Sum
To 5.9	2	3	3	4	0	12
6.0 to 6.4	2	2	0	1	2	7
6.5 to 7.0	1	3	4	5	3	16
7.0 to 7.4	8	3	5	7	3	26
7.5 to 7.9	13	12	14	4	12	55
8.0	2	5	7	3	4	21
8.1	4	3	3	1	4	15
8.2	1	1	1	2	4	9
8.3	8	8	10	3	3	32
8.4	4	5	5	3	1	18
8.5	6	13	11	5	4	39
8.6	6	11	2	4	5	28
8.7	11	3	8	2	3	27
8.8	11	7	3	8	5	34
8.9	16	8	8	3	1	39
9.0	19	26	27	11	9	95
9.1	21	8	12	11	4	56
9.2	31	9	6	16	6	68
9.3	37	22	22	16	11	108
9.4	30	29	14	19	18	110
9.5	46	31	35	18	19	149
9.6	46	52	57	25	10	190
9.7	72	67	53	27	28	247
9.8	26	38	18	46	39	167
9.9	80	88	91	61	63	383
10.	297	343	381	492	537	2050

In estimates of this class observers generally show a well-marked bias, entire numbers having a preference, .5's, .3's or .7's coming next in order, and .9's and .1's last of all. In the present case this bias is much less marked than usual, though evident in the case of the 0's. There is, however, an evident bias affecting not special decimals so much as special magnitudes. $8^{M}.2$ is discriminated against throughout, and $9^{M}.8$ in the first three zones, where it occurs not half so often as $9^{M}.7$. The progression of the numbers seems to show a paucity of estimates from $8^{M}.5$ to $8^{M}.9$, followed by a sudden increase at $9^{M}.0$. The number $9^{M}.9$ shows a marked preponderance through the whole series of zones, the opposite of what one might expect when we have above it only 10^{M}.

Most remarkable and regrettable is the progressive increase in the last mentioned class as the work goes on, exceeding one-half the whole at the southern limit. This is yet more marked in the galactic regions. In those strips of the southernmost zones which fall in this region, say −50° and −51°, more than two-thirds of the stars are assigned the magnitude 10 simply, while less than one-third, little more than one-fourth in fact, have other magnitudes assigned.

We may suppose that the work in question stops at some fairly uniform limit of magnitude. But the deficiency in question makes it impossible to determine what that limit is, unless from extraneous considerations.

II.

The next step is to reduce the Cordoba system of magnitudes to the photometric scale. This is best done by means of the Southern Harvard Photometry (*Annals, H.C.O.*, Vol. 24). Besides the magnitudes of all the lucid stars this work gives those of the fainter stars to about $10^{M}.0$ in certain narrow zones of declination 5° apart. I have compared these magnitudes with those of some 480 stars of the Cordoba DM., Parts I and II, about 300 stars being distant from the galaxy, and 180 in the galactic region. The mean differences were taken for groups of three-tenths of a magnitude each, from $7^{M}.0$ to $9^{M}.9$, $7^{M}.0$, $7^{M}.1$ and $7^{M}.2$ being combined as $7^{M}.1$, etc. The mean results are shown in Table II for the two regions separately: P, that distant from the galaxy; G, that in or near it. Here columns Σ give the algebraic sum of the differences, N the number of stars compared, P and G the means of the differences.

The two regions are considered separately because it is known that, in a field bright with stars, the observer is apt to make too high an estimate of the magnitudes of faint stars. The table seems to show that the effect is small in the present case — only about $0^{M}.15$.

TABLE II.

Mean Excesses of the Harvard Phometric Magnitudes over those of the Cordoba DM., I and II, for Different Values of the Cordoba Magnitudes.

Cord. Mag.	Non-galactic regions		Galactic regions		Means		
	Σ	N	Σ	N	P	G	$G-P$
6.5	− 0.4	10	. .	. .	−.01	. .	. .
6.8	− 1.1	11	. .	. .	−.10	. .	. .
7.1	− 0.7	13	+ 0.3	7	−.05	+.04	+.09
7.4	+ 5.7	31	− 0.1	12	+.18	−.01	−.19
7.7	+ 5.1	20	+ 0.8	10	+.26	+.08	−.18
8.0	+10.2	39	− 0.4	19	+.26	−.02	−.28
8.3	+ 9.3	30	+ 3.0	15	+.31	+.20	−.11
8.6	+24.4	50	+ 9.7	29	+.49	+.33	−.16
8.9	+25.9	51	+17.0	40	+.50	+.42	−.08
9.2	+ 7.9	17	+ 7.4	25	+.47	+.30	−.17
9.5	+ 0.8	4	+ 2.6	12	+.20	+.22	+.02

It will be seen that the numbers progress with fair regularity up to mag. $9^{M}.0$, after which there is a falling off, both in the number of stars compared and the amount of the correction. The cause of the latter falling off is this: — the group of Cordoba stars of any given magnitude, say $9^{M}.5$, owing to the unavoidable errors of the estimates, really consists of stars ranging through a certain series of magnitudes, say from $9^{M}.1$ to $9^{M}.9$. The brighter the star is, the more likely it is to be included in the Harvard work, which, nominally stopping at about $10^{M}.4$, must omit many stars near this limit. Hence we compare only the brighter stars of each Cordoba class, to which a fainter than the normal Cordoba magnitude has been assigned, and thus the cor-

rection comes out too small. I therefore regard the corrections as unreal beyond 9M.0, and proceed by the reverse process, taking the Harvard magnitudes as the argument. Since all the Harvard stars are contained in the C.DM., we shall thus avoid the error in question. The result of this process is shown in Table III, which is formed and arranged on the same plan as Table II.

TABLE III.

Excesses of the Cordoba over the Harvard Magnitudes in the Galactic Regions, G, and the Non-galactic Regions, P, for Harvard Magnitudes from 5M.6 to 10M.4.

Harv. Mag.	Non-galactic regions Σ	N	Galactic regions. Σ	N	Means P	G	G−P	Wt.
5.6	+ 2.2	11	+ 0.6	1	+.20	+.60	+.40	1
5.9	+ 0.4	2	+ 0.3	1	+.20	+.30	+.10	1
6.2	+ 0.6	3	+ 0.5	1	+.20	+.50	+.30	1
6.5	+ 2.3	15	+ 2.1	4	+.15	+.52	+.37	3
6.8	+ 0.3	8	+ 0.4	3	+.04	+.13	+.09	2
7.1	+ 0.4	9	+ 2.0	6	+.04	+.33	+.29	4
7.4	0.0	12	+ 1.7	8	.00	+.21	+.21	5
7.7	− 3.4	25	− 0.7	13	−.14	−.05	+.09	9
8.0	− 2.7	25	− 1.0	8	−.11	−.12	−.01	6
8.3	− 7.2	26	− 1.7	20	−.28	−.08	+.20	11
8.6	−11.4	33	− 1.5	11	−.34	−.14	+.20	8
8.9	−12.7	32	− 2.7	15	−.40	−.18	+.22	10
9.2	−11.1	30	− 8.7	27	−.37	−.32	+.05	14
9.5	−17.8	33	−16.8	40	−.54	−.42	+.12	18
9.8	−16.6	22	−10.0	18	−.75	−.56	+.19	10
10.1	− 6.9	7	− 1.4	2	−.99	−.70	+.29	1
10.4	− 0.8	1	− 0.7	1	−.80	−.70	+.10	0

Mean, $G-P = +0^{M}.155$.

The numbers in the columns "Means" are, it will be noted, completely independent of each other, each depending on a different set of stars. Their regularity seems to show that little would be gained by extending the comparison to more stars in non-galactic regions.

It is clear, however, that a more extended reduction of the fainter magnitudes to the photometric scale is required. To do this in the simplest way it is necessary to select a few stars marked 9M.9 and 10M, from Parts I and II (50 of each would be enough), and determine their magnitudes with the photometer. After the mean correction for 9M.9 is thus determined, the correction for values from 9M.0 to 9M.9 can readily be interpolated. The measures of stars noted as 10M will show over what part of the photometric scale these stars range.

The mean systematic error of estimate in the galactic regions, −0M.155, seems to be nearly constant through the whole series of values, the tendency being, possibly, to diminish. Dividing the series into three divisions, we have the following separate means:

5M.6 to 7M.4	$G-P =$	+0M.25	Wt. 17
7.7 to 8.9		+0.18	44
9.2 to 10.1		+0.12	43

The probable error for Wt. 1 is about ±0M.2. The differences from the mean exceed the probable errors but little, and the diminution of the error with the fainter stars is the opposite of what we should expect. I therefore consider that we should regard the error as having the constant value 0M.15.

It must be remarked, however, that the stars *G* were selected from the galactic region generally, and not, as they should have been, from the denser galactic agglomerations. A comparison with the latter class of stars must be made before we can decide upon the systematic errors of their estimated magnitudes. But, even if the error should come out twice as great for the richest regions, it will still, I believe, be below the corresponding error of the Northern DM.

I have combined the *P* and *G* corrections after correcting the latter by −0.15, with results shown in the second column, *A*, of the following table. Then, I have smoothed off the results in column *B*, and applied them to the arguments, so as to show the mean Cordoba magnitude corresponding to individual values of the Harvard magnitude.

TABLE IV.

Reduction of the Harvard to the Cordoba Magnitudes for Different Values of the Former.

Harv. Mag.	Reduction A	B	Cord. Mag.
5.6	+0.22	+0.22	5.82
5.9	+ .20	+ .23	6.13
6.2	+ .25	+ .21	6.41
6.5	+ .20	+ .18	6.68
6.8	+ .03	+ .12	6.92
7.1	+ .10	+ .04	7.14
7.4	+ .02	− .01	7.39
7.7	− .16	− .10	7.60
8.0	− .15	− .19	7.81
8.3	− .26	− .25	8.05
8.6	− .33	− .32	8.28
8.9	− .37	− .38	8.52
9.2	− .42	− .46	8.74
9.5	− .56	− .58	8.92
9.8	− .73	− .70	9.10
10.1	− .96	− .82	9.28
10.4	−0.80	−0.94	9.46

Most remarkable is the great and rapidly increasing difference between the two scales from 9M.0 upwards, which is rendered yet more remarkable by the fact that, up to 9M.0, the Cordoba scale seems to be nearly identical with that of ARGELANDER. Yet the results to which the comparison leads us are, as we shall see, both plausible and self-consistent.

The comparison as I have given it does not enable us to determine separately the part of the photometric scale through which each $0^{M}.1$ of the Cordoba magnitudes is to be considered as extending. But a smoothed-off result is easily derived from Table IV. For example, the range of Cordoba mag. 8.5 is that part of the Harvard scale lying between the limits corresponding to the values 8.45 and 8.55 of the Cordoba scale, namely 8.81 and 8.91.

The investigation of the individual tenths can be more profitably undertaken after the comparison can be carried to higher magnitudes.

III.

I have been careful to confine the preceding discussion of the Cordoba magnitudes to the first two volumes of the *Durchmusterung*, extending from −22° to −42°, for the following reason. In the Introduction to Part III, Thome gives a comparison of its magnitudes with those of the Harvard photometry, published four years before, showing an almost perfect agreement, the mean systematic deviation through the whole length of the scale being only $\pm 0^{M}.05$. I have verified this agreement by an independent comparison of 84 stars common to the two works, which brings out the additional notable fact that the deviations of the individual values from the Harvard work are much smaller than the deviations from the corrected means in Vols. I and II. Omitting three stars of the U.A. and one doubtful case, we have left 80 comparisons, with the following differences:

	1	Difference (C.−H.) of	$-0^{M}.8$
	6		−0.3
	7		−0.2
	21		−0.1
	32		0.0
	9		+0.1
	3		+0.2
	1		+0.3
Total	80	Mean	±0.10

This mean deviation is just half the mean accidental residual in the comparisons already given of Parts I and II with the Harvard work, which is $\pm 0^{M}.20$. There is evidently a complete change of system in passing from Part II to Part III which no one but the author can explain, and which seems to demand a detailed statement of the way in which the magnitudes of Part III were derived. Whatever the explanation may be it is not clear that we have a more satisfactory system in Part III than in Parts I and II, and the question is suggested whether, in the preparation of the two remaining parts, which all must hope to see speedily completed, a return to the former system is not to be desired.

IV.

One of the important results to be derived from such a work as the one under examination is not only a knowledge of the richness of the various regions of the sky in stars, but their richness in light emanating from these stars. This depends on the ratio of progression in the numbers of stars, as the count is extended to fainter magnitudes. The value of this ratio is, for the brighter magnitudes, between 3 and 4. It is different for different parts of the sky, and as Seeliger has shown, exceptionally large for the agglomerations of the Milky Way. It must ultimately diminish with the fainter magnitudes, so as to fall below 2.5, else the totality of star-light would have no limit. Yet, for the sky in general, and for each region of the sky outside star-clusters and the galactic masses, the researches of Seeliger seem to show that it varies but slowly as we ascend in the scale of magnitudes.

It follows that if we put R_{M} for the richness of the sky, or any region of it, up to magnitude M, understanding by the term richness the number of stars in some unit of sky-surface, say 1° square; and if we express that quantity in the form

$$R_{M} = h\rho^{M}$$

we may choose h and ρ so that they shall vary but little with M through a wide range of the fainter magnitudes, and shall accurately represent the true richness for two magnitudes chosen at pleasure. Moreover, if we put h' for another constant, approximately equal to $h \log \rho$, and call r_{M} the richness in stars between the close limits M and $M + \Delta M$, we may express this quantity in the form

$$r_{M} = h' \rho^{M} . \Delta M$$

It must be understood that the values of h, h' and ρ will vary with the adopted light-scale. Taking any one scale, the method of proceeding will be this:

Put r_0 for the richness in stars of magnitude M; r_n for that in stars of magnitude $M + n$. We shall then have the two equations

$$h' \rho^{M} \; \Delta M = r_0$$
$$h' \rho^{M+n} \; \Delta M = r_n$$

from the quotient of which we find

$$n \log \rho = \log r_n - \log r_0$$

which gives us the value of ρ for the special light-scale of the catalogue used. The value of h' for any small value of ΔM, say 0.1, can then be formed from either equation.

If we now take a fresh light-scale, such that the difference of magnitude n in the one scale is expressed by $(1 + \epsilon)\, n$ in the other scale, we shall have a different value of ρ, say ρ', which will be given by the equation

$$\log \rho' = \frac{\log \rho}{1 + \epsilon}$$

This method may be illustrated by some counts of stars

from the C.DM. I took a number of strips lying within 30° of the galactic pole, and counted the magnitudes within them, extending the count to a larger area for the brighter magnitudes, so as to get a number of stars large enough to form a good basis for the series.

The following are some details of the count. That already given for the magnitudes in zones −23°, −32°, and −40°, was taken so far as it goes. It was supplemented by a count of certain strips in zones −24° and −25° for all the magnitudes; then by additional strips for magnitudes 7ᴹ.0 to 8ᴹ.9, and yet additional ones for magnitudes 7ᴹ.0 to 7ᴹ.9 only. The abnormally small number of the latter led to a recount and extension for magnitudes 6ᴹ.0 to 7ᴹ.9. The following is a conspectus of the strips and trapezia counted with the areas and the numbers of stars. Under mag. 7ᴹ.0 . . we put the stars from 7ᴹ.0 to 7ᴹ.4, etc.

A. For Magnitudes to 9.9.

Zone	0ʰ R.A. to	Number of Stars 7.0 . .	7.5 . .	8.0 . .	8.5 . .	9.0 . .	9.5 . .
°	h m						
−23	2 4.3	8	13	19	50	138	270
−24	2 11.6	5	8	30	44	157	427
−25	0 34.1	2	3	11	13	53	103
−32	2 2.0	3	12	22	42	94	276
−40	3 0.3	5	14	26	32	81	254
Total Number		23	50	108	181	523	1330

Areas: 28°.6 + 30°.1 + 7°.7 + 25°.8 + 34°.2 = 126°.4.

B. For Magnitudes 8.0 to 8.9 Inclusive.

I.	Zone −26°	R.A. 0ʰ to 1ʰ 17ᵐ	Area 17°.2
II.	" −27	" 0 to 1 31	" 20.2
III.	" −28	" 0 to 2 0	" 26.4
IV.	" −33	" 23 to 1 30	" 31.2
V.	Trap. −23 to −29	" 23 to 24	" 80.9

	8.0 . .	8.5 . .
Total Number of Stars,	163	271
Total Area,	175°.9.	

C. For Magnitudes 6.0 to 7.9 Inclusive.

			6.0 . .	7.0 . .	7.5 . .
Trap. I.	−23° to −35°	23ʰ to 2ʰ 30ᵐ	97	93	184
II.	−35 to −41	0 to 1 30	19	25	41
Total number of stars,			116	118	225

Areas: 550°.2 + 123°.2 = 673°.4.

The computation of the richness per square degree for the several classes, with the resulting ratios, is now as follows:

Mag.	Area °	Stars	Richness	log R	ρ	ρ'
6.0 to 6.9	673.4	116	0.171	9.236	3.0	
7.0 to 7.4	"	118	0.175	9.244	3.6	3.0
7.5 to 7.9	"	225	0.334	9.524	7.2	
8.0 to 8.4	302.3	271	0.897	9.953	2.8	4.7
8.5 to 8.9	"	453	1.50	0.176	7.6	
9.0 to 9.4	126.4	523	4.14	0.617	6.5	6.1
9.4 to 9.9	"	1330	10.52	1.022		

The values of ρ are those which result from the ratios of the successive numbers; those of ρ' from the ratios of numbers differing by a whole magnitude. Both sets are too discordant to lead to any useful conclusion. The discordance arises principally from the discrimination already pointed out against magnitudes 8.6 to 8.9, and in favor of 9ᴹ.9. We shall therefore reduce the numbers to the photometric scale by the data already given, and start from the richness of the lucid stars. Schiaparelli has plotted the richness of every part of the sky in stars to 6ᴹ.0, using the Harvard Photometry for the northern hemisphere, and the Uranometria Argentina for the southern. The richness of the two hemispheres from these two authorities he finds to be substantially equal. Since Schiaparelli's work was published, the Southern Harvard Photometry has appeared, giving 63 more stars south of −30° than Schiaparelli, the numbers being

Schiaparelli–Gould	1190
Harvard Photometry	1253

This excess is smaller than should result from the difference of the two scales of magnitude, but I have not the data at command for explaining the discrepancy.

I also find, from Schiparelli's planisphere, that the mean richness within 10° of the S. galactic pole is

$$R_{6.05} = 0.0836$$

To fix the starting point for our comparisons we must reduce this number to $R_{5.73}$, which will include the stars to 5ᴹ.9 of the C DM. scale, and multiply it by the ratio Harvard ÷ Schiaparelli. We have

$$\rho^{-0.32} = 3.7^{-0.32} = 0.6586$$

and then for the Harvard scale,

$$R_{5.95} = R_{6.05} \times 0.6586 \times 1.055 = 0.0581$$

Instead of finding the ratios for the successive magnitudes individually, I shall find them for all the stars up to each given magnitude. This is done in the following table.

Limits of Magnitude.

Cordoba Scale	Harvard Photom. Scale	Richness between limits	Richness 0 to limit.
0. to 5.95	0. to 5.73	0.0581	0.058
5.95 to 6.95	5.73 to 6.84	0.171	0.229
6.95 to 7.45	6.84 to 7.49	0.175	0.404
7.45 to 7.95	7.49 to 8.18	0.334	0.738
7.95 to 8.45	8.18 to 8.82	0.897	1.635
8.45 to 8.95	8.82 to 9.56	1.50	3.14
8.95 to 9.45	9.56 to 10.46	4.14	7.28
9.45 to 9.95	10.46 to 11.?	10.52	17.80

Using the formula already given, we have the equations

$$h\rho^{5.73} = 0.0581$$
$$h\rho^{6.84} = 0.229$$
$$h\rho^{7.49} = 0.404$$
$$h\rho^{10.46} = 7.28$$

Taking the logarithms the several equations, with the values of ρ which result from each two consecutive equations, are:

To mag. 5.73	$\log 5 + 5.73 \log \rho$	$= 8.764$	
" " 6.84	" $+ 6.84$ "	$= 9.360$	$\rho = 3.14$
" " 7.49	" $+ 7.49$ "	$= 9.606$	2.39
" " 8.18	" $+ 8.18$ "	$= 9.868$	2.40
" " 8.82	" $+ 8.82$ "	$= 0.214$	3.48
" " 9.56	" $+ 9.56$ "	$= 0.497$	2.42
" " 10.46	" $+ 10.46$ "	$= 0.862$	2.55

These values of ρ are now smaller than any hitherto found, and show a diminution as we proceed to fainter stars, a result of capital importance. They are not very different from the light-ratio of the photometric scale, from which it follows that the totality of light from all the stars of each successive order of magnitude does not go on increasing, as it would if $\rho > 2.51$.

The irregularities are caused by those of the estimates of magnitude which, in the comparison with the Harvard magnitudes, we smoothed out, so as to make the correction continuous.

Most noteworthy is the deviation of the richness to $9^{M}.0$ from that found by SEELIGER from SCHÖNFELD's *Durchmusterung*. From the preceding investigation we should find, for the region within 10° of the S. galactic pole

Richness to $9^{M}.0$ = 1.93
While SEELIGER found 3.19

This great difference must be attributed to a difference between the scale of magnitude of the S. H. Photometry, and that to which SEELIGER reduced his results. If this is the sole cause, then, to $8^{M}.5$ of SEELIGER's standard corresponds $9^{M}.0$ of the Harvard scale. This singular discrepancy I am not in a position to clear up. The difference is slightly greater than that between the Harvard and Cordoba scales; since to $9^{M}.0$ of the Harvard scale corresponds $8^{M}.6$ of the Cordoba scale. We might, therefore, anticipate a close agreement between the Cordoba scale and that of SCHÖNFELD. But, when we pass from mag. $8^{M}.5$ to $9^{M}.0$, we find the Cordoba richness for $8^{M}.9$, which is 3.14, to be almost equal to that of SCHÖNFELD for $9^{M}.0$. It would seem, therefore, that the interval $8^{M}.5$–$9^{M}.0$ of the SCHÖNFELD scale corresponds to $8^{M}.6$ to $8^{M}.9$ of the Cordoba scale.

A desideratum of stellar astronomy at the present time is a determination of the ratio of progression in the number and light of the stars as we ascend in the scale of magnitude. The present paper may, it is hoped, pave the way for more complete researches on this subject.

EPHEMERIS OF COMET a 1901.

By C. J. MERFIELD.

The appended ephemeris of this comet has been prepared for the purpose of assisting observers in finding this object. There seems to be a chance of again observing this comet with large telescopes, and further observations will be of great value in the determination of the definitive orbit elements:

EPHEMERIS.

Greenwich Mean Noon.

1901	App. α	App. δ	Log Δ
	h m s	° ′ ″	
Sept. 16	8 46 34.81	+10 50 14.0	0.5403
18	47 25.70	49 21.9	
20	48 13.42	48 36.3	0.5407
22	48 57.89	47 57.7	
24	8 49 39.04	+10 47 26.9	0.5407
Sept. 26	8 50 16.82	+10 47 4.6	
28	50 51.17	46 51.1	0.5402
30	51 22.00	46 47.2	
Oct. 2	51 49.24	46 53.7	0.5393
4	52 12.81	47 11.3	
6	52 32.60	47 40.8	0.5381
8	52 48.52	48 22.7	
10	53 0.47	49 17.9	0.5365
12	53 8.33	50 27.1	
14	53 12.02	51 51.0	0.5345
16	53 11.44	53 30.3	
18	53 6.51	55 25.6	0.5322
20	52 57.12	10 57 37.6	
22	8 52 43.19	+11 0 6.8	0.5297

Sidney, 1901 *August* 12.

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MICROMETRICAL OBSERVATIONS OF THE SATELLITE OF *NEPTUNE*, AND OF STARS NEAR THE PLANET,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY, 1899 AND 1900,

BY E. E. BARNARD.

In 1892 I began a systematic measurement of the position of the satellite of *Neptune*. The satellite had been neglected, and much needed measurement. These measures were made at the Lick Observatory, and were as continuous as the circumstances permitted. They were kept up until I left Mt. Hamilton in 1895.

In 1897, when the 40-inch here was ready for observations, I again took up the work. Through the kindness of Professor HALE, who gave me every opportunity to observe the planet with the great telescope, I succeeded in getting as complete a series of measures of the satellite as the atmospheric conditions would permit.

Professor HALL, in *A.J.* 441, has called attention to the fact that only continuous measures of this object are of much value.

It was intended to make a long series of careful measures of the position of the satellite at the oppositions of 1899–1900 and 1900–1901, but unavoidable absence from the observatory during a part of the winter of 1899 prevented observations from December 4 to March 30, 1900. Absence on the eclipse expedition to Sumatra prevented observations in 1901 from February 5. On several occasions sickness also interfered with the work.

The micrometer is illuminated by a small electric lamp. The amount of illumination is conveniently controlled at the lamp, so that the wires can be given any intensity of light down to the faintest visibility. This is very important and necessary in the measurement of a very faint object, such as the satellite of *Neptune* frequently was. For this object, though bright under the best conditions with the large telescope, was often at the limit of vision through the blurring of its light by bad definition.

In nearly every case the measures have been made with 700 diameters. On a number of dates the satellite was difficult from bad seeing. During the work, *Neptune* several times passed near considerable stars; at such times the position of the star was carefully measured with reference to the planet. On three dates the seeing permitted micrometer measures of the diameter of *Neptune*. These measures are in good accord with my previous determination of the diameter of this planet with the 36-inch at the Lick Observatory in 1894. See *A.J.*, Vol. 15, p. 41.

MEASURES OF THE DIAMETER OF *Neptune*.

	Apparent	At dist. 30.0551
	"	"
1899 Feb. 13	2.36	2.365
Aug. 15	2.53	2.495
Oct. 7	2.41	2.449
		2.436

The value of the diameter of *Neptune* obtained with the 36-inch in 1894 and 1895 from ten nights' measures, reduced to the mean distance of *Neptune* (30.0551), was 2″.133.

When best seen the planet has always appeared round and free from markings.

OBSERVATIONS OF THE SATELLITE.

1899 *August.*				
	h m s	°	"	
12	15 45 7	27.9	. .	2
13	15 49 25	302.66	. .	4
	15 57 52	. . .	12.15	2
14	15 48 15	255.35	. .	5
	15 54 33	. . .	15.77	4
	15 58 54	. . .	15.72	4

August.				
	h m s	°	"	
15	15 55 41	203.51	. .	5
	16 1 4	. . .	11.50	4
	16 4 27	. . .	11.09	1
16	15 57 31	116.32	. .	4
18	15 33 53	197.30	. .	5
	15 42 21	. . .	10.96	4

August.				
	h m s	°	"	
18	15 46 10	. . .	10.94	4
	15 53 1	197.63	. .	4
	15 56 40	. . .	10.98	3
	16 0 51	. . .	11.28	3
19	15 26 50	294.26	. .	5
	15 32 42	. . .	12.92	4
	15 36 53	. . .	12.68	4

August.

	h m s	°	″	
20	15 24 52	251.62	. .	6
	15 32 30	. . .	16.28	4
	15 38 35	. . .	15.75	4
	16 0 6	250.00	. .	5
	16 4 57	. . .	16.09	3
	16 7 52	. . .	15.99	3
21	15 40 1	192.65	. .	5
	15 49 17	. . .	10.55	5
	15 55 32	. . .	10.58	5
22	15 19 13	112.05	. .	5
	15 26 10	. . .	13.17	4
	15 31 28	. . .	13.60	5
26	15 21 27	216.70	. .	5
	15 28 11	. . .	15.34	4
	15 33 16	. . .	15.18	4
27	15 46 10	181.19	. .	4
	15 51 35	. . .	10.22	4
	15 55 8	. . .	10.27	4
28	14 51 59	104.72	. .	7
	15 4 37	. . .	14.01	4
	15 9 4	. . .	14.09	4
	16 13 52	101.83	. .	4
	16 20 9	. . .	14.62	4
	16 24 1	. . .	14.59	4
29	15 33 28	58.98	. .	5
	15 39 38	. . .	15.30	4
	15 42 18	. . .	15.34	4

1899 *September.*

	h m s	°	″	
3	15 45 25	97.93	. .	4
	15 51 18	. . .	15.15	4
	15 57 30	. . .	15.19	4
4	15 11 45	57.55	. .	5
	15 25 10	. . .	15.64	6
	15 31 23	. . .	14.95	5
6	14 48 56	278.15	. .	5
	14 55 48	. . .	14.85	4
	15 0 15	. . .	14.73	4
8	15 25 35	158.70	. .	5
	15 32 55	. . .	10.13	4
	15 38 9	. . .	10.55	4
	16 4 27	156.77	. .	6
	16 10 28	. . .	10.09	5
	16 17 12	. . .	10.30	5
10	15 49 56	50.60	. .	6
	15 56 36	. . .	14.03	4
	16 0 2	. . .	14.22	4
11	15 37 51	332.77	. .	6
	15 45 52	. . .	10.29	4
	15 50 26	. . .	10.55	4
12	15 38 40	270.35	. .	5
	15 44 32	. . .	15.46	4
	15 48 59	. . .	15.69	4
18	14 35 13	269.35	. .	5
	14 47 53	. . .	16.11	4
	14 52 19	. . .	16.06	5
	14 58 35	267.17	. .	5
24	13 52 32	263.25	. .	4
	13 56 43	. . .	16.32	3
	13 59 24	. . .	16.32	3
25	14 47 33	216.40	. .	5
	14 54 4	. . .	12.84	3
	14 57 45	. . .	12.78	3
26	14 30 15	129.53	. .	4
	14 35 9	. . .	12.00	4
	14 39 38	. . .	11.87	4
30	14 34 53	258.12	. .	5
	14 43 1	. . .	16.78	5
	14 48 51	. . .	16.40	5

1899 *October.*

	h m s	°	″	
1	14 11 0	209.18	. .	6
	14 23 40	. . .	11.93	4
	14 27 20	. . .	11.93	4
2	14 46 44	119.92	. .	4
	14 54 15	. . .	12.89	4
	14 59 7	. . .	12.82	4
7	14 11 25	199.52	. .	4
	14 16 13	. . .	11.30	4
	14 18 55	. . .	11.47	4
8	14 50 37	112.42	. .	4
	14 55 26	. . .	13.55	4
	14 59 0	. . .	13.77	4
9	15 11 46	69.29	. .	5
	15 18 30	. . .	16.45	4
	15 23 35	. . .	15.78	3
14	15 42 3	104.07	. .	5
	15 52 58	. . .	11.71	6
	15 56 56	. . .	11.99	5
	16 8 10	103.25	. .	5
15	12 14 19	67.39	. .	5
	12 20 17	. . .	16.30	4
	12 24 6	. . .	16.03	4
17	13 33 8	286.36	. .	5
	13 37 50	. . .	11.43	4
	13 41 1	. . .	11.24	4
21	15 9 22	58.88	. .	6
	15 15 25	. . .	15.20	4
	15 19 11	. . .	15.10	4
	15 24 26	58.32	. .	4
23	12 44 9	282.00	. .	4
	12 52 29	. . .	14.83	4
	12 56 55	. . .	14.76	4
28	14 2 0	340.13	. .	5
	14 6 22	. . .	10.19	4
	14 9 33	. . .	10.51	4
29	12 0 51	277.42	. .	5
	12 5 53	. . .	15.62	4
	12 9 59	. . .	15.51	4
30	13 30 37	233.61	. .	5
	13 36 6	. . .	14.78	4
	13 40 20	. . .	14.95	4

1899 *November.*

	h m s	°	″	
4	12 7 48	271.63	. .	5
	12 12 54	. . .	15.70	4
	12 17 11	. . .	15.88	4
5	13 40 0	226.39	. .	5
	13 45 1	. . .	14.17	4
	13 49 30	. . .	13.94	4
6	11 46 12	150.81	. .	5
	11 52 52	. . .	10.66	4
	11 56 55	. . .	10.96	4
7	12 39 6	88.31	. .	6
	12 45 35	. . .	16.60	4
	12 49 13	. . .	16.48	4
11	11 11 10	225.60	. .	5
	11 17 28	. . .	14.04	4
	11 20 30	. . .	13.89	4
12	13 35 35	134.76	. .	5
	13 42 7	. . .	11.65	4
	13 46 18	. . .	12.09	4
18	11 53 55	130.25	. .	5
	11 59 25	. . .	12.28	4
	12 3 43	. . .	12.25	4
19	11 55 54	79.25	. .	5
	12 0 42	. . .	16.89	4
	12 4 37	. . .	16.79	4
25	10 23 34	76.49	. .	6
	10 29 1	. . .	17.16	4
	10 32 6	. . .	17.06	4
26	10 35 52	27.37	. .	5
	10 42 38	. . .	12.18	4
	10 45 55	. . .	12.19	4

November.

	h m s	°	″	
27	10 45 47	301.23	. .	5
	10 51 11	. . .	12.64	4
	10 54 25	. . .	12.79	4

1899 *December.*

4	9 45 13	251.57	. .	6
	9 49 48	. . .	17.20	4
	9 52 47	. . .	16.98	4

1900 *March.*

30	8 18 16	346.29	. .	5
	8 22 33	. . .	10.17	4
	8 25 3	. . .	10.22	4
31	7 41 18	275.20	. .	5
	7 45 5	. . .	15.00	5
	7 49 6	. . .	14.97	5

1900 *April.*

2	7 54 15	160.87	. .	5
	7 58 56	. . .	10.35	5
	8 0 50	. . .	10.17	5
3	7 45 41	. . .	15.07	5
	7 48 23	. . .	15.45	5
	7 52 6	92.74	. .	5

April.

	h m s	°	″	
4	8 10 34	51.02	. .	6
	8 16 3	. . .	14.28	5
	8 18 45	. . .	14.37	5
	8 21 10	51.02	. .	3
6	7 27 57	270.06	. .	7
	7 31 35	. . .	15.27	5
	7 34 7	. . .	15.30	6
7	7 30 20	228.23	. .	5
	7 35 18	. . .	14.31	5
	7 39 7	. . .	14.46	5
9	7 25 14	88.62	. .	6
	7 29 15	. . .	16.07	6
	7 34 25	. . .	15.93	8
10	7 27 49	46.88	. .	7
	7 34 41	. . .	13.65	6
	7 40 8	. . .	13.66	7
18	7 49 7	259.28	. .	5
	7 54 2	. . .	15.68	6
	8 1 26	. . .	15.94	7
19	7 34 54	214.40	. .	6
	7 41 54	. . .	12.67	5
	7 45 54	. . .	12.90	5
	7 51 50	214.51	. .	5

April.

	h m s	°	″	
22	8 0 42	32.51	. .	6
	8 4 45	. . .	12.11	4
	8 8 21	. . .	11.81	4
	8 12 46	31.50	. .	6
	8 16 45	. . .	11.73	3
	8 18 51	. . .	11.82	3
24	7 48 35	255.54	. .	6
	7 53 47	. . .	15.80	4
	7 56 54	. . .	15.89	4
26	7 46 35	120.34	. .	5
	7 51 51	. . .	12.07	4
	7 56 33	. . .	12.24	5
27	7 46 26	74.21	. .	5
	7 51 14	. . .	16.25	4
	7 53 57	. . .	16.43	4
30	7 50 27	251.37	. .	6
	7 55 28	. . .	16.09	5
	7 59 29	. . .	15.89	5

1900 *May.*

4	7 54 51	13.29	. .	4
	7 59 8	. . .	10.33	3
	8 2 41	. . .	10.25	4

Positions of the Satellite.

1900 *September.*

	h m s	°	″	
6	14 51 52	253.84	. .	5
	14 59 57	. . .	16.30	5
	15 12 52	. . .	15.80	3
10	15 22 38	7.36	. .	7
	15 28 49	. . .	10.81	5
	15 32 56	. . .	10.85	5
11	15 35 17	288.08	. .	5
	15 40 52	. . .	14.53	5
	15 46 37	. . .	14.73	5
13	15 35 33	179.81	. .	6
	15 41 6	. . .	10.39	5
	15 44 15	. . .	10.23	5
19	14 26 20	172.61	. .	5
	14 49 13	. . .	10.01	4
	14 54 33	. . .	10.33	4
24	13 34 27	240.04	. .	6
	13 39 25	. . .	11.69	4
	13 48 14	. . .	14.76	4
25	13 50 38	162.41	. .	8
	13 59 13	. . .	10.31	4
	14 3 9	. . .	10.31	4

1900 *October.*

	h m s	°	″	
1	14 7 51	119.18	. .	5
	14 11 2	. . .	10.23	4
	14 18 0	. . .	10.02	4
2	14 11 20	90.14	. .	5
	14 15 54	. . .	16.49	4
	14 19 9	. . .	16.36	4
3	14 14 35	16.98	. .	6
	14 19 46	. . .	13.85	4
	14 23 22	. . .	13.79	4
4	13 50 57	327.69	. .	6
	13 57 31	. . .	11.37	5
	14 2 0	. . .	11.69	5
5	15 9 34	266.95	. .	6
	15 15 28	. . .	16.26	5
	15 19 52	. . .	16.26	5
8	14 53 9	84.52	. .	6
	14 59 18	. . .	16.72	4
	15 3 0	. . .	16.42	4
9	14 52 25	38.43	. .	5
	14 58 28	. . .	12.73	5
	15 2 16	. . .	12.68	5

October.

	h m s	°	″	
10	14 0 35	316.54	. .	6
	14 6 6	. . .	11.79	4
	14 9 17	. . .	11.70	4
11	15 7 39	262.16	. .	5
	15 12 14	. . .	16.74	4
	15 15 24	. . .	16.48	4
16	13 15 47	309.62	. .	5
	13 21 33	. . .	12.35	5
	13 25 26	. . .	12.45	5
17	12 35 54	261.39	. .	5
	12 40 5	. . .	16.67	5
	12 43 29	. . .	16.75	5
18	14 24 26	207.37	. .	5
	14 29 36	. . .	11.64	4
	14 31 0	. . .	11.62	4
25	12 11 23	119.96	. .	5
	12 17 9	. . .	13.65	4
	12 21 7	. . .	13.69	4
26	12 34 44	74.22	. .	5
	12 39 57	. . .	16.44	4
	12 43 49	. . .	16.51	4

October.

	h m s	°	″	
27	13 34 59	14.96	. .	5
	14 5 54	. . .	14.02	5
	14 12 22	. . .	14.53	7
30	14 29 34	184.26	. .	5
	14 37 44	. . .	10.83	4
	14 41 22	. . .	10.77	4

1900 *November.*

	h m s	°	″	
1	15 48 23	63.43	. .	6
	15 53 17	. . .	15.70	4
	15 56 10	. . .	15.84	4
2	15 53 55	355.35	. .	5
	16 0 22	. . .	10.48	5
	16 4 0	. . .	10.70	5
3	12 17 20	289.06	. .	5
	12 21 47	. . .	14.34	4
	12 24 17	. . .	14.34	4
4	11 23 26	248.85	. .	5
	11 27 15	. . .	16.41	4
	11 29 42	. . .	16.15	4
5	12 43 31	180.52	. .	5
	12 50 26	. . .	10.35	5
	12 53 58	. . .	10.63	5
8	15 42 42	164.78	. .	6
	15 48 55	. . .	10.51	5
	15 52 38	. . .	10.66	5
13	12 22 40	58.69	. .	6
	12 27 24	. . .	15.76	4
	12 29 58	. . .	15.52	4

November.

	h m s	°	″	
22	10 8 30	234.62	. .	6
	10 14 57	. . .	14.94	5
	10 19 42	. . .	14.74	5
26	9 44 2	334.74	. .	6
	9 47 55	. . .	11.52	5
	9 53 21	. . .	11.58	6

1900 *December.*

	h m s	°	″	
8	11 9 53	307.87	. .	5
	11 15 18	. . .	12.79	5
	11 19 54	. . .	12.67	5
11	9 4 3	129.31	. .	6
	9 9 21	. . .	12.51	4
	9 12 31	. . .	12.26	4
18	9 28 23	74.04	. .	5
	9 33 47	. . .	16.87	4
	9 37 44	. . .	16.72	4
19	12 18 18	10.68	. .	7
	12 24 6	. . .	11.10	4
	12 27 37	. . .	10.95	4
28	12 56 31	171.22	. .	6
	13 2 37	. . .	11.18	4
	13 5 44	. . .	11.20	4
29	10 24 30	103.48	. .	5
	10 29 13	. . .	15.02	4
	10 32 43	. . .	15.19	4
31	11 49 39	158.10	. .	5
	11 55 20	. . .	10.58	4
	11 58 32	. . .	10.73	4

1901 *January.*

	h m s	°	″	
1	8 16 4	287.40	. .	6
	8 22 10	. . .	14.57	4
	8 25 54	. . .	14.28	4
11	12 56 47	222.00	. .	7
	13 5 22	. . .	13.66	4
	13 9 36	. . .	13.49	4
16	8 48 31	89.95	. .	5
	8 53 57	. . .	16.60	5
	8 58 3	. . .	16.49	5
17	7 53 16	49.00	. .	5
	7 59 6	. . .	15.11	5
	8 2 24	. . .	15.32	5
19	8 5 28	269.18	. .	5
	8 9 52	. . .	16.21	4
	8 12 35	. . .	16.19	4
21	11 25 12	132.27	. .	5
	11 30 19	. . .	11.91	4
	11 33 30	. . .	12.02	4
25	8 58 32	262.73	. .	6
	9 4 57	. . .	16.62	5
	9 8 34	. . .	16.54	6
28	8 27 23	81.11	. .	5
	8 34 59	. . .	16.95	4
	8 39 22	. . .	16.89	4

1901 *February.*

	h m s	°	″	
5	8 36 59	300.48	. .	5
	8 41 20	. . .	12.96	5
	8 48 15	. . .	12.89	5

On the 18th and 19th of April (1900) *Neptune* was near the star Σ 777. This object passed some 3′ north of *Neptune*. On the above dates the following measures were made of the double:

	°	″
1900.297	84.8	4.84
.300	85.4	4.52
1900.298	85.1	4.68

There is no change in the star. It was measured in 1830 by STRUVE, 85°.4, 4″.55.

MICROMETER MEASURES OF NEPTUNE AND NEIGHBORING STARS.

(Measures referred to the center of *Neptune.*)

1899 *August.*

*	d	h m s	°	″	
1	26	15 3 5	167.08	. .	4
		15 8 4	. . .	37.36	4
		15 12 59	. . .	37.72	4
1	27	16 1 42	237.99	. .	4
		16 7 18	. . .	69.32	3
		16 11 54	. . .	69.20	3
1	28	15 27 54	252.98	. .	4
		15 34 4	. . .	125.25	4
		15 39 36	. . .	125.32	4

August

*	d	h m s	°	″	
2	28	15 47 39	89.08	. .	4
		15 51 30	. . .	149.63	4
		15 59 52	. . .	149.35	4
	29	15 50 38	212.08	. .	2
		15 57 31	. . .	13.75	3
2		16 19 20	88.18	. .	4
		16 25 51	. . .	87.53	3
		16 30 5	. . .	87.49	3
1		16 42 31	258.80	. .	4
		16 47 34	. . .	187.47	3
		16 52 23	. . .	187.28	3

1899 *September.*

*	d	h m s	°	″	
3	3	15 30 27	96.58	. .	5
		15 35 45	. . .	9.12	4
		15 39 55	. . .	8.81	4
2		16 17 30	271.70	. .	4
		16 22 59	. . .	188.82	3
		16 27 56	. . .	189.12	3
1	18	14 14 56	112.35	. .	4
		14 21 0	. . .	12.35	3
		14 25 30	. . .	12.57	3

September.

*	d	h	m	s	°	″	
5	24	13	32	46	274.38	. .	4
		13	37	29	. . .	82.49	3
		13	40	57	. . .	82.47	3
5	25	14	16	48	274.98	. .	4
		14	22	3	. . .	91.57	3
		14	26	23	. . .	91.27	3
5	26	14	9	50	275.93	. .	4
		14	18	5	. . .	97.94	3
		14	22	9	. . .	98.08	3
5	30	14	58	9	280.19	. .	4
		15	3	1	. . .	104.72	4
		15	7	2	. . .	104.67	4

1899 *October.*

*	d	h	m	s	°	″	
5	1	14	40	32	281.61	. .	5
		14	45	52	. . .	101.62	3
		14	50	11	. . .	101.45	3
5	2	15	13	15	283.80	. .	4
		15	19	3	. . .	96.09	3
		15	22	47	. . .	96.19	3
5	7	14	29	40	314.42	. .	4
		14	33	7	. . .	49.15	3
		14	35	16	. . .	49.17	3
5	8	14	38	26	335.15	. .	4
		14	42	13	. . .	40.91	3
		14	45	13	. . .	41.15	3

October.

*	d	h	m	s	°	″	
5	9	15	29	53	4.79	. .	4
		15	34	47	. . .	39.60	3
		15	38	5	. . .	39.88	3
5	15	12	34	53	71.07	. .	4
		12	39	30	. . .	168.40	3
		12	43	5	. . .	168.80	3
6	22	13	25	43	246.28	. .	6
		13	32	12	. . .	77.89	3
		13	36	0	. . .	78.09	3
6	23	13	2	11	222.20	. .	4
		13	6	59	. . .	38.67	3
		13	10	0	. . .	38.66	3

1900 *September.*

*	d	h	m	s	°	″	
7	19	15	2	18	27.61	. .	5
		15	5	48	. . .	21.86	4
		15	9	50	. . .	21.69	5
7	21	13	13	54	288.40	. .	5
		13	20	36	. . .	90.80	4
		13	25	38	. . .	91.10	4
7	25	14	13	51	287.03	. .	4
		14	20	49	. . .	105.24	4
		14	25	28	. . .	105.26	4

1900 *October.*

*	d	h	m	s	°	″	
7	2	14	28	49	287.88	. .	4
		14	33	4	. . .	144.60	1
		14	36	55	. . .	145.00	4
7	3	14	29	5	289.08	. .	5
		14	35	8	. . .	143.23	1

October.

*	d	h	m	s	°	″	
7	5	15	24	52	292.23	. .	4
		15	44	10	. . .	133.34	4
		15	48	37	. . .	133.12	4
7	8	15	8	17	301.93	. .	5
		15	14	21	. . .	107.16	5
		15	20	14	. . .	107.16	5
7	9	15	7	7	307.70	. .	4
		15	12	10	. . .	96.36	1
		15	16	10	. . .	96.46	4
7	10	14	12	42	315.25	. .	4
		14	16	57	. . .	85.64	4
		14	20	44	. . .	85.79	4
7	11	15	19	41	327.32	. .	4
		15	23	14	. . .	74.86	3
		15	26	0	. . .	74.82	3
7	16	13	30	39	47.91	. .	4
		13	35	5	. . .	108.99	4
		13	38	36	. . .	108.81	4
8	26	12	50	44	25.56	. .	2
		12	55	4	. . .	11.75	2
9		13	4	28	314.54	. .	4
		13	8	47	. . .	103.81	4
		13	12	32	. . .	103.64	4
9	27	14	18	18	313.31	. .	4
		14	21	41	. . .	78.05	4
		14	24	36	. . .	78.25	4

These measures have been corrected for refraction.

On 1899 Aug. 27, star 2 was 0′ 1″.58 (2 obs.) north of the center of *Neptune* and following some 15^s.

		M M
Star 1	=	8–9
" 2		8.9
" 3		12–13
" 4		10

	M M
Star 5	9.5
" 6	9–10
" 7*	9.8
" 8	15
" 9†	

* There is a small star s.f. 15″.

† Magnitude not noted; will be a considerable star.

NOTES ON THE SATELLITE.

1899 *Aug.* 12th, Seeing excessively bad and measures of distance not possible. — 13th, Observations stopped by clouds. — 14th, Seeing fair and observations good. — 16th, Satellite lost in bad seeing before distances could be measured. — 18th, Seeing very poor and observations repeated. — 20th, The second set believed to be the better. — 21st, Seeing bad. — 22d, Satellite faint through dense haze. — 28th, Seeing very bad; satellite seen only once in awhile. *Sept.* 3d, Star between satellite and planet; it is ½^M brighter than the satellite; a star of 15^M in 190°± and distance 14″±. — 4th, Through dense clouds, with very bad seeing. — 6th, Difficult. — 8th, Very faint through very thick sky. — 10th, Magnifying power = 1340 diameters. — 12th, Images very bad; waited long for it to steady enough to measure. — 18th, Through breaks in clouds; excessively difficult. — 25th, Satellite very faint; seeing very bad. — 30th, Satellite very difficult. *Oct.* 2d, Faint and difficult in very bad seeing. — 14th, Excessively difficult from bad seeing; the last position-angle better. — 15th, Satellite very difficult; a south gale blowing. — 17th, Very difficult. — 21st, Difficult; gale of wind blowing. — 23d. A faint star = the satellite in magnitude, n.p. and a little fainter than the satellite. — 29th, A very small star fol. and a little south; distance 5″. *Nov.* 4, Two small stars near south form a trapezoid with *Neptune* and the satellite — a little brighter than the satellite. — 5th, Observations good: satellite well seen. — 12th, Satellite very difficult; seeing very bad. — 26th, Seeing good. *Dec.* 4th, Seeing so bad can only glimpse the satellite. 1900 *Apr.* 2d, Satellite faint and difficult. — 7th, A faint star n.p. a little more distant than the satellite. — 9th, Excessively faint and difficult. — 10th, Faint and difficult. — 18th, Difficult and very faint in haze. *Sept.* 6th, Through dense clouds. — 19th, Through clouds; seeing bad. *Oct.* 1st, Very difficult. — 3d, Excessively faint; through clouds. — 5th, Satellite unusually bright and easy. — 25th, Faint from bad seeing. — 27th, Faint and difficult through thin places in clouds. — 30th, Excessively faint and difficult; but observations good. *Nov.* 3d, Seeing excessively bad. — 5th, Very faint from thick sky and bad seeing. — 8th, Very faint from bad seeing. — 26th, Very faint; very bad seeing. *Dec.* 8, Very faint in thick sky. — 18th, Satellite bright. — 19th, Satellite bright. 1901 *Jan.* 1st, Satellite very difficult; seeing excessively bad. — 16th, A 12^M star 5″–6″ due n. of the satellite. — 25th, Satellite bright.

On Oct. 2, 1900, *Neptune* was near the star B.DM. +22°1248, whose position for 1855.0 is

$$\alpha = 5^h\ 54^m\ 5^s.3 \quad , \quad \delta = +22^\circ\ 16'.6$$

This star is an unrecorded double, the following measures were made of it

		°	″	M M
1900 Oct.	8	193.7	1.88	9 : 9.5
	9	193.2	1.94	9 : 9.1

The proper quadrant was not recorded.

A 13^M.5 star was compared with the north component of the pair.

		°	″
1900 Oct.	8	13±	4.85
	9	13±	4.84

This small star was in the same line with the other two, and north. Hereafter it will not be possible to secure as continuous observations of the satellite of *Neptune*.

Yerkes Observatory, Williams Bay, Wisconsin.

OBSERVATIONS OF *EROS*,

MADE WITH THE 2[illegible]-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY OF THE UNIVERSITY OF VIRGINIA,

BY HERBERT R. MORGAN.

During these observations the value of one revolution of the micrometer-screw was determined by transits of equatorial stars, as follows:

1900	1 rev.	No. transits	Distance	Seeing	Focus	Temp.
Oct. 17	9.847	80	40 rev.	good	.50 in.	50
18	9.845	66	60 "	bad	.48 "	58
19	9.847	138	65 "	. .	.50 "	52
26	9.847	162	60 "	bad	.22 " (?)	62

The mean, 9″.847, was adopted as the value of one revolution of the screw. In observing *Eros*, the recommendations of the Paris Astrophotographic Conference were followed as closely as possible, and in general, 6 *I*δ's, 12 *I*α's and 6 *I*δ's were taken. In only two cases did the difference in time of *I*α's and *I*δ's amount to three minutes. Corrections for refraction have been applied.

Charlottesville M.T.				$\Delta\alpha$	$\Delta\delta$	No. Settings	Star	$\log p\Delta\alpha$	$\log p\Delta\delta$	Approx. App. Place Star α	δ
1900	h	m	s	s	′ ″		M			h m s	° ′
Sept. 21	12	38	32	− 3.69	+ 21.11	10 , 10	8.9	n9.460	n9.256	2 42 6	+ 12 24.4
25	11	51	59	− 0.57	+5 11.30	10 , 10	11.0	n9.584	n9.324	42 29	12 11.3
Oct. 9	11	59	43	+ 1.01	−2 36.13	10 , 9	9.0	n9.423	n0.063	12 47	47 59.2
15	11	56	28	+11.03	+ 17.92	10 , 10	10.0	n9.310	n0.213	38 59	49 56.3
16	11	13	5	+ 8.27	−3 25.62	8 , 8	9.0	n9.505	n0.130	38 11	50 18.2
17	9	18	21	− 0.11	+4 33.40	12 , 12	10.5	n9.770	9.404	37 27	50 27.4
18	9	10	35	+ 2.95	+1 5.11	8 , 8	9.4	n9.777	9.437	36 24	50 15.3
19	8	11	2	− 2.00	−2 36.81	12 , 12	11.0	n9.809	9.864	35 25	51 9.0
20	7	56	3	+ 1.17	−2 15.31	12 , 12	9.8	n9.847	0.195	34 13	51 25.5
24	11	33	52	−34.08	−1 35.22	12 , 12	. .	n9.311	n0.306	29 0	52 27.8
26	9	53	58	+29.44	+3 53.17	12 , 12	10.0	n9.621	n0.146	25 6	52 17.9
29	8	27	12	+ 8.09	−2 15.81	12 , 12	. .	n9.779	n9.530	20 24	53 27.6
Nov. 5	9	38	21	+18.32	+3 45.74	12 , 12	. .	n9.479	n0.316	2 6 32	54 10.8
12	8	23	49	+25.73	− 4.22	12 , 12	.	n9.606	n0.234	1 52 11	54 15.8
13	8	24	4	+36.19	+ 1.97	12 , 12	10.0	n9.585	n0.251	50 2	54 11.7
14	8	9	18	+ 9.86	−5 30.11	12 , 12	10.0	n9.614	n0.220	48 33	54 12.4
15	9	3	1	− 1.45	+ 13.85	12 , 12	. .	n9.346	n0.349	46 46	54 0.0
16	9	0	34	− 0.28	+ 6.75	12 , 12	. .	n9.321	n0.350	44 55	53 53.5
21	8	22	3	−35.21	+ 4.91	12 , 12	9.3	n9.381	n0.310	37 19	53 3.9
22	8	14	1	−10.13	−1 38.73	12 , 12	8.7	n9.176	n0.338	35 29	52 52.6
27	6	21	54	+22.75	−2 59.94	12 , 12	7.4	n9.681	n9.921	29 27	51 39.8
Dec. 1	8	14	22	+ 9.16	− 34.18	12 , 12	9.3	n8.995	n0.271	27 7	50 22.6
8	10	19	22	+ 7.42	−1 19.82	12 , 12	10.5	9.546	n9.914	27 9	47 53.1
10	6	10	31	− 2.12	+ 59.18	12 , 12	10.0	n9.420	n0.014	28 7	47 7.7
11	6	13	23	+ 1.55	− 36.08	12 , 12	10.5	n9.612	n9.874	28 41	46 46.3
13	8	33	40	+10.90	+1 12.16	12 , 12	9.0	8.974	n0.076	1 30 11	45 51.3
1901 Jan. 5	6	35	17	+ 0.70	− 39.76	12 , 12	11.0	n8.992	9.498	2 14 28	+36 12.3

NEW SOUTHERN *ALGOL*-VARIABLE.

BY ALEXANDER W. ROBERTS.

C.P.D. − 41 4511.

$\alpha = 10^h 16^m 14^s$, $\delta = -41° 43'.8$ 1875.

In May of this year, my attention was called to this star by Mr. INNES, who suspected it of variation of the *Algol*-type.

Observations made at Lovedale during May, June and July confirm this.

The star is an *Algol*-variable with the following elements:

Period,	$1^d\ 20^h\ 30^m\ 2^s$
Epoch of min.,	1900 Jan. $1^d\ 15^h\ 10^m$ (G.M.T.)
Limits,	$10^M.0 - 10^M.9$

The light-changes are completed in three hours and twenty minutes. There is no stationary period at minimum.

The ascending and descending phases are equal, each occupying one hour and forty minutes.

MICROMETRICAL OBSERVATIONS OF MÖESTING A, PTOLEMAUS A AND TRIESNECKER B,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

The following measures of the relative positions of three spots on the moon's surface, Möesting A, Ptolemaus A and Triesnecker B, were made at the request of Professor S. A. SAUNDER of England. They have been discussed by Professor SAUNDER in *M.N.* for May, 1900, from material privately communicated to him. The measures themselves, however, have not been published. It is thought they may be useful to others interested in the moon, and are therefore printed here.

It must be understood that measures of the positions of lunar craters are not subject to the accuracy attained in the measures of stellar points because of the bigness of the objects bisected. This will readily be seen from the measures of the diameters of these three craters made at the time of the position observations on April 7, 1900.

$8^h 46^m 39^s$	App. Diameter	Möesting A	$6''.38$ (3) = 7.77 miles
8 49 55	"	Ptolemaus A	4.66 (3) = 5.67 "
8 51 41	"	Triesnecker B	2.91 (3) = 3.51 "

Yerkes Observatory, Williams Bay, Wis., 1901 *Aug.* 12.

The linear values have been kindly supplied me by Professor SAUNDER.

Following are the measures of the craters:

Pt. A and Mö. A.

1900 April $7^d 8^h 8^m 0^s$	$51^\circ.00$ (4)	$8^h 13^m 7^s$	$108''.77$ (8)
9 7 59 43	58.65 (5)	8 7 42	106.99 (9)

Pt. A and Tr. B.

1900 April $7^d 8^h 18^m 36^s$	$4^\circ.47$ (4)	$8^h 27^m 25^s$	$153''.14$ (9)
9 8 29 0	11.63 (5)	8 37 48	150.43 (8)

Mö. A and Tr. B.

1900 April $7^d 8^h 34^m 20^s$	$319^\circ.07$ (4)	$8^h 40^m 19^s$	$110''.91$ (8)
9 8 16 2	326.50 (5)	8 22 41	109.95 (8)

These are corrected for refraction.

The method of double distances was used.

The times are $6^h 0^m 0^s$ slow of Greenwich.

On both dates the seeing was bad.

Observations with the full apertures of the 40-inch.

PROBABLE VARIATION OF Z.C. XVIII^h, 1913,

By ALEXANDER W. ROBERTS.

The star Arg. Z.C. XVIII^h, 1913,

$$\alpha = 18^h 32^m 45^s \quad , \quad \delta = -37^\circ 35'.8 \quad 1875,$$

is probably variable within the limits $8^M.0$ and $9^M.0$. It has been observed as a comparison-star to Ch. 6686, *U Coronae Australis*, when that star approaches its maximum.

A period of about 185 days would satisfy the most of the observations made. With this period the probable elements would be

Period,	185 days
Maximum,	1900 Jan. 30
Limits,	$8^M.0 - 9^M.0$
$M - m$,	75 days

OBSERVATIONS OF COMET *b* 1900 (BROOKS),

MADE AT THE CHAMBERLIN OBSERVATORY, UNIVERSITY PARK, COLORADO,

By N. B. HELLER.

1900	Univ. Park M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
July 30	$11^h 51^m 15^s$	1	20 , 10	$+ 0^m 25^s.14$	$+ 6' 41''.2$	$2^h 51^m 32^s.95$	$+33° 40' 13''.1$	n9.630	0.365	$+3^s.29$	$+ 5''.0$
Aug. 6	11 13 17	2	20 , 6	+ 1 51.19	− 1 16.6	3 4 2.60	54 3 9.3	n9.877	0.648	+4.22	− 1.6
11	11 39 42	3	20 , 6	− 9 14.81	+16 19.0	3 20 14.43	66 36 6.3	n0.055	0.139	+4.79	− 6.3
	12 13 34	4	20 , 6	−10 59.48	+ 1 56.1	3 20 26.85	66 39 12.4	n0.057	0.397	+5.21	− 6.5
15	9 13 48	5	10 , 6	−12 20.94	+ 6 48.8	3 43 4.77	74 28 24.2	n0.090	0.776	+6.59	− 9.7
17	10 7 34	6	10 , 6	− 3 38.39	+ 7 55.7	4 2 30.56	+77 57 17.5	n0.261	0.674	+7.76	−11.1

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	$2^h 51^m 4^s.52$	$+33° 33' 26''.9$	Weisse's Bessel II,1159	4	$3^h 31^m 21^s.10$	$+66° 37' 22''.5$	Christiania, A.G. 617
2	2 59 7.19	54 1 27.6	Cambr. (Eng.) A.G. 1373	5	3 55 19.03	74 21 45.1	Lewitzky, DM. 74 186
3	3 29 54.03	+66 19 53.6	Christiania, A.G. 612	6	4 6 1.19	+77 19 32.9	Kasan, A.G. 657

OBSERVATIONS OF MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

By ALICE E. DAVIS.

[Communicated by the Director, MARY W. WHITNEY.]

1901 Greenwich M.T.	✳	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(27) *Euterpe.*						
Jan 16 14 17 4	1	10, 9	+0 42.91	+4 50.1	7 19 13.75	+23 11 58.0	*n*9.424	0.506	+2.71	− 9.8
18 12 25 40	2	7, 7	+0 17.45	−0 25.0	7 17 18.07	+23 17 27.5	*n*9.615	0.606	+2.74	− 9.8
22 13 8 58	3	10, 8	−2 19.61	+1 59.7	7 13 29.29	+23 28 33.3	*n*9.519	0.538	+2.77	− 9.6
26 12 35 53	4	8	. . .	−2 31.3	. . . .	+23 38 2.9	. . .	0.550	+2.76	− 9.1
12 13 33	4	7	+0 34.50	. .	7 10 5.25	. . . .	*n*9.532	. .	+2.58	−17.0
				(78) *Diana.*						
Feb. 7 13 48 53	5	10, 7	+2 7.91	−1 34.1	10 14 37.87	+13 34 25.6	*n*9.606	0.697	+2.72	−17.3
8 14 52 15	5	10, 7	+1 5.73	−2 6.6	10 13 35.70	+13 33 53.1	*n*9.512	0.663	+2.72	−17.2
18 13 6 43	6	6, 7	−0 5.20	−8 8.1	10 3 14.67	+13 28 13.8	*n*9.593	0.692	+2.72	−17.2
19 12 26 14	7	10, 6	−0 6.25	−2 11.3	10 2 13.85	+13 27 30.1	*n*9.625	0.710	. .	. .
21 12 8 17	8	9, 8	−0 13.97	−0 20.9	10 0 8.38	+13 25 56.8	*n*9.624	0.709	. .	. .

Mean Places of Comparison-Stars for the beginning of the year.

✳	α	δ	Authority	✳	α	δ	Authority
1	7 18 28.10	+23 7 17.4	Berlin (B) A.G. 2932	5	10 12 27.38	+13 36 17.0	Leipzig I. A.G. 3983
2	7 16 57.88	+23 18 2.3	" " 2916	6	10 3 17.15	+13 36 39.5	" " 3955
3	7 15 46.13	+23 26 13.2	" " 2907	7	10 2 17.38	+13 30 28.6	" " 3950
4	7 9 27.99	+23 40 13.3	" " 2852	8	10 0 19.63	+13 26 34.9	" " 3912

SOUTHERN VARIABLE STARS,

By ALEXANDER W. ROBERTS.

I am able to confirm the variation of the following stars. They are entered in the catalogue of Southern Variable Stars (*Astronomical Journal*, Nos. 491, 492) as suspected variables.

CH. 851. *S Horologii.*

$\alpha = 2^h\ 22^m\ 22^s$, $\delta = -60°\ 1'.2$ 1900.

The elements of this star are,

Maximum,	1900 May 29
Period,	330 days
Limits,	$9^M.7 - <11^M.2$

The rise to a maximum is very rapid compared with the decreasing phase. In 1901 the star rose from $11^M.2$ to $9^M.7$ in about 35 days; it takes five months to fall through the same range.

CH. 6101. *RT Scorpii.*

$\alpha = 16^h\ 56^m\ 48^s$, $\delta = -36°\ 40'$ 1900.

A maximum of this star, $9^M.8$, was observed in 1901 April 18. By the beginning of June it had descended below $11^M.2$, and so I could not follow it further. Its descending phase seems rapid. I am unable, as yet, to determine, with certainty, the period.

CH. 8302. *Y Sculptoris.*

$\alpha = 23^h\ 3^m\ 40^s$, $\delta = -30°\ 40'.5$ 1900.

Observations of this star, made during 1899, 1900 and 1901, seem to indicate a full period of about eighteen months, with irregular intermediate secondary maxima.

The star is very irregular in its variation. The limits, however, are not far from $7^M.5$ and $8^M.5$.

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DEFINITIVE ORBIT OF COMET 1896 IV.

By HENRY A. PECK.

This comet was discovered by W. E. Sperra of Randolph, Ohio, on August 31, although first announced by W. R. Brooks of Geneva. It was already nearly two months past perihelion, and the values of Δ and r were 1.7 and 1.4 respectively. It had the appearance of a weak, hazy nebula of the twelfth magnitude, with no nucleus, and faded from sight quite rapidly. The recorded observations begin on Sept. 6, and with the exception of three made nearly a month later under rather difficult circumstances, all fall within a period of nine days. The entire heliocentric arc traversed while under observation, was less than eighteen degrees. Under these conditions, it is evident that the margin of uncertainty must be very large, and but little more than an approximation to the true orbit can be expected.

About two years ago, Mr. E. H. Shepard, then a student in the Holden Observatory, collected the scattered observations, and compared them with Lamp's elements. These elements are found in *A.N.* 3381, and are as follows:

$$T = 1896 \text{ July } 10.94119 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 41^\circ\ 2'\ 7''.5 \\ \Omega &= 151\ \ 2\ \ 0.8 \\ i &= 88\ 25\ 35.7 \end{aligned}\right\} 1896.0$$

$$\log q = 0.057853$$

To which correspond the equatorial coördinates,

$$\begin{aligned} x &= [9.942010]\, r \sin(v + 310^\circ\ 9'\ 52''.7) \\ y &= [9.786255]\, r \sin(v + 174\ 25\ \ 2.6) \\ z &= [9.967420]\, r \sin(v +\ 53\ \ 1\ 36.0) \end{aligned}$$

The comparison-stars are either all well determined A.G. stars, or are equatorial comparisons with such, and are as follows:

No.	α 1896.0	δ 1896.0	Authority
	h m s	° ′ ″	
1	13 53 51.60	+55 23 39.6	Hels. A.G. 7767
2	13 59 10.27	55 9 59.4	Hels. A.G. 7805
3	14 0 44.82	55 18 55.1	Hels. A.G. 7820
4	14 5 6.67	55 8 16.4	Comp. Hels. A.G. 7856
5	14 10 33.52	55 7 21.2	Hels. A.G. 7884
6	14 13 26.61	54 58 24.8	Hels. A.G. 7905 / Camb. A.G. 4487
7	14 16 0.64	54 55 33.1	Comp. Hels. A.G. 7906
8	14 16 50.47	54 59 4.6	Hels. A.G. 7929
9	14 23 50.44	54 23 28.3	Camb. A.G. 4530
10	14 25 22.84	54 31 55.7	Comp. Camb. A.G. 4529
11	14 29 50.60	54 26 26.4	Comp. Camb. A.G. 4577
12	14 40 5.99	54 14 1.5	Camb.,(Mass.) A.G.4592
13	14 45 47.76	53 56 20.1	Camb.,(Mass.) A.G.4617
14	14 48 2.81	53 40 3.3	Camb.,(Mass.) A.G.4624
15	14 51 55.02	53 37 18.4	Comp. Camb. A.G. 4615 / Comp. Camb. A.G. 4636
16	16 12 25.84	48 12 23.9	Bonn A.G. 10437
17	16 42 4.88	45 17 34.4	Bonn A.G. 10713
18	16 51 55.47	+44 1 21.2	Comp. Bonn A.G.10804

The observations have been collected from the usual sources. Dr. Wilson and Dr. Palisa have furnished corrections in manuscript to the Northfield observation of Sept. 28 and that of Vienna for Oct. 6. In dealing with material of this character, the weighting of the observations must be somewhat arbitrary. For the two first normals an approximate value was found and the weight of the various observations estimated by their departure from this mean. The Northfield observation in declination was excluded after careful consideration and correspondence with Dr. Wilson. The residuals are in the sense O—C.

Observations of Comet 1896 IV.

Place		Time	App. α	Par.	$\Delta\alpha \cos\delta$	Wt.	App. δ	Par.	$\Delta\delta$	Wt.	*
			h m s	s	″		° ′ ″	″	″		
Mt. Hamilton	Sept.	6.82582	13 51 44.06	+0.33	— 0.8	3	+55 25 1.4	+2.8	+ 6.4	3	1
Munich		7.37568	55 33.47	.39	—13.1	1	21 1.3	3.2	10.7	3	1
Toulouse		7.44266	56 4.85	.38	+13.5	1	20 41.1	3.7	22.4	1	2
Munich		7.52552	37.69	.42	— 5.8	2	19 39.9	5.0	1.7	2	1
Arcetri		8.32323	14 2 13.81	.14	+ 3.8	2	13 7.0	2.0	9.9	3	2
Arcetri		8.32323	13.60	.44	+ 2.0	3	7.4	2.0	10.3	3	3

Place	Time	App. α	Par.	Δα cos δ	Wt.	App. δ	Par.	Δδ	Wt.	*
		h m s	s	"		° ' "	"	"		
Hamburg	Sept. 8.37531	14 2 36.01	+0.35	+ 5.0	2	+55 12 33.6	+3.4	+ 6.0	3	3
Kiel	8.39942	45.35	.33	− 1.8	3	26.9	3.5	12.7	2	2
Munich	8.39978	46.36	.37	+ 5.4	2	16.1	4.1	2.3	2	3
Mt. Hamilton	8.70794	4 54.58	.49	− 3.6	2	9 35.0	2.1	11.0	3	4
Munich	9.34170	9 19.49	.44	−18.0	1	3 24.0	2.5	13.5	2	5
Toulouse	9.41243	51.65	.42	+ 6.6	2	2 38.1	3.2	+14.6	2	5
Oxford	9.41256	52.48	.36	+11.5	1	18.0	3.2	− 5.3	1	5
	8.2			+ 0.6	25			+ 9.0	30	
Teramo	10.31124	16 8.39	.45	+ 6.3	2	54 52 37.5	1.8	+ 3.3	3	7
Bordeaux	10.39783	43.79	.43	− 1.5	2	51 44.6	2.8	11.7	2	6
Toulouse	10.40719	47.83	.42	− 0.4	3	45.9	3.0	19.8	1	8
Toulouse	11.40321	23 45.73	.42	+16.9	1	39 19.7	2.9	8.3	3	9
Mt. Hamilton	11.68597	25 39.10	.47	−21.2	1	35 42.7	2.5	13.6	2	10
Teramo	12.29746	29 56.58	.44	+ 8.9	2	27 12.2	1.5	+ 3.9	3	11
Strassburg	13.36667	37 17.71	.39	+ 1.8	3	11 20.1	2.7	− 0.8	2	12
Edinburg	14.42770	44 33.90	.32	+ 9.9	2	53 54 16.8	3.3	0.3	2	13
Pulkowa	15.34014	50 43.47	.28	− 5.7	2	38 25.8	3.4	− 6.3	1	14
Oxford	15.48619	51 44.46	.28	+10.1	1	35 56.5	4.1	+ 1.7	2	15
	12.4			+ 2.4	19			+ 5.3	21	
Northfield	28.68590	16 13 23.11	.36	− 2.3	3	48 9 23.8	3.1	+ 8.9	0	16
Vienna	Oct. 4.25562	42 37.36	.27	−10.1	3	45 12 36.4	1.3	−28.0	3	17
Vienna	6.30727	52 35.09	+0.30	−23.8	3	+44 4 45.0	+2.0	−24.6	3	18
	5.3			−17.0	6			−26.3	6	

The equations of condition were formed by Schönfeld's method, and tested by arbitrary variations of the elements, the coefficients being expressed by their logarithms.

$$\begin{array}{lllllllll}
1) & +9.8297\,\partial k & -9.6432\,k\sqrt{_2}\,\partial T & -9.6794\,\partial q & -9.3255\,\partial\lambda & -9.5060\,\partial\nu & -9.7782 & = & 0 \\
2) & 9.8655 & 9.6601 & 9.7389 & 9.1818 & 9.4097 & -0.3802 & & \\
3) & 9.9265 & 9.6504 & 9.8505 & +8.4384 & +8.8529 & +0.3617 & & \\
4) & 9.9240 & 9.6198 & 9.8581 & 8.8161 & 9.3090 & +1.2304 & & \\
5) & 9.3880 & 9.3463 & 9.0215 & 9.6304 & 9.8109 & -0.9542 & & \\
6) & 9.1869 & 9.1904 & 8.6163 & 9.6272 & 9.8551 & -0.7243 & & \\
7) & -9.5625 & +8.8284 & +9.5927 & 9.4379 & 9.9308 & +1.4200 & &
\end{array}$$

Multiplying each equation by the square root of its weight, the normal equations are

$$\begin{array}{llllllll}
+1.4928\,\partial k & -1.2813\,k\sqrt{_2}\,\partial T & -1.3599\,\partial q & -0.1547\,\partial\lambda & -0.4158\,\partial\nu & -1.9678 & = & 0 \\
-1.2813 & +1.0888 & +1.1373 & -9.6794 & -9.9320 & +1.8174 & & \\
-1.3599 & +1.1373 & +1.2390 & +0.4316 & +0.6826 & +1.6854 & & \\
-0.1547 & -9.6794 & +0.4316 & +1.0522 & +1.2692 & -2.0094 & & \\
-0.4158 & -9.9320 & +0.6826 & +1.2692 & +1.5021 & -1.9136 & &
\end{array}$$

From these is derived the system of elimination equations,

$$\begin{array}{rrrrrrrr}
\partial k & -9.7885\,k\sqrt{_2}\,\partial T & -9.8671\,\partial q & -8.6619\,\partial\lambda & -8.9230\,\partial\nu & -0.4750 & = & 0 \\
 & k\sqrt{_2}\,\partial T & -9.8308\,\partial q & -0.4123\,\partial\lambda & -0.6704\,\partial\nu & +1.2157 & & \\
 & & \partial q & +0.5006\,\partial\lambda & +0.7271\,\partial\nu & -1.7847 & & \\
 & & & \partial\lambda & +0.4827\,\partial\nu & -0.8656 & & \\
 & & & & \partial\nu & +1.9460 & &
\end{array}$$

Passing from logarithms

$$\begin{aligned}
\partial k &= +\ 58''.63 \\
k\sqrt{_2}\,\partial T &= -\ \ 7.24 \\
\partial q &= +\ 82.82 \\
\partial\lambda &= +141.84 \\
\partial\nu &= -\ 88.30
\end{aligned}$$

And the corresponding corrections of the elements are

$$\begin{aligned}
\partial T &= -0.00144 \\
\partial\omega &= +\ 63''.1 \\
\partial\Omega &= -165.0 \\
\partial i &= +\ 26.6 \\
\log\partial q &= +\ \ 6.6037
\end{aligned}$$

On comparing the results obtained by substitution in the differential equations with the residuals between a new ephemeris and the observations we have

Substitution	From Elements
″	″
−0.9	−1.0
+0.4	0.0
+6.1	+5.6
−0.8	−1.0
−1.5	−1.9
+1.7	+1.9
+0.6	+0.4

After computing the probable errors, and collecting the various results, the definitive elements are found to be

$$T = 1896 \text{ July } 10.94275 \pm 0.00468 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 41^{\circ}\ 3'\ 10''.6 \pm 31''.2 \\ \Omega &= 150\ \ 59\ \ 15.8 \pm 21.5 \\ i &= 88\ \ 26\ \ 2.3 \pm 4.4 \end{aligned}\right\} 1896.0$$

$$q = 1.112895 \pm 0.000143$$

Syracuse University, 1901 *September* 16.

EQUATORIAL COÖRDINATES.

$$x = [9.941819]\, r \sin(310^{\circ}\ 11'\ \ 4''.6 + v)$$
$$y = [9.786535]\, r \sin(174\ \ 23\ \ \ 9.9 + v)$$
$$z = [9.967472]\, r \sin(\ 53\ \ \ 3\ \ 37.4 + v)$$

In view of the discordance of the last three observations in right-ascension, it has seemed best to investigate how far a variation in the assumed values would affect the resulting elements. At the same time a term has been introduced to show the effect of any departure of the orbit from the parabolic form. If we represent the error of the normal places by $\Delta\alpha'$ and $\Delta\alpha''$ respectively, and regard the variation of the eccentricity as expressed in seconds of arc, the elements need the corrections,

$\delta T =$	$+0.000061\,\Delta\alpha'$	$+0.000447\,\Delta\alpha''$	$+0.0001801\,\delta e$
$\delta\omega =$	-0.71	-0.87	-0.401
$\delta\Omega =$	$+0.48$	$+1.92$	$+2.680$
$\delta i =$	$+0.05$	$+0.02$	-0.382
$\delta q =$	-0.0000065	-0.0000159	-0.00000470

RESULTS OF OBSERVATIONS WITH THE ZENITH TELESCOPE, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA,

By C. L. DOOLITTLE.

The results which follow are a continuation of those given in this Journal of 1901 Feb. 5.

An asterisk in the margin indicates that the corresponding observations were made by Mr. ERIC DOOLITTLE. There being no adequate data for investigating the relative personal equation, it is an open question to what extent these results should be combined with my own. In this discussion only those of May 20–29 inclusive have been employed in the adjustments and in the investigation of the aberration.

The observations from 1890 May 5 to 1891 Aug. 30, give for this constant

20″.560.

This value has been employed in the final latitude determination.

The results for the latitude which follow are to be regarded as definitive.

$\varphi = 39^{\circ}\ 58'+$

1900	I	No.	II	No.
	″		″	
Jan. 23	2.45	10	2.12	10
31*	2.19	9	2.05	9
Feb. 1*	2.02	9	2.12	9
2*	2.27	9	..	..
3*	2.30	9	..	..
5*	2.20	9	2.14	3
6*	2.28	3	..	..
9*	2.34	7	..	..
13*	2.42	10	1.98	5
18*	2.30	10	2.05	7
19*	2.28	10	2.18	9
Feb 20*	2.26	1	..	..
25*	2.05	6	..	..
26*	2.23	10	2.58	8
27*	2.11	10	..	..
Mar. 2*	2.03	7	2.22	8
3*	2.30	10	2.00	9
7*	2.52	7	..	..
8*	2.20	10	..	..
10*	2.34	2	..	..
12*	2.43	9	..	..
14*	2.73	3	..	..

1900	II	No.	III	No.
	″		″	
May 5	2.09	10	2.02	9
9	2.20	8	1.82	8
10	2.17	10	..	..
12	1.99	4	..	..
13	1.99	9	1.98	10
14	1.86	10	1.92	10
15	1.64	4	..	..
16	2.26	9	..	..
17	1.95	10	1.99	10
20*	2.30	8	..	..
21*	1.80	1	2.16	10
22*	2.03	6	1.83	10
27*	1.78	8	..	..
29*	2.01	5	..	..
June 1	1.73	10	1.94	10
3	2.28	10	2.25	2
5	1.93	10	1.92	10
6	1.93	9	1.89	10
7	1.85	8	2.00	4
9	2.26	10	2.25	10
10	1.71	10	1.87	4
11	1.92	4	..	..
19	..	..	2.13	10
20	..	..	2.03	10
21	..	..	2.03	10
22	..	..	1.91	10
27	..	..	1.77	4
30	..	..	1.97	10
July 1	..	..	2.06	10

1900	III	No.	IV	No.
July 2	1.88	10	2.02	7
3	..	..	2.08	8
4	1.91	8	..	..
8	2.01	5	1.74	1
9	2.09	10	2.13	7
10	2.06	10	2.01	9
11	2.06	10	..	..
13	2.05	10	2.10	9
14	2.02	10	..	..
15	1.92	10	1.95	9
16	1.96	10	1.96	9
17	1.98	9	..	..
20	2.06	9	..	..
21	..	..	2.05	9
24	1.95	10	..	..
27	2.10	10	2.16	9
28	1.96	10	2.12	9
29	2.18	9	2.12	6
30	2.03	9	..	..
31	1.99	10	2.09	9
Aug. 1	1.92	10	2.08	8
2	2.16	10	2.00	6
3	1.90	10	1.98	9

1900	III	No.	IV	No.
Aug. 4	1.94	10	2.06	9
5	2.00	10	2.01	9
9	2.25	10	2.05	9
10	1.92	4	1.99	9
13	2.11	9	2.16	5
14	2.01	10	2.21	2
15	1.90	10	..	..
17	..	..	2.01	7
22	2.19	10	2.20	9
25	2.11	10	2.15	7
Sept. 2	..	..	2.01	8
4	..	..	2.12	9
5	..	..	2.07	9
9	..	..	2.18	9
11	..	..	1.91	9
12	..	..	2.11	9
13	..	..	2.06	2
16	..	..	2.03	8
18	..	..	2.35	9
19	..	..	2.12	9
22	..	..	2.13	9
23	..	..	2.06	9

1901	I	No.	II	No.
Feb. 18	2.31	3	..	..
19	2.10	9	..	..
20	..	..	2.30	10
21	2.06	10	..	..
22	..	..	2.40	10
24	2.26	9	..	..
25	1.99	10	2.01	5
26	2.49	10	2.35	10
27	2.32	10	2.38	10
28	2.23	10	..	..
Mar. 5	..	..	2.36	10

1901	I	No.	II	No.
Mar. 6	2.22	10	2.28	10
7	1.92	7	2.16	10
12	2.11	10	2.29	2
16	..	..	2.11	10
17	..	..	2.25	10
18	..	..	2.24	9
29	..	..	2.31	10
31	..	..	2.32	10
Apr. 16	..	..	2.29	10
28	..	..	2.21	8
29	..	..	2.11	6

1901	II	No.	III	No.
May 1	2.06	8	1.93	7
5	2.56	8	2.19	10
6	1.93	8	2.15	10
12	1.99	7	..	..
13	..	..	2.04	9
14	2.14	10	2.14	10
15	2.01	10	..	..
16	1.96	10	2.00	10
21	2.01	6	..	..
23	1.95	7	1.97	10

1901	II	No.	III	No.
May 31	2.03	9	2.25	10
June 3	2.11	10	2.11	10
4	1.93	10	2.13	10
5	1.81	8	1.98	10
6	1.83	5	1.86	2
7	2.33	3	..	..
8	2.21	10	2.16	10
9	2.06	10	2.10	10
10	2.08	10	2.06	10
11	..	..	1.95	8

1900	IV	No.	I	No.
Oct. 10	2.10	9	2.17	10
11	2.35	9	2.20	10
14	..	..	2.20	10
15	2.15	9	2.09	10
17	2.10	9	2.07	10
20	2.13	9	2.11	10
21	2.13	9	..	..
22	2.27	9	2.08	10
24	2.32	9	2.43	10
29	2.25	9	2.23	10
Nov. 2	2.19	9	..	..
5	2.31	9	2.19	10
6	2.06	9	2.16	10
7	2.19	1	..	..
9	2.11	9	2.16	9
10	2.11	9	..	..
11	2.33	8	..	..
12	2.11	9	1.99	10
13	2.13	9	..	..
14	2.23	9	2.12	9
15	2.05	7	2.21	10
16	..	..	2.25	1

1900	IV	No.	I	No.
Nov. 17	2.20	3	..	..
21	2.00	9	..	..
22	2.11	3	..	..
23	2.18	9	..	..
28	2.20	9	..	..
30	2.32	9	2.29	10
Dec. 2	2.13	8	2.35	9
5	..	..	2.20	10
16	..	..	2.33	10
17	..	..	2.21	10
19	..	..	2.36	10
20	..	..	2.38	10
21	..	..	2.35	10
24	..	..	2.10	7
26	..	..	2.19	10
29	..	..	2.05	10
1901 Jan. 1	..	..	2.35	10
3	..	..	2.16	9
4	..	..	2.11	1
5	..	..	2.24	10
7	..	..	2.21	10

1901	III	No.	IV	No.
June 30	2.11	10	2.08	7
July 1	2.11	9	1.95	7
5	2.01	7	2.10	9
10	2.06	10	1.96	9
11	1.89	4	..	..
15	2.01	10	..	..
16	1.87	10	..	..
19	2.12	10	2.17	9
20	2.13	10	1.91	9
21	2.07	10	1.86	5
23	2.00	10	1.90	7
24	2.06	10	1.93	9
28	1.92	10	..	..
29	..	..	1.88	9
31	1.90	8	2.18	9
Aug. 1	2.26	8	2.13	7
2	1.97	10	..	..

1901	III	No.	IV	No.
Aug. 3	2.02	10	1.98	7
4	2.01	4	..	..
7	1.92	10	1.84	9
8	1.84	9	1.86	9
9	2.14	8	..	..
10	2.10	8	..	..
17	1.89	8	1.91	4
20	2.06	7	1.87	8
21	1.86	7	..	..
22	..	..	2.25	1
23	..	..	1.97	5
26	1.95	1	2.05	9
27	1.95	10	..	..
28	2.09	10	1.90	9
29	2.01	10	2.16	9
30	2.13	10	1.97	8

1901	I	No.	II	No.
Jan. 20	2.04	10	2.16	8
22	..	..	2.33	6
25	2.32	10	2.33	9
27	2.26	10	1.98	2
31	2.42	10	2.07	10
Feb. 1	2.35	1	..	..
2	2.26	2	..	..
4	..	..	2.15	7
5	2.29	10	2.20	10

1901	I	No.	II	No.
Feb. 6	2.25	10	2.15	10
7	2.24	10	..	..
8	2.38	9	..	..
9	..	..	2.21	8
10	2.22	9	2.24	10
12	2.33	10	2.45	10
13	2.22	10	..	..
14	2.32	10	..	..
15	2.22	10	2.23	10

Weighted Mean Date	Latitude	No.
1900 Feb. 27*	39° 58′ 2.″174	111
27*	2.236	136
May 11	2.012	156
June 5	1.973	131
21	2.007	61
July 10	2.006	161
29	2.018	162
Aug. 13	2.070	119
Sept. 13	2.102	99
Oct. 15	2.156	114
30	2.226	108
Nov. 18	2.171	171

Weighted Mean Date	Latitude	No.
1900 Dec. 17	39° 58′ 2.″319	67
1901 Jan. 2	2.200	60
30	2.225	135
Feb. 14	2.280	128
Mar. 1	2.270	143
Apr. 2	2.279	73
May 12	2.070	140
June 6	2.065	155
July 12	2.031	115
Aug. 1	1.982	171
25	2.000	119
Whole number		2904

Flower Observatory, Upper Darby, Pa.

THE PERIOD OF *ALGOL*.

By S. C. CHANDLER.

The examination of Prof. MÜLLER's important contribution in *A.N.* 3732–33 has led me to renew inquiry into the minor inequalities of the period of *Algol* not represented in the elements in *A.J.* 166–167 (Vol. VII). The existence of such irregularities is rendered probable by the deviations O—C in the table on pp. 174–181, *l.c.*, but the order of their magnitude is so near that of the errors of observation that their character is not certainly recognizable. The subject was scrutinized when that investigation was made without arriving at definite conclusions, and the matter was therefore passed in silence. But the comparison of MÜLLER's observations with others now seems to throw some light on the question for the particular interval embraced by them, and the results may be of some significance and interest.

If we reduce MÜLLER's observations, *A.N.* 3732, s. 179–186 by means of his light-curve, s. 194, using only observations lying symmetrically on both sides of a minimum, and giving reduced weight to those near minimum, we find the local times of observed minimum given in the second column of the table below. Reducing them to the sun, and comparing with my elements and with those employed by MÜLLER for the purpose of deducing his light-curve, we have the columns O—CH. and O—MÜLLER.

OBSERVED MINIMA FROM POTSDAM OBSERVATIONS.

E	Geocentric Potsdam M.T. d h m	O—CH. m	O—MÜLLER m
9959	1878 Mar. 9 8 7	—27.8	— 4.3
10042	Nov. 2 7 43	—15.9	— 0.3
10057	Dec. 15 8 7	— 5.6	+ 8.7
10065	1879 Jan. 7 6 51	+ 5.7	+19.0
10139	Aug. 7 11 5	— 1.1	+ 5.1
10154	Sept. 19 11 20	+ 6.6	+11.2
10185	Dec. 17 8 22	— 3.5	— 1.8
10200	1880 Jan. 29 8 43	+ 0.2	+ 0.5
10215	Mar. 12 8 56	— 5.0	— 6.2
10259	July 16 12 45	— 5.8	—11.2
10274	Aug. 28 12 57	— 1.2	— 8.1
10275	Aug. 31 10 8	+21.4	+14.4
10282	Sept. 20 11 34 ::	+ 7.7 ::	+ 0.1 ::
10327	1881 Jan. 27 12 12 ::	+ 5.3	— 7.7
10329	Feb. 2 5 54	+ 8.9 ::	— 3.3 ::
11172	1887 Sept. 16 8 31 ::	+27.0 ::	—66.8 ::

Combining the data that I have given on pp. 180–182, *l.c.*, to form normal minima, we have the following table:

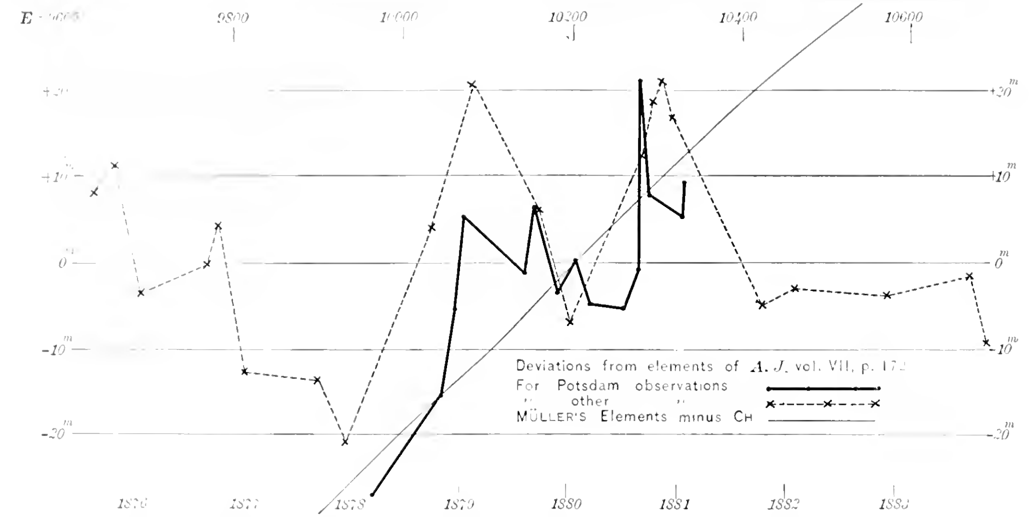
NORMAL OBSERVED MINIMA FROM OTHER OBSERVATIONS.

E	Heliocentric Paris M.T. d h m	O—CH. m	O—MÜLLER m
9638	1875 Aug. 31 22 35.3	+ 8.3	+ 62.2
9660	Nov. 3 0 33.5	+11.6	+ 63.6
9690	1876 Jan. 28 0 43.8	— 3.7	+ 45.5
9766	Sept. 1 22 40.4	0.0	+ 42.1
9779	Oct. 9 5 15.5	+ 4.5	+ 40.8
9809	1877 Jan. 3 5 28.3	—12.8	+ 25.2
9897	Sept. 12 13 6.4	—13.6	+ 16.0
9931	Dec. 19 0 40.9	—12.9	+ 5.3
10031	1878 Oct. 1 18 30.4	+ 4.3	+ 21.0
10080	1879 Feb. 19 6 40.4	+20.7	+ 32.6
10158	Sept. 30 21 56.0	+ 5.9	+ 10.2
10192	1880 Jan. 6 9 24.1	— 6.9	— 5.9
10278	Sept. 8 23 44.9	+12.7	+ 5.4
10289	Oct. 10 12 48.2	+18.7	+ 10.3
10302	Nov. 16 19 25.6	+21.1	+ 14.4
10315	Dec. 24 1 56.1	+16.7	+ 5.9
10419	1881 Oct. 18 6 15.0	— 5.3	— 26.0
10457	1882 Feb. 4 5 15.7	— 3.0	— 25.4
10566	Dec. 13 17 57.5	— 4.2	— 37.2
10659	1883 Sept. 6 9 42.9	— 2.0	— 45.5
10680	Nov. 5 11 41.1	— 9.6	— 55.2
10836	1885 Jan. 25 21 55.6	+ 4.0	— 56.6
10933	Oct. 31 0 35.2	—14.9	— 84.9
11072	1886 Dec. 3 14 7.8	— 7.8	— 76.0
11197	1887 Nov. 26 23 40.1	— 5.8	—102.1

It is clear that MÜLLER's assumption of a period about six seconds longer than mine, for the interval 1878–1881, does not distinctly conform better to his own observed minima, except for the first two; nor is such a progressive deviation indicated by the other observations. But both series of observations accord satisfactorily in showing that there was a pronounced negative deviation about the beginning of 1878, a rise to the first part of 1879, a decline to the beginning of 1880, and a sudden rise to the end of that year. The other observations show a decline from 1875 to 1878, and also during 1881. (See diagram, p. 40.)

The result of the whole comparison is to strengthen the probability of the real existence of such short minor irregularities. An extended examination of the deviations since 1840 gives no satisfactory evidence that they possess any regular periodic character.

It is to be regretted that the observations of the minima have in recent years fallen much into neglect. A persistent and continuous record of at least a dozen minima annually is needed for the elucidation of this and kindred questions relating to this important star.



With regard to the period in general, the diagram on p. 42 representing its variations will be instructive. The heavy broken line gives its values according to observation, found from the data on p. 172, *A.J.*, Vol. VII, and from such minima as have been published since, so far as they are accessible to me at this writing. The dotted curve represents the elements given in the place cited, and the continuous curve the new elements which will be found below.

It will be seen that the actual course of observation has satisfactorily verified the prediction that the protracted interval of general decrease which began about 1830 and had continued up to the time of the publication of that paper would soon be followed by a long interval of increase. The beginning of this increase indeed set in immediately after, rather earlier than the elements indicated. It is manifest that the length of the principal inequality, which was there assumed as 18000 periods (coefficient $\frac{1}{50}° = 0°.020$) is considerably shorter than that. The new value now adopted, 15000 periods (coefficient $0°.024$), cannot be very far from the truth.

The new elements are

$$1888 \text{ Jan. } 3^d\,8^h\,11^m.2 \text{ (Gr. M.T.)} + 2^d\,20^h\,48^m\,55^s.60\,E'$$
$$+117^m \sin(0°.024\,E' + 226°) + 22^m \sin(\tfrac{1}{13}°\,E' + 216°)$$

where $E' = E - 11210$,

E being reckoned, according to established usage, from the minimum on 1800 Jan. 1.

The value of the period by these elements is

$$2^d\,20^h\,48^m\,55^s.60 + 3^s.694 \sin(133° - 0°.024\,E)$$
$$+ 1^s.784 \sin(16° - \tfrac{1}{13}°\,E)$$

The values given by this expression are represented in the continuous curve of the diagram on p. 42.

The following tables of observed normal epochs and periods give the comparison of observation with the above elements. The first two columns are copied from the similar table on p. 172, *l.c.*, the additional epochs found from recent observations, as hereafter described, being marked with asterisks. The column v_1 is the deviation, O—C, from the epoch and mean period given above, without the periodical inequalities, and is found by subtracting from the column v_1 of the similar table in Vol. VII the difference between the two sets of elements without the periodical terms, or

$$17^m.0 + 0^s.175\,E = +49^m.7 + 0^s.175\,E'$$

The 4th and 6th columns give the periodical terms, and the columns v_2, v_3, are the residuals after their successive subtraction. The comparison of each sine-term with the column immediately preceding it will leave little reasonable doubt as to the reality of these inequalities. The third periodical term of the former elements has been omitted, because, whether real or not, its very small coefficient ($3^m.5$) is so nearly masked by accidental errors of observation and occasional temporary irregularities, like those made palpable earlier in this article, that it is practically insignificant.

The second part of the table, giving the observed and computed periods, is found from the data of the first portion by taking differences of alternate epochs, except when the intervals are very large when differences of adjacent epochs are used. It should be noted that the column

of computed periods gives the *average* period for each interval according to the elements. For short intervals it will correspond exactly with the period computed for the middle epoch in column *E*, but will necessarily differ slightly when the interval is large. This will account for any small differences which appear between the computed values in the table and the chart.

Comparison of Observed Epochs and Periods with Elements.

Mean Epoch	Wt.	v_1 m	First Sine-term m	v_2 m	Second Sine-term m	O—C v_3 m
−2058.2	12.5	−178.9	−146.9	−32.0	−21.9	−10.1
−1612.5	4.0	−155.6	−145.5	−10.1	−16.9	+ 6.8
−1194.3	4.5	−135.6	−139.5	+ 3.9	− 6.9	+10.8
− 608.8	3.0	−106.9	−124.2	+17.3	+10.0	+ 7.3
− 118.9	3.5	− 85.3	−105.5	+20.2	+19.8	+ 0.4
+ 2100.2	3.0	+ 22.6	+ 18.9	+ 3.7	−18.1	+21.8
3150.0	1.0	+ 58.2	+ 79.1	−20.9	−15.3	− 5.6
5212.9	13.0	+154.3	+145.6	+ 8.7	+20.0	−11.3
5512.8	5.0	+160.1	+147.0	+13.1	+11.8	− 1.7
6055.2	11.5	+134.1	+143.6	− 9.5	+ 0.2	− 9.7
6486.4	10.5	+119.7	+135.7	−16.0	−11.9	− 4.1
6856.7	16.0	+106.0	+125.3	−19.3	−19.3	0.0
7019.2	36.5	+102.6	+119.8	−17.2	−21.1	+ 3.9
7252.9	20.0	+ 89.3	+110.9	−21.6	−22.0	+ 0.4
7477.1	35.0	+ 79.3	+101.3	−22.0	−20.8	− 1.2
7823.9	18.5	+ 67.3	+ 84.9	−17.6	−15.4	− 2.2
8225.3	9.0	+ 54.8	+ 63.6	− 8.8	− 5.1	− 3.7
8435.1	28.5	+ 50.9	+ 51.7	− 0.8	+ 1.0	− 1.8
8825.1	27.5	+ 41.3	+ 28.6	+12.7	+11.8	+ 0.9
9084.7	24.5	+ 29.5	+ 13.0	+16.5	+17.5	− 1.0
9267.6	17.5	+ 23.0	+ 1.6	+21.4	+20.2	+ 1.2
9451.2	23.0	+ 16.6	− 9.7	+26.3	+21.7	+ 4.6
9712.9	12.0	− 7.7	− 25.7	+18.0	+21.6	− 3.6
10035.1	10.5	− 34.1	− 45.0	+10.9	+17.9	− 7.0
10175.0*	14.0	− 47.3	− 53.1	+ 5.8	+15.2	− 9.4
10473.0	15.5	− 62.2	− 69.7	+ 7.5	+ 7.8	− 0.3
11103.0	8.0	−115.5	−101.1	−14.4	−10.2	− 4.2
11427.0*	3.5	−129.9	−114.5	−15.4	−17.5	+ 2.1
12084.0*	4.0	−160.5	−135.3	−25.2	−21.4	− 3.8
12209.0*	9.0	−153.1	−137.2	−15.9	−20.3	+ 4.4
12322.0*	4.0	−153.5	−140.3	−13.2	−18.7	+ 5.5
12460.0*	7.0	−155.0	−142.6	−12.4	−16.3	+ 3.9
+12963.0*	1.0	−161.6	−146.9	−14.7	− 3.6	−11.1

E	Wt.	Period $2^d\ 20^h\ 48^m+$ Obs'd s	Comp'd s	O—C s
− 1626	3	58.6	57.2	+1.4
− 1111	2	58.5	58.5	0.0
− 657	2	58.4	59.0	−0.6
+ 991	4	58.5	57.9	+0.6
+ 2625	1	57.6	59.2	−1.6
+ 4181	2	58.4	58.6	−0.2
5634	5	54.2	54.1	+0.1
6000	3	53.1	53.3	−0.2
6456	5	53.5	52.8	+0.7
6753	4	53.7	52.8	+0.9
7055	4	53.1	53.0	+0.1
7248	8	52.6	53.2	−0.6
7538	5	53.3	53.6	−0.3
7851	5	53.6	53.8	−0.2
8130	7	54.0	54.0	0.0
8525	4	54.3	53.8	+0.5
8760	8	53.6	53.6	0.0
9046	6	53.1	53.1	0.0
9268	4	53.5	52.6	+0.9
9490	3	51.5	52.1	−0.6
9743	5	50.4	51.6	−1.2
9944	3	50.5	51.2	−0.7
10254	3	51.7	50.8	+0.9
10639	5	51.2	50.9	+0.3
10950	3	51.3	51.2	+0.1
11593	3	52.9	52.8	+0.1
11818	2	53.6	53.6	0.0
12203	½	57.6	55.0	+2.6
12335	1	55.1	55.4	−0.3
12642	½	54.9	56.4	−1.5

It remains to give the additional later observations used to form the normals indicated by an asterisk in the foregoing table. Those of Müller have already been given above. Besides these I have used the following:

E	Heliocentric Greenw. M.T. h m	O—Ch. m	Observer
11223	1888 Feb. 9 12 45.6	− 1.0	Sawyer
11577	1890 Nov. 20 13 30.0	+32.0	Yendell
11607	1891 Feb. 14 13 7.8	−15.7	"
12066	1894 Sept. 22 15 20.5	+11.6	"
12082	Nov. 7 12 47.1	+36.0	"
12089	27 14 23.9	+31.0	"
12097	Dec. 20 12 40.6	+18.7	"
12195	1895 Sept. 27 12 39.1	+25.4	Nijland
12196	30 9 38.6	+36.0	"
12202	Oct. 17 11 42.1	+46.2	"
12203	20 11 21.5	+36.7	"
12204	1895 Oct. 23 8 22.2	+48.6	Nijland
12205	26 5 1.1	+38.6	"
12225	Dec. 22 13 23.2	+43.0	"
12235	1896 Jan. 20 5 42.1	+53.1	"
12257	Mar. 23 7 42.4	+58.0	"
12308	Aug. 16 13 1.0	+43.5	"
12323	Sept. 28 13 20.7	+49.8	"
12324	Oct. 1 10 14.5	+44.8	"
12330	18 15 10.9	+57.8	"
12453	1897 Oct. 6 7 9.2	+42.2	"
12458	20 15 37.8	+66.4	"
12459	23 12 12.4	+52.0	"
12460	26 9 0.9	+51.6	"
12461	29 5 44.9	+46.7	"
12470	Nov. 15 10 49.6	+58.0	"
12473	Dec. 2 15 21.6	+36.6	"
12963	1901 Oct. 8 14 58.0	+45.7	Chandler

The comparison, O — CH., is with the elements of *A.J.*, Vol. VII. Combining into means we have the following corrections:

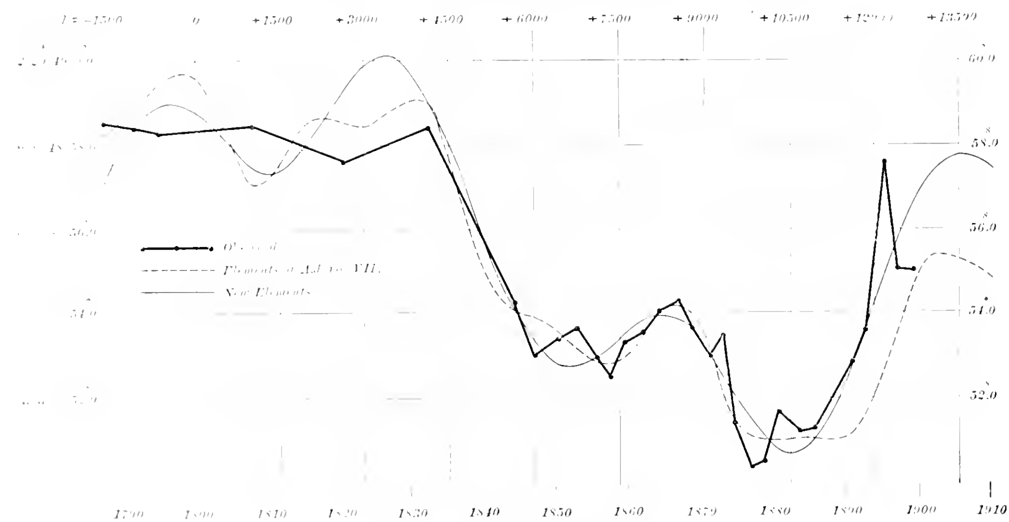
E	O—Ch.	System. Corr.	Dev. from New Elements
	m	m	m
10175	— 1.3	—5.0	— 9.4
11427	+ 7.2	0.0	+ 2.1
12084	+21.1	0.0	— 3.8
12209	+11.7	— 4.0	+ 1.1
12322	+16.0	— 4.0	+ 5.5
12427	+18.9	— 4.0	+ 3.9
12963	+15.7	0.0	—11.1

The systematic corrections are those required to reduce to the zero of SCHÖNFELD'S light-curve, and have been obtained by comparison of that curve with those of MÜLLER and NIJLAND. Those of YENDELL require no such modification, since SCHÖNFELD'S curve was employed in the reduction of the observations.

It should be added that it is not practicable at the time of this writing to secure completeness in the list of observations made or published since 1887, as the results of PLASSMAN, DUNÉR and others are not accessible.

In conclusion it may be well to note a correction of some importance in NIJLAND'S valuable paper in *A.N.* 3695, s. 422. By a slight oversight the correction of $+2^s.545$ which he obtained for the period between the epoch of my elements, $E = 11210$, and that of the mean of his observations, is there applied to the mean period $2^d\,20^h\,48^m\,55^s.425$ of my elements. But it should have been applied to the average period for that particular interval by those elements. Thus his equation for the observed average period between 1888 and 1896 should have been $2^d\,20^h\,48^m\,51^s.503 + 2^s.545 = 2^d\,20^h\,48^m\,54^s.05$, instead of $2^d\,20^h\,48^m\,57^s.97$.

PERIOD OF *Algol*. $2^d\,20^h\,48^m\,55^s.6\,E + 3^s.691 \sin(133^\circ.0 - 0^\circ.024\,E) + 1^s.784 \sin(16^\circ - \frac{1}{13}{}^\circ E)$



CORRIGENDUM.

In the observations of *Neptune*, *A.J.* 508, the time used is $6^h\,0^m\,0^s$ slow of Greenwich.

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A GENERAL METHOD OF DETERMINING THE ELEMENTS OF ORBITS OF ALL ECCENTRICITIES FROM THREE OBSERVATIONS.

BY F. R. MOULTON.

1. INTRODUCTION. In 1801 GAUSS devised a very general and elegant method of determining the elements of elliptic, parabolic and hyperbolic orbits. Very few improvements have been made upon his method, as expounded in the *Theoria Motus*, in form of demonstration, or clearness, or even in the notation employed. The reason is that the work is particularly perfect both theoretically and practically, as well as the fact that the determination of orbits presents few questions of much mathematical interest compared to those arising in the problems of Celestial Mechanics. Notwithstanding the perfection of this method, in the case of comets' orbits which are provisionally assumed to be parabolic, the method of OLBERS, which was developed a little previous to that of GAUSS, has been in almost universal use for over a century with no essential changes except in the treatment of certain critical cases which sometimes arise.

The method of OLBERS is very convenient in practice, but can be applied only when it is known that the orbit is a parabola. Since it lacks that generality which is desirable it will not be discussed further here.

The method of GAUSS consists of two distinct parts: (a) The determination of two heliocentric distances at the epochs of the first and last observations; and (b), the determination of the elements from the two positions which are known when (a) has been solved, and from the interval of time which it has taken the body to move from one to the other.

The solution of (a) is given in the first section of the second book of the *Theoria Motus*, but the notation of WATSON with only slight changes will be adopted here as being more generally accessible. Let r and ρ with the subscripts 1, 2 and 3, represent the heliocentric and geocentric distances respectively at the epochs of the three observations. Then the following equation determines z.*

$$m_0 \sin^4 z = \sin(z \mp \zeta) \tag{1}$$

where m_0 and ζ are given by the observations.

* WATSON's *Theoretical Astronomy*, p. 236, eq. (40).

After z has been found r_2 and ρ_2 are given by the equations*

$$r_2 = \frac{R_2 \sin\psi_2}{\sin z}\ , \quad \rho_2 = \frac{R_2 \sin(\psi_2 + z)}{\sin z} \tag{2}$$

where R_2 is the geocentric distance of the sun at t_2, and ψ_2 is given by the observations. The heliocentric distances at the epochs t_1 and t_3 are next computed from†

$$\left.\begin{aligned} \rho_1 &= M_1 \frac{\rho_2}{n} + M_2\left(1 - \frac{N}{n}\right) \\ \rho_3 &= M_1'' \frac{\rho_2}{n''} + M_2''\left(1 - \frac{N''}{n''}\right) \end{aligned}\right\} \tag{3}$$

in which the right members are entirely known.

Then the heliocentric coordinates are found by solving the triangle formed by the earth, sun, and body at the epochs t_1 and t_3.

Equation (1) depends upon the fact that the orbit of the unknown body is in a plane passing through the center of the sun, and that the law of areas with respect to the sun as origin is fulfilled. Equations (2) depend upon the fact that the earth, sun and body form a triangle. Equations (3) depend upon the same things as one. Therefore, *the work so far is equally valid and convenient for all species of conic sections.*

It is in part (b), after the node and inclination have been found, that the chief difficulties in the method of GAUSS arise. The processes are explained in the *Theoria Motus*, Arts. 88–109, pp. 114–151, in CARL HAASE's translation into the German, and in WATSON's *Theoretical Astronomy*, pp. 247–264. The formulas are very complicated, and the eliminations of unknowns are tedious. Moreover, the processes of computation are different for each of the three species of conics. This must be regarded as being a serious fault in the method, since there is dynamically no singularity for the eccentricity equal to unity.

* WATSON's *Theoretical Astronomy*, pp. 235-6, eqs. (37) and (41).
† WATSON's *Theoretical Astronomy*, p. 228, eqs. (18).

The method which follows in this paper involves only very simple formulas, is mathematically direct and simple, is very convenient for computation, and is precisely the same for all species of conics. The only objection that can be raised against it is that it can not be applied when the intervals between the observations are indefinitely great. This is, however, of no practical importance, because from the nature of the problem the observations will be invariably near together, and besides part (a), which is employed in the method of GAUSS, requires that they shall be not very distant from each other.

2. METHOD OF COMPUTING AN APPROXIMATE VALUE OF THE PARAMETER. In the method of GAUSS the coordinates at the epoch t_2 were used only in getting those at t_1 and t_3. In the following method the coordinates at all three of the epochs, all of which are computed in part (a), will be used.

Three points on the conic and one of its foci are known. They determine the conic uniquely, and the most obvious method would be to find the elements from these geometrical considerations. There is no theoretical difficulty whatever, but, as the points would be in practice near together, the elements would be poorly defined numerically. This is an essential difficulty which cannot be overcome by any artifice in dealing with the equations.

Instead of making the determination depend upon the geometry of the problem alone, the dynamical conditions will be introduced by employing the law of areas. It will be observed that this involves the same things fundamentally as the method of GAUSS, and that it cannot be less exact, therefore, unless the peculiar artifices employed are less appropriate to the problem.

Suppose the node and inclination and arguments of latitude have been computed by the usual methods. Let v represent the true anomaly, and u the argument of the latitude, or the longitude from the ascending node. The element p, and later the elements e, ω, and T, are to be determined from the data t_1, t_2, t_3, r_1, r_2, r_3, u_1, u_2 and u_3. In the method of GAUSS t_2, r_2 and u_2, were not used.

The expression for the law of areas is

$$k\sqrt{p}\,dt = r^2 dv = r^2 du$$

whence

$$(4)\qquad k\sqrt{p}\,(t_3-t_1) = \int_{u_1}^{u_3} r^2 du$$

If r^2 were expressed in terms of u the integral in the right member could be found, giving the value of p directly. The value of r^2 when $u = u_2$ (viz. $r^2 = r_2^2$) is known, and for values of u not too far from this r^2 may be expanded in a converging series of the form

$$(5)\qquad r^2 = r_2^2 + c_1(u-u_2) + c_2(u-u_2)^2 + c_3(u-u_2)^3 + c_4(u-u_2)^4 + \dots$$

where

$$\left.\begin{aligned} c_1 &= \left[\frac{\partial(r^2)}{\partial u}\right]_{u=u_2} & c_4 &= \tfrac{1}{24}\left[\frac{\partial^4(r^2)}{\partial u^4}\right]_{u=u_2} \\ c_2 &= \tfrac{1}{2}\left[\frac{\partial^2(r^2)}{\partial u^2}\right]_{u=u_2} & c_5 &= \tfrac{1}{120}\left[\frac{\partial^5(r^2)}{\partial u^5}\right]_{u=u_2} \\ c_3 &= \tfrac{1}{6}\left[\frac{\partial^3(r^2)}{\partial u^3}\right]_{u=u_2} & c_6 &= \tfrac{1}{720}\left[\frac{\partial^6(r^2)}{\partial u^6}\right]_{u=u_2} \end{aligned}\right\}\qquad (6)$$

In an unknown orbit the coefficients of series (5) are unknown, but it will now be shown how a sufficient number may be easily obtained in order to define the integral (4) with the desired degree of accuracy in the problems which actually arise. Let the questions of the realm and rapidity of convergence be left aside until the formal developments have been completed. By hypothesis the radii and arguments of latitude at the epochs t_1 and t_3 are known. Hence (5) becomes for these epochs

$$(7)$$
$$r_1^2 = r_2^2 + c_1(u_1-u_2) + c_2(u_1-u_2)^2 + c_3(u_1-u_2)^3 + c_4(u_1-u_2)^4 + \dots$$
$$r_3^2 = r_2^2 + c_1(u_3-u_2) + c_2(u_3-u_2)^2 + c_3(u_3-u_2)^3 + c_4(u_3-u_2)^4 + \dots$$

For abbreviation let

$$\begin{aligned} \sigma_1 &= u_3-u_2 \qquad\qquad (8)\\ \sigma_2 &= u_3-u_1 \\ \sigma_3 &= u_2-u_1 \\ \epsilon_1 &= c_3(u_1-u_2)^3 + c_4(u_1-u_2)^4 + \dots = -c_3\sigma_3^3 + c_4\sigma_3^4 - c_5\sigma_3^5 + c_6\sigma_3^6 - \dots \\ \epsilon_3 &= c_3(u_3-u_2)^3 + c_4(u_3-u_2)^4 + \dots = c_3\sigma_1^3 + c_4\sigma_1^4 + c_5\sigma_1^5 + c_6\sigma_1^6 + \dots \end{aligned}$$

Then equations (7) may be written

$$\left.\begin{aligned} -c_1\,\sigma_3 + c_2\,\sigma_3^2 &= r_1^2 - r_2^2 - \epsilon_1 \\ +c_1\,\sigma_1 + c_2\,\sigma_1^2 &= r_3^2 - r_2^2 - \epsilon_3 \end{aligned}\right\}\qquad (9)$$

Solving these equations for c_1 and c_2, it is found that

$$\left.\begin{aligned} c_1 &= \frac{-(r_1^2-\epsilon_1)\,\sigma_1^2 + (r_3^2-\epsilon_3)\,\sigma_3^2 - r_2^2(\sigma_3^2-\sigma_1^2)}{\sigma_1\,\sigma_2\,\sigma_3} \\ c_2 &= \frac{(r_1^2-\epsilon_1)\,\sigma_1 + (r_3^2-\epsilon_3)\,\sigma_3 - r_2^2\,\sigma_2}{\sigma_1\,\sigma_2\,\sigma_3} \end{aligned}\right\}\qquad (10)$$

These equations become, on substituting the values of ϵ_1 and ϵ_3 and reducing,

$$(11)$$
$$c_1 = \frac{-r_1^2\sigma_1^2 + r_3^2\sigma_3^2 - r_2^2(\sigma_3^2-\sigma_1^2)}{\sigma_1\,\sigma_2\,\sigma_3} - c_3\,\sigma_1\,\sigma_3 - c_4\,\sigma_1\,\sigma_3\,(\sigma_1-\sigma_3)$$
$$-c_5\,\sigma_1\,\sigma_3\,(\sigma_2^2-3\sigma_1\,\sigma_3) - c_6\,\sigma_1\sigma_3\,(\sigma_1^2+\sigma_3^2)(\sigma_1-\sigma_3) - \dots$$

$$c_2 = \frac{r_1^2\sigma_1 + r_3^2\sigma_3 - r_2^2\sigma_2}{\sigma_1\,\sigma_2\,\sigma_3} - c_3\,(\sigma_1-\sigma_3) - c_4\,(\sigma_2^2-3\sigma_1\,\sigma_3)$$
$$-c_5(\sigma_1^2+\sigma_3^2)(\sigma_1-\sigma_3) - c_6(\sigma_1^4-\sigma_1^3\sigma_3+\sigma_1^2\sigma_3^2-\sigma_1\sigma_3^3+\sigma_3^4) - \dots$$

Let the series (5) be substituted for r^2 in (4), and the integration performed. It is easily found that, in the notation of (8),

$$(12)$$
$$k\sqrt{p}\,(t_3-t_1) = r_2^2\,\sigma_2 + \frac{c_1}{2}(\sigma_1^2-\sigma_3^2) + \frac{c_2}{3}(\sigma_1^3+\sigma_3^3) + \frac{c_3}{4}(\sigma_1^4-\sigma_3^4)$$
$$+\frac{c_4}{5}(\sigma_1^5+\sigma_3^5) + \frac{c_5}{6}(\sigma_1^6-\sigma_3^6) + \frac{c_6}{7}(\sigma_1^7+\sigma_3^7) + \dots$$

Substituting the expressions for c_1 and c_3 given in (11), this equation reduces to

(13)

$$k\sqrt{p}\,(t_3-t_1) = \frac{r_2^2\,\sigma_2^3}{6\sigma_1\,\sigma_3} + \frac{r_1^2\,\sigma_2}{6\sigma_3}(2\sigma_3-\sigma_1) + \frac{r_3^2\,\sigma_2}{6\sigma_1}(2\sigma_1-\sigma_3)$$

$$-\frac{c_3\sigma_2^3}{12}(\sigma_1-\sigma_3) - \frac{c_4\sigma_2^3}{30}\left\{4(\sigma_1-\sigma_3)^2+\sigma_1\sigma_3\right\} - \frac{c_5}{6}\sigma_2^2(\sigma_1-\sigma_3)(\sigma_1^2+\sigma_3^2)$$

$$+\frac{c_6}{42}\sigma_2\left\{\sigma_1^5\sigma_3+6\sigma_1^4\sigma_3^2-6\sigma_1^3\sigma_3^3+6\sigma_1^2\sigma_3^4+\sigma_1\sigma_3^5-8(\sigma_1^6+\sigma_3^6)\right\}+\ldots$$

If the second observation divides the whole interval into two nearly equal parts, as generally will be the case in practice, σ_1 and σ_3 will be nearly equal. Let their difference be ϵ. Then

$$(14)\quad \left\{\begin{array}{ll} \sigma_1-\sigma_3 = \epsilon & \sigma_1 = \dfrac{\sigma_2+\epsilon}{2} \\ \sigma_1+\sigma_3 = \sigma_2 & \sigma_3 = \dfrac{\sigma_2-\epsilon}{2} \end{array}\right.$$

Equation (13) becomes considerably simplified if these expressions for σ_1 and σ_3 are substituted in its right member, the result being

(15)

$$k\sqrt{p}\,(t_3-t_1) = \frac{r_2^2\,\sigma_2^3}{6\sigma_1\sigma_3} + \frac{r_1^2\,\sigma_2}{6\sigma_3}(2\sigma_3-\sigma_1) + \frac{r_3^2\,\sigma_2}{6\sigma_1}(2\sigma_1-\sigma_3)$$

$$-\frac{c_3\,\sigma_2^3\,\epsilon}{12} - \frac{c_4\,\sigma_2^3}{120}(\sigma_2^2+15\,\epsilon^2) - \frac{c_5\,\sigma_2^3\,\epsilon}{24}(\sigma_2^2+3\epsilon^2)$$

$$-\frac{c_6\,\sigma_2}{336}(\sigma_2^6+28\,\sigma_2^4\epsilon^2+35\,\sigma_2^2\epsilon^4)+\ldots$$

An approximate value of p may be obtained by neglecting in (15) the unknown terms in the right member. The discussion of the degree of the approximation under various possible conditions will be taken up later. The known terms of (15) give

$$(16)\quad k\sqrt{p_0}\,(t_3-t_1) = \frac{r_2^2\,\sigma_2^3}{6\sigma_1\,\sigma_3} + \frac{r_1^2\,\sigma_2}{6\sigma_3}(2\sigma_3-\sigma_1) + \frac{r_3^2\sigma_2}{6\sigma_1}(2\sigma_1-\sigma_3)$$

The right member is numerically well determined, since it is the sum of positive numbers. It may be remarked that this is a generalized form of COTES's method of finding the approximate value of a definite integral. In COTES's formula the interval is divided into equal parts with respect to the independent variable, but here the intervals are in general unequal, being determined by the observations. To obtain COTES's formula suppose $\sigma_1 = \sigma_3$, when (16) becomes

$$(17)\qquad k\sqrt{p_0}\,(t_3-t_1) = \tfrac{1}{6}(r_1^2+r_3^2+4r_2^2)\,\sigma_2$$

which is the correct form for the function under consideration. When σ_1 and σ_3 are very nearly equal (17) may be used in place of (16) if it is desired.

3. Method of Computing Approximate Values of the Eccentricity and Longitude of the Perihelion.

Let e_0 and ω_0 represent approximate values of the eccentricity, and the longitude of the perihelion from the ascending node. They are defined by the polar equation of the conic at the epochs t_1 and t_3, which may be written

$$\left.\begin{array}{l} e_0\cos(u_1-\omega_0) = \dfrac{p_0-r_1}{r_1} \\ e_0\cos(u_3-\omega_0) = \dfrac{p_0-r_3}{r_3} \end{array}\right\}\quad(18)$$

But $\qquad u_3-\omega_0 = (u_3-u_1)+u_1-\omega_0$

which, substituted in the second equation of (18), gives, after expansion and easy reductions by means of the first equation,

(19)

$$e_0\sin(u_1-\omega_0) = \frac{1}{\sin(u_3-u_1)}\left\{\frac{p_0-r_1}{r_1}\cos(u_3-u_1)-\frac{p_0-r_3}{r_3}\right\}$$

$$e_0\cos(u_1-\omega_0) = \frac{p_0-r_1}{r_1}$$

whence

(20)

$$\tan(u_1-\omega_0) = \frac{1}{\sin(u_3-u_1)}\left\{\cos(u_3-u_1)-\frac{p_0-r_3}{p_0-r_1}\cdot\frac{r_1}{r_3}\right\}$$

$$e_0 = \frac{p_0-r_1}{r_1}\sec(u_1-\omega_0)$$

Or, taking the sum and difference of equations (18), and reducing by well-known trigonometrical formulas, it is found that

(21)

$$2e_0\left\{\cos\tfrac{1}{2}(u_1+u_3-2\omega_0)\cos\tfrac{1}{2}(u_3-u_1)\right\} = \frac{p_0-r_1}{r_1}+\frac{p_0-r_3}{r_3}$$

$$2e_0\left\{\sin\tfrac{1}{2}(u_1+u_3-2\omega_0)\sin\tfrac{1}{2}(u_3-u_1)\right\} = \frac{p_0-r_1}{r_1}-\frac{p_0-r_3}{r_3}$$

which define ω_0 and e_0 uniquely since $e_0>0$.

4. Correction of the Approximate Elements.

With the approximate values of p, e, and ω the unknown coefficients c_3, c_4, c_5, and c_6 of equation (15) may be computed. From the equation

$$r_2 = \frac{p^2}{[1+e\cos(u-\omega)]^2}$$

and equations (6) it is found that

$$
(22)\quad
\begin{cases}
\dfrac{c_3}{p^2} = \dfrac{-e\sin(u-\omega)}{3[1+e\cos(u-\omega)]^3} + \dfrac{3e^2\sin(u-\omega)\cos(u-\omega)}{[1+e\cos(u-\omega)]^4} + \dfrac{4e^3\sin^3(u-\omega)}{[1+e\cos(u-\omega)]^5} \\[2ex]
\dfrac{c_4}{p^2} = \dfrac{-e\cos(u-\omega)}{12[1+e\cos(u-\omega)]^3} - \dfrac{e^2\sin^2(u-\omega)}{[1+e\cos(u-\omega)]^4} + \dfrac{3e^2\cos^2(u-\omega)}{4[1+e\cos(u-\omega)]^4} \\
\qquad + \dfrac{6e^3\sin^2(u-\omega)\cos(u-\omega)}{[1+e\cos(u-\omega)]^5} + \dfrac{5e^4\sin^4(u-\omega)}{[1+e\cos(u-\omega)]^6} \\[2ex]
\dfrac{c_5}{p^2} = \dfrac{e\sin(u-\omega)}{60[1+e\cos(u-\omega)]^3} - \dfrac{3e^2\sin(u-\omega)\cos(u-\omega)}{4[1+e\cos(u-\omega)]^4} - \dfrac{2e^3\sin^3(u-\omega)}{[1+e\cos(u-\omega)]^5} \\
\qquad + \dfrac{3e^3\sin(u-\omega)\cos^2(u-\omega)}{[1+e\cos(u-\omega)]^5} + \dfrac{10e^4\sin^3(u-\omega)\cos(u-\omega)}{[1+e\cos(u-\omega)]^6} + \dfrac{6e^5\sin^5(u-\omega)}{[1+e\cos(u-\omega)]^7} \\[2ex]
\dfrac{c_6}{p^2} = \dfrac{e\cos(u-\omega)}{360[1+e\cos(u-\omega)]^3} + \dfrac{2e^2\sin^2(u-\omega)}{15[1+e\cos(u-\omega)]^4} - \dfrac{e^2\cos^2(u-\omega)}{8[1+e\cos(u-\omega)]^4} - \dfrac{5e^3\sin^2(u-\omega)\cos(u-\omega)}{2[1+e\cos(u-\omega)]^5} \\
\qquad + \dfrac{e^3\cos^3(u-\omega)}{2[1+e\cos(u-\omega)]^5} - \dfrac{10e^4\sin^4(u-\omega)}{3[1+e\cos(u-\omega)]^6} + \dfrac{15e^4\sin^2(u-\omega)\cos^2(u-\omega)}{2[1+e\cos(u-\omega)]^6} \\
\qquad + \dfrac{15e^5\sin^4(u-\omega)\cos(u-\omega)}{[1+e\cos(u-\omega)]^7} + \dfrac{7e^6\sin^6(u-\omega)}{[1+e\cos(u-\omega)]^8}
\end{cases}
$$

When c_3, c_4, c_5 and c_6 have been computed from these equations (15) and (19) or (21) should be used again to compute p, ω, and e. It will be noticed that the series (15) has been developed in such a way that the terms used in the first computation require no alterations.

The time of perihelion passage, T, is to be computed from the law of areas in the usual manner.

5. CONVERGENCY OF THE SERIES EMPLOYED. Up to the present time astronomers have been generally contented if the series which they have found it convenient to employ give satisfactory results in practice, without entering into the question of their convergence. In accordance with this custom two examples are appended to this paper illustrating the method for both asteroid and cometary orbits. Nevertheless, a direct investigation of the limits of convergence and the accuracy attained when only the first terms are taken, makes a solid foundation for the work, and enables one to proceed with entire confidence within the prescribed limits. Therefore these questions will be investigated.

It will be shown first that series (15), which is the one used in practice, has as great a range of convergence as (5), and then the radius of convergence of (5) will be investigated. Suppose it has been found that series (5) is convergent for $|u-u_2| < \sigma$; then series (12) has a radius of convergence equally great. For, if $\sigma_1 < \sigma$, $\sigma_2 < \sigma$, then every term of (12) is numerically greater than, or at the least equal to, the numerical value of the corresponding term of the series

$$
(23)\qquad \tfrac{1}{1}c_0'\sigma + \tfrac{1}{2}c_1\sigma^2 + \tfrac{1}{3}c_2\sigma^3 + \tfrac{1}{4}c_3\sigma^4 + \ldots
$$

If $u-u_2$ be replaced by σ in (5), and the nth term multiplied $\frac{2\sigma}{n}$ the nth term of (23) will be obtained. For every finite value of σ there is always a finite number N such that, for every $n > N$, $\frac{2\sigma}{n} < 1$. Consequently, after the first N terms, the terms of (23) are respectively numerically less than the absolute values of the corresponding terms of (5). By hypothesis (5) converges for $|u-u_2| \leqq \sigma$: therefore (23), and *à fortiori* (12), also converges for at least as large a value of σ. Q.E.D.

It will be shown that series (15) has the same radius of convergence as (12). Series (13) is just another form of (15), and may be considered instead of it. The question is then how (12) and (13) are related. The auxiliaries ϵ_1 and ϵ_3, defined in (8), are series in (u_1-u_2) and (u_3-u_2) and, being the higher terms of (5), are, by hypothesis, convergent if $|u_1-u_2| \leqq \sigma$, $|u_3-u_2| \leqq \sigma$. Then c_1 and c_2, defined in (10), are linear functions of the series ϵ_1 and ϵ_3 and, therefore, are convergent if $\sigma_1 \leqq \sigma$, $\sigma_3 \leqq \sigma$. Series (13), beyond the first three terms, is the sum of (12), beyond the first three terms, and linear functions of the series ϵ_1 and ϵ_3. Since these series are all convergent if $\sigma_1 \leqq \sigma$, $\sigma_3 \leqq \sigma$, (13), and consequently (15), is convergent under the same conditions. Q.E.D.

Consider, therefore, the series (5). The quantity r^2 is expressed in terms of u by the polar equation of the conic,

$$
r^2 = \frac{p^2}{[1+e\cos(u-\omega)]^2}
$$

Let $u-\omega = v_i$, from which it follows that v_i is the true anomaly at the epoch t_i. Then the polar equation becomes

$$
r^2 = \frac{p^2}{[1+e\cos v]^2} \tag{24}
$$

The question is for what values of v is r^2 expressible as a converging power series in $v-v_2 = u-u_2$. The limit of this region of convergency is evidently a function of e and v_2, but independent of p, and it is proposed to discuss it as a function of these parameters.

r^2 is a *uniform* analytic function of v having consequently no branch points, and it follows from the way in which v enters that there are no essential singularities. It remains to discover the location of the poles as functions of e. Then, according to the researches of CAUCHY in the Theory of Functions, the series will converge for all values of $v-v_2$ whose moduli are less than the least modulus of those values of $v-v_2$ for which the function has a pole.

The condition for a pole of (24) is

$$0 = 1 + e\cos v \tag{25}$$

For values of e less than unity there is no real value of v satisfying this equation. Even if this were not so it would be necessary to consider complex values of the argument of the expansion in order to find the true radius of convergency. Suppose v has the form

$$v = x + \sqrt{-1}\ y$$

where x and y are real numbers. Substituting in (25) and expanding, it is found that

$$0 = 1 + e\cos x\cos(\sqrt{-1}\ y) - e\sin x\sin\sqrt{-1}\ y \tag{26}$$

In order that this equation may be fulfilled the real and imaginary parts must be separately equal to zero. Since the sine of a pure imaginary is a pure imaginary while the cosine is real, and e being real, it follows that

$$(27)\left\{\begin{aligned} 0 &= 1 + e\cos x\cos(\sqrt{-1}\ y) \\ 0 &= \qquad e\sin x\sin(\sqrt{-1}\ y)\end{aligned}\right.$$

The poles are given by those values of x and y which satisfy these equations simultaneously. Then the radius of convergence of (5) is

$$R = \sqrt{(x-v_2)^2+y^2} \tag{28}$$

where x and y satisfy (27) and give the smallest $(x-v_2)^2$ and y^2.

Consider the second equation of (27). If e is not zero it is satisfied only by either

(a) $y = 0$, or

(b) $x = n\pi$,

where n is zero or any integer.

(a) *Case* $y = 0$. In this case the first equation of (27) becomes

$$0 = 1 + e\cos x \tag{29}$$

There is no solution of this equation if $e<1$. If $e = 1$ the solution is

$$x = (2n+1)\pi$$

where n is zero or any integer. It will be convenient to let v_2 run from $-\pi$ to $+\pi$, and, the function being even, the radius of convergence is the same for numerically equal positive and negative values of v_2; hence it will be sufficient to consider the former. As that pole is sought whose corresponding $v-v_2$ has the smallest modulus, n must be put equal to zero, giving

$$x = \pi \quad . \quad y = 0$$

Consider the case where $e>1$. Equation (29) may be written

$$0 = 1 + e\cos(x+2n\pi)$$

which gives, on solving,

$$x = -2n\pi + \cos\left(\frac{-1}{e}\right) = -(2n-1)\pi - \cos^{-1}\left(\frac{1}{e}\right)$$

The principal positive value of $\cos^{-1}\left(\frac{1}{e}\right)$ is to be taken, and n is to be chosen so that the modulus of $x-v_2$ shall be the least possible. It follows that it is necessary to take $n = 0$, whence

$$\left.\begin{aligned} x - v_2 &= \pi - v_2 - \cos^{-1}\left(\frac{1}{e}\right), \quad y = 0 \\ R &= \pi - v_2 - \cos^{-1}\left(\frac{1}{e}\right)\end{aligned}\right\} \tag{30}$$

In the hyperbola v_2 varies only from

$$-\cos^{-1}\left(\frac{-1}{e}\right) = -\pi+\cos^{-1}\left(\frac{1}{e}\right) \text{ to } \cos^{-1}\left(\frac{<1}{e}\right) = \pi-\cos^{-1}\left(\frac{1}{e}\right)$$

Hence, as v_2 varies from 0 to $\cos^{-1}\left(\frac{-1}{e}\right)$, R varies from $\pi-\cos^{-1}\left(\frac{1}{e}\right)$ to 0, the sums of v_2 and R being $\pi-\cos^{-1}\left(\frac{1}{e}\right)$. It is not known whether these poles give the radii of convergence until those defined by (b) have been examined.

(b) *Case* $x = n\pi$. In this case the first equation of (27) becomes

$$0 = 1 + (-1)^n e\cos(\sqrt{-1}\ y) \tag{31}$$

Since $\cos(\sqrt{-1}\ y)$ is a real positive number for all values of y this equation can be satisfied only if n is odd, and the pole whose corresponding $v-v_2$ has the smallest modulus is the one for which $n = 1$. Hence (31) becomes

$$0 = 1 - e\cos(\sqrt{-1}\ y) = 1 - e\cosh y \tag{32}$$

The minimum value of $\cos(\sqrt{-1}\ y)$ is unity, therefore equation (32) can be satisfied only if $e\leq 1$. When $e = 1$ equation (32) is satisfied only by $y = 0$, which is the same result as found in case (a). Hence the two series of poles given by (a) and (b) merge for $e = 1$, but do not overlap. It follows that the poles of (a) give the true radii of convergence for $e\geq 1$, and that the poles of (b) give them for $e<1$.

As y^2 varies from zero to infinity $\cos(\sqrt{-1}\ y)$ is an increasing monotonic function whose lower limit is unity and upper limit infinity. There is, therefore, one, and only one, solution of (32) for every $e<1$. This solution is

$$(33)\qquad y = \cos h^{-1}\left(\frac{1}{e}\right) = \log\left(\frac{1+\sqrt{1-e^2}}{e}\right)$$

hence the radius of convergence for $e<1$ is given by

$$(34)\qquad R = \sqrt{(\pi-v_2)^2+\left[\cos h^{-1}\left(\frac{1}{e}\right)\right]^2}$$
$$= \sqrt{(\pi-v_2)^2+\left[\log\left(\frac{1+\sqrt{1-e^2}}{e}\right)\right]^2}$$

It follows from (33) that the value of y satisfying (32) is independent of v_2. Consequently, if v_2 be given such a value that the numerical value of $x-v_2$ shall be a minimum, the minimum radius of convergence will be found. Since in this case $x=\pi$ it follows that the minimum radius of convergence occurs for $v_2=\pi$, and, in a similar manner, the maximum for $v_2=0$; hence the theorem:

The series (5), in the case of an elliptic orbit, is convergent for the largest values of $u-u_2$ if the body is at perihelion at t_2 and for the smallest if it is at aphelion, decreasing continuously from the first position to the second.

It should be remarked that this is a specially favorable condition of affairs, for in the orbits of the periodic comets, where the radius of convergence is the least, the method will be applied always when the comet is not far from perihelion.

The radii of convergence are given by equations (30) and (34), and depend upon the two parameters e and v_2, (30) being used when $e>1$ and (34) when $0<e\overline{<}1$. If R, v_2 and e be taken as running coordinates these equations define the portion of a surface contained between the planes $v_2=0$ and $v_2=\pi$, and $e=0$ and $e=\infty$. The distance of this surface from the v_2e-plane at any point is the radius of convergence for the corresponding values of the parameters. An idea of the shape of this surface can be best obtained by intersecting it with the planes $e=$ constant. When this constant is less than unity equation (34), in which R and v_2 are the running coordinates, gives the corresponding curve. When this equation is rationalized, it becomes

$$(35)\qquad R^2-(v_2-\pi)^2-\left[\log\left(\frac{1+\sqrt{1-e^2}}{e}\right)\right]^2=0$$

This is the equation of an equilateral hyperbola whose major-axis coincides with the line $v_2=\pi$, whose center is $R=0$, $v_2=\pi$, and whose vertices are distant $\log\left(\frac{1+\sqrt{1-e^2}}{e}\right)$ from the v_2-axis. The branch below the v_2-axis was introduced in rationalizing (34) and is foreign to the problem. When $e=0$ the vertex of the hyperbola is at $R=\infty$, the distance decreasing continually as e increases, becoming zero when $e=1$. For this value of e the hyperbola degenerates into two orthogonal lines, intersecting at $R=0$, $v_2=\pi$, and making angles of 45° and 135° with the v_2-axis.

When $e>1$ equation (30) defines the curve of intersection of the plane $e=$ constant and the surface. This is the equation of a straight line making an angle of 135° with the v_2-axis. A direct computation from (30) and (34) gives the following table of radii of convergence:

TABLE I.

e	$v_2=0$	$v_2=60°$	$v_2=120$	$v_2=180$
0.0	$R=\infty$	$R=\infty$	$R=\infty$	$R=\infty$
0.02	225°	181°	148°	135°
0.1	204	154	113	95
0.2	196	143	98	78
0.3	192	138	90	67
0.4	190	134	85	60
0.6	187	130	77	48
0.8	184	126	72	38
1.0	180	120	60	0
1.2	147	87	27	–
1.4	134	74	14	–
1.6	129	69	9	–
∞	90	30	–	–

When $e=\infty$ $R=0$ for $v_2=90°$

From the table it is seen that in all cases which actually arise in practice the limits of convergence of series (5), and consequently of (15), are very large. In the case of an asteroid orbit with the exceptionally high eccentricity of 0.3, and in the most unfavorable case when the asteroid is at aphelion, the series converges for values of u 67° each side of u_2, giving a range of 134° between the first and third observations for which the series are valid. Since the asteroids move only about one-fourth of a degree daily, the series is convergent when the whole interval of time is not more than 536 days. It is needless to remark that this is many times the interval which would be used in actual practice.

In the case of the orbits of comets the second observation is rarely made when the comet is more than 90° from perihelion, and it is seen from the table that, unless the perihelion distance is exceedingly small, or the eccentricity much greater than that in any orbit yet computed, the region of convergence is many times greater than the heliocentric motion would be in the short interval covered by the observations for a preliminary orbit.

6. THE ACCURACY ATTAINED IN USING A GIVEN NUMBER OF TERMS. It is a question of importance whether the elements computed from a given number of terms of (15) have the desired degree of accuracy. It is clearly useless to carry the computation to more decimals than the

precision of the observations would warrant. The probable errors of micrometrical measurements, and the still greater uncertainties in the positions of the comparison-stars are such that, as seems to be shown by experience in computation, a preliminary orbit is seldom exact in the fifth place in the logarithms of the elements. Even if the data would admit of greater accuracy in the determination of the elements there would be no object in carrying the computation further. Therefore, the question will be raised within what limit σ_1 and σ_3 must be in order that the preliminary orbit shall be exact to five places, and likewise the same limit when the first two correction terms of (15) have been added. That is, σ_1 and σ_3 must be determined so that the remainder of the series beyond the first terms shall not be large enough to change p by one unit in the fifth place. The remainder of the series is divided by $k(t_3-t_1)$ and the whole right member squared in order to get p. If $k(t_3-t_1)$ should be near unity, as will be found to be true, the remainder will change p by twice its amount in consequence of the squaring. Consequently, the superior limit to the remainder for the proposed degree of accuracy should be five units in the sixth place.

It is proposed to develop the explicit formulas for the determination of the limit to σ_1 and σ_3 in order to insure any degree of accuracy, and then to apply them numerically to a sufficient number of examples to enable one to find from the preliminary orbit and the tables to follow, by mere inspection, save in exceptional cases, how many terms of (15) must be employed. Suppose $\sigma_1 = \sigma_3$, then the fourth and sixth terms of (15) vanish. Nevertheless, it follows from the way that (15) was built up that the remainder is larger than it would be if one of σ_1 and σ_3 were smaller than their supposed common value. Suppose the remainder of the series must be η. Then the condition is

$$\eta = a_5\, \sigma_2^5 + a_7\, \sigma_2^7 + \ldots \tag{36}$$

where

$$a_5 = -\frac{c_4}{120}\ , \quad a_7 = \frac{-c_6}{336}\ , \ \ldots$$

When $\eta = 0$ $\sigma_2^5 = 0$ is a solution. Consequently, when η is different from zero but sufficiently small σ_2 may be expressed in a convergent power series in $\lambda\eta^{1/5}$, where λ is any fifth root of unity.* Only one λ is real, viz., $\lambda = 1$. Therefore, there is but one real series, which is the one having meaning in the problem. The series will have the form

$$\sigma_2 = b_1\, \eta^{1/5} + b_2\, \eta^{2/5} + b_3\, \eta^{3/5} + \ldots \tag{37}$$

where $b_1, b_2, b_3 \ldots$ are coefficients to be determined. To find them, substitute (37) in (36), which gives

$$\eta = a_5 b_1^5 \eta + 5a_5 b_1^4 b_2 \eta^{6/5} + 5a_5 b_1^4 b_3 \eta^{7/5} + 10a_5 b_1^3 b_2^2 \eta^{7/5} + a_7 b_1^7 \eta^{7/5} + \ldots$$

Since (37) has a finite realm of convergency, this expression is an identity in $\eta^{1/5}$ with the coefficients of corresponding powers equal. Comparing them, it is found that

$$b_1 = \frac{1}{a_5^{1/5}}\ , \quad b_2 = 0\ , \quad b_3 = -\frac{a_7}{5a_5^{8/5}}\ , \quad \ldots$$

Then (37) becomes

$$\sigma_2 = \left(\frac{\eta}{a_5}\right)^{1/5} - \frac{a_7}{5a_5}\left(\frac{\eta}{a_5}\right)^{3/5} + \ldots$$

or

$$\sigma_2 = \left(\frac{120\eta}{-c_4}\right)^{1/5} + \frac{c_6}{-14c_4}\left(\frac{120\eta}{-c_4}\right)^{3/5} + \ldots \tag{38}$$

To apply this formula and make the corrections to (15), the following tables were computed from (22).

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* Jordan, *Cours d'Analyse*, 2d edition, Vol. I, p. 342.

Table II.

e	$v_2 = 0$	$v_2 = 30°$	$v_2 = 60°$	$v_2 = 90°$	$v_2 = 120°$	$v_2 = 150°$	$v_2 = 180°$
0.0	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$	$\frac{c_3}{p^2} = 0$
0.2	0	+.0086	+.0050	− .0346	− .1232	− .1598	0
0.4	0	+.0432	+.1002	+ .1227	− .2255	− 1.1099	0
0.6	0	+.0725	+.2361	+ .6640	+ .8863	− 5.4615	0
0.8	0	+.0922	+.3795	+1.7813	+ 9.6226	− 4.3803	0
1.0	0	+.1090	+.5132	+3.6667	+ 80.8293	+ 7474.0	–
1.2	0	+.1091	+.6290	+1.400	+ 359.92	–	–
0.0	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$	$\frac{c_4}{p^2} = 0$
0.2	+.0048	+.0036	−.0080	− .0320	− .0421	+ .0269	+ .1058
0.4	+.0191	+.0235	+.0289	− .0320	− .3521	− .4705	+ 1.0802
0.6	+.0290	+.0467	+.1241	+ .2880	− .5640	− 5.1686	+ 11.3282
0.8	+.0313	+.0656	+.2613	+1.4080	+ 7.2531	− 66.3584	+308.3333
1.0	+.0365	+.0795	+.4197	+1.0000	+ 99.3333	+21910.0	–
1.2	+.0367	+.0884	+.5826	+8.9280	+1013.30	–	–

Table III.

e	$v_2=0$	$v_2=90$	$v_2=180$	$v_2=0$	$v_2=90$	$v_2=180$
0.3	$\frac{c_4}{p^2}=0$	$\frac{c_4}{p^2}=-.0344$	$\frac{c_4}{p^2}=0$	$\frac{c_6}{p^2}=+.0001$	$\frac{c_6}{p^2}=-.0099$	$\frac{c_6}{p^2}=+.1247$
1.0	0	+4.0167		+.0082	+3.8000	–

Suppose, as agreed above, η be put equal to five units in the sixth place, $\eta=.000005$, and let equation (38) be applied. Before the actual values of c_4 and c_6 can be substituted, values of p, must be assumed. For the asteroid orbits $p=2.4115$ will be a fair assumption, and in the case of the orbits of comets $p=2$, though in the latter there is a much wider range of variation. It is seen that, with the value of η assumed above, the second term of (38) is very small compared to the first: hence, in the practical determination of σ_2, p enters in the inverse two-fifths power. When p is not remote from unity the value of σ_2 will change, therefore, very slowly as p varies, and the table which follows, depending upon the two other arguments, e and v_2, will give a good idea of the limits to σ_2 in any case which will actually arise. Expressing σ_2 in degrees, equation (38) and Table II gives the following table:

Table IV.

	e	$v_2=0$	$v_2=30$	$v_2=60$	$v_2=90°$	$v_2=120°$	$v_2=150°$	$v_2=180°$
	0.0	$\sigma_2=\infty$	$\sigma_2=\infty$	$\sigma_2=\infty$	$\sigma_2=\infty$	$\sigma_2=\infty$	$\sigma_2=\infty$	$\sigma_2=\infty$
$p=2.4115$	0.2	26.6	28.2	24.0	18.2	17.2	18.8	14.3
	0.4	20.2	19.3	18.6	18.2	11.3	10.6	9.0
	0.6	20.0	18.2	14.9	12.6	11.1	7.1	6.1
	0.8	19.3	17.0	12.9	9.2	6.6	1.3	3.1
$p=2.0$	1.0	19.1	16.3	11.7	7.5	3.9	1.3	–
	1.2	19.1	16.0	11.0	6.4	2.5	–	–

There are some slight irregularities in the numbers of this table on account of the manner in which c_4 changes sign, and from the fact that the second term in (38) was neglected.

Suppose the first two corrective terms in (15) are added, and let the limit of σ_2 be found such that the resulting expression given by the first five terms of the series shall have a given degree of accuracy. Letting η represent the remainder as before, it is found that the equation corresponding to (38) is

$$\sigma_2=\left(\frac{336\,\eta}{-c_6}\right)^{1/7}+\ldots \tag{39}$$

Putting η equal to .000005 and making use of Table III, the following table was computed:

Table V.

	$v_2=0$	$v_2=90$	$v_2=180$
$e=0.3$, $p=2.4115$	$\sigma_2=66.7$	$\sigma_2=34.6$	$\sigma_2=21.1$
$e=1.0$, $p=2.0$	37.5	15.6	

The last two tables enable one to determine at a glance when the approximate elements have been computed, whether any corrective terms are required, and if so, if the first two will suffice. If the first two are required it will rarely be necessary to compute c_3 and c_4 from (22), as their values can be interpolated from Table II with sufficient accuracy. It was for this purpose that Table II was computed for so many values of the arguments.

It is seen from Tables IV and V that the arc of convergence is greater at perihelion than at aphelion, but, on the other hand, the body moves more slowly at aphelion. It will be of interest to express this radius of sufficiently rapid convergence in days. The integral of areas is

$$r^2\frac{dv}{dt}=k\sqrt{p}$$

It will be sufficiently accurate for present purposes to take as the integral of this equation

$$r_2^2\,(v_3-v_1)=k\sqrt{p}\,(t_3-t_1)$$

whence

$$t_3-t_1=\frac{r_2^2\,\sigma_2}{k\sqrt{p}} \tag{40}$$

This equation will give too great a value of t_3-t_1 when $v_2=180$ and too small a value when $v_2=0$. Of course, the rigorous formula could be used, but it would require a great deal more work, and the results would be scarcely more valuable here. Equation (40) and Table IV give

TABLE VI.

		$v_2 = 0$	$v_2 = 30°$	$v_2 = 60°$	$v_2 = 90°$	$v_2 = 120°$	$v_2 = 150°$	$v_2 = 180°$
		d	d	d	d	d	d	d
$p = 2.4115$	$e = 0.0$	∞	∞	∞	∞	∞	∞	∞
	$e = 0.2$	70.2	77.9	75.4	69.1	80.7	104.5	84.9
	$e = 0.4$	39.2	40.4	49.0	69.1	67.1	94.3	95.0
$p = 2$	$e = 0.6$	22.4	22.6	25.3	36.2	65.0	88.3	109.4
	$e = 0.8$	17.1	17.1	18.9	26.4	52.6	131.0	222.0
	$e = 1.0$	13.7	13.4	14.9	21.5	44.7	208.0	–
	$e = 1.2$	11.3	11.0	12.3	18.4	44.8	–	–

Equation (40) and Table V give

TABLE VII.

	$v_2 = 0$	$v_2 = 90°$	$v_2 = 180°$
$e = 0.3$, $p = 2.4115$	150.^{d}	131.^{d}	131.^{d}
$e = 1.0$, $p = 2.0$	21.5	40.8	–

The value of the Gaussian constant k is about $\frac{1}{60}$. Hence it follows from Table VI (remembering that the numbers are too small when $v_2 = 0$) that, in the case of an asteroid orbit, the first three terms of (15) will give p accurate to sixth place when the interval of time covered by the observations is not greater than 40 days, and that the first five terms will give the same accuracy when the interval is not greater than 100 days. The average eccentricity of the asteroid orbits is about 0.14; hence the limits for the average orbit would be very much greater than those just given, which are computed for the most unfavorable cases that have yet arisen in about 500 orbits.

In the case of the orbits of comets the first three terms of (15) will give p accurate to the sixth place when the interval of time covered by the observations does not exceed 10 days, unless p should happen to be very small, and the first five terms will give the same accuracy when the interval is not greater than 20 days. These intervals are sufficiently large for all practical purposes in the computation of preliminary orbits.

7. ILLUSTRATIVE EXAMPLE — ELLIPTIC ORBIT. It was intended to compare the method developed above with that of GAUSS upon an example given in one of the standard works on the Theory of Orbits, as that of WATSON, OPPOLZER, or TISSERAND, but they have all selected orbits with such small eccentricities (0.19, 0.196, and 0.12 respectively), and such short intervals of time (14, 33, and 47 days respectively), that their examples furnish only a mild test for the theory. In every case, starting where this method deviates from that of GAUSS, equations (16) and (20) gave the correct elements with a few minutes' computation.

In order to avoid the work which is common to the two methods let the start be made with an assumed orbit, and let the radii and arguments of latitude be computed by the usual methods, and let the inverse process of finding the elements be carried out by the methods explained above. This will be in every respect as good a test as an actual orbit, and will save considerable work.

Assume the exceptionally high eccentricity 0.3, and take the intervals of time 30 and 35 days. Then

$$a = 2.65 \qquad t_1 - T = 30 \text{ days}$$
$$e = 0.3 \qquad t_2 - T = 60 \text{ ''}$$
$$p = 2.4115 \qquad t_3 - T = 90 \text{ ''}$$
$$\log p = 0.382287$$

Let M represent the mean anomaly, E the eccentric anomaly, and v the true anomaly; then it is found by the usual formulas that

$$M_1 = 6°\ 51'\ 15''.1 \qquad E_1 = 9°\ 46'\ 17''.2 \qquad v_1 = 13°\ 17'\ 19''.3$$
$$M_2 = 13\ 42\ 30.2 \qquad E_2 = 19\ 25\ 29.5 \qquad v_2 = 26\ 15\ 31.1$$
$$M_3 = 21\ 42\ 17.8 \qquad E_3 = 30\ 24\ 14.7 \qquad v_3 = 40\ 38\ 24.2$$

$$\log r_1 = 0.271036 \qquad \log \sigma_1 = 9.399679 \qquad \sigma_1 = 0.251003$$
$$\log r_2 = 0.278811 \qquad \log \sigma_2 = 9.678856 \qquad \sigma_2 = 0.477371$$
$$\log r_3 = 0.293215 \qquad \log \sigma_3 = 9.354815 \qquad \sigma_3 = 0.226368$$

Suppose $\omega = 30°$, then the arguments of latitude are given by $u_i = v_i + \omega$ ($i = 1, 2, 3$). Then equation (16) gives

$$k\sqrt{p_0}\,(t_3 - t_1) = 1.152245 + 0.247024 + 0.337128$$

whence

$$\log p_0 = 0.382308$$

Then equations (20) give

$$\omega_0 = 29°\ 59'\ 43''.4 \quad , \quad e_0 = 0.300058$$

It is seen from Table IV that it will be necessary to add the fourth and fifth terms as corrections. It is found from equations (22), or Table II, that $c_3 = 0.130840$, $c_4 = 0.057310$. Then the fourth and fifth terms of (15) are respectively -0.000029 and -0.000013, whence it is found by (15) and (20) that

$$\log p = 0.382287 \quad , \quad \omega = 30°\ 0'\ 0''.0 \quad , \quad e = 0.300000$$

agreeing to the sixth place with the elements which were assumed on the start.

8. Illustrative Example. Parabolic Orbit. In the example of determining the elements of a parabolic orbit in Watson the whole interval of time is 6 days, and in Oppolzer 6 days. It follows from Table VI that no corrective terms would be required in this case. While 6 days is a fair interval for a preliminary orbit let the method be given a more complete test by using an interval of 22 days. Suppose $q = 1$, $p = 2$, $t_1 - T = 20$ days, $t_3 - T = 30$ days, $t_2 - T = 12$ days. Then it is found from the usual formulas that

$v_1 = 26°\ 51'\ 17''.1$	$\log r_1 = 0.024073$	$\log \sigma_1 = 9.327902$
$v_2 = 38\ \ 38\ \ 11.2$	$\log r_2 = 0.050336$	$\log \sigma_2 = 9.621587$
$v_3 = 50\ \ 19\ \ 37.4$	$\log r_3 = 0.088399$	$\log \sigma_3 = 9.313085$

$\sigma_{12} = 0.212766$ Suppose $\omega = 30°$

$\sigma_{13} = 0.418395$

$\sigma_{23} = 0.205629$

The University of Chicago, 1901.

Then equation (16) gives

$$k\sqrt{p_0}\,(t_3 - t_1) = 0.351798 + 0.075201 + 0.108284$$

whence

$$p_0 = 2.000612$$

Equations (20) give

$$\omega = 30°\ 0'\ 40''.3 \quad , \quad e = 1.000550$$

The values of c_3 and c_1 are found from (22), or Table II, to be respectively 0.651730 and 0.183972. The fourth and fifth terms of (15) are respectively -0.000028 and -0.000052. Then equations (15) and (20) give

$$\log p = 0.301031 \quad , \quad \omega = 30°\ 0'\ 1''.0 \quad , \quad e = 1.000018$$

which is more than accurate enough for practical purposes.

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CORRIGENDUM.

No. 508, p. 34: The last four reductions to apparent place belong opposite Feb. 7, 18, 19 and 21. That for Feb. 8 is 2s.59, —17″.0. All the App. α's and App. δ's are correct as given.

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ON THE USE OF THE SPHERO-CONIC IN ASTRONOMY.*

By G. W. HILL.

If a planet circulating about the *Sun* in an elliptic orbit is viewed from a fixed point its apparent path as projected on the celestial sphere is a curve named a sphero-conic. This curve is divided into symmetrical quadrants by two great circles at right-angles to each other, and intersecting at a point called the center. The subject is especially interesting as showing what would be the apparent course of any planet as viewed from the *Earth* if the latter were stopped at any point of its orbit. The two envelopes of all these curves, when the *Earth* is made to take all positions in its orbit, evidently embrace between them the zodiac of the planet. A valuable application is also found in the secular perturbations of one planet by another.

The paths of the superior major planets, as seen from any point of the *Earth's* orbit, do not greatly deviate from a great circle; but those of *Mercury* and *Venus* take a spindle-shaped form which is of great interest. Hence, we propose to illustrate the matter in the case of *Venus*, as it would be viewed from the point of the *Earth's* orbit where the eccentric anomaly is 90°. The necessary values of the elements of the problem are taken from Prof. NEWCOMB'S Tables of the *Sun* and *Venus* for the epoch 1900. The distance of the *Sun* from the point of view is unity in this case, but we may always adopt this distance as the linear unit. The solar eccentricity being 0.01675104, and the longitude of the perigee 281° 13′ 15″.0, the *Sun's* longitude as seen from this point is 12° 10′ 50″.31. Let u_0 denote the angular distance of the *Sun* from the ascending node of its orbit on that of *Venus*, ω the angular distance of this node from the perihelion of *Venus*, i the inclination of the orbits, a the semi-axis major, and e the eccentricity of *Venus*. The problem involves no more than these five quantities, which have the following values:

$$u_0 = 116°\,24'\,3''.58,\quad \omega = 125°\,36'\,56''.93,\quad i = 3°\,23'\,37''.07,$$
$$\log a = 9.859333781 \quad , \quad e = 0.00682069$$

If we imagine a system of rectangular axes having its origin at the point of view, the axis of x being directed towards the perihelion of *Venus*, the axis of y to a point in the plane of the orbit a quadrant in advance of the perihelion, and the axis of z towards the north pole of the orbit, the coordinates of the *Sun* will have the following expressions:

$$\begin{aligned} x_0 &= \cos\omega\,\cos u_0 - \cos i\,\sin\omega\,\sin u_0 \\ y_0 &= \sin\omega\,\cos u_0 + \cos i\,\cos\omega\,\sin u_0 \\ z_0 &= \sin i\,\sin u_0 \end{aligned}$$

By employing the auxiliary quantities k, k', K, K', determined from the equations

$$\begin{aligned} k\cos K &= \cos\omega & k'\cos K' &= \cos i\cos\omega \\ k\sin K &= \cos i\,\sin\omega & k'\sin K' &= \sin\omega \end{aligned}$$

the expressions for x_0 and y_0 become

$$x_0 = k\cos(u_0 + K) \qquad y_0 = k'\sin(u_0 + K')$$

In our example

$$\log k = 9.9994966 \quad , \quad \log k' = 9.9997418,$$
$$K = 125°\,39'\,48''.37 \quad , \quad K' = 125°\,34'\,5''.60$$

The formulas give these values of the coordinates of the *Sun*:

$$x_0 = -0.4679356\ ,\ y_0 = -0.8821706\ ,\ z_0 = +0.05302156$$

Let $b = a\sqrt{1-e^2}$ denote the semi-axis minor of *Venus*. The elements of the sphero-conic constituting the apparent orbit of *Venus* in the heavens are found through the solution of a certain cubic. Let A, B, C denote the rectangular coordinates of the center of the elliptic orbit of *Venus*; then

$$A = x_0 - ae \quad , \quad B = y_0 \quad , \quad C = z_0$$

also let r denote the distance of this center from the origin, so that $\mathrm{r}^2 = A^2 + B^2 + C^2$.

Then the cubic will be thus expressed:

$$G^3 + (a^2 + b^2 - \mathrm{r}^2)G^2 - [b^2(\mathrm{r}^2 - a^2) + a^2(e^2B^2 + C^2)]G - a^2b^2C^2 = 0$$

Accurate computation of the coefficients is facilitated by using the equations

$$a^2 + b^2 - \mathrm{r}^2 = 2b^2 - 1 + 2ae\,x_0 \quad , \quad \mathrm{r}^2 - a^2 = 1 - b^2 - 2ae\,x_0$$

* This article is intended as supplementary to one in the *American Journal of Mathematics* Vol. XXIII, p. 317.

In the present example we have

$$G^3 + 0.01175306\,G^2 - 0.2533680\,G - 0.0007695186 = 0$$

The roots of this cubic are always real; we will name them in the order of their algebraic magnitude, G_x, G_y, G_z. The value of G_x is nearly $-a^2$; in fact, neglecting quantities of the fourth order with respect to e and i, the expression for G_x is

$$G_x = -a^2\left(1 - e^2\,\frac{i^2}{e^2+i^2}\right)$$

In our example this gives a sufficiently accurate value, the error probably not exceeding a unit in the 8th decimal; thus $G_x = -0.52320107$. The quadratic containing the remaining roots is

$$G^2 - 0.4814501\,G - 0.001470838 = 0$$

Thus, $G_y = -0.00303587$ and $G_z = +0.4814868$.

The equation of the cone having its vertex at the origin and the orbit of *Venus* as directrix, when referred to the axes of symmetry, has the form

$$\frac{X^2}{G_x} + \frac{Y^2}{G_y} + \frac{Z^2}{G_z} = 0$$

the axis of z being in the body of the cone, that of x in the direction of longitude, and that of y in the direction of latitude. Now let η denote latitude measured from the major-axis of the sphero-conic, and λ longitude measured from the minor axis. Then, making

$$X = \rho\cos\eta\sin\lambda \quad , \quad Y = \rho\sin\eta \quad , \quad Z = \rho\cos\eta\cos\lambda$$

we obtain, as the equation connecting the variables η and λ in the sphero-conic

$$\frac{\sin^2\lambda}{G_x} + \frac{\tan^2\eta}{G_y} + \frac{\cos^2\lambda}{G_z} = 0$$

The greatest longitude λ_0 and the greatest latitude η_0 of the planet moving on the sphero-conic will be given by the equations

$$\tan\lambda_0 = \sqrt{-\frac{G_x}{G_z}} \quad , \quad \tan\eta_0 = \sqrt{-\frac{G_y}{G_z}}$$

The equation of the sphero-conic can be put in the form

$$\tan^2\eta = \frac{\tan^2\eta_0}{\sin^2\lambda_0}\sin(\lambda_0+\lambda)\sin(\lambda_0-\lambda)$$

In the present example

$$\eta_0 = 4°\ 31'\ 33''.75 \quad , \quad \lambda_0 = 46°\ 6'\ 3''.50$$

As it is interesting we will derive the equation of the stereographic projection of this curve. Taking the radius of the projected sphere as unity, and placing the pole of projection at the center of the sphero-conic, the equations connecting the variables λ and η with the projected coordinates x and y are

$$\tan\lambda = \frac{2x}{1-x^2-y^2} \quad , \quad \sin\eta = \frac{2y}{1+x^2+y^2}$$

The inverse of these formulas, which may be used for plotting the projection of the curve, are

$$x = \frac{\sin\lambda\cos\eta}{1+\cos\lambda\cos\eta} \quad , \quad y = \frac{\sin\eta}{1+\cos\lambda\cos\eta}$$

Thus the equation of the projected sphero-conic is

$$\frac{1}{G_x}\,\frac{4x^2}{(1-x^2-y^2)^2+4x^2} + \frac{1}{G_y}\,\frac{4y^2}{(1+x^2+y^2)^2-4y^2} + \frac{1}{G_z}\,\frac{(1-x^2-y^2)^2}{(1-x^2-y^2)^2+4x^2} = 0$$

Taking two constants α and β this may be put in the form

$$\frac{y^2}{(1+x^2+y^2)^2-4y^2} = \frac{\alpha(1-x^2-y^2)^2+\beta x^2}{(1-x^2-y^2)^2+4x^2}$$

The projected curve is therefore an octic.

To have the position of the sphero-conic, let Ω be the longitude of the ascending node of the major-axis of that curve on the orbit of the planet measured from the perihelion of the latter, and ι the inclination (always between 0° and 180°), and τ the angular distance of the center of the sphero-conic from the node measured in the direction of increasing longitudes; the following equations can be used for the determination of these quantities:

$$\tan\iota\sin\Omega = \frac{A}{C}\,\frac{G_y}{G_y+a^2} \quad , \quad \tan\iota\cos\Omega = -\frac{B}{C}\,\frac{G_y}{G_y+b^2}$$

$$\sin^2\theta = \frac{G_y-G_x}{G_z-G_x} \quad , \quad \tan^2\tau = -\tan^2\theta\tan^2\lambda_0\,\frac{(G_z+a^2)(G_z+b^2)}{(G_x+a^2)(G_x+b^2)}$$

where θ and τ are taken in the first quadrant. In our example we get

$$\iota = 6°\ 17'\ 14''.56 \quad , \quad \Omega = 151°\ 48'\ 30''.62$$
$$\theta = 15°\ 55'\ 11''.70 \quad , \quad \tau = 89°\ 59'\ 58''.07$$

That τ should so nearly be equal to a quadrant is due to the smallness of the eccentricity of *Venus*.

In order to have the position of the ecliptic referred to the axes of the sphero-conic, let ψ denote the distance of the ascending node of the ecliptic on the major-axis of the sphero-conic from the similar point of the ecliptic on the orbit of *Venus*, and χ the distance of the same point from the ascending node of the major-axis on the orbit of *Venus*, and let I denote the inclination of the ecliptic to the mentioned axis. Then these quantities are determined by the equations:

$$\begin{aligned}
\sin I\cos\psi &= \cos\iota\sin i - \sin\iota\cos i\cos(\Omega-\omega)\\
\sin I\sin\psi &= -\sin\iota\sin(\Omega-\omega)\\
\sin I\cos\chi &= -\sin\iota\cos i + \cos\iota\sin i\cos(\Omega-\omega)\\
\sin I\sin\chi &= -\sin i\sin(\Omega-\omega)
\end{aligned}$$

$$v = u_0 - \psi \quad , \quad \chi - \tau = \sigma$$

In the present example we get

$$I = 3^\circ 34' 12''.94 \;,\; \psi = 230^\circ 55' 7''.07 \;,\; \chi = 204^\circ 48' 29''.24$$
$$\nu = 245^\circ 28' 56''.51 \;,\; \sigma = 114^\circ 48' 31''.17$$

We can now get the coordinates of the *Sun* as referred to the axes of the sphero-conic: calling the longitude λ_0 and the latitude η_0, we have

$$\tan(\lambda_0 - \sigma) = \cos I \tan \nu \quad , \quad \sin \eta_0 = \sin I \sin \nu$$

where $\lambda_0 - \sigma$ is taken in the same semicircle as ν. In the example

$$\lambda_0 = 0^\circ 14' 56''.29 \quad , \quad \eta_0 = -3^\circ 14' 52''.73$$

We now make application to the question of the secular perturbations of the *Earth* by *Venus*. For brevity put $m^2 = G_z - G_x$. In the example $\log m = 0.0016637$. Derive the *nome* q from

$$\frac{q + q^9 + q^{25} + \ldots}{1 + 2(q^4 + q^{16} + q^{36} + \ldots)} = \left(\frac{\sin\frac{1}{2}\theta}{1 + \sqrt{\cos\theta}}\right)^2$$

or from the equations

$$\cos\theta = x^2 \;,\; q = \frac{1}{2}\,\frac{1-x}{1+x} + 2\left(\frac{1}{2}\,\frac{1-x}{1+x}\right)^5 + 30\left(\frac{1}{2}\,\frac{1-x}{1+x}\right)^9 + \ldots$$

we obtain $\log q = 8.6556780$. Next compute K and L from the equations

$$K = \frac{4}{\cos^2\theta(1 + \sqrt{\cos\theta})^2}\left[1 + 2(q^4 + q^{16} + \ldots)\right]^2$$
$$L = \frac{1 + \sqrt{\cos\theta}}{4\cos^2\frac{\theta}{2}\cos^3\theta}\;\frac{1 - 4q^3 + 9q^8 - 16q^{15} + \ldots}{\left[1 + 2(q^4 + q^{16} + \ldots)\right]^2}$$

The results in the present example are

$$\log K = 0.3906003 \quad , \quad \log L = 0.1270540$$

The components of the action of the ring formed on the orbit of *Venus* on the origin are

$$X = -\frac{L\cos^2\theta}{m^3}\cos\eta_0 \sin\lambda_0 = -\frac{M}{m^3}\sin^2 K \cos\eta_0 \sin\lambda_0$$
$$Y = -\frac{K - L}{m^3}\sin\eta_0 = -\frac{M}{m^3}\cos^2 K \sin\eta_0$$
$$Z = \frac{K - L\sin^2\theta}{m^3}\cos\eta_0 \cos\lambda_0 = -\frac{M}{m^3}\cos\eta_0 \cos\lambda_0$$

Let R denote the magnitude of the resultant of these components, Λ and H severally the longitude and latitude of the point in the heavens towards which it is directed, the circles of reference being the axes of the sphero-conic. We have the equations

$$R\cos H \sin\Lambda = X \;,\; R\sin H = Y \;,\; R\cos H\cos\Lambda = Z$$

The numerical results are

$$R = 1.7446004 \;,\; \Lambda = 359^\circ 54' 31''.10 \;,\; H = +2^\circ 3' 26''.84$$

In order to have the components of the force referred to the ecliptic, it is necessary to convert Λ and H to a longitude and latitude referred to that plane. Let α and δ severally denote this longitude counted from the descending node of the major-axis of the sphero-conic on the ecliptic and the latitude. Then the equations are

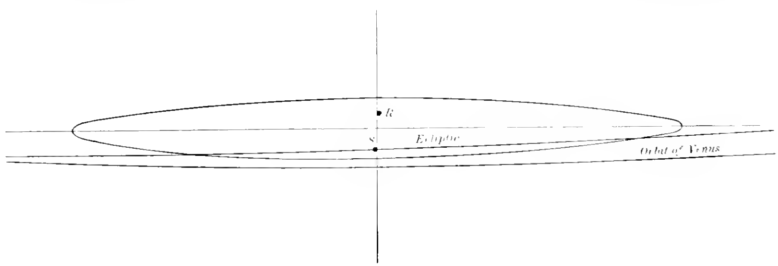
$$\sin\delta = \cos I \sin H - \sin I \cos H \sin(\Lambda - \sigma)$$
$$\cos\delta \sin\alpha = \sin I \sin H + \cos I \cos H \sin(\Lambda - \sigma)$$
$$\cos\delta \cos\alpha = \cos H \cos(\Lambda - \sigma)$$

Then, if R_0 denote the component directed towards the *Sun*, S_0 the component perpendicular to this and lying in the plane of the ecliptic, and W_0 the component directed to the north pole of that plane, we shall have

$$R_0 = R\cos\delta\cos(\alpha - \nu) \;,\; S_0 = R\cos\delta\sin(\alpha - \nu) \;,\; W_0 = R\sin\delta$$

The numerical results are

$$\alpha = 245^\circ 0' 11''.53,\; \alpha - \nu = 359^\circ 31' 15''.02,\; \delta = +5^\circ 17' 11''.12$$
$$R_0 = +1.7370960 \;,\; S_0 = -0.0145277,\; W_0 = +0.1609908$$



The diagram exhibits the form of the sphero-conic in the example. The projection used is the stereographic, the pole being at the center of the sphero-conic. The radius of the sphere projected is five inches. The point R is that towards which the resultant of the attraction of the elliptic ring is directed, and the point S shows the position of the *Sun*.

In the application to secular perturbations almost the sole aim is to obtain the values of R_0, S_0, W_0; but the preceding method involves the computation of many auxiliary quantities, hence it may be well to substitute the following, in which λ_0, η_0, Λ, H, a, δ, are not used, but the desired values are expressed in terms of the quantities which precede. We have

$$R = \frac{M}{m^3}\left[\cos^2\eta_0\cos^2\lambda_0 - \sin^2\kappa\cos^2\eta_0\sin^2\lambda_0 - \cos^2\kappa\sin^2\eta_0\right]$$

But
$$\begin{aligned}\cos\eta_0\cos\lambda_0 &= \cos\sigma\cos v - \cos I\sin\sigma\sin v\\ \cos\eta_0\sin\lambda_0 &= \sin\sigma\cos v + \cos I\cos\sigma\sin v\\ \sin\eta_0 &= \sin I\sin v\end{aligned}$$

By substituting these values in the preceding equation, and putting

$$\begin{aligned}N\cos\nu &= (1+\sin^2\kappa)(1-\tfrac{1}{2}\sin^2 I)\cos 2\sigma + \tfrac{1}{2}\cos^2\kappa\sin^2 I\\ N\sin\nu &= (1+\sin^2\kappa)\cos I\sin 2\sigma\end{aligned}$$

the expression for R_0 becomes

$$R_0 = \frac{M}{m^3}\left[\cos^2\sigma - \sin^2\kappa\sin^2\sigma - N\sin v\sin(v+\nu)\right]$$

There is no need for the direct elaboration of the values of S_0 and W_0, since, provided the quantities v and I are left evident in the expression for R_0, they are determined by the partial differential equations

$$S_0 = \tfrac{1}{2}\frac{\partial R_0}{\partial v} \quad , \quad W_0 = \frac{1}{2\sin v}\frac{\partial R_0}{\partial I}$$

Thus
$$S_0 = -\tfrac{1}{2}\frac{M}{m^3}N\sin(2v+\nu)$$

Moreover, if we put

$$\begin{aligned}P\cos\pi &= [(1+\sin^2\kappa)\cos 2\sigma - 3\cos^2\kappa]\cos I\\ P\sin\pi &= (1+\sin^2\kappa)\sin 2\sigma\end{aligned}$$

we shall have

$$W_0 = \tfrac{1}{2}\frac{M}{m^3}P\sin I\sin(v+\pi)$$

In making use of the values of R_0, S_0, W_0 as obtained for different points in the orbit of the attracted planet it will be necessary to reduce them to a common linear unit; when we bear in mind that they are of the dimension -2 in reference to this unit, the procedure to be followed will be obvious.

As the example from *Venus* is abnormally simple in some respects on account of the smallness of the eccentricity of that planet, another may be given of the orbit of *Mercury* seen from the same point. Here the elements have the values

$$u_0 = 145^\circ\ 2'\ 1''.91\ ,\quad \omega = 151^\circ\ 14'\ 46''.19\ ,\quad i = 7^\circ\ 0'\ 10''.37$$
$$\log a = 9.5878217\quad ,\quad e = 0.20561421$$

As before, these are from Prof. Newcomb's Tables, and for the epoch 1900. From these elements the coordinates of the *Sun* are

$$x_0 = +0.4148295\ ,\quad y_0 = -0.8928855\ ,\quad z_0 = +0.06986952$$

and the cubic in G is

$$G^3 - 0.6121687\,G^2 - 0.1185352\,G - 0.0001049789 = 0$$

By the approximate formula $G_x = -0.1489421$, and this needs only the small correction of 7 units in the last decimal, so that $G_x = -0.1489428$. The quadratic containing the remaining roots is

$$G^2 - 0.7911115\,G - 0.0007048269 = 0$$

Whence $G_y = -0.0008899314\quad ,\quad G_z = +0.7920014$

The dimensions of the sphero-conic, in this case, are

$$\lambda_0 = 23^\circ\ 26'\ 39''.57\quad ,\quad \eta_0 = 1^\circ\ 55'\ 11''.59$$

Its position in the heavens is established by the values

$$\iota = 4^\circ\ 53'\ 11''.32\ ,\quad \Omega = 201^\circ\ 23'\ 16''.94\ ,\quad \tau = 89^\circ\ 16'\ 43''.53$$

It will be seen that, in spite of the great eccentricity of *Mercury*, τ does not differ much from a quadrant.

EXPLANATION OF THE INCLINATION OF THE PLANETARY AXES,

By W. H. PICKERING.

Suppose a uniform spheroid to be revolving in its orbit about the sun, and to present always the same face to a star. If this spheroid is covered with liquid an annual tide will be produced, which in the process of time will cause the spheroid to rotate upon its axis so as to present always the same face to the sun.

Suppose now that this spheroid possesses an original rotation about its minor axis, and that this axis lies in the plane of its orbit, as is for instance approximately the case with the planet *Uranus*. We shall thus have two independent rotations about the two axes placed at right-angles to one another. When these motions are combined, however, as may be clearly illustrated by means of the gyroscope, the effect produced is to shift the minor axis of the planet out of its original plane, so that the plane of the planet's equator shall approach the plane of its orbit, and in such a manner that the rotation and revolution shall take place in the same direction.

This shifting of the axis is not to be confused with that producing precession, which is due to a different cause, and is periodic. The present shifting is continuous in its action, and its direction lies at right-angles to that causing precession.

According to Laplace's nebular hypothesis, as is well known, when the rings break up into planets, these should rotate in a retrograde direction. Owing to the tidal action

above described, however, the plane of their rotation will gradually shift, so that from being at first nearly parallel to the plane of their orbits, it becomes later perpendicular to them, and finally again parallel, but this time with the rotation direct. Successive satellites formed by the contracting mass would thus originally revolve in different planes, but would all finally approach the plane of rotation of their primary through the attraction of their equatorial regions.

Such a progressive change of plane is found in the orbits of the satellites of the four major planets. Thus, for *Neptune* the approximate angle is 145°, for *Uranus* 98°, for *Saturn* (inner satellites) 27°, and for *Jupiter* 2°. It is also found in the case of the four inner planets, as far as is known, as determined by the inclination of their equators to the plane of their orbits. Thus, for *Mars* the angle is 25°, for the *Earth* 23°, for *Venus* the angle is unknown, and for *Mercury*, while undetermined, it is certainly very small, as is indicated by the drawings of surface detail made at Milan, Arequipa and Flagstaff. While the force producing this change must at the present time be almost infinitesimal, yet such would not have been the case in the past, when the planets were perhaps one hundred or more times their present dimensions.

If we take the great nebula in *Andromeda* as a type of our earlier existence, the planets being merely condensed masses revolving within and with the solar atmosphere, this atmosphere itslf would excite a frictional force which would tend always to keep the same side of the planet towards its primary, while gravitation acting on the mass before it had separated itself from the spiral would tend to cause it to revolve in a retrograde direction. Thus in the earlier times it is very certain that these forces would be much more active than we find them to be at the present day.

Harvard College Observatory, 1901 *Nov.* 7.

THE GREENWICH REFLEX ZENITH-TUBE,

By S. C. CHANDLER.

Nothing in modern astronomy is so melancholy a record of futile result from a promising project, pursued with admirable perseverance under disheartening circumstances, as the history of the Greenwich Reflex Zenith-Tube. All investigations of the observations made with it have been absolutely nugatory, and the memoirs communicating them have candidly confessed failure in arriving at any acceptable astronomical conclusions or any determinable reason therefor. The persistently anomalous results, apparently pointing to undiscoverable instrumental sources of error, finally discouraged further employment of the instrument, and the apparently hopeless undertaking was abandoned in 1882 after thirty years' continuous trial.

But, although the immediate objects for which this series of observations was instituted have been frustrated, it is proposed in this article to show that it has accomplished, unwittingly, an even higher service to astronomy than that for which it was intended, and that, in place of a fallacious aberration-constant and an impossible stellar-parallax, it in reality contains a body of evidence of the highest value in relation to the constants of the latitude-variation during the period which the observations cover. Immediately upon the discovery of the approximate laws of the polar motion about ten years ago the writer was forcibly impressed with the importance of this series as a contribution to their further development, and through a provisional investigation it was drafted into that service. This use of it was criticized at the time by reputable authority, but I never thought it worth while to notice this or other like strictures, since they were of the kind that time could be relied upon to answer. They are only mentioned now as curious illustrations of the tenacity of prejudice, once respectable, but no longer so when the cause for it has disappeared. It is now manifest that astronomers up to the last decade had incurred a habit of resorting to the hypothesis of subjective error whenever they met unexplained residuals in series of zenith-distance measurements, and that the inertia of this habit led to a certain want of alertness to perceive that the verdict of discredit that had fallen on many of these series required reconsideration, in view of a *vera causa* suddenly thrust upon their attention. In the case of the series now under discussion, for example, however and by whomever treated the outcome had always been a large negative parallax of γ *Draconis*, and an aberration-constant impossible to accept. It is indeed true that the palpably large accidental error of the individual measures intensified the impression as to the untrustworthiness of the series as a whole, and it is to be regretted that a higher degree of refinement in this regard was not striven for, since there is no discernible reason why it could not have been reached. The theory of the instrument is perfect, its construction is of the last simplicity, and the quantity measured is obtained with a degree of directness and thorough elimination of instrumental error as high as it is easy to conceive.

It is needless here to devote any space to description of the instrument or the method of observation. It suffices to refer to Airy's account in the first appendix to the Greenwich volume for 1854, to Main's paper in the *R.A.S. Memoirs*, Vol. XXIX, and to Poor's discussion of the theory and errors of the instrument in *Astr. Journ.*, Vol. IX, p. 153 ff. The results of the observations from 1852–59 will be found in Main's paper above referred to; from 1857–75 in Downing's paper in the *Monthly Notices, R.A.S.*, Vol. XLII.

Date	No.	n
1852		$''$
Aug. 27	12	+0.30
Sept. 20	4	— .22
Oct. 14	2	.82
1853		
Apr. 19	2	— .21
May 27	4	.18
June 17	6	.47
July 14	4	— .97
Aug. 9	4	.93
22	2	+ .13
Sept. 15	2	— .54
1854		
Jan. 25	1	— 0.77
Mar. 15	1	(—3.23)
Apr. 22	2	—1.09
May 15	3	—0.36
30	5	— .53
June 15	1	+ .65
29	2	— .98
July 24	9	— .32
Aug. 16	5	— .37
Sept. 5	11	—0.92
27	2	—1.14
1855		
Mar. 27	1	—0.07
Apr. 21	1	— .16
May 9	1	+ .52
June 5	4	— .90
25	6	— .61
July 10	5	+ .23
21	3	— .81
Aug. 15	5	— .52
31	8	+ .04
Sept. 21	3	— .69
1856		
May 23	2	+ .31
June 6	5	+1.01
16	3	+0.13
July 1	5	+ .27
22	8	+ .03
31	6	— .06
Aug. 9	8	— .27
Sept. 2	7	— .13
26	4	— .82
Oct. 21	5	+ .11
Nov. 5	2	— .20
21	1	— .42
Dec. 17	4	— .31
1857		
Jan. 1	1	+ .03
Feb. 13	5	— .36
Mar. 15	4	— .08
Apr. 17	6	+ .15
May 7	7	+ .13
31	6	+ .11
June 14	7	+ .01
26	6	+ .66
July 12	9	+ .17
27	8	+ .12
Aug. 21	8	.02
Sept. 6	5	+ .31
23	8	— .32
Oct. 8	3	— .11
28	3	— .71
Nov. 11	1	—0.36

Date	No.	n
1857		$''$
Nov. 29	3	—0.17
Dec. 10	2	—0.30
31	4	—1.11
1858		
Jan. 10	3	—0.64
28	4	— .82
Feb. 22	6	— .27
Mar. 9	3	— .46
23	1	— .79
Apr. 18	4	+ .41
May 8	7	— .12
22	5	+ .36
June 13	11	+ .74
29	6	+ .28
July 16	7	+ .25
Aug. 1	9	+ .45
13	5	+ .55
31	5	+ .26
Sept. 17	8	+ .17
Oct. 7	3	— .11
Nov. 1	3	— .03
18	1	— .04
Dec. 2	2	— .79
21	2	+ .20
1859		
Jan. 31	3	— .56
Feb. 23	5	— .01
Mar. 11	5	— .81
Apr. 5	4	— .96
22	1	— .19
May 10	5	— .82
29	3	—0.89
June 25	6	—1.08
July 11	12	—0.14
Aug. 2	1	— .08
21	7	+ .63
Sept. 2	4	— .07
21	3	— .17
Oct. 7	3	+ .12
27	3	+ .51
Nov. 16	1	— .07
Dec. 1	5	— .05
26	1	— .32
1860		
Jan. 7	1½	— .25
23	2	— .75
Feb. 25	6	— .28
Mar. 5	2	— .88
17	4	— .84
Apr. 1	1	— .09
May 19	6	— .92
June 5	3	— .20
16	4	+ .29
July 10	3	+ .13
20	2	+ .51
Aug. 7	5	+ .18
Sept. 10	5	+0.34
29	3	+1.28
Oct. 16	4	+0.32
Nov. 3	4	+ .25
21	2	+ .14
Dec. 24	3	+0.04

Date	No.	n
1861		$''$
Jan. 9	2	+0.01
Feb. 20	3	+ .03
Mar. 21	2	+ .06
Apr. 19	5	— .88
June 14	7	— .17
25	4	+ .13
July 11	5	+ .96
29	4	+ .17
Aug. 12	4	+ .46
25	10	— .09
Sept. 6	3	+ .30
Oct. 4	2	— .89
27	6	+ .17
Dec. 7	3	+ .45
1862		
Jan. 9	1	+ .77
Mar. 18	1	—0.69
Apr. 20	2	+1.71
May 20	1	—0.41
June 3	1	+ .29
28	2	+ .13
July 12	1	— .20
23	5	— .58
Aug. 6	5	+ .37
26	5	—0.73
Sept. 15	4	—1.35
Oct. 8	5	+0.02
1863		
Jan. 6	1	— .86
Feb. 1	2	—0.21
11	3	—1.01
Mar. 2	1	—0.44
13	1	+ .31
May 13	1	— .19
26	2	+ .02
June 3	4	— .59
23	7	— .01
July 11	11	+ .01
28	6	+0.66
Aug. 12	5	+1.03
20	1	+0.04
Sept. 23	7	— .46
Oct. 16	3	— .08
Nov. 29	4	— .30
1864		
Jan. 29	3	— .13
Feb. 9	0	(—3.48)
Mar. 19	3	—1.05
May 18	4	—0.88
June 4	4	— .29
13	3	— .01
July 2	4	— .83
18	5	— .03
Aug. 4	1	— .27
13	6	+ .11
Sept. 4	3	+ .76
25	6	.00
Oct. 4	2	— .11
26	3	— .18
Dec. 2	5	—0.06
1865		
Jan. 12	½	+1.12
Feb. 19	1	—0.85
Mar. 19	1	—0.55

Date	No.	n
1865		$''$
May 12	1	+0.01
27	3	—1.32
June 10	4	—0.84
22	5	— .27
July 9	6	— .26
21	5	— .09
Aug. 5	4	+0.26
25	3	—1.20
Sept. 5	6	+0.02
22	9	— .56
Oct. 5	7	— .19
1866		
Jan. 16	2	—0.64
Feb. 21	2	—1.20
Mar. 6	4	—0.58
May 4	2	—1.68
21	8	—1.35
June 8	1	—1.11
24	5	—0.73
July 10	5	—0.25
19	2	—1.28
Aug. 5	3	—0.43
19	5	—0.26
Sept. 10	1	+0.30
27	½	+1.94
Oct. 12	3	+0.26
28	2	—0.88
Nov. 30	1	—1.22
1867		
Feb. 6	1	—0.34
Mar. 15	1	— .40
May 30	2	+ .17
June 13	5	— .01
26	5	— .85
July 10	6	— .23
29	3	— .60
Aug. 13	7	— .46
30	5	— .25
Sept. 23	6	—0.97
Oct. 17	½	—1.80
Nov. 6	1	—1.84
1868		
Feb. 5	1	+0.55
27	1	—1.71
Mar. 11	4	—0.53
May 18	1	— .40
26	5	+ .86
June 6	6	+0.70
17	5	+1.40
25	7	+0.52
July 9	1	— .31
15	5	+ .39
23	5	— .03
Aug. 1	6	+ .08
Sept. 5	8	— .21
22	3	+ .29
Oct. 9	1	+ .15
29	1	+ .10
1869		
Jan. 23	2	+ .68
Feb. 12	2	+ .01
Mar. 12	1	+0.51
Apr. 1	1	+1.09
May 5	2	+0.12

Date	No.	n
1869		$''$
May 27	3	—0.79
June 11	7	+0.30
26	3	—1.35
July 18	7	+0.36
Aug. 3	4	+ .11
17	7	— .21
29	1	— .99
Oct. 9	2	— .29
Nov. 16	2	+0.60
Dec. 1	0	(—2.67)
1870		
Jan. 20	2	—0.30
Feb. 24	1	— .25
Mar. 13	1	— .84
May 21	8	—0.09
June 7	4	—1.03
19	8	+0.09
July 2	4	— .33
20	13	— .26
Aug. 13	9	— .32
Sept. 1	5	— .14
26	10	— .30
Oct. 10	2	— .28
Nov. 1	2	— .57
12	1	—0.63
1871		
Feb. 16	½	—1.54
May 24	2	—0.91
June 5	1	— .03
July 6	4	— .31
17	3	— .80
31	4	— .35
Aug. 11	8	+ .01
1872		
Jan. 18	2	—0.68
Feb. 12	1	—1.13
Aug. 15	1	—0.78
Dec. 6	1	— .84
27	3	+0.10
1873		
Jan. 27	2	—1.05
May 17	3	—0.61
June 20	3	—0.32
July 1	1	—1.83
23	8	—0.56
Aug. 5	8	—1.34
18	8	—0.09
Sept. 5	3	+ .72
24	5	— .18
Oct. 17	1	+ .23
Nov. 15	1	—0.16
25	1	—2.23
1874		
May 22	5	+0.60
June 8	9	+1.02
23	2	+1.47
July 1	6	+0.56
17	8	+ .70
29	3	+ .20
Aug. 9	1	+ .79
19	4	+ .13
31	4	+ .10
Sept. 13	3	+ .80
25	2	—0.33
Oct. 10	2	—1.03

Date	No.	n
1874		$''$
Oct. 24	1½	+0.58
Nov. 20	1	— .50
1875		
May 22	3	— .30
June 1	2	+ .27
14	5	+ .79
24	3	—0.97
July 7	5	—1.62
30	5	—1.39
Aug. 20	5	—1.80
Sept. 3	6	—0.81
17	5	—1.23
1876		
May 27	4	+0.42
June 23	4	— .60
July 17	6	—0.48
Aug. 6	6	—1.04
14	5	—0.14
Sept. 17	4	—1.12
1877		
Feb. 2	1	—1.35
May 28	3	—0.86
June 21	4	—1.24
July 9	5	—0.97
28	5	—0.02
Aug. 16	5	—0.62
29	4	—1.51
Sept. 30	3	—2.18
1878		
June 15	4	+1.34
24	4	—0.04
July 1	5	+1.00
19	5	+0.45
31	4	+ .61
Sept. 8	5	— .36
26	5	— .42
1879		
May 17	3	— .54
June 2	3	— .02
24	4	— .21
July 6	3	— .50
26	4	+ .05
Aug. 28	6	+ .22
Sept. 22	3	— .24
1880		
May 23	4	—0.85
June 6	3	—2.70
29	5	—1.42
July 10	5	—1.07
26	5	—1.34
Aug. 10	5	+0.50
28	6	—0.27
1881		
May 13	3	—0.59
June 1	7	—1.29
22	4	—0.47
July 9	1	—0.56
27	1	—1.36
Aug. 17	5	—1.14
Sept. 23	2	—1.80
1882		
May 18	5	—0.60
June 5	6	+ .30
25	5	— .46
July 15	6	+0.27
Aug. 3	6	—1.21
14	5	—1.13

Those from 1875 to 1882, when the series was abandoned, I have collected from the Greenwich annual volumes.

In the preceding table the results for the whole series from 1852 to 1882 are given, homogeneously reduced. Each line contains the result of a group of observations, the mean date and number of observations being given in the first and second columns. The third column, n, contains the mean value of the residual of the zenith-distance of γ *Draconis*, expressed in the sense $\Delta\varphi\,(O-C) = \varphi - \varphi_0$. These have been referred to the value of the aberration-constant 20″.505, which seemed to me to be the best at the time these computations were made, although evidence has been since accumulating that would seem to make a somewhat larger value more probable. Besides this correction for aberration it was necessary to refer the portion before 1857 to the same zero as for the subsequent series, on account of the change of location of the instrument in 1856 to a point 53 feet south of its previous position, and to correct for the difference of annual proper motion in declination assumed by MAIN (−0″.01), and that used by DOWNING and myself (−0″.028). Thus, MAIN's residuals, after change of sign, required the correction, +0″.16 +0″.012 $(t-1852)$; and both MAIN's and DOWNING's the reduction from the aberration-constant assumed by them (20″.400).

An inspection of the table of observations will show at once that the unequal distribution as regards time of year makes it impossible to get an adequately independent determination of the constants of the latitude-variation except for the interval from 1857 to 1870. Before and after this the measurements are practically confined to five months of the year, from May to September, which makes futile the attempt at an independent elimination of the fourteen-months' and annual terms. The following condensed count of the observations makes this clear.

	Jan.–Apr.	May–Oct.	Nov. & Dec.
1852–56	10	200	7
1857–63	98	345	41
1864–70	34	340	12
1871–77	6	213	7
1878–82	0	153	0

Pertinent results for both terms can therefore be expected only for the two seven-year cycles 1857–63 and 1864–70.

The process of finding the constants was that of successive approximation, by alternate elimination of each term. Thus, the circumference of the fourteen-months' term was divided into sixteen parts; the data of the table, for each of the intervals 1857–63 and 1864–70, were classified accordingly; and the mean values of n thus found furnished a first approximation to the 428-day term. The original values of n were then corrected for this and the residuals were classified by calendar months, giving a first approximation to the annual term. Correcting the original n's for this, a second approximation to the 428-day term was found. A repetition of the process for the annual term gave values differing so little from the previous approximation that it did not seem necessary to carry the process further.

In this manner we have the values of $\varphi - \varphi_0$.

$$1857\text{–}63:\ -0''.216\cos(t-2400541)0.84 - 0''.166\cos(\odot-323^\circ)$$

$$1864\text{–}70:\ -0.222\cos(t-2403238)0.84 - 0.134\cos(\odot-313)$$

which have been used in the investigations in *A.J.* 489, p. 71 (note), and in *A.J.* 494, p. 109, as there stated. The following tables give the comparison of the computed and observed values, the latter having been corrected by the constants +0″.09 and +0″.34, which are the reductions to arbitrary zero given by the computations for the series 1857–63 and 1864–70, respectively.

428d-TERM.

1857–63 t	O	C	1864–70 t	O	C
	″	″		″	″
2400413	+0.06	+0.07	2403013	−0.08	+0.22
0440	+ .03	− .03	3040	+ .26	+ .21
0467	− .11	− .12	3067	+ .17	+ .18
0494	− .29	− .19	3094	+ .07	+ .11
0521	− .13	− .21	3121	+ .32	+ .03
0548	− .31	− .24	3148	− .17	− .06
0575	− .03	− .22	3175	− .34	− .13
0602	− .05	− .15	3202	− .17	− .19
0629	− .34	− .07	3229	− .37	− .22
0656	− .14	+ .02	3256	− .38	− .21
0683	+ .21	+ .12	3283	+ .04	− .18
0710	+ .45	+ .19	3310	+ .17	− .11
0737	+ .16	+ .21	3337	+ .07	− .03
0764	+ .27	+ .21	3364	− .11	+ .06
0791	+ .14	+ .22	3391	+ .03	+ .13
2400818	+0.16	+0.15	2403418	+0.20	+0.19

ANNUAL TERM.

	1857–63 O	C		1864–70 O	C
	″	″		″	″
Jan.	−0.25	−0.15	Jan.	+0.20	−0.13
Feb.	− .15	− .17	Feb.	− .27	− .13
Mar.	− .29	− .14	Mar.	− .16	− .10
Apr.	+ .29	− .08	Apr.	−	− .04
May	− .25	+ .01	May	− .15	+ .03
June	+ .12	+ .09	June	+ .24	+ .09
July	+ .18	+ .15	July	+ .13	+ .13
Aug.	+ .31	+ .17	Aug.	+ .06	+ .13
Sept.	+ .01	+ .14	Sept.	+ .09	+ .10
Oct.	+ .10	+ .08	Oct.	+ .11	+ .04
Nov.	− .01	− .01	Nov.	−0.16	− .03
Dec.	−0.06	−0.09	Dec.	−	−0.09

I do not think the above results could be essentially varied by different method of reduction, and see no reason to doubt that they are real, and afford a tolerably correct

idea of the elements of the latitude-variation during the period embraced by the series. The outcome of the discussion ought to dissipate the prejudice that has so long prevailed against these observations, since the anomalies are now traced to the intervention of a phenomenon not suspected at the time they were made. In the installation and persistent use of this instrument AIRY was building better than he knew, and in an unforeseen way conferred on astronomy a higher benefit than his actual aim would have been had it been attained.

THE OBSERVATIONS OF *ALGOL* BY ARGELANDER, SCHMIDT AND SCHÖNFELD,

BY S. C. CHANDLER.

The long series of minima of *Algol* observed by ARGELANDER, SCHMIDT and SCHÖNFELD supply means that are too important to be overlooked, for examining the question of the existence of minor irregularities in its period. SCHMIDT's series overlaps ARGELANDER's at one end and SCHÖNFELD's at the other, and all three are long enough to give series of observed values of the period that are independent of each other and entirely free from the effects of personal differences between the three observers in the determination of the time of minimum.

On p. 171 of Vol. VII, *A.J.*, I have given a table of yearly mean epochs deduced separately for the different observers. By combining these into normals, each embracing one or two years' observations, and taking differences for epochs separated by several hundred periods, independently for ARGELANDER, SCHMIDT and SCHÖNFELD, we have the appended table. The column O contains the observed periods, and column C the periods computed from the elements in *A.J.* 509.

The differences $O-C$ display very clearly a systematic deviation from the elements that must be entirely independent of systematic personal differences in deducing the times of minima, as has been stated above. The accordance of the three sets of observations in this regard enables us to conclude with reasonable certainty that there was an actual irregularity, consisting of a rather sudden diminution of the period, with respect to the elements, between 1851 and 1857 of about 1".5, and a subsequent increase, gradual from 1863 to 1872, of the same amount.

Thus we have additional evidence, of a cumulative character, of the reality of such minor irregularities, similar to that adduced in *A.J.* 509 from a comparison of MÜLLER's results with those of other observers for the interval 1875 to 1878.

Date	Epoch	Period $2^d\,20^h\,48^m+$ O	C	$O-C$	Wt.	Observer
		s	s	s		
1811.4	5661	53.9	53.9	+0.0	3	Argelander
1817.4	6042	53.2	53.0	+0.2	2½	"
1850.1	6380	53.3	52.8	+0.5	1½	"
1851.4	6548	53.2	52.7	+0.5	3	"
1851.7	6583	53.3	52.6	+0.5	1½	Schmidt
1854.0	6880	53.2	52.8	+0.4	1	Argelander
1854.1	6891	53.0	52.8	+0.2	3½	Schmidt
1856.0	7135	54.1	53.1	+1.0	2	Argelander
1856.6	7212	52.6	53.1	−0.5	2	Schönfeld
1856.8	7233	53.3	53.2	+0.1	3½	Schmidt
1857.4	7299	52.2	53.3	−1.1	1	Argelander
1859.2	7534	52.6	53.5	−0.9	2	Schönfeld
1859.4	7556	52.5	53.6	−1.1	1	Argelander
1859.5	7586	53.0	53.6	−0.6	1½	Schmidt
1862.1	7913	52.8	53.9	−1.1	1	Argelander
1862.3	7933	53.6	53.9	−0.3	4½	Schmidt
1862.9	8023	54.2	53.9	+0.3	3½	Schönfeld
1864.5	8218	54.1	54.0	+0.1	2½	"
1864.6	8229	53.8	54.0	−0.2	6	Schmidt
1867.4	8580	53.9	53.8	+0.1	4	"
1867.8	8635	54.0	53.6	+0.4	4	Schönfeld
1869.4	8836	53.9	54.0	−0.1	2½	"
1869.9	8902	53.6	53.3	+0.3	4	Schmidt
1871.4	9081	53.8	53.0	+0.8	2½	Schönfeld
1872.6	9213	52.9	52.6	+0.3	5	Schmidt
1872.8	9278	53.1	52.6	+0.8	2½	Schönfeld

CHANGE IN NEBULA SURROUNDING *NOVA PERSEI*,

The following telegraphic dispatches have been received *via* Harvard College Observatory:

Nov. 10. From Crossley-photograph PERRINE finds that four principal condensations faint nebula surrounding *Nova Persei* moved southeast one minute of arc in six weeks.

Nov. 11. RITCHEY states photograph Yerkes November ninth confirms large motion nebula near *Nova*.

Nov. 12. From photographs *Nova Persei* November RITCHIE finds Nebula probably expanding in all directions. This certainly true of southern half.

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ON THE ROTATORY MOTION OF A BODY OF VARIABLE FORM,

BY KURT LAVES.

The differential equations of motion of a rotating solid of variable form were first derived by LAGRANGE. They were published after his death in an appendix to the second volume of his *Mécanique Analytique* (Courier publishers). His investigations in this subject were however little known to the mathematical public in the first half of this century, and in 1858 LIOUVILLE published an article in which he derived the same equations LAGRANGE had obtained, although in an entirely different manner (*Journal de Mathématiques, 2d série*, t. III).

LAGRANGE has employed different methods to derive the differential equations of motion. Among them there is one which foreshadows in a peculiar manner the essentials of HAMILTON's principle. It is the purpose of this paper to show that LAGRANGE's equations may be advantageously and elegantly derived from this fundamental principle. KIRCHHOFF, in his *Vorlesungen über Mechanik*, has first shown how HAMILTON's principle may be used to derive the differential equations of motion for a rotating body of invariable form.

Let X_ι, Y_ι, Z_ι designate the components of the type of forces acting upon the body. Write for abbreviation $U' = \Sigma(X_\iota\delta x_\iota + Y_\iota\delta y_\iota + Z_\iota\delta z_\iota)$ and call T the kinetic energy of the body, then we have HAMILTON's principle by writing

$$\int_{t_0}^{t_1}(\delta T + U')\,dt = 0$$

Introducing two systems of rectangular axes with the same origin in an arbitrary point O of the body, we call Ox, Oy, Oz axes fixed in space, Ox_1, Oy_1, Oz_1 axes fixed in the body. The coordinates x_1, y_1, z_1 of a point of the body are constants when the body is of invariable form.

We have

$$\begin{aligned} x &= ax_1 + by_1 + cz_1 \\ y &= a'x_1 + b'y_1 + c'z_1 \\ z &= a''x_1 + b''y_1 + c''z_1 \end{aligned}$$

Let OI designate the direction of the instantaneous axis of rotation, and p, q, r the components of rotation with respect to the moving axes. They are defined as follows:

$$\begin{aligned} -p &= b\frac{dc}{dt} + b'\frac{dc'}{dt} + b''\frac{dc''}{dt} \\ -q &= c\frac{da}{dt} + c'\frac{da'}{dt} + c''\frac{da''}{dt} \\ -r &= a\frac{db}{dt} + a'\frac{db'}{dt} + a''\frac{db''}{dt} \end{aligned}$$

$2T = \sum m\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right\}$ when expressed in x_1, y_1, z_1 and p, q, r takes the form:

$$\begin{aligned} 2T = &\sum m\left\{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dy_1}{dt}\right)^2 + \left(\frac{dz_1}{dt}\right)^2\right\} \\ &+[Ap^2 + Bq^2 + Cr^2 - 2Dqr - 2Erp - 2Fpq] \\ &+ 2q\sum m\left(z_1\frac{dx_1}{dt} - x_1\frac{dz_1}{dt}\right) + 2r\sum m\left(x_1\frac{dy_1}{dt} - y_1\frac{dx_1}{dt}\right) \\ &+ 2p\sum m\left(y_1\frac{dz_1}{dt} - z_1\frac{dy_1}{dt}\right) \end{aligned}$$

where $A, B, \ldots F$ have their usual meaning. It must be remembered that they are not any more constants but functions of the time t.

In applying HAMILTON's principle we have to select independent displacements of such a nature that the instantaneous axis will change its position during the element of time dt from OI to OI', the position it will take at the time $t + dt$. Let I and I' be the points of intersection of the vectors representing the amount of rotation with the unit sphere having O for its center. Using KIRCHHOFF's notation we call p', q', r' the three independent displacements to be used and define them by the equations:

$$\begin{aligned} -p' &= b\,dc + b'dc' + b''dc'' \\ -q' &= c\,da + c'da' + c''da'' \\ -r' &= a\,db + a'db' + a''db'' \end{aligned}$$

δT and U' are to be expressed in terms of p', q', r'. U' is representing work. The work of a system of forces acting on a system of material points during an element of

time dt is equal to the sum of elementary work for the components of the infinitesimal displacement. We may, therefore, write $U' = Lp' + Mq' + Nr'$, where L, M, N are the moments of rotation with respect to the x_1, y_1, z_1 axes.

From the expression of T we obtain for the rotatory displacement

$$\delta T = \frac{\partial T}{\partial p}\cdot\delta p + \frac{\partial T}{\partial q}\cdot\delta q + \frac{\partial T}{\partial r}\cdot\delta r$$

LAGRANGE has shown in his second paper that the following relations hold between the p, q, r and p', q', r' quantities:

$$\delta p = \frac{dp'}{dt} + qr' - q'r$$
$$\delta q = \frac{dq'}{dt} + rp' - r'p$$
$$\delta r = \frac{dr'}{dt} + pq' - p'q$$

Substituting the values for U' and δT (when expressed by p', q', r') into HAMILTON's equation we obtain:

$$\int_{t}^{t}\Big[\frac{\partial T}{\partial p}\frac{dp'}{dt} + \frac{\partial T}{\partial q}\cdot\frac{dq'}{dt} + \frac{\partial T}{\partial r}\frac{dr'}{dt} + \left(N + \frac{\partial T}{\partial p}\cdot q - \frac{\partial T}{\partial q}\cdot p\right)r'$$
$$+\left(L + \frac{\partial T}{\partial q}\cdot r - \frac{\partial T}{\partial r}\cdot q\right)p'$$
$$+\left(M + \frac{\partial T}{\partial r}\cdot p - \frac{\partial T}{\partial p}\cdot r\right)q'\Big]dt = 0$$

The University of Chicago, 1900 *March* 10.

The quantities p', q', r' are of the nature of KIRCHHOFF's quantities ϵ, and the foregoing equation will fall under the general form

$$\int_{t_0}^{t_1}\sum\left(R.\epsilon + S.\frac{d\epsilon}{dt}\right)dt = 0 \text{ which leads to}$$

$$\int_{t_0}^{t_1}dt\sum\left(R - \frac{dS}{dt}\right)\epsilon = 0$$

The ϵ quantities being entirely independent of each other, this last equation leads to the system of equations:

$$R - \frac{dS}{dt} = 0$$

Identifying the ϵ quantities with p', q', r', we obtain at once:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial p}\right) + q\frac{\partial T}{\partial r} - r\frac{\partial T}{\partial q} = L$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial q}\right) + r\frac{\partial T}{\partial p} - p\frac{\partial T}{\partial r} = M$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial r}\right) + p\frac{\partial T}{\partial q} - q\frac{\partial T}{\partial p} = N$$

These are LAGRANGE's differential equations of rotatory motion for a body of variable form.

NOTE BY THE EDITOR. — For the long delay in the appearance of the foregoing, as well as of the following article, the Editor is entirely responsible.

NOTE ON THE ARTICLE CONCERNING INTERIOR EFFECTIVE FORCES IN No. 445 OF THE *ASTRONOMICAL JOURNAL*.

By KURT LAVES.

It is the purpose of this note to correct an oversight in the definition of the V-function used in *A.J.*, No. 445. On page 98 it is stated, that V should be an arbitrary function of the mutual distances and relative velocities of the n bodies which does not contain the time explicitly. This definition is not general enough, inasmuch as V may contain also the first differential quotients of the mutual distances. My attention was called to this fact by a paper of Professor A. MAYER of Leipzig, who has solved the problem proposed in No. 445 in a broader and more elegant manner.*

Keeping the notations of the former paper, the definition of V depends mainly upon the integration of a system of three partial differential equations:

$$\left.\begin{aligned}
&\sum_{1}^{n}\left\{y_i\frac{\partial V}{\partial z_i} - z_i\frac{\partial V}{\partial y_i} + y_i'\frac{\partial V}{\partial z_i'} - z_i'\frac{\partial V}{\partial y_i'}\right\} = 0\\
&\sum\left\{z_i\frac{\partial V}{\partial x_i} - x_i\frac{\partial V}{\partial z_i} + z_i'\frac{\partial V}{\partial x_i'} - x_i'\frac{\partial V}{\partial z_i'}\right\} = 0\\
&\sum\left\{x_i\frac{\partial V}{\partial y_i} - y_i\frac{\partial V}{\partial x_i} + x_i'\frac{\partial V}{\partial y_i'} - y_i'\frac{\partial V}{\partial x_i'}\right\} = 0
\end{aligned}\right\} \quad (a)$$

The six partial differential equations (3) and (4), page 100 of my former paper, are not rewritten, since they bear not directly upon our present problem. They restrict V in such a manner that only differences of the absolute coordinates and velocities may enter in it. Integrals of the $4n - 1$ total differential equations of the first order, which can be regarded to replace the first equation (a), are

*See *Berichte der math.-physikalischen Classe der Koenigl. Saechs. Gesellschaft der Wissenschaften, Sitzung vom 9 Jan. 1899;* "*Zur Theorie der Bewegung von Punktsystemen unter dem Einfluss von Potentialkraften.*"

$$z_i^2+y_i^2;\ z_i z_k+y_i y_k;\ y_i'^2+z_i'^2;\ z_i' z_k'+y_i' y_k'$$

as given on page 101. Besides there are integrals of the form

$$z_i z_i'+y_i y_i' \quad \text{and} \quad z_i z_j'+z_i' z_k+y_i y_k'+y_i' y_i$$

which were overlooked in the former investigation. It is evident that when we construct an arbitrary function V, which is a solution of system (a) we may select the following functions to enter as arguments into V:

$$\begin{aligned}
r_h^2 &\equiv x_h^2+y_h^2+z_h^2\\
r_{hk}^2 &\equiv (x_h-x_k)^2+(y_h-y_k)^2+(z_h-z_k)^2\\
r_h\frac{dr_h}{dt} &\equiv x_h\frac{dx_h}{dt}+y_h\frac{dy_h}{dt}+z_h\frac{dz_h}{dt}\\
r_{hk}\frac{dr_{hk}}{dt} &\equiv (x_h-x_k)(x_h'-x_k')+(y_h-y_k)(y_h'-y_k')+(z_h-z_k)(z_h'-z_k')\\
v_{hk}^2 &\equiv (x_h'-x_k')^2+(y_h'-y_k')^2+(z_h'-z_k')^2\\
v_h^2 &\equiv x_h'^2+y_h'^2+z_h'^2
\end{aligned}$$

Thus the necessary number of independent solutions can be secured by varying h and k between proper limits. But since V has besides to fulfill six partial differential equations [(3) and (4) of the former paper] it follows that the solutions r_h^2, $r_h\frac{dr_h}{dt}$ and v_h^2 are not admissible since they are functions of the absolute coordinates and velocities alone. Thus it appears that V must be an arbitrary function of the relative distances, their first differential quotients and the relative velocities of the n material points of the system. The time must not enter explicitly in it in order that the tenth integral may hold. It will be observed that in this corrected form a close relationship is established between the V-function of this paper and the V-function defined by Professor A. MAYER in the XIII[th] volume of the *Mathematische Annalen*. When the relative velocities do not enter in our V-function we see at once that this special case will lead us to Professor MAYER's V-function. WEBER's electro-dynamical potential falls now under both forms of the V-function, whereas RIEMANN's potential is outside the domain of Professor MAYER's V-function. The fact that the problem of two bodies could yet be solved completely, engaged first my interest in this subject: the former lack of connection is now properly removed, and a wider grasp of the conception of the V-function is secured. In conclusion a few remarks are in place concerning the method which Professor MAYER has employed in his recent contribution to the Leipzig Academy. Instead of a system of n material points one of $n+1$ points is considered: the absolute coordinates x_i, y_i, z_i, $i=0,\ldots n$, are replaced by the absolute coordinates of the center of gravity ξ, η, ζ and the $3n$ relative coordinates of the last n points with reference to the first point: $\xi_h=x_h-x_0$, $\eta_h=y_h-y_0$, $\xi_h=z_h-z_0$, $h=1\ldots n$. It is found that the effective Potential — fulfilling the ten integrals — has the form

$$V=\frac{a_0}{2}(\xi'^2+\eta'^2+\zeta'^2)+\left(\xi'\frac{d\lambda}{dt}+\eta'\frac{d\mu}{dt}+\zeta'\frac{d\nu}{dt}\right)+V'=V^0+V'$$

a_0 is a constant; λ, μ, ν are defined in the following manner: Call $\theta, \theta_1, \theta_2$ arbitrary functions of

$$\begin{aligned}
r_{0h} &\equiv \xi_h^2+\eta_h^2+\zeta_h^2\\
r_{hk} &\equiv (\xi_h-\xi_k)^2+(\eta_h-\eta_k)^2+(\zeta_h-\zeta_k)^2
\end{aligned}$$

these we put

$$\begin{aligned}
\lambda &= (\eta_1\zeta_2-\eta_2\zeta_1)\theta+\xi_1\theta_1+\xi_2\theta_2\\
\mu &= (\zeta_1\xi_2-\zeta_2\xi_1)\theta+\eta_1\theta_1+\eta_2\theta_2\\
\nu &= (\xi_1\eta_2-\xi_2\eta_1)\theta+\zeta_1\theta_1+\zeta_2\theta_2
\end{aligned}$$

V' is an arbitrary function of r_{0h}, r_{hk}, $\frac{dr_{hk}}{dt}$, it is therefore of the same character as the V-function formerly discussed. The function V^0 is of a character essentially different from V', and does not fulfill in the common way of definition the properties of the Potential.

When we keep in mind the dynamical side of our problem it will be conceded that we shall have to bar the assumption of a potential function that depends upon the components of velocity of the center of gravity. Indeed, it is evident that for the simpler problem, where the velocities of the coordinates of the $n+1$ points do not enter, we may as well construct a function $V=V^0+V'(r_{hk})$ that will leave the ten integrals in force. The function V^0 would be here the same as above. It will be observed, that the introduction of the system of coordinates, which Professor MAYER has chosen, makes the definition of the V'-function simple and satisfactory. It depends upon a system of three partial differential equations, a complete solution of which can readily be obtained. This is not so easily achieved when the absolute coordinates are kept, since in this case the function is defined by a system of nine partial differential equations.

ON THE EFFECT OF SINGLE AND DOUBLE LINES UPON PERSONAL ERROR IN TRANSIT OBSERVATIONS,

BY W. V. BROWN.

In the determination of the longitude of McKim Observatory, there arose the question of the effect on the personal equation in changing from a double to a single line reticle: and the following investigation was carried out. Increased interest is given to the results by the recent publication, in *Harvard Annals*, Vol. XLI, No. VII, of the effects on personal equation when a system of spider lines was substituted for a glass plate ruled with double lines.

The reticle, on which was taken the observations given below, was a ruled glass plate with the lines disposed in three tallies of five, seven, and five respectively, and each line double, that is, composed of two close parallel ones five and a half arc-seconds apart, the point midway between which was ordinarily chosen for noting transits. The interval between lines, measuring from center to center of doubles, was 2ˢ.914 ± 0ˢ.002, and the interval between the single lines that formed each pair was the exact one-eighth of this.

On this reticle transits were now taken, so that with each star half of the transit was on double lines in the ordinary way, and the other half was observed on the first line of each pair. On the first night eight lines were taken single, the middle line omitted, and eight taken double. On the second night the conditions were reversed, being eight double, middle omitted, eight single. On the third night the outside tallies were taken single, and the middle tally double. The means of the single and double lines were separately found, when one mean was reduced to the other by the use of the known line-interval of the reticle, and so directly compared. The accuracy with which this interval was known was within the requirements of the problem, and any effect from error here, or from inequality of spacing on the reticle, etc., would show itself doubled when the conditions were reversed on the second night.

In observing over a single line of a pair, there was danger of a personal equation being produced by the close presence of the other line. It was decided that this would be less with the first than with the second line of a pair, since in using the second line, one would necessarily have the warning of the transit over the first. It is evident that any effect that might come from this source would also be shown in the reversal on the second night. No marked difference appears, however, between the three dates.

The diagonal eye-piece was kept in its normal position, so that the apparent path of the star was parallel to the horizon, and the observer sat squarely in front of the instrument facing either due north or south. The stars range from $-30°$ to $+80°$, the distribution fairly uniform, but with a slight excess south of the zenith. The magnitudes ranged between 2.3 and 5.3. The telescope has a three-inch glass, and power of 124.

In the reduction it was assumed that most of the factors which enter into the ordinary problem of personal equation between different observers could here be ignored, since all of the observations were to be taken by the same person. It was assumed that the result would depend entirely upon the difference in estimating when a star was bisected by a line, and when it was midway between two close parallel lines.

This estimation would be affected by the velocity of the star's motion, and each difference found was reduced to the corresponding equatorial interval by multiplying by $\cos\delta$.

In the following tables, the differences of personal error are all given in the sense of the mean of the transits over the double lines *minus* those over the single lines. In each group, the sums of the positive and negative values are given, as well as the mean values. The results for stars north and south of the zenith are separately given.

Date 1888	No. Stars	e			$e\cos\delta$		
		South of Zenith					
		s	s	s	s	s	s
Dec. 20	10	+0.32	−0.11	+0.021	+0.30	−0.11	+0.019
21	16	+0.48	−0.13	+0.022	+0.47	−0.13	+0.021
23	14	+0.66	−0.05	+0.014	+0.63	−0.04	+0.042
	40	Mean		+0.029	Mean		+0.028
		North of Zenith					
		s	s	s	s	s	s
Dec. 20	9	+1.02	−0.05	+0.108	+0.56	−0.02	+0.060
21	11	+0.98	−0.09	+0.081	+0.42	−0.05	+0.034
23	10	+0.88	−0.01	+0.087	+0.46	−0.01	+0.045
	30	Mean		+0.091	Mean		+0.045
	70	Mean of all		+0.056	Mean of all		+0.035

It will be seen that there appears to be a marked difference between the results for stars moving from left to right, and those moving from right to left. Thus, we have

North *minus* South

1888	e	$e\cos\delta$	1888	e	$e\cos\delta$
	s	s		s	s
Dec. 20	+0.087	+0.041	Dec. 23	+0.013	+0.003
21	+0.059	+0.013	Mean	+0.062	+0.017

From the above results it may be inferred that the difference in the personal errors in observing transits over single and double lines is composite; one part being dependent on the direction of the motion of the star, and the other being due to erroneous bisection of the interval between the double lines. If we denote the equatorial value of these elements by x and y, we have

$$x + y = +0^s.015 \qquad x - y = +0^s.028$$

whence

$$x = +0.036 \qquad y = +0.009$$

There was then mounted another ruled plate having a central tally of seven single lines, and two side tallies of four double lines each. The spaces were very closely the same as on the former plate.

In the series following, each star was observed over all the lines; the single and double groups were separately reduced to the center, and directly compared.

The algebraic difference is taken in the sense of double *minus* single lines.

Date 1889	No. Stars	ϵ			$\epsilon\cos\delta$		
		South of Zenith					
June 24	9	+0.06	−0.15	−0.010	+0.06	−0.15	−0.010
25	11	+0.11	−0.26	−0.014	+0.11	−0.23	−0.011
July 4	13	+0.17	−0.21	−0.003	+0.17	−0.20	−0.002
5	12	+0.14	−0.47	−0.027	+0.13	−0.14	−0.026
	45	Mean		−0.014	Mean		−0.014
		North of Zenith					
June 24	8	+0.39	0.00	+0.019	+0.23	0.00	+0.029
25	8	+0.22	−0.30	−0.010	+0.12	−0.09	+0.004
July 4	10	+0.70	−0.17	+0.053	+0.33	−0.09	+0.024
5	12	+0.26	−0.52	−0.022	+0.16	−0.20	−0.003
	38	Mean		+0.015	Mean		+0.012
	83	Mean of all		0.000	Mean of all		−0.002

North *minus* South

1889	ϵ	$\epsilon\cos\delta$
June 24	+0.059	+0.039
25	+0.004	+0.015
July 4	+0.056	+0.026
July 5	+0.005	+0.023
Mean	+0.029	+0.026

Letting x and y represent the same elements as before, we have

$$x + y = +0^s.012 \qquad x - y = -0^s.014$$

whence

$$x = -0.001 \qquad y = +0.013$$

It is apparent that the second element — the erroneous bisection of the interval between the double lines — is persistent, with practically the same value; while the effect of the direction of the star's motion is not appreciable.

ON THE ORBIT OF η *CASSIOPEIAE*.

$\alpha = 0^h\ 43^m.0 \ ; \ \delta = +57°\ 17'$.

By GEORGE C. COMSTOCK.

The material available for a determination of the orbit of this star consists of a fairly continuous series of observations by all the principal double-star observers from the epoch 1820 to the present time, during which period the companion has moved through an arc of a little more than 140°. This is supplemented by a few scattering observations during the forty years preceding 1820 which, if they may be trusted, add ten or twelve degrees to the observed motion. But at best the observed motion of the satellite falls considerably short of 180°, and would therefore leave room for some doubt as to the measure of confidence to be given an orbit based upon such data even though the observations themselves were unimpeachable. But the earlier observations are far from being of this character. Scanty in number they are hopelessly discordant among themselves, so that the considerable discrepancies that exist among the half dozen or more orbits that have been computed for η *Cassiopeiae* are chargeable in great part to the varying interpretations accorded these data.

The most recent, and presumably the best, of these orbits is that of SEE, based upon observations to 1895 inclusive (*Evolution of the Stellar Systems*, p. 72), and its inadequacy is sufficiently shown by a comparison of the ephemeris there given and reproduced below, with the following annual means of recent observations of this star, which include all data subsequent to 1895 that I have been able to find.

EPHEMERIS (SEE'S Elements).

Epoch	p	s
1896.5	207.6	4.73
1897.5	210.1	4.68
1898.5	213.7	4.62
1899.5	217.2	4.55
1900.5	221.1	4.46

OBSERVATIONS OF *η Cassiopeiae.*

Epoch	p	s	n	Observers
1896.84	207.9	5.10	6	Hussey, Aitken
1897.64	210.4	5.07	18	Hus., Ait., Dob., Doo.
1898.70	215.1	5.04	6	Aitken, Bryant
1900.73	219.0	5.16	6	Doberck, Comas Sola
1901.84	224.0	5.28	1	Comstock

By extrapolation from the ephemeris I find for the computed distance at the last date given above, $s = 4''.33$, indicating by comparison with the observations that an error of a second of arc has accumulated during the last six years, a period within which the motion in position-angle has been only 20°.

This failure of the orbit is probably due to errors in the early observations by Sir W. HERSCHEL, and may serve to illustrate the danger of treating these notoriously crude measures as adequate data upon which to base an orbit-determination. But without the use of these observations the orbit is at the present time indeterminate since a number of very different ellipses can be made to fit the observations subsequent to 1820, within the limits of their probable errors. For example, I find by plotting upon SEE's apparent orbit, p. 76 of the work cited, the observations above given, that a circle of radius 10″.0 with its center at the point $p = 10°.7$, $s = 5''.2$, satisfies the observations from 1820 to 1895 quite as well as does SEE's ellipse, and also satisfies very well the recent observations without introducing into the representation of those prior to 1820 errors of improbable magnitude. Upon the whole this circle makes a very fair representation of the observed data, and from it I have derived the following orbit with which are printed for comparison SEE's elements from which the preceding ephemeris was computed:

	COMSTOCK	SEE
P	500 ± years	195.76 years
T	1892	1907.84
a	11″.4	8″.2128
e	0 .49	0 .5112
☊	101°	16°.4
i	29	15 .95
λ	90	217 .87

I do not regard my own elements as furnishing more than a very crude approximation to the star's real orbit, and perhaps not even that. They correspond to one out of many apparent orbits that fit reasonably well to the existing data and at present there are no means for deciding which among these orbits is the right one. It appears, however, entirely safe to say that both the periodic time and the major axis of the real orbit are considerably greater than has hitherto been supposed, even when due allowance is made for the margin of uncertainty attributed by SEE to the periodic time, "which may possibly differ several years from the value here derived."

OBSERVED MINIMA OF 320 *U CEPHEI*,

BY P. S. YENDELL.

The following are all the times of minima of *U Cephei* observed by me since 1895. The reductions are made in the same manner as those of this and other variables of the *Algol* type published by me in former numbers of this Journal, excepting that I have given no determinations by the mean light-curve, as I am at present engaged upon a study of the light-variations of this star, with a view to the construction of a definitive mean curve.

There are nine minima in all, as follows:

1896 May 13, twenty-two observations, from $8^h\ 26^m$ to $15^h\ 10^m$, Local Mean Time.

Time of minimum by single curve, $13^h\ 8^m$, wt. 4.

Time of minimum by equal brightness:

M	Before h m	After h m	Mean h m
8.2	10 37	14 57	12 47.0
8.4	11 0	14 43	12 51.5
8.6	11 12	14 44	12 58.0
8.8	11 28	14 40	13 4.0
9.0	11 46	14 36	13 11.0
		Mean	12 58.9

Least observed light, $9^M.4$.

1899 September 4, twenty-three observations, from $10^h\ 33^m$ to $16^h\ 1^m$.

Time of minimum by single curve, $14^h\ 0^m$, wt. 4.

Time of minimum by equal brightness:

M	Before h m	After h m	Mean h m
8.4	12 30	15 55	14 12.5
8.6	12 36	15 41	14 8.5
8.8	12 41	15 31	14 6.0
9.0	12 45	15 21	14 3.0
		Mean	14 7.5

Least observed light, $9^M.3$.

1899 September 9, twenty-three observations, from $10^h\ 46^m$ to $16^h\ 2^m$.

Time of minimum by single curve, $13^h\ 52^m$, wt. 4.

Time of minimum by equal brightness:

M	Before h m	After h m	Mean h m
8.4	12 4	15 36	13 50.0
8.6	12 19	15 21	13 50.0
8.8	12 32	15 17	13 54.5
9.0	12 37	15 12	13 54.5
		Mean	13 52.3

Least observed light, $9^M.2$.

1899 September 14, fifteen observations, from $9^h\ 56^m$ until stopped by haze at $15^h\ 5^m$. Least observed light, $9^M.2$.

Time of minimum by single curve, $13^h\ 34^m$, wt. 3.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
9.0	12 11	14 52	13 31.5

1899 September 24, thirty observations, from $8^h\ 28^m$ to $15^h\ 15^m$. Least observed light, $9^M.2$.

Time of minimum by single curve, $12^h\ 48^m$, wt. 1.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
8.4	10 55	14 55	12 55.0
8.6	11 11	14 29	12 50.0
8.8	11 28	14 17	12 52.5
9.0	11 39	14 2	12 50.5
		Mean	12 52.0

1899 October 4, twenty-six observations, from $8^h\ 23^m$ to $14^h\ 21^m$. Least observed light, $9^M.2$.

Time of minimum by single curve $11^h\ 49^m$, wt. 1.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
8.4	10 9	13 47	11 58.0
8.6	10 18	13 30	11 54.0
8.8	10 36	13 19	11 57.5
9.0	10 50	13 2	11 56.0
		Mean	11 56.4

1899 October 14, thirty-two observations, from $7^h\ 18^m$ to $14^h\ 22^m$.

Time of minimum by single curve, $11^h\ 35^m$, wt. 5.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
8.0	8 58	13 54	11 26.0
8.2	9 23	13 44	11 33.5
8.4	9 39	13 13	11 26.0
8.6	9 43	13 0	11 21.5
8.8	10 0	12 55	11 27.5
9.0	10 18	12 47	11 32.5
		Mean	11 27.8

Least observed light, $9^M.2$.

1900 November 2, sixteen observations, from $6^h\ 30^m$ to $11^h\ 15^m$. Least observed light, $9^M.3$.

Time of minimum by single curve, $9^h\ 1^m$, wt. 5.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
8.2	6 57	11 7	9 2.0
8.4	7 20	10 50	9 5.0
8.6	7 50	10 33	9 11.5
8.8	8 0	10 28	9 14.0
9.0	8 3	10 23	9 13.0
		Mean	9 9.1

1901 October 7, eighteen observations, from $7^h\ 13^m$ to $12^h\ 3^m$. Least observed light, $9^M.3$.

Time of minimum by single curve, $9^h\ 58^m$, wt. 4.

Time of minimum by equal brightness:

	Before	After	Mean
M	h m	h m	h m
8.4	8 29	11 39	10 4.0
8.6	8 34	11 30	10 2.0
8.8	8 40	11 22	10 1.0
9.0	8 47	11 19	10 3.0
		Mean	10 2.5

ON THE RELATIVE ACCURACY OF CERTAIN METHODS FOR REDUCING STELLAR PHOTOGRAPHS.

By FRANK SCHLESINGER.

In *A.J.* No. 475, Mr. ARTHUR R. HINKS, of Cambridge, England, has a paper on "The Methods of Reduction and Publication of Measures of Celestial Photographs of Isolated Star Groups; with a New Reduction of the Rutherfurd *Praesepe* Plates." These plates were taken by RUTHERFURD in 1873 and 1877, and were measured in rectangular coordinates by Dr. KRETZ, Mr. HAYS and myself at Columbia University in 1898.* Mr. HINKS's new reduction consists chiefly in applying TURNER's method to these measures, whereas my reduction had been made according to the method of JACOBY. Many papers regarding the relative merits of the these two methods have already appeared in this *Journal*, and I have been reluctant to write what might be considered an addition to these. But Mr. HINKS arrives at one conclusion which is so important and of such general interest that I must permit myself to call attention to it. The necessity for this is all the greater since the point seems to have been misunderstood in more than one quarter.

In comparing the separate results from the eight plates, Mr. HINKS finds a better agreement in his reduction than in mine, and concludes that TURNER's method is more accurate than JACOBY's. Now between the two methods there is only one essential difference, and this is in the way the effects of refraction are eliminated from the measures. All other differences are matters of mere convenience in computation; were it not for refraction, the two methods would lead to precisely the same numerical results. In TURNER's method, the effects of refraction, along with those of aberration, precession, etc., are determined by means of "comparison-stars": that is, the measured coordinates of certain stars are compared with their known positions and the required corrections are thus deduced. Six unknown quantities must be determined, and at least three com-

*Contributions from the Observatory of Columbia University, No. 15.

comparison-stars are necessary. In JACOBY's method, the corrections for refraction are computed by differentiation of BESSEL's well known expression. After applying these to the measured coordinates, the other corrections (aberration, precession, etc.) are obtained, as in TURNER's method, from comparison-stars. Only four unknowns must now be found, and thus two stars are just sufficient.

From these considerations it would appear that if any preference as regards accuracy be made, it should be in favor of JACOBY's method. There are, however, two possible escapes from this conclusion. The first is that the effects of refraction may be more accurately determined from a single photograph than from a differentiation of BESSEL's well tried formula. The second is that a photograph may be so distorted as to make the "scale-value" vary with the position-angle. This special kind of distortion would be allowed for in TURNER's method, but not in JACOBY's. If, however, the distortion take any other form (and equally probable ones suggest themselves), neither method would take it into account. Moreover, so far as I know, no astronomer has yet published a well-grounded statement of the actual discovery of a distortion which followed a systematic law over the entire surface of a plate. Professor TURNER himself has declared emphatically against the existence of systematic distortion.*

The fact that Mr. HINKS has secured a better agreement between the plates than in my reduction is capable of a simple explanation. He has used thirty-two comparison-stars, while I have used only five. The constants of the plates are therefore more accurately determined by him, and a slightly better agreement was to be expected. My reasons for using so small a number of comparison-stars, as well as the fact that a larger number would lead to somewhat better results, are indicated on pages 231 and 235 of my paper on the *Praesepe* group. It should be remarked that whatever objections may be urged against TURNER's method fall away when the number of comparison-stars is large. As Mr. HINKS used thirty-two stars he is justified in employing his convenient method. On the other hand, as I used only five stars, JACOBY's method was to be preferred, in spite of the slight additional labor involved.

* *Mon. Not. of the R.A.S.*, Vol. LXI, No. 5, page 312.

THE NOVEMBER LEONIDS, 1901.

BY THEO. I. KING, ASSISTANT ASTRONOMER.

[Communicated by the Superintendent of the U.S. Naval Observatory.]

On the nights of November 13, 14 and 15, observers at the U.S. Naval Observatory watched for *Leonids*, beginning about $12^h\ 30^m$ of the astronomical day. The sky for the most part was reasonably free from clouds, but the worst interference terminated the count on the second morning, when the shooting-stars were most numerous. Prof. F. B. LITTELL, who was alone at the time, counted 70 *Leonids* between $14^h\ 50^m$ and $16^h\ 5^m$ (75th Meridian Time), and states that "nearly all the *Leonids* were bright, with tails of bluish color lasting from one to ten seconds." A careful projection of the position of the radiant on the first night seemed to place it a little to the south and east of 15 H *Leonis*, but an equally careful estimate of its position from the more numerous meteors of the last night transferred it to about the same distance north and west of that star, making its approximate right-ascension $9^h\ 57^m$, approximate declination $+22\frac{1}{2}°$.

The record for the three nights is as follows:

Date	75th Meridian Time		*Leonids*	Not *Leonids*
	h m	h m		
Nov. 13	12 30 to	16 10	14	23
14	12 45	14 30	21	2
14	14 50	16 30	73	18
15	12 30	15 15	45	35
	Total		153	

Professor LITTELL was with me the greater part of the first night, and observed alone the latter part of the second night. Computer E. I. YOWELL also voluntarily rendered assistance during a portion of the first and last nights, and Mr. H. R. MORGAN for a short time on the first night.

CORRIGENDA.

No. 510, p. 43, col. 2: In eq. (3) insert the multipliers $\sec\beta_1$ and $\sec\beta_2$ in right hand members for ρ_1 and ρ_2 respectively.
" p. 45, col. 1: In eq. (13), term in ϵ_2, *for* $\sigma_1\ \sigma_2$ *put* $\sigma_1 - \sigma_2$.
" p. 46: In eq. (22), third term of 2d eq., in denominator, *for* $1 - e\cos(u - \omega)$ *put* $1 + e\cos(u - \omega)$.
" p. 46, col. 1, bottom line: *put* by *after* multiplied.
No. 510, p. 46, col. 2, line 21 from bottom: *for* the same *put* as great a.
" p. 47, col. 1, line 12 from bottom: *for* and y^2 *put* $+y^2$.
" p. 47, col. 2, line 20 from bottom: *for* <1 *put* -1.
No. 511, p. 56, col. 1, line 3 from bot.: *for* about the two *put* about two.
" p. 57, col. 1, line 8 from top: *for* their *put* its.

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No. 513.

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NOTES ON VARIABLE STARS, — No. 36,

By HENRY M. PARKHURST.

Aperture. From July until October, 1901, the observations were made in Ohio with 4 inches aperture, using the photometric apparatus belonging to the telescope of 9 inches aperture in Brooklyn, N.Y.

Results of Observations.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors		d	Remarks
				1901								
1717	*V Tauri*	Max.	5449	Mar. 5	61	+23	9	9.63	2.92	0.95	27	
2625	*V Geminorum*	Max.	5479	Apr. 4	28	— 3	6	8.37	—	—	—	
2684	*S Canis min.*	Max.	5512	May 7	42	+ 8	6	7.56	—	—	—	Possibly later
2689	*Z Puppis*	Max.	5510	May 5	—	—	1	7.5	—	—	—	A regular period not established
2735	*U Canis min.*	Max.	5521	May 16	19	—29	5	—	—	—	—	Probably later
2742	*S Geminorum*	Max.	5474	Mar. 30	61	— 6	5	9.13	—	—	—	
3264	*W Cancri*	Max.	5528	May 23	10	+24	5	8.40	—	—	—	Brighter than usual
3994	*S Leonis*	Max.	5514	May 9	78	—74	6	—	—	—	—	Large correction continues
4315	*R Comae*	Max.	5585	July 19	45	+ 1	6	8.75	—	—	—	Elements, *A.J.* 384
4377	*T Virginis*	Max.	5505	Apr. 30	43	+16	7	9.73	—	—	—	
4665	*RT Virginis*	Max.	5510	May 5	—	—	2	—	—	—	—	Period prob. more than a year
5194	*V Bootis*	Max.	5633	Sept. 5	24	+70	9	7.22	1.58	1.67	19	
5405	*RT Librae*	Max.	5556	June 20	7	+ 7	9	9.37	1.60	1.78	48	
5494	*S Librae*	Max.	5499	Apr. 24	51	—	E	—	—	—	—	Diminishing in May
5501	*S Serpentis*	Max.	5610	Aug. 13	73	+63	9	8.41	2.04	2.06	33	
5566	*RU Librae*	Max.	5560 :	June 24	15	+49 :	1	8.6	—	—	—	Reduction uncertain
5677	*R Serpentis*	Max.	5637	Sept. 9	76	+11	9	6.46	1.27	0.89	23	
5704	*RR Librae*	Max.	5500 :	April :	—	—	—	—	—	—	—	Probably earlier
5798	*RU Herculis*	Min.	5600 :	August :	—	—	—	—	—	—	—	Invisible August to October
5887	*V Ophiuchi*	Max.	5634	Sept. 6	33	—18	9	6.93	1.15	0.75	22	
5950	*W Herculis*	Max.	5674	Oct. 16	29	+ 6	5	—	—	—	—	
6044	*S Herculis*	Min.	5699	Nov. 10	54	—	E	—	—	—	—	Tr. Elements, *A.J.* 388
6160	*RT Herculis*	Max.	5619	Aug. 22	6	—38	3	—	—	—	—	Elements, *A.J.* 421
6225	*RS Herculis*	Max.	—	—	—	—	—	—	—	—	—	Max. before August
6624	*T Serpentis*	Max.	—	July :	—	—	—	—	—	—	—	Probably earlier
6849	*R Aquilae*	Max.	5650	Sept. 22	19	+26	9	6.12	0.81	0.56	9	Elements, *A.J.* 490

Individual Observations.

Including Observations by Arthur C. Perry

1717 *V Tauri.*

(Continued from 487.)

Julian	Calendar	Mag.
	1900	
5317.6	Oct. 21	13]
5345.5	Nov. 21	12]
5374.5	Dec. 20	12]
	1901	
5407.6	Jan. 22	11.72_2
5423.5	Feb. 7	9.65_2
5424.5	8	9.48_2
5426.5	10	10.21_2

1717 *V Tauri.*—Cont.

Julian	Calendar	Mag.
	1901	
5429.5	Feb. 13	9.37_2
5434.5	18	9.65_2
5437.5	21	10.09_2
5451.5	Mar. 7	9.27_2
5456.5	12	9.39_2
5460.5	16	10.15_2
5471.5	27	11.21

2625 *V Geminorum.*

(Cont. from 468. Comp. Stars 468)

Julian	Calendar	Mag.
	1901	
5472.5	Mar. 28	8.6
5473.5	Mar. 29	8.68_2
5476.5	Apr. 1	8.73_2
5478.5	3	8.00_2
5487.6	12	8.78_2
5502.5	27	9.91_2

2684 *S Canis min.*

(Cont. from 468. Comp. Stars 441)

Julian	Calendar	Mag.
	1901	
5471.5	Mar. 27	8.63_2
5474.5	30	8.44_2
5476.5	Apr. 1	8.34
5487.5	12	8.32_2
5502.5	27	7.84_2
5508.5	May 3	7.95_2
5511.5	6	7.17_2

2684 *S Canis min.*—Cont.

Julian	Calendar	Mag.
	1901	
5517.5	May 12	7.97_2
5519.6	14	7.30
5521.6	16	7.66_2

2689 *Z Puppis.*

(Cont. from 487. Comp. Stars 403)

Julian	Calendar	Mag.
	1901	
5441.5	Feb. 25	9.9
5472.5	Mar. 28	10.3

2689 *Z Puppis*. — Cont.

Julian	Calendar	Mag.
	1901	
5473.6	Mar. 29	10.41_2
5501.6	Apr. 26	7.85_2
5502.5	27	8.21
5503.5	28	7.61_2
5504.5	29	7.54_2

2735 *U Canis min.*

(Cont. from 470. Comp. Stars 470.)

Julian	Calendar	Mag.
	1901	
5471.5	Mar. 27	9.27_2
5475.5	31	9.93
5487.6	Apr. 12	9.53_2
5502.6	27	9.69_2
5508.5	May 3	9.65_2
5511.5	6	8.89_2
5517.5	12	9.61_2
5519.6	14	8.08
5521.6	16	8.32_2

2742 *S Geminorum*.

(Continued from 403.)

Julian	Calendar	Mag.
	1901	
5471.5	Mar. 27	9.5
5473.5	29	9.81
5474.5	30	9.38_2
5475.5	31	9.49_2
5476.5	Apr. 1	9.85
5487.6	12	9.57_2

3264 *W Cancri*.

(Cont. from 487. Comp. Stars 384.)

Julian	Calendar	Mag.
	1901	
5473.5	Mar. 29	11.0]
5474.5	30	11.0]
5475.5	31	11.0]
5487.5	Apr. 12	10.13_2
5492.5	17	10.06_2
5501.6	26	11.0
5502.5	27	9.77_3
5506.5	May 1	9.23_2
5509.5	4	9.25_2
5517.6	12	8.55_2
5521.6	16	8.85_2
5536.6	31	8.40_2
5539.6	June 3	8.71_2

3994 *S Leonis*.

(Cont. from 487. Comp. Stars 470.)

Julian	Calendar	Mag.
	1901	
5473.6	Mar. 29	11.0]
5501.6	Apr. 26	11.0
5508.6	May 3	10.22
5509.6	4	10.23_2
5517.6	12	10.72_2
5536.6	31	10.77_2

4315 *R Comae*.

(Cont. from 487. Comp. Stars 415.)

Julian	Calendar	Mag.
	1901	
5511.6	May 6	12.6]
5528.6	23	11.0]
5545.6	June 9	10.7]
5548.6	12	11.6
5572.6	July 6	9.1
5576.6	10	8.9
5577.6	11	9.0
5578.6	12	8.9
5579.6	13	8.8
5580.6	14	8.8
5585.6	19	8.7
5591.6	25	8.8

4377 *T Virginis*.

(Cont. from 487. Comp. Stars 470.)

Julian	Calendar	Mag.
	1901	
5474.6	Mar. 30	10.3
5475.6	31	10.60_2
5501.6	Apr. 26	9.30_2
5502.6	27	9.69_2
5508.6	May 3	9.61_2
5511.6	6	9.85_2
5517.6	12	9.88_2
5519.6	14	10.06_2

4665 *RT Virginis*.

(Continued from 487.)

Julian	Calendar	Mag.
	1901	
5510.5	May 5	8.33_2
5536.6	31	8.64_2

5191 *V Bootis*.

(Cont. from 470. Comp. Stars 333.)

Julian	Calendar	Mag.
	1901	
5511.6	May 6	8.70_2
5575.6	July 9	8.8
5580.6	14	9.0
5601.6	Aug. 4	9.0
5610.6	13	7.33_2
5621.6	24	7.07_2
5625.5	28	7.87_2
5630.5	Sept. 2	6.99_2
5632.5	4	6.92_2
5635.5	7	7.80_2
5648.5	20	6.83_2
5651.5	23	7.97_2
5658.5	30	8.79_2

5405 *RT Librae*.

(Continued from 487.)

Julian	Calendar	Mag.
	1901	
5510.6	May 5	11.3]
5517.6	12	11.7
5521.6	16	10.58_2

5405 *RT Librae*. — Cont.

Julian	Calendar	Mag.
	1901	
5539.6	June 3	9.21_2
5546.6	10	9.1
5574.6	July 8	9.4
5576.6	10	9.8
5579.6	13	9.6
5585.6	19	10.8

5491 *S Librae*.

(Continued from 487.)

Julian	Calendar	Mag.
	1901	
5517.6	May 12	9.60_2
5521.6	16	10.03_2

5501 *S Serpentis*.

(Cont. from 490. Comp. Stars 388.)

Julian	Calendar	Mag.
	1901	
5510.5	May 5	10.5]
5512.6	7	11.23
5528.6	23	10.7
5546.6	June 10	10.5
5575.6	July 9	9.1
5578.6	12	9.1
5585.6	19	8.3
5604.6	Aug. 7	8.05_2
5608.6	11	7.98
5618.6	21	8.16_2
5624.6	27	8.18_2
5631.5	Sept. 3	8.80_2
5634.5	6	8.25_2
5643.5	15	7.89_2

5566 *RU Librae*.

(Continued from 490.)

Julian	Calendar	Mag.
	1901	
5511.6	May 6	10.4 :
5521.6	16	8.8
5539.6	June 3	8.61_2
5546.6	10	8.6
5577.6	July 11	8.6
5585.6	19	8.7
5591.6	25	9.4]
5601.6	Aug. 4	9.5

5677 *R Serpentis*.

(Cont. from 490. Comp. Stars 476.)

Julian	Calendar	Mag.
	1901	
5512.6	May 7	12.0
5528.6	23	10.2]
5546.6	June 10	10.4 :
5575.6	July 9	8.6
5578.6	12	7.9
5585.6	19	7.6
5604.6	Aug. 7	6.64_2
5608.6	11	6.88_2

5677 *R Serpent.* — Cont.

Julian	Calendar	Mag.
	1901	
5618.6	Aug. 21	6.66_2
5624.6	27	6.98_2
5631.5	Sept. 3	7.05_2
5634.5	6	5.84_2
5641.5	13	5.73_2
5649.5	21	7.86_2
5651.5	23	6.97_2
5655.5	27	7.42_2
5661.5	Oct. 3	8.07_2

5701 *RR Librae*.

(Continued from 421.)

Julian	Calendar	Mag.
	1901	
5511.6	May 6	8.6
5512.6	7	8.91_2
5517.6	12	8.89_2
5539.6	June 3	9.48_2
5546.6	10	9.7
5575.6	July 9	9.9

5798 *RU Herculis*.

(Continued from 490.)

Julian	Calendar	Mag.
5604.6	Aug. 7	12]
5630.6	Sept. 2	12]
5664.5	Oct. 6	12]

5887 *V Ophiuchi*.

(Cont. from 490. Comp. Stars 456.)

Julian	Calendar	Mag.
	1901	
5577.6	July 11	8.3
5585.6	19	7.5
5601.6	Aug. 4	8.50_2
5608.6	11	7.67_2
5618.6	21	7.9
5621.6	24	7.46_2
5630.6	Sept. 2	6.13_2
5632.6	4	7.20_2
5641.5	13	7.05_2
5643.5	15	6.55_2
5650.5	22	8.77_2
5655.5	27	9.31_2
5658.5	30	8.22_2

5950 *W Herculis*.

(Continued from 476.)

Julian	Calendar	Mag.
	1901	
5618.6	Aug. 21	10]
5630.6	Sept. 2	10]
5663.5	Oct. 5	8.7
5664.5	6	8.63_2
5665.5	7	8.11_4
5668.5	10	8.41_2
5671.5	13	7.82_2
5676.5	18	7.75_2
5679.5	21	8.22_2

6044 *S Herculis*.

(Cont. from 490. Comp. Stars 388.)

Julian	Calendar	Mag.
	1901	
5608.6	Aug. 11	10.6]
5631.6	Sept. 3	10.6
5632.6	4 to	
5673.5	Oct. 15	10.8]
	7 dates	

6160 *RT Herculis*.

(Cont. from 490. Comp. Stars 421.)

Julian	Calendar	Mag.
	1901	
5607.6	Aug. 10	9.9
5618.6	21	9.9
5630.6	Sept. 2	9.9
5637.5	7	11.23
5664.5	Oct. 6	11.3

6225 *RS Herculis*.

(Cont. from 490. Comp. Stars 490.)

Julian	Calendar	Mag.
	1901	
5604.6	Aug. 7	9.34_2
5618.6	21	9.8
5630.6	Sept. 2	10.3

6624 *T Serpentis*.

(Continued from 490.)

Julian	Calendar	Mag.
	1901	
5607.6	Aug. 10	10.9
5630.6	Sept. 2	11]
5643.6	15	11]

6849 *R Aquilae*.

(Cont. from 490. Comp. Stars 456.)

Julian	Calendar	Mag.
	1901	
5577.6	July 11	9.1
5585.6	19	8.9
5601.6	Aug. 4	7.4
5609.6	12	6.38_2
5621.6	24	6.57_2
5631.6	Sept. 3	5.85_2
5633.5	5	6.69_2
5643.5	15	5.53_2
5646.5	18	7.50
5649.5	21	5.43_2
5650.5	22	5.95_2
5651.5	23	6.22_2
5652.5	24	6.55_2
5655.5	27	6.61_2
5663.5	Oct. 5	6.65_2
5669.5	11	6.63_2
5676.5	18	6.99_2
5678.5	20	6.92_2

Comparison Stars, 1893–1901.

1717 *V Tauri*.

Star	DM.	Mag.	n
T	+17°797	8.78	25
Y	+17°801	9.52	33
1*Y*	+17°796	9.79	21
Z	+17°799	9.51	34
a	3s1f *W*	10.26	31
j	5s1p *Y*	11.32	21

4665 *RT Virginis*.

Star	DM.	Mag.	n
P	+5°2710	8.85	22
1*R*	+5°2709	8.80	23
1*T*	+5°2707	9.66	10
Z	+4°2680	10.10	14
1*Z*	+4°2689	10.44	13
2*Z*	+5°2706	10.18	6

5405 *RT Librae*.

Star	DM.	Mag.	n
Q	−18°3969	8.83	29
S	−18°3973	8.98	3
1*S*	−18°3974	9.03	4
T	−18°3965	9.39	18
X	−18°3977	10.22	22
Y	−18°3976	10.78	17

5491 *S Librae*.

Star	DM.	Mag.	n
I	−20°4196	7.68	13
J	−20°4206	7.86	17
K	−20°4198	7.82	13
1*W*	−19°4082	10.01	12
X	−19°4083	10.62	13
a	6s11p 1*W*	10.85	5

VARIATION OF LATITUDE FROM MOLYNEUX'S AND BRADLEY'S OBSERVATIONS.

By S. C. CHANDLER.

The following article presents the determination of the constants of the polar motion from the observations by MOLYNEUX at Kew with his Zenith-Tube, and by BRADLEY at Wanstead with his Zenith-Sector. The results of my discussion of these series, in *A.J.*, Vol. XI, p. 85, have been taken as the basis of this investigation. The accompanying table gives the values of n there derived, with the weights taken in proportion to the number of observations.

First the question must be examined whether there is any evidence of changes in the line of collimation. The instruments being non-reversible, the only means of answering it is by scrutiny of the observations and comparison with the record, in BRADLEY'S notes, of any alteration in the instrumental adjustments which could influence this element. Any change in the relation of the plumb-line to the tube is of vital consequence in this regard. Now, on several occasions the wire sustaining the plummet was broken and replaced by new ones. I have examined the observations in each of these cases to ascertain if there was any visible effect, with a negative result except in one instance where I find positive proof of a difference of nearly a second of arc in the measurements made before and after.

The first case of the kind at Wanstead during the observations here used was on 1727 Dec. 13, when, after observing ten stars, BRADLEY has this note:

"As I went to set the telescope to observe π *Cass.*, I somehow or other touched the plummet and broke the wire; it broke this time just at the notch at the top. I did not fix on another till after ten of the clock, just soon enough to observe *Capella*."

He then observes four stars, and on Dec. 14 and 15 eleven stars, followed by the remarks:

"By these observations 't is evident that there is no sensible difference in rectifying the instrument with the present wire and the former that was broke; for the observations made on the 13th and 14th days differ some one way and some another, whereas did that arise from the plumb line, the difference ought to have been all the same way."

An independent examination, like those made hereafter in the other cases, confirms BRADLEY'S conclusion.

The second case is on 1728 March 20:

"As I was letting down the plummet to rectify the spot to it, after the passage as usual, the wire catched in something and broke about a yard from the bottom."

"Mar. 20. I put on another wire this evening. Memorand. 'T is sufficient to pinch the wire with the head of the small screw at top (after 't is in the notch and drawn up with the plummet on) without turning the wire round the screw."

BRADLEY resumed observations on Mar. 21, but does not record whether he noticed any effect on his results, although it is manifest that the matter gave him some solicitude, as will appear later. In order to settle the point I have reduced anew all the observations from February to April, inclusive, and taken the means of the difference for each star before and after the time of the break, with the following results, where the column Δ is the difference, before *minus* after, expressed in polar distance; the observations having been reduced to a common date for precession, nutation and aberration; and the weights being the product divided by the sum of the number of observations in the two groups.

	Δ	Wt.		Δ	Wt.
	$''$			$''$	
β *Draconis*	+1.26	2	γ *Ursae Maj.*	+2.14	1
α *Cassiop.*	+1.15	2	ϵ *Ursae Maj.*	+1.51	1
α *Persei*	−0.05	2	21 *Can. Ven.*	+2.10	1
α *Aurigae*	−0.32	3	ζ *Ursae Maj.*	+0.97	1
θ *Ursae Maj.*	+0.22	1	η *Ursae Maj.*	+1.66	1
β *Ursae Maj.*	+0.81	3	β *Draconis*	+1.12	2
ψ *Ursae Maj.*	+0.88	3	ι *Herculis*	+0.99	2
χ *Ursae Maj.*	+1.79	1	γ *Draconis*	+0.84	2

The mean difference is +0″.90, with a probable error of ±0″.11. Classified according to position we have.

Stars	R.A.	Δ		Stars	Zen. Dist.	Δ	
	h h	$''$	$''$			$''$	$''$
8	0–12	+0.65	±0.19	8	North	+1.02	±0.11
8	12–24	+1.22	±0.12	8	South	+0.72	±0.20

This seems to leave no reasonable doubt of the reality of the disturbance, but it does not demonstrate the exact date of it. To do this I subtracted the mean zenith-distance of each star from the individual values and arranged the residuals chronologically; and then took the averages for each date. It is manifest that these will show the date of the break by a discontinuity at that point, before and after which the results will be affected in opposite directions by the change in collimation. Thus we find:

		Obs.			Obs.
	$''$			$''$	
Mar. 2	+0.11	2	Mar. 21	−1.09	7
5	+ .19	2	22	−0.57	5
6	+ .38	1	23	− .42	3
7	+ .41	4	24	− .57	6
12	+ .54	4	Apr. 2	−0.81	3
13	+ .32	4	6	−1.45	6
17	+ .09	4	7	− .65	8
18	− .02	1	10	− .78	1
19	+ .85	1	15	− .97	2
20	−0.08	3	16	−1.12	8

The demonstration could scarcely be more direct or complete that the breaking and replacing the wire on Mar. 20 caused a change in collimation. I adopt this conclusion without hesitation, and have accordingly subtracted the quantity $+0''.90$ from the values of n in the table for the observations at Wanstead before Mar. 21, Old Style = Apr. 1, New Style; *i.e.*, for the groups 2352090–2270, inclusive. But in order to examine the effect on the latitude-constants I have made two solutions, one including and one excluding these observations.

Another breakage of the wire occurred 1728 June 21, O.S., and again BRADLEY makes no comment as to the possibly sensible effect. In this instance I find that there was none, an examination similar to the preceding giving $+0''.11 \pm 0''.06$ for the difference before and after; thus practically insensible.

Subsequently BRADLEY appears to have noticed that a change had somewhere occurred, for we find the following notes:

"1728 Sept. 3. I took off the wire of the plummet and fixed it on again about a quarter of an inch shorter, so that the bent part (that rested in the notch at top) might be cut off after 't was again fastened. This I did in order to try whether it would hang exactly the same after as before; because I suspected that there might be some difference, the observations of last year not exactly agreeing with those of this. But upon my adjusting the same spot to the plummet after I had altered it, as I had done before, I could not perceive any difference; so that I judge this difference cannot arise from the uncertainty of hanging of the wire truly in the same place, but may possibly proceed from some small alteration in the tube itself, &c., if the succeeding observations shall be found to confirm the small disagreement that those already made of γ *Drac.* seem to show."

"Sept. 10. I took off the wire after having very well rectified a spot to it, and immediately fixed on a new one, and then compared the same spot with the new wire as before with the old one; but I could not perceive that there was the least difference; I am sure, had there been a quarter of a second odds, I could have discerned it.

The reason why I have been thus curious in trying whether different wires agree with each other is, that if there should appear any little difference in the different years' observations, I might be satisfied that it cannot arise from the different wires, but must proceed from some other cause. And though the experiment of Sept. 3d might seem sufficient, yet because that was made with the same wire (which was also pieced near the bob) I chose to repeat the experiment in the manner I did this day; which has not only confirmed the former, but also makes it evident that the tying a knot, &c., near the bob or ball, does no harm."

I have given these passages at length because they have a significant historical interest quite apart from the point we are examining. It is very clear that the differences in the observations which were troubling BRADLEY, while they were partially due to the instrumental change in March above demonstrated, were in reality largely due to what we now know as the variation of latitude. If we compare the diagram accompanying this article with BRADLEY's remarks just cited, my meaning will be manifest. In virtue of the character of the harmonic curve flowing from the two terms of the polar motion, in a seven-year cycle, the latitude up to near the end of 1727 was almost stationary. It declined in the spring of 1728, and then rapidly rose, reaching a maximum in the middle of September when its influence became visible and led him to make the experiments just recorded. The anomalies that disturbed him, and which he thus unsuccessfully endeavored to trace to an instrumental source, had their origin in the phenomenon with which we are now familiar. Its vibrations were palpable under his hand. So near did BRADLEY come to the discovery of the polar motion! Thus confidently can we now trace our first knowledge of it to the same immortal work that gave us the aberration and the nutation.

BRADLEY's conclusion from these experiments, that in this case there was no appreciable change in collimation, is borne out by my own examination of the case, which gives a difference of $+0''.25$ to be sure, but with a probable error of $\pm 0''.15$.

The only other accident of the kind during our portion of the series was on 1729 Sept. 30.

"This morning as I was moving the telescope to set it for *Capella*, the wire broke just at the notch at top. I afterwards fixed on the same wire again, it being long enough without piecing. I had used this wire almost twelve months without breaking, for it was put on Sept. 28, 1728."

BRADLEY seems to have been reassured by his experiments of the previous year; at least he does not mention examining this case. In fact we find that no disturbance occurred, for the difference between the observations before and after is only $+0''.05 \pm 0''.10$.

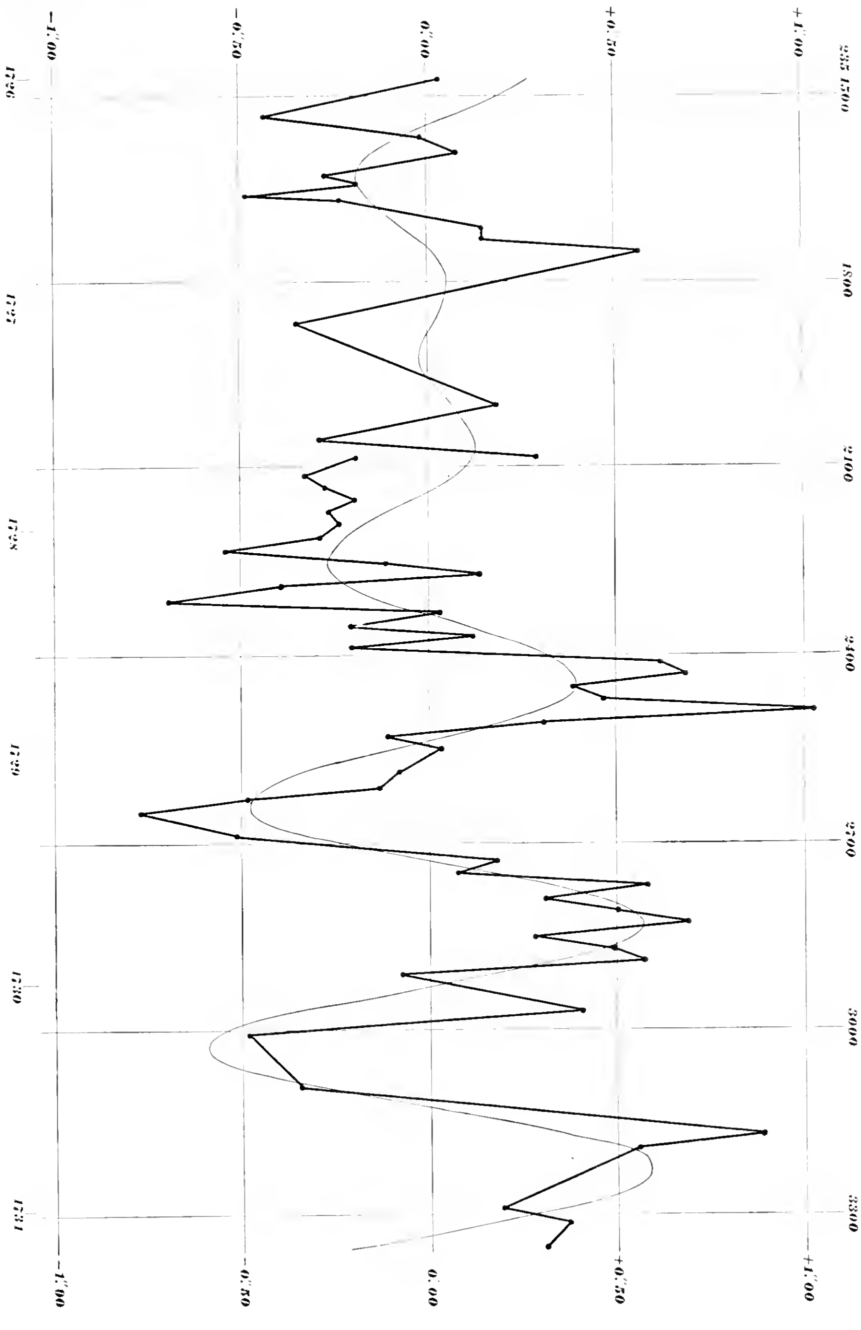
As to the Kew instrument, the plumb-line broke on 1726 May 31 and was replaced by a thicker one, and on June 8 a heavier plummet was put on. I can discern no effect on the observations.

In neither series is there any evidence of disturbance of the collimation, either from this or any other cause, except the one above established, on 1728 Mar. 20 (O.S.).

The results of the solutions, using the values of n in the table, will now be given. Unknowns were introduced for the epochs and semi-amplitudes of the 14-mos. and annual terms, and for the corrections to the arbitrary zero of each series. The weights were employed as given. First, using the whole data, with the correction necessary between 2352090 and 2270 ($+0''.90$ subtracted algebraically from the n's) we find

$$\varphi - \varphi_0 = -0''.300 \cos(t - 2352164)\,0^\circ.84 - 0''.303 \cos(\odot - 27^\circ)$$

The values from this direct solution are those given in the "Computed" column of the table, and are represented



Variation of Latitude from Molyneux's and Bradley's Observations at Kew and Wanstead, 1726–1731.

as the curve in the diagram. The corrections to arbitrary zero were, for Kew $+0''.16$, for Wanstead $-0''.28$; which, subtracted algebraically from the n's, give the "Observed" values of $\varphi - \varphi_0$ in the table and the diagram.

t 2350000+	p	n	$\varphi - \varphi_0$ Obs'd	$\varphi - \varphi_0$ Comp'd	O—C
			"	"	
1474	1	+0.20	+0.04	+0.28	−0.24
1532	1	.28	.44	+ .07	.51
1569	1	+ .15	.04	− .07	+ .06
1594	1	+ .24	+ .08	− .13	+ .21
1629	2	.11	− .27	.18	− .09
1642	2	− .03	− .19	− .18	− .01
1659	2	− .33	.49	− .17	− .32
1671	1	− .07	− .23	− .15	− .08
1712	2	+ .31	+ .15	− .07	+ .22
1732	3	+ .31	+ .15	− .02	+ .17
1752	2	+ .72	+ .56	+ .01	+ .55
1865	1	− .19	− .35	+ .01	− .36
2000	1	+ .35	+ .19	+ .06	+ .13
2053	1	− .13	− .29	+ .12	− .11
2082	1	+ .45	+ .29	+ .12	+ .17
2090	7	+ .42	− .20	+ .12	− .32
2110	6	+ .29	− .33	+ .08	− .41
2130	5	+ .34	− .28	+ .04	− .32
2150	2	+ .42	− .20	− .02	− .18
2170	3	+ .36	− .26	− .09	− .17
2190	2	+ .38	− .24	− .15	− .09
2210	6	+ .33	− .29	− .21	− .08
2230	3	+ .07	− .55	− .25	− .30
2250	6	+ .51	− .11	− .27	+ .16
2270	1	+ .76	+ .14	− .25	+ .39
2290	4	− .68	− .40	− .21	− .19
2310	2	− .98	− .70	− .14	− .56
2330	2	− .25	+ .03	− .05	+ .08
2350	2	− .49	− .21	+ .05	− .26
2370	2	− .16	+ .12	+ .16	− .04
2390	7	−0.49	−0.21	+0.25	−0.46

t 2350000+	p	n	$\varphi - \varphi_0$ Obs'd	$\varphi - \varphi_0$ Comp'd	O—C
		"	"	"	"
2410	1	+0.34	+0.62	+0.33	+0.29
2430	9	+ .41	+ .69	+ .38	+ .31
2450	4	+ .10	+ .38	+ .39	− .01
2470	6	+ .18	+0.46	+ .35	+ .11
2490	2	+ .74	+1.02	+ .28	+ .74
2510	3	+ .02	+0.30	+ .18	+ .12
2530	1	− .39	− .11	+ .05	− .16
2550	7	− .25	+ .03	− .09	+ .12
2590	1	− .37	− .09	− .35	+ .26
2610	6	− .42	− .14	− .44	+ .30
2630	5	−0.77	− .49	− .48	− .01
2650	3	−1.05	− .77	− .48	− .29
2690	2	−0.79	− .51	− .33	− .18
2730	5	− .10	+ .18	− .03	+ .21
2750	3	− .21	+ .07	+ .14	− .07
2770	2	+ .30	+ .58	+ .30	+ .28
2790	6	+ .02	+ .30	+ .43	− .13
2810	7	+ .22	+ .50	+ .52	− .02
2830	3	+ .41	+ .69	+ .56	+ .13
2850	2	− .01	+ .27	+ .54	− .27
2870	3	+ .21	+ .49	+ .47	+ .02
2890	2	+ .28	+ .56	+ .35	+ .21
2910	4	− .36	− .08	+ .19	− .27
2970	1	+ .12	+ .40	− .35	+ .75
3010	2	− .77	− .49	− .57	+ .08
3090	1	− .63	− .35	− .34	− .01
3170	6	+ .61	+ .89	+ .38	+ .51
3190	4	+ .27	+ .55	+ .51	+ .04
3290	3	− .09	+ .19	+ .31	− .12
3310	3	+ .08	+ .36	+ .17	+ .19
3350	1	+0.02	+0.30	−0.22	+0.52

In order to ascertain what effect variations in the constants of the annual term would have on those for the 14-mos. term, an indeterminate elimination gave y and z in terms of η and ζ (see *A.J.*, Vol. XII, p. 177), as follows:

$$\left.\begin{aligned} y &= +0''.266 \;-0.598\,\eta \;+0.200\,\zeta \\ z &= 0''.219 \;-0.094\,\eta \;-0.597\,\zeta \end{aligned}\right\} (A)$$

These equations will enable us to make various hypotheses as to r_2 and G, and find the corresponding values of r_1 and T_1.

If we examine the table of observed constants, *A.J.*, Vol. XIV, p. 73, we see that from 1828 to 1892, G ranged approximately between 270° and 360°, and r_2 between about 0''.15 and 0''.04; the average values, as found on p. 74, I.c., being 317° and 0''.136. It has elsewhere been shown that these limits of r_2 correspond to the major and minor semi-axes of the annual ellipse. If now we make different hypotheses I–IV below, corresponding to various combinations of these limits; also V ignoring the annual-term, and VI with the mean values; we find by the above equations for y and z the corresponding semi-amplitudes and epochs of the 14-mos. term under "A" below.

Hyp.	G	r_2	A r_1	A T_1	B r_1	B T_1
	°	"	"		"	
I	360	0.15	0.27	2188	0.23	2216
II	270	.15	.29	2216	.31	2236
III	360	.04	.35	2205	.30	2215
IV	270	.04	.33	2204	.32	2221
V	. .	.00	.34	2200	.32	2215
VI	317	0.136	0.25	2203	0.25	2230

A similar solution, excluding the dates 2352090–2270 which were affected by the change in collimation, gave

$$\left.\begin{aligned} y &= +0''.201 \;-0.603\,\eta \;+0.371\,\zeta \\ z &= -0''.253 \;-0.273\,\eta \;-0.182\,\zeta \end{aligned}\right\} (B)$$

and thence the values of r_1 and T_1 under "B" above.

It will be seen that all the values of T_1 range within the limits 2352188 and 2352236, and those of r_1 between 0″.23 and 0″.35.

The purpose of this calculation is two-fold and the inferences that may be drawn from it, in connection with the result of the direct computation of all four constants, are important. First, whatever the uncertainty as to the constants of the annual term at this epoch may be, the radius of the 14-mos. term, which according to the investigation in *A.J.* 494 ranges between 0″.05 and 0″.24, was in 1728 near the maximum. Secondly, this uncertainty cannot affect the average period of the 14-mos. term which we get by comparison of the result from Bradley with the modern observations, a range of 174 years, by more than a quarter, or probably by more than a tenth of a day; so that this element is fixed within very precise limits.

Inversely we may draw similar inferences as to the influence of possible uncertainty in the constants of the 14-mos. term upon the annual term. From (A) and (B) we have,

$$\left.\begin{aligned}\eta &= +0''.306 \;-1.589\,y \;-0.532\,z\\ \zeta &= -0''.416 \;+0.251\,y \;-1.592\,z\end{aligned}\right\}(A)$$

$$\left.\begin{aligned}\eta &= +0''.008 \;-1.230\,y \;-0.917\,z\\ \zeta &= -0''.529 \;+0.697\,y \;-1.537\,z\end{aligned}\right\}(B)$$

If now we make various hypotheses as to T_1 and r_1 covering their probable ranges we find the corresponding values of r_2 and G for the two solutions (A) and (B), as below:

			A		B	
Hyp.	T_1	r_1 ″	r_2 ″	G °	r_2 ″	G °
I	2150	0.24	0.38	11	0.48	38
II	2150	.05	.47	331	.50	6
III	2200	.24	.16	323	.18	25
IV	2200	.05	.16	326	.15	1
V	...	.00	.52	326	.52	359
VI	2197	0.14	0.31	327	0.36	9

From these results we draw the important conclusion that the major-axis of the annual ellipse in 1728 lay somewhere near the Greenwich meridian, under any probable assumption that we can make.

All the conclusions just indicated will be valuable in assisting us to a knowledge of the law of the anomalies in both terms, which were approximately developed in *A.J.* 489, 490 and 494, when the results of the observations of the next few years are at hand.

ON THE VARIABLE STAR 6684 *U VULPECULAE*,

By P. S. Yendell.

Since making the observations of this star published in *A.J.* 477, I have observed it fifty-five times during the season of 1900, and nineteen times in 1901.

At the close of the season of 1900, I made a preliminary reduction of my observations, and finding the negative residuals of the previous year continued, I assembled the observations of both years, one hundred and eleven in number, and formed from them a mean light-curve, using the period $7^d.98$, published by Pickering in H.C.O. Circular, No. 41. This was done in preparation for an examination of my results, with a view to obtaining corrections to the star's elements of variation; but the work was interrupted at the completion of the mean light-curve, and no opportunity of resuming it occurred until the present month.

My observations of the star for the present year were taken up late, at the beginning of October, and, as seen above, are few in number, in view of which, the very satisfactory mean light-curve already completed was adopted as a basis for the reduction of the observations, without incorporating them with it.

The readings from this mean curve are given in the subjoined table.

It will be noted that the character of the curve is that of the curves of most of the variables of short period, and especially resembles that of *T Vulpeculae*, the increase occupying only 2.13 days of the 7.98 days of the star's period of variation. In this it corresponds in character with the curve of Pickering, already mentioned, and distinctly differs from the symmetrical light-curves for this same star, published by Muller and Kempf, *A.N.* 3483, and Luizet, *A.N.* 3570. The conclusive manner in which this characteristic of the curve is indicated by the normals is shown by their mean departure from the curve as drawn, which is only $0^M.037$ for the twenty-three normals.

The mean light-range shown is from $7^M.00$ to $7^M.68$, as compared with Müller and Kempf's $6^M.94$ to $7^M.61$.

Readings from Mean Light-Curve.

Decrease				Increase	
d	M	d	M	d	M
0.00	7.00	+3.25	7.40	−1.98	7.67
+0.25	7.01	3.50	7.43	1.73	7.61
0.50	7.04	3.75	7.45	1.48	7.52
0.75	7.09	4.00	7.48	1.23	7.42
1.00	7.15	4.25	7.51	0.98	7.32
1.25	7.21	4.50	7.54	0.73	7.22
1.50	7.26	4.75	7.57	0.48	7.17
1.75	7.31	5.00	7.60	−0.23	7.03
2.00	7.33	5.25	7.63	0.00	7.00
2.25	7.33	5.50	7.66		
2.50	7.34	5.75	7.68		
2.75	7.35	+5.85	7.682		
+3.00	7.37				

All my observations of the star for the years 1899, 1900, and 1901 were reduced by the use of this curve, all phases

indicated by isolated single observations at a magnitude corresponding nearly to the mean maximum or minimum light being rejected, and no determinations by single curves being included.

The reductions resulted in twenty-one maxima and fifteen minima, which are shown in the following table.

OBSERVED MAXIMA AND MINIMA.

MAXIMA.

E	Local M.T.	Obs.	O−C	O−C′
80	1899 July 2.19	3	0.00	−0.02
82	18.38	1	−0.07	−0.09
83	26.43	2	0.00	−0.02
84	Aug. 3.31	3	−0.10	−0.12
85	11.84	1	+0.15	+0.43
87	27.51	1	+0.16	+0.11
88	Sept. 1.33	3	0.00	−0.02
89	11.97	1	−0.31	−0.36
90	20.24	3	−0.05	−0.07
91	28.34	1	+0.07	+0.05
96	Nov. 7.16	4	+0.29	+0.27
122	1900 June 1.97	3	−0.67	−0.70
124	18.16	2	−0.11	−0.17
129	July 28.66	1	+0.16	+0.13
131	Aug. 14.03	3	+0.57	+0.51
136	Sept. 22.83	4	+0.47	+0.44
142	Nov. 9.32	1	+0.09	+0.05
183	1901 Oct. 2.51	3	+0.10	+0.06
184	10.40	2	+0.01	−0.03
187	Nov. 2.44	3	−0.89	−0.93
188	11.38	1	+0.07	+0.03

MINIMA.

E	Local M.T.	Obs.	O−C	O−C′
86	1899 Aug. 16.65	1	−0.59	−0.61
87	25.34	1	+0.12	+0.10
90	Sept. 17.42	2	−0.74	−0.76
125	1900 June 24.18	3	−0.27	−0.30
127	July 10.91	1	+0.53	+0.50
129	26.86	2	+0.49	+0.46
136	Sept. 20.98	2	+0.75	+0.72
137	28.42	2	+0.21	+0.18
	Oct. 22.35	1	+0.20	+0.17
184	1901 Oct. 8.24	2	−0.02	−0.06
185	16.75	1	+0.51	+0.17

Dorchester, Mass., 1901 November 28.

In addition to the above maxima, the observations of MÜLLER and KEMPF, published with their announcement of the star's variability, were reduced by the use of the same curve, affording six additional maxima, available for the discussion of the elements of variation.

It was found impossible to include the nine maxima of LUIZET (*A.N.* 3570) in the discussion, as they are entirely discordant, showing residuals uniformly positive, none of them less than a day, and their mean value being $+1^d.48$.

From a comparison of the above times of maximum with the computed times resulting from a combination of PICKERING's period of $7^d.98$ with MÜLLER and KEMPF's principal epoch of 1897 Oct. 2.17 Greenwich M.T., approximate elements were assumed as follows:

$$1897 \text{ Oct. } 2.303 \text{ Greenwich M.T., } + 7^d.9798\,E$$

The residuals resulting from a comparison of the observed times of maximum with these elements are shown in the column O−C. A discussion of these residuals resulted in the following corrections:

To the Epoch, $+0^d.008$, $\pm 0^d.0009$

To the Period, $+0^d.00017$ $\pm 0^d.00005$

Applying these corrections, the resulting elements are

$$\text{Max., } 1897 \text{ Oct. } 2.311 \text{ Greenwich M.T., } + 7^d.97997\,E$$

The minimum precedes the maximum $2^d.13$.

These elements appear to be the best at present obtainable. The residuals found by a comparison with them will be found in the column O−C′.

The comparison-stars used were as follows: the magnitude-scale is that of the Potsdam Photometric *Durchmusterung*:

COMPARISON-STARS FOR *U Vulpeculae*.

		M	Light-Scale
a =	DM. +20°1210	6.64	15.3
f =	21°3489	6.97	11.0
c =	18°4137	7.04	10.1
b =	20°1179	7.25	7.4
d =	21°3863	7.49	4.1
e =	20°1215	7.79	0.0

FURTHER CHANGES IN NEBULA AROUND *NOVA PERSEI*.

The following telegraphic dispatch, received Dec. 5 from Harvard College Observatory, was sent by Prof. CAMPBELL:

"CROSSLEY photograph Dec. 4, PERRINE finds for *Nova* nebula motions condensations *A* and *B* continue. Condensation *C* motion apparently continued but form changed. Condensation *D* apparently unchanged."

CONTENTS.

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No. 514.

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CATALOGUE OF STARS CERTAINLY RECOGNIZED AS VARIABLE SINCE THE APPEARANCE OF CHANDLER'S THIRD CATALOGUE.

As preliminary to the intended publication of a new catalogue of variable stars, the Committee appointed by the Council of the *Astronomische Gesellschaft* has prepared a list of those stars which have been recognized as certainly variable since the appearance of CHANDLER'S last catalogue.* Thirty-six of these stars have already been provided by CHANDLER in the *Astronomical Journal* with their definitive designation; they have here been included for the sake of completeness. For nine other stars Mr. A. W. ROBERTS has introduced names in his recently published list of Southern Variables (*Astr. Journ.* Nos. 491–492): we have accepted the same except for two which were erroneous. From the unusually large number of the remaining stars announced as variable within the last five years, those have been first chosen whose variability has been independently confirmed by at least two observers. This condition has been waived in a few cases, and then only because the particulars communicated by a single observer placed the variability so completely beyond question that the control by a second observer could be dispensed with. For the many photographic variables discovered recently at the Harvard College Observatory we have assumed the variability to be sufficiently substantiated if the plates were examined by several persons, and also if the indicated amplitude of the fluctuation amounted to at least a full magnitude.

In the following catalogue the first column contains CHANDLER'S number. In all cases where two stars stand so near in right-ascension as to require the same number, the immediately preceding or following number has not been assigned to the star last discovered, as in CHANDLER'S catalogue, but the correct number with the letter *a*, *b* . . . . appended.

The second column contains the provisional notation recently used in the *Astr. Nachrichten*.

Column 3 gives the definitive names of the variables. The boundaries of the constellations have been taken in conformity with ARGELANDER'S Uranometry for the northern hemisphere, and with GOULD'S for the southern.

Columns 4 and 5 give R.A. and Decl. for 1900.0, and the two following columns the precession for 1900.

Columns 8 and 9 contain, for the convenience of observers, the chart-positions: that is, for the stars from the north pole to declination $-22°$ incl., the places for 1855 (*Nördliche und Südliche Bonner Durchmusterung*), and for stars from $-23°$ to the south pole the places for 1875 (Cordoba charts). When the variable is contained in no star-catalogue the place given depends on the data, more or less certain, furnished by the observer. Very doubtful positions are indicated as such by giving the declinations only to whole minutes. An accurate determination of the place of these stars is much to be desired.

The columns 10 and 11 give the maximum and minimum magnitudes so far as they are known with moderate certainty. The letters in the last column, v. (visual) and ph. (photographic), allow it to be seen whether the foregoing magnitudes were obtained by direct observation or depend on the photographic plates.

In general it is to be remarked with regard to this list that it has been prepared in duplicate independently by Messrs. HARTWIG and MULLER. Both drafts were then compared star by star with one another, and all doubtful cases subjected to joint scrutiny. This furnishes a reasonable assurance of the completeness of the catalogue and the trustworthiness of the numerical data.

In publishing the following catalogue, we make the earnest request that observers of variable stars will devote especial attention to the objects therein contained, in order that we may be in position to provide in the new catalogue accurate information as to the period and course of the light-variations.

The Committee on Publication of a Catalogue of Variable Stars,

DUNÉR, HARTWIG, MULLER, OUDEMANS.

* *A.J.*, Vol. XVI, pp. 145–172.

Number Ch.	Provis. Notation A.N.	Name	Position for 1900.0 R.A.	Position for 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-place north of −23 : eq. 1855, south of −23 : eq. 1875 R.A.	Chart-place Decl.	Magnitude Max.	Magnitude Min.	
			h m s	° ′	s	′	h m s	° ′			
65	3.1900	X Andromedae	0 10 50	+46 27	+3.11	+0.33	0 8 30	+46 12	8.9	–	v
147	. .	RR Sculptoris	21 32	−38 36.5	+2.96	+0.33	23 18	−38 44.7	9.10	<11	v
154	. .	T Phoenicis	25 35	−46 57.7	+2.91	+0.33	24 23	−47 6.1	9	<12	ph
169	. .	W Sculptoris	28 15	−33 25.6	+2.96	+0.33	27 1	−33 33.9	8.9	<11	ph
188	9.1900	Y Cephei	31 19	+79 18.4	+4.08	+0.33	28 18	+79 33.5	9	12	v
210	. .	Z Sculptoris	35 2	−34 30.2	+2.93	+0.33	33 49	−34 38.5	6.7	7.8	v
268	. .	V Andromedae	44 39	+35 6.5	+3.25	+0.33	42 13	+34 51.8	8.9	14	v
268a	. .	X Sculptoris	44 42	−35 27.8	+2.88	+0.33	43 30	−35 36.0	9	<13	v
276	69.1901	RR Andromedae	45 56	+33 50	+3.24	+0.33	43 30	+33 35	10	<11	v
562	23.1900	Y Andromedae	1 33 45	+38 50.2	+3.50	+0.31	1 31 8	+38 36.3	9.10	<12	v
787	. .	W Andromedae	2 11 14	+43 50.5	+3.77	+0.28	2 8 25	+43 37.8	8	<11	v
854	. .	S Horologii	22 22	−60 1.2	+1.72	+0.27	21 39	−60 8.0	9.10	<12	ph
1066	. .	T Horologii	57 40	−51 2.2	+1.92	+0.24	56 52	−51 8.2	8	<11	v
1205	68.1901	Y Persei	3 20 55	+43 19.6	+1.05	+0.21	3 17 52	+43 40.0	8.9	9.10	v
1332	. .	S Fornacis	41 56	−24 42.3	+2.56	+0.19	40 52	−24 47.0	5.6	9	v
1375	. .	X Persei	49 8	+30 45.1	+3.71	+0.18	46 20	+30 36.8	6	7	v
1438	. .	V Eridani	59 45	−15 59.9	+2.74	+0.17	57 42	−16 7.5	8	9	ph
1484	. .	W Eridani	4 7 19	−25 23.5	+2.51	+0.16	4 6 17	−25 27.5	8	<12	ph
1752	. .	U Leporis	52 1	−21 22.7	+2.57	+0.10	50 5	−21 27.2	9	10	v
1913	. .	S Doradus	5 18 55	−69 21.1	−0.42	+0.06	5 19 6	−69 22.5	8	9.10	ph
1921	. .	W Aurigae	20 9	+36 48.9	+4.06	+0.06	17 6	+36 46.2	9	<12	v
1929	71.1901	Y Aurigae	21 32	+42 21.2	+4.27	+0.06	18 20	+42 18.5	8.9	9.10	v
2000	6.1900	RR Tauri	33 17	+26 19.0	+3.73	+0.04	30 30	+26 17.1	9	<12	ph
2038	. .	Y Tauri	39 42	+20 39.2	+3.57	+0.03	37 1	+20 37.8	6.7	8.9	v
2080a	4.1900	Z Tauri	46 41	+15 46	+3.45	+0.02	44 6	+15 45	9	<11	v
2186	8.1900	X Aurigae	6 4 25	+50 14	+4.68	0.00	6 0 54	+50 14	8	11	v
2328	. .	Z Monocerotis	28 2	− 8 18.2	+2.87	−0.04	25 53	− 8 46.4	9	10	ph
2335	. .	W Geminorum	29 14	+15 24.5	+3.41	−0.04	26 39	+15 26.3	7	8	v
2376	. .	S Lyncis	35 56	+58 0.5	+5.19	−0.05	32 3	+58 2.8	9.10	<12	v
2404	. .	X Geminorum	40 43	+30 22.6	+3.85	−0.06	37 50	+30 25.2	8	<11	v
2468	21.1900	Y Monocerotis	51 19	+11 22.4	+3.33	−0.07	48 49	+11 25.6	8.9	<11	ph
2475	.	X Monocerotis	52 25	− 8 56.0	+2.87	−0.08	50 16	− 8 52.7	8	10	ph
2554	. .	S Can. maj.	7 5 43	−32 45.9	+2.25	−0.09	7 4 46	−32 43.6	9	10	v
2565	. .	R Volantis	7 26	−72 51.3	−1.07	−0.10	7 53	−72 48.9	8	<10	v
2690	19.1900	Z Puppis	28 18	−20 26.7	+2.61	−0.12	26 21	−20 21.1	8.9	<11	v
2781	. .	RR Puppis	43 32	−41 7.5	+2.02	−0.15	42 41	−41 4.0	9.10	10.11	v
2890	. .	RT Puppis	8 1 44	−38 29.4	+2.16	−0.17	8 0 50	−38 25.2	8	10	ph
2899	. .	RU Puppis	3 10	−22 37.4	+2.59	−0.17	1 14	−22 29.7	9.10	11.12	ph
2933	. .	Y Puppis	8 51	−34 50.3	+2.30	−0.18	7 53	−34 45.9	8.9	10	v
2935	. .	RS Puppis	9 11	−34 16.6	+2.30	−0.18	8 16	−34 12.2	7	8	v
3021	1.1901	R Chamaeleontis	21 2	−76 1.8	−1.27	−0.20	24 33	−75 56.9	–	–	–
3028	. .	RT Hydrae	21 45	− 5 59.1	+2.96	−0.20	22 32	− 5 50.3	8	9.10	ph
3089	52.1901	RV Hydrae	34 53	− 9 14.0	+2.90	−0.21	32 43	− 9 4.6	7.8	9	ph
3179	. .	X Cancri	49 45	+17 36.7	+3.39	−0.22	47 13	+17 46.8	6.7	7.8	v
3241	. .	S Pyxidis	9 0 41	−24 41.4	+2.64	−0.24	59 35	−24 35.5	8.9	11	ph
3247	70.1901	V Urs. maj.	1 7	+51 32	+1.28	−0.24	57 54	+51 42	9.10	10.11	v
3321	. .	RU Carinae	13 27	−65 18.9	+1.10	−0.25	9 13 0	−65 42.6	11	12	ph
3349	5.1901	RW Carinae	18 12	−68 20	+0.89	−0.25	17 50	−68 14	–	–	–
3418a	. .	T Antliae	29 43	−36 10.4	+2.48	−0.26	28 41	−36 3.8	8.9	9.10	ph
3482	. .	RR Hydrae	40 24	−23 33.5	+2.74	−0.27	39 16	−23 26.7	8.9	<12	ph
3519	. .	Y Hydrae	46 27	−22 33.0	+2.77	−0.28	44 22	−22 20.4	8	10	ph
3548	6.1901	X Velorum	51 21	−41 6.8	+2.45	−0.28	50 19	−40 59.7	–	–	–
3573	. .	RV Carinae	55 33	−63 25.0	+1.69	−0.29	54 51	−63 17.9	9.10	<11	ph
3649	. .	U Urs. maj.	10 8 11	+60 28.9	+1.19	−0.29	10 5 5	+60 42.1	7	8	ph
3669	. .	W Velorum	11 31	−53 58.9	+2.23	−0.30	10 35	−53 51.5	9	<11	v
3785	7.1901	U Antliae	30 47	−39 2.7	+2.66	−0.31	29 40	−38 55.0	–	–	–
3799	8.1901	RX Carinae	33 12	−61 48	+2.15	−0.31	32 18	−61 40	10	<12	ph
3845	. .	RT Carinae	40 54	58 53.5	+2.32	−0.31	39 56	−58 15.7	9.10	10.11	ph
3879	. .	RS Hydrae	10 46 33	−28 6.1	+2.85	−0.32	10 45 22	−27 58.2	8.9	<11	v

Number Ch.	Provis. Notation A.N.	Name	Position for 1900.0 R.A.	Position for 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-place north of −23°: eq. 1855 south of −23°: eq. 1875 R.A.	Chart-place Decl.	Magnitude Max.	Magnitude Min.	
			h m s	° ′	s	′	h m s	° ′			
3978	9.1901	RW Centauri	11 2 56	−54 34.8	+2.61	−0.32	11 1 51	−54 26.8	–	–	–
4057	. .	RS Centauri	16 6	−61 19.6	+2.61	−0.33	15 1	−61 11.3	9	<12	v
4216	. .	Z Hydrae	42 37	−32 42.8	+3.00	−0.33	41 22	−32 34.5	8.9	9.10	v
4318	. .	RX Virginis	59 38	− 5 12.7	+3.07	−0.33	57 20	− 4 57.7	7	9	ph
4333	. .	RW Virginis	12 2 6	− 6 12.2	+3.07	−0.33	59 48	− 5 57.2	7	8	ph
4345	. .	RU Centauri	4 12	−44 52.1	+3.10	−0.33	12 2 55	−44 43.7	9	10	ph
4471	. .	T Canum Venat.	25 15	+32 3.4	+2.98	−0.33	23 1	+32 18.3	8.9	12	v
4481	. .	U Crucis	26 50	−57 1.7	+3.31	−0.33	25 27	−56 53.4	10	<13	ph
4573	. .	RU Virginis	42 13	+ 4 41.5	+3.05	−0.33	39 56	+ 4 56.3	8	12	v
4665	. .	RT Virginis	57 33	+ 5 43.4	+3.04	−0.32	55 17	+ 5 58.0	8.9	10	ph
4696	7.1900	RV Virginis	13 2 40	−12 37.8	+3.15	−0.32	13 0 18	−12 23.3	10	<14	ph
4754	25.1900	U Octantis	12 20	−83 42.0	+6.83	−0.32	9 32	−83 34.1	7.8	10	v
4867	. .	RV Centauri	31 8	−55 57.9	+3.84	−0.31	29 32	−55 50.2	9	<13	ph
4898	10.1901	RY Virginis	36 19	−18 37.7	+3.26	−0.31	33 53	−18 24.0	–	–	–
4935	. .	RT Centauri	42 30	−36 21.8	+3.50	−0.30	41 2	−36 14.2	8	<12	v
5013	. .	θ Apodis	55 35	−76 18.8	+5.69	−0.29	53 13	−76 11.5	5.6	6.7	v
5050	. .	Z Bootis	14 1 40	+13 58.5	+2.90	−0.29	59 29	+14 11.5	10	13	ph
5075	. .	RU Hydrae	5 48	−28 24.8	+3.45	−0.29	14 4 22	−28 17.6	8	12	v
5099	. .	RR Centauri	9 55	−57 23.3	+4.19	−0.28	8 10	−57 16.2	7.8	8	v
5221	. .	RV Librae	30 14	−17 35.9	+3.33	−0.27	27 45	−17 23.9	8	9.10	ph
5355	11.1901	V Lupi	52 33	−53 0.4	+4.28	−0.24	50 46	−52 54.3	–	–	–
5451	12.1901	W Lupi	15 8 30	−50 25	+4.26	−0.23	15 6 44	−50 19	–	–	–
5520	13.1901	R Circini	20 0	−57 22	+4.67	−0.21	18 3	−57 17	–	–	–
5572	. .	R Normae*	28 45	−49 10.4	+4.29	−0.21	26 58	−49 5.2	7	11	v
5608	. .	U Normae	34 37	−54 59.4	+4.61	−0.20	32 42	−54 54.4	8.9	9.10	v
5618	. .	T Normae	36 21	−54 40.0	+4.59	−0.20	34 27	−54 35.1	7	<11	v
5727	. .	U Lupi	54 29	−29 38.3	+3.72	−0.17	52 56	−29 33.9	9	11	v
5752	. .	RZ Scorpii	58 37	−23 49.5	+3.58	−0.17	57 8	−23 45.2	8.9	<10	v
5775	. .	U Serpentis	16 2 31	+10 11.9	+2.86	−0.17	16 0 23	+10 19.4	9	<12	ph
5776a	14.1901	V Normae	2 39	−48 58.2	+4.41	−0.16	0 49	−48 54.1	–	–	–
5796	. .	RX Scorpii	5 56	−24 38.4	+3.61	−0.16	4 26	−24 34.4	9	<12	ph
5796a	. .	RU Herculis	6 2	+25 19.8	+2.52	−0.16	4 9	+25 27.0	7	<12	v
5814	15.1901	W Normae	9 0	−52 21.1	+4.60	−0.16	7 5	−52 17.2	–	–	–
5867	16.1901	X Normae	17 45	−51 41.7	+4.60	−0.14	15 50	−51 38.1	11	<12	ph
5941	. .	ST Scorpii	30 14	−31 1.8	+3.82	−0.13	28 39	−30 58.5	8	10	v
5965	. .	SU Scorpii	34 12	−32 11.0	+3.85	−0.12	32 36	−32 7.9	8	9	v
5999	17.1901	V Triang. austr.	39 51	−67 36.0	+6.12	−0.11	37 18	−67 33.3	–	–	–
6019	. .	RR Ophiuchi	43 11	−19 17.3	+3.51	−0.11	40 33	−19 12.3	8	<11	v
6053	. .	SS Scorpii	48 46	−32 27.6	+3.89	−0.10	47 9	−32 25.0	7.8	9.10	v
6086	. .	T Arae	54 22	−54 55.4	+4.90	−0.10	52 20	−54 53.0	10	11	ph
6100	. .	RV Herculis	56 45	+31 22.3	+2.29	−0.09	55 2	+31 26.4	9	<12	v
6161	. .	RT Herculis	17 6 46	+27 10.8	+2.40	−0.08	17 4 58	+27 14.3	9	<12	v
6229	18.1901	SW Scorpii	18 8	−43 43.8	+4.33	−0.06	16 20	−43 42.3	–	–	–
6328	. .	V Pavonis	34 41	−57 40.4	+5.17	−0.04	32 32	−57 39.4	8	10	ph
6364	19.1901	SX Scorpii	40 45	−35 39.7	+4.03	−0.03	39 4	−35 39.0	–	–	–
6366	. .	W Pavonis	41 5	−62 22.4	+5.62	−0.03	38 45	−62 21.6	9	<13	ph
6370	. .	SV Scorpii	41 36	−35 39.9	+4.03	−0.03	39 56	−35 39.1	9	<11	v
6386	. .	RY Scorpii	44 16	−33 40.5	+3.96	−0.02	42 37	−33 39.9	7.8	9	v
6389	20.1901	RS Ophiuchi	44 50	− 6 40.1	+3.23	−0.02	42 25	− 6 39.1	–	–	–
6394	. .	U Arae	45 41	−51 39.8	+4.76	−0.02	43 42	−51 39.2	9	<12	ph
6404a	21.1901	V Arae	47 17	−48 16.8	+4.57	−0.02	45 23	−48 16.3	9.10	<12	ph
6416	22.1901	W Arae	49 15	−49 46.8	+4.65	−0.02	47 19	−49 46.4	–	–	–
6429	24.1900	S Arae	51 27	−49 25.2	+4.63	−0.01	49 32	−49 24.9	9.10	11	v
6452	. .	RY Herculis	55 25	+19 29.3	+2.60	−0.01	53 28	+19 29.7	9	<12	v
6458	1.1900	V Draconis	56 17	+54 52.8	+1.17	−0.01	55 24	+54 53.0	9	11	v
6469	23.1901	W Coron. austr.	58 11	−39 20.5	+4.17	0.00	56 30	−39 20.4	–	–	–
6490	. .	RW Herculis	18 1 42	+22 3.8	+2.53	0.00	59 48	+22 3.8	9	<12	v
6496	24.1901	X Coron. austr.	2 36	−45 26	+4.43	0.00	18 0 45	−45 26	–	–	–
6523	25.1901	Y Coron. austr.	18 7 12	−42 53	+4.31	+0.01	18 5 24	−42 53	–	–	–

* In accordance with GILL's suggestion (A.J. 442), the name correctly assigned by GOULD is retained.

Number Ch.	Provis. Notation A. N.	Name	Position for 1900.0 R.A.	Position for 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-place (north of −23°: eq. 1855; south of −23°: eq. 1875) R.A.	Chart-place Decl.	Magnitude Max.	Magnitude Min.	
			h m s	° ′	s		h m s	° ′			
6549	. .	W Lyrae	18 11 28	+36 38.4	+2.08	+0.02	18 9 51	+36 37.4	8.9	12	v
6594	26.1901	T Telescopii	19 0	−49 12	+4.61	+0.03	17 1	−49 43	11	<13	ph
6628	27.1901	SS Sagittarii	21 39	−16 58.0	+3.48	+0.04	22 2	−16 59.5	9.10	–	v
6636a	. .	RX Herculis	26 1	+12 32.5	+2.78	+0.04	23 56	+12 30.9	7	7.8	v
6676	10.1900	RZ Herculis	32 44	+25 57.9	+2.43	+0.05	30 55	+25 55.8	9	12	ph
6685	12.1900	Y Lyrae	34 12	+43 51.8	+1.80	+0.05	32 51	+13 49.6	10.11	<12	ph
6686	. .	U Coron. austr.	34 18	−37 55.6	+4.10	+0.05	32 35	−37 56.8	8.9	11	v
6721	. .	V Coron. austr.	40 12	−38 15.7	+4.11	+0.06	38 59	−38 17.2	9	<10	v
6749	28.1901	S Scuti	44 55	− 8 1.3	+3.26	+0.06	42 28	− 8 1.1	6	8	v
6773	73.1901	U Scuti	48 52	−12 43.8	+3.37	+0.07	46 20	−12 46.9	9	9.10	v
6780	29.1901	T Scuti	50 0	− 8 18.4	+3.26	+0.07	47 34	− 8 21.6	9.10	–	v
6815	30.1901	ST Sagittarii	55 55	−12 54.1	+3.37	+0.08	53 22	−12 57.6	7.8	<10	v
6816	15.1900	Z Lyrae	56 0	+34 19.1	+2.17	+0.08	54 22	+34 15.5	9	11	ph
6826	34.1901	SU Sagittarii	57 43	−22 51.3	+3.62	+0.08	55 0	−22 55.0	8.9	–	v
6843	32.1901	U Telescopii	19 0 30	−49 1	+4.56	+0.09	58 36	−49 6	–	–	–
6894	. .	X Lyrae	8 58	+26 36.1	+2.43	+0.10	19 7 9	+26 31.7	8.9	9.10	v
6896	72.1901	RS Lyrae	9 18	+33 14.6	+2.24	+0.10	7 37	+33 10.2	10	<12	v
6900a	. .	U Draconis	9 57	+67 6.6	+0.05	+0.10	9 54	+67 2.1	9.10	<12	v
6900b	. .	RY Sagittarii	10 1	−33 41.9	+3.92	+0.10	8 23	−33 44.3	6	<11	v
6903a	33.1901	V Telescopii	10 34	−50 37.5	+4.62	+0.10	8 38	−50 40.0	9	10.11	ph
6920	2.1901	TZ Cygni	13 22	+50 0	+1.56	+0.10	12 12	+49 55	9.10	11	v
6974	34.1901	RR Lyrae	22 17	+42 35.5	+1.92	+0.12	20 54	+42 30.2	7	8	ph
7008	. .	UV Cygni	28 3	+43 25.5	+1.90	+0.12	26 38	+43 19.9	7.8	9	v
7019	1.1901	TY Cygni	29 54	+28 6.2	+2.41	+0.13	28 2	+28 0.5	10	<14	ph
7034	. .	U Vulpeculae	32 16	+20 6.6	+2.62	+0.13	30 18	+20 0.8	7	7.8	v
7040	. .	RT Aquilae	33 17	+11 29	+2.82	+0.13	31 10	+11 23	9	<13	v
7056	17.1900	RV Aquilae	35 57	+ 9 41.5	+2.86	+0.14	33 48	+ 9 35.4	9	<12	v
7063	. .	TT Cygni	37 7	+32 23.1	+2.30	+0.14	35 24	+32 17.0	7.8	<9	v
7085a	. .	SU Cygni	40 19	+29 1.1	+2.40	+0.11	39 0	+28 55.0	6.7	7.8	v
7096	. .	SY Cygni	42 43	+32 27.6	+2.31	+0.14	41 0	+32 21.1	10	<12	ph
7099	35.1901	W Telescopii	43 6	−50 15	+4.52	+0.15	41 12	−50 19	–	–	–
7100	13.1900	TU Cygni	43 19	+18 19.3	+1.70	+0.15	42 3	+18 12.7	9	<13	v
7190	. .	S Telescopii	58 25	−55 50.1	+4.78	+0.16	56 25	−55 54.2	9	11	ph
7220a	. .	X Pavonis	20 3 21	−60 13.8	+5.09	+0.17	20 1 14	−60 18.1	9	10	ph
7223	. .	SW Cygni	3 50	+46 0.6	+1.89	+0.17	2 25	+45 52.9	8.9	11.12	v
7239	. .	SV Cygni	6 28	+47 31.6	+1.83	+0.17	5 5	+47 26.7	8	9	v
7244	. .	RW Aquilae	7 16	+15 45.7	+2.75	+0.17	5 42	+15 37.8	8.9	9.10	v
7248	. .	RU Aquilae	8 3	+12 41.8	+2.82	+0.18	5 56	+12 33.8	9	11	v
7251	. .	RZ Sagittarii	8 30	−44 42.6	+4.19	+0.18	6 45	−44 47.0	9	<11	ph
7267	36.1901	X Telescopii	11 12	−52 56	+4.56	+0.18	9 18	−53 1	10.11	13	ph
7268	. .	RT Capricorni	11 15	−21 37.5	+3.52	+0.18	8 37	−21 45.6	7.8	10	v
7269	. .	SX Cygni	11 33	+30 46.0	+2.40	+0.18	9 45	+30 37.9	8.9	10	v
7277	37.1901	Y Telescopii	12 52	−51 0.9	+4.45	+0.18	11 0	−51 5.5	8	10	ph
7331	. .	T Microscopii	21 50	−28 35.1	+3.67	+0.19	20 18	−28 40.2	7.8	8.9	ph
7336	. .	U Microscopii	22 36	−40 45.0	+4.01	+0.19	20 55	−40 49.7	8.9	<12	ph
7378	2.1900	SZ Cygni	29 38	+46 15.6	+1.96	+0.20	28 10	+46 6.5	8	9.10	v
7379	. .	ST Cygni	29 55	+54 37.6	+1.58	+0.20	28 44	+54 28.5	9	14	v
7380	16.1900	TV Cygni	30 2	+46 13.3	+1.97	+0.20	28 34	+46 4.2	9	9.10	v
7538	.	RR Capricorni	56 23	−27 29.0	+3.57	+0.23	54 54	−27 34.8	9	<10	v
7539	22.1900	TX Cygni	56 25	+42 12.1	+2.20	+0.23	54 46	+42 2.0	8.9	10	v
7570	. .	RS Capricorni	21 1 10	−16 49.1	+3.36	+0.24	59 9	−17 0.0	8	9.10	ph
7571a	20.1900	TW Cygni	1 45	+29 0.3	+2.55	+0.24	59 50	+28 49.6	9	<12	ph
7582	. .	X Cephei	3 39	+82 39.9	−1.15	+0.24	21 6 39	+82 29.0	10	<12	v
7594	. .	RS Aquarii	5 45	− 4 26.6	+3.14	+0.24	3 23	− 4 37.1	9.10	<14	v
7610	18.1900	R Equulei	8 24	+12 23.4	+2.87	+0.24	6 15	+12 12.1	8	<10	v
7619	. .	RR Aquarii	9 49	− 3 18.6	+3.12	+0.25	7 28	− 3 29.6	8.9	13	v
7641	. .	T Indi	13 34	−45 26.6	+3.98	+0.25	11 55	−45 32.8	7	9	ph
7658	. .	X Pegasi	16 46	+14 1.6	+2.85	+0.25	14 8	+13 50.3	9	<12	v
7685	. .	S Microscopii	21 20 48	−30 17.0	+3.57	+0.26	21 19 19	−30 23.4	8	<11	–

Number Ch.	Provis. Notation A.N.	Name	Position for 1900.0 R.A.	Position for 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-place north of −23°: eq. 1855, south of −23°: eq. 1875 R.A.	Chart-place Decl.	Magnitude Max.	Magnitude Min.	
			h m s	° ′	s	′	h m s	° ′			
7774	38.1901	UU Cygni	21 35 12	+42 45	+2.34	+0.27	21 33 57	+42 33	–	–	–
7793	. .	SS Cygni	38 47	+43 7.6	+2.35	+0.27	37 1	+42 55.4	7	11	v
7961	14.1900	Y Pegasi	22 6 47	+13 51	+2.92	+0.29	22 4 36	+13 38	9.10	<10	v
8026	39.1901	RT Aquarii	17 41	−22 34.0	+3.31	+0.30	15 12	−22 47.5	9.10	–	v
8027	. .	T Lacertae	17 54	+33 52.3	+2.69	+0.30	15 53	+33 38.8	9	<12	v
8043	40.1901	T Pisc. austr.	20 31	−29 35.4	+3.39	+0.30	19 7	−29 42.9	–	–	–
8302	. .	Y Sculptoris	23 3 40	−30 40.5	+3.27	+0.32	23 2 18	−30 48.6	8	9	v
8395	66.1901	RU Aquarii	19 10	−17 52.1	+3.15	+0.33	16 48	−18 6.9	8.9	9.10	ph
8453	41.1901	Z Andromedae	28 51	+48 16.0	+2.87	+0.33	26 43	+48 1.1	–	–	–
8518	. .	Z Cassiopeae	39 40	+56 1.6	+2.90	+0.33	37 30	+55 46.6	9.10	<10	v
8562	. .	Z Aquarii	47 5	−16 24.7	+3.09	+0.33	44 45	−16 39.7	8	9.10	ph
8584	5.1900	RR Cassiopeae	50 38	+53 10	+3.00	+0.33	48 24	+52 55	9	11	v
8610	42.1901	Z Pegasi	54 59	+25 20.6	+3.06	+0.33	52 41	+25 5.6	9	–	v
8629	. .	Y Cassiopeae	23 58 10	+55 7.3	+3.06	+0.33	23 55 53	+54 52.3	9.10	<13	ph
					New Stars.						
1226	3.1901	Nova Persei	3 24 24	+43 33.7	+1.06	+0.21	3 21 22	+43 24.2	1	–	v
6817	.	Nova Sagittarii	18 56 13	−13 18.4	+3.38	+0.08	18 53 41	−13 22.0	1.5	–	v
6932	11.1901	Nova Aquilae	19 15 16	− 0 19.2	+3.08	+0.11	19 12 57	− 0 21.1	7	–	ph

COMPARISON OF METHODS FOR THE REDUCTION OF STAR-PHOTOGRAPHS.

By HAROLD JACOBY.

The appearance in *A.J.* 512 of Dr. SCHLESINGER's reply to Mr. HINKS's article in *A.J.* 475 has led me to communicate certain calculations that seem to throw additional light on the question at issue. Mr. HINKS has certainly taken a step in the right direction in applying TURNER's method to a set of plates that had already been reduced by another method. Surely, when we have to decide between two methods of reducing astronomical observations, the best thing to do is to use both methods on the same example. But this proceeding will be much more instructive, if we can take as the example a set of observations that have already been reduced completely by a third method quite different from either of the other two.

We can apply this desirable test to my method and to TURNER's by reducing the plate discussed by MM. HENRY in *Bull. Com. Perm.*, Tome II, fasc. 2. This will furnish the desired solution of one and the same problem by three different independent methods; whereas in Mr. HINKS's paper the problem discussed is perhaps not strictly identical with that treated by Dr. SCHLESINGER. For Mr. HINKS has made a direct inter-comparison of the plates themselves, whereas Dr. SCHLESINGER sought in the first place a comparison of the photographic results with meridian observations. In discussing MM. HENRY's plate here I shall therefore endeavor to solve, both by TURNER's method and my own, exactly the same problem.

The plate in question was made at Paris (latitude 48° 50′) 1891 Dec. 2, at an hour-angle of $-0^h\,9^m$. The position of the center was

Right-ascension, $1^h\,4^m$: Declination, $+24°$

Eleven stars were used; their positions from meridian observations and their measured rectangular coordinates (reduced to seconds of arc with an approximate scale-value) are given in the following Table I.

TABLE I. DATA OF THE PLATE.

Star	From Meridian Observations Right-Ascen.	From Meridian Observations Declination	From the Plate Meas'd X	From the Plate Meas'd Y
	h m s	° ′ ″	″	″
1	0 59 31.38	24 25 18.2	−3665.36	+1517.35
2	0 59 59.37	23 56 51.1	−3295.56	− 191.70
3	1 1 1.05	24 22 9.7	−2443.12	+1320.37
4	1 1 41.71	24 18 22.6	−1888.50	+1093.05
5	1 2 4.61	24 26 21.5	−1572.99	+1569.09
6	1 3 52.92	23 55 30.1	− 95.11	− 283.20
7	1 4 14.86	23 45 42.1	+ 208.92	−2671.86
8	1 4 16.12	23 59 5.7	+ 222.84	− 68.32
9	1 6 57.04	24 0 2.9	+2427.12	− 5.19
10	1 7 55.00	24 55 49.6	+3199.59	+3347.08
11	1 8 7.14	24 28 32.3	+3376.76	+1710.60

With the data of Table I, plate-constants were determined by least-squares both by TURNER's method and mine. The eleven stars were then treated as unknown stars, and

their right-ascensions and declinations computed from the photographic measures with the plate-constants thus determined. These right-ascensions and declinations from the photographs are therefore directly comparable with each other, and with those published by MM. Henry. Such a comparison is set down in Table II.

Table II. Comparison of Reduction by MM. Henry with Reductions by Methods of Turner and Jacoby.

Star	Right-Ascension Computed from photographic measures					Declination Computed from photographic measures				
	Henry	Jacoby	Turner	H.–J.	H.–T.	Henry	Jacoby	Turner	H.–J.	H.–T.
		"	"	"	"	° ′ ″	"	"	"	"
1	14 52 51.66	51.67	51.69	−0.01	−0.03	24 25 17.66	17.65	17.72	+0.01	−0.06
2	14 59 50.84	50.85	50.88	− .01	− .04	23 56 51.30	51.30	51.31	.00	− .01
3	15 15 14.97	14.97	15.01	.00	− .04	24 22 8.68	8.69	8.73	− .01	− .05
4	15 25 24.84	24.85	24.86	− .01	− .02	21 18 23.91	23.90	23.94	+ .01	− .03
5	15 31 9.36	9.37	9.37	− .01	− .01	24 26 21.17	21.16	21.22	+ .01	− .05
6	15 58 13.00	13.01	13.03	− .01	− .03	23 55 31.01	31.01	30.99	.00	+ .02
7	16 3 44.16	44.14	44.20	+ .02	− .04	23 15 41.90	41.90	41.81	.00	+ .09
8	16 4 1.05	1.05	1.05	.00	.00	23 59 5.86	5.85	5.83	+ .01	+ .03
9	16 44 14.60	14.58	14.56	+ .02	+ .04	24 0 2.38	2.37	2.34	+ .01	+ .04
10	16 58 45.81	45.80	45.72	+ .01	+ .09	21 55 49.79	49.78	49.82	+ .01	− .03
11	17 1 47.70	47.68	47.63	+0.02	+0.07	24 28 32.14	32.13	32.13	+0.01	+0.01

If these three methods are all correct mathematically, and differ only in convenience of application, they should all produce the same results, just as a transit observation gives the same right-ascension by Mayer's or Bessel's transit-formulas. Since identical meridian places and measured coordinates have been used in all three methods, it becomes merely a question of correct mathematics and correct arithmetic to bring out similarly identical photographic right-ascensions and declinations.

The columns H.–J. and H.–T. of Table II show that Henry's method and mine agree very much better than Henry's method and Turner's. Indeed, the discordances between Henry's results and mine are no larger than might properly be ascribed to dropping decimals at various points in these rather complicated calculations; and Prosper Henry has himself stated * that my method agrees exactly with his in its results. So we are irresistibly driven to the conclusion that Henry's method and mine are both mathematically correct, or very nearly so; for though this conclusion is not strictly logical, because Henry's method and mine might agree exactly, and still both be wrong, yet it is certainly a conclusion possessing probability of a very high order.

The cause of these differences between Turner's method and mine is not at all mysterious; it becomes apparent at once if we compare formulas. In both methods the first step is the computation from the meridian places of rectangular coordinates, such as they would be on a perfectly oriented plate. These coordinates Turner calls "standard coordinates"; his formulas for computing them give identical results with mine, and differ only in facility of application. The next step in my method is the computation of refraction-coefficients M_x, N_x, M_y, N_y, by formulas in *A.J.* 387. Now let

X, Y, be the measured coordinates,

x, y, be the computed or "standard" coordinates,

and put

$$X' = X + M_x X + N_x \cos\delta\, Y \quad , \quad Y' = Y + M_y \sec\delta\, X + N_y Y$$

so that X' and Y' are the measured coordinates corrected for refraction. Turner now finds his six plate-constants a, b, c, d, e, f: and I find my four plate-constants p, r, k, c', from the equations

Turner (1)	Jacoby (2)
$aX + bY + c + (X - x) = 0$	$pX + rY + k + (X' - x) = 0$
$dX + eY + f + (Y - y) = 0$	$-rX + pY + c' + (Y' - y) = 0$

Now if both methods are correct, so that the two pairs of equations can co-exist, it is clear that we must have

$$a = p + M_x \qquad d = -r + M_y \sec\delta$$
$$b = r + N_x \cos\delta \qquad e = p + N_y$$
$$c = k \qquad f = c'$$

From these equations we derive the following relations between Turner's constants:

$$\left.\begin{aligned} a - e &= M_x - N_y \\ b + d &= M_y \sec\delta + N_x \cos\delta \end{aligned}\right\} \quad (3)$$

The right-hand members of these equations are functions of the refraction-constant, the latitude, and the hour-angle and declination of the plate center. All these are known with sufficient precision to compute the small differential refraction coefficients M_x, etc., practically free from error; so that we may regard $a - e$ and $b + d$ as given quantities in our problem. The numerical values of a, b, d and e, resulting from the least-squares solution by Turner's method, ought to satisfy exactly the relations (3); but

* *Bull. Com. Perm.*, Tome III, p. 18, last line.

owing to errors of observation they will not in general do this. In the present case the numerical amounts by which TURNER's constants fail to satisfy the relations (3) are

For $a-e$, .000039 ; For $b+d$, .000077

These numbers are small, as, indeed, are the differences in Table II between TURNER's results and mine; and it is easy to explain why they are not larger. If the errors of observation in the photographic measures were absolutely *nil*, the least-squares solutions would produce values of the plate-constants giving identical residuals from equations (1) and (2). These values would then satisfy equations (3) exactly, and TURNER's method would give final results agreeing exactly with mine. It follows that the high accuracy with which the plate measures have been made at Paris is favorable to making TURNER's discordances small in the present instance. I shall return to this point further on.

It may be urged that the relations (3) need not be satisfied exactly by TURNER's constants, because my refraction formulas may be incorrect. But if this be the case, there must exist some other values for M_r, etc., which are correct; and it is certain that if any relations exist similar to (3), having right-hand members containing known quantities only, there is a theoretical objection to TURNER's results. A legitimate application of the method of least-squares to TURNER's observation-equations requires that the unknowns a, b, c, d, e, f, shall be independent variables. Any relation of the form (3) involving a mutual inter-dependence of these unknowns is forbidden by theory.* If such a relation exists, a solution by the method of correlates is in order; and such a solution should give results rigidly accordant with mine.

Nor is it sufficient to defend TURNER's method on the ground that it leads to errors small enough to be negligible. As we have seen, these errors will always be of the same order of magnitude as the errors of the photographic observations themselves; and it is an impregnable principle of astronomy that methods of reduction should never introduce errors of the same order as the errors of observation. Moreover, the amount of labor involved in TURNER's least-squares solution with six unknowns is considerably more than the corresponding labor in my solution with but four unknowns, especially when we consider the peculiar nature of my coefficients, which permit an appreciable abbreviation of the least-squares computations.† This fully balances the labor of applying special refraction-corrections; so that TURNER's method cannot even be recommended as a quick way of getting approximate results. Correct results can be computed quite as quickly.

* CHAUVENET's *Astronomy*, Vol. 2, p. 511, line 13.

† *Contrib. Col. Univ. Obs'y*, 10, p. 106.

Columbia University, New York, 1901 *December*.

TURNER's method may be compared to an adjustment of the measured angles in a geodetic triangulation in which no attention is given to the rigid condition that in every adjusted triangle the three angles must sum up to 180° *plus* the spherical excess. In such a case, if the errors of observation were absolutely *nil*, the adjusted angles would sum up correctly; but if not, the adjustment would always introduce fresh errors of the same order of magnitude as the errors of observation. Just so TURNER's method would give correct results, in strict accord with my method, if the errors of observation were *nil*. But so long as this is not possible, his adjustment must introduce fresh errors of the same order of magnitude as the errors of observation.

So far I have treated TURNER's method from the point of view of results: it must now be considered as to weights and probable errors. Mr. HINKS prefers his reduction to SCHLESINGER's because of a better agreement in the intercomparison of the plates. SCHLESINGER ascribes this to the fact that in the HINKS reduction a larger number of stars were used for the determination of the plate-constants. This has doubtless contributed to the closer agreement obtained by HINKS; but in the light of my theorem given in the following article, a smaller value of $[vv]$ was certainly to be expected from the HINKS reduction, and this would mean a better agreement. TURNER's method corresponds exactly to mine, except that it introduces two new unknowns: for disregarding relations of the form (3) is equivalent to allowing the photograph to have separate scale-value and orientation constants for the two co-ordinates; that is to say, separate constants that differ by amounts other than can be caused by refraction.

In the reduction of the HENRY plate given in the present paper, the sum of the squared residuals came out,

From TURNER's method	$10''.2348$
From my method	10.2874

The difference in favor of TURNER's method is due simply to the operation of my theorem of the preceding article; which brings out clearly the inadvisability of basing a decision between two methods of reduction on the size of residuals alone.

My conclusions reached in the present paper may be finally stated as follows: TURNER's reduction of photographs with six plate-constants leads to results that are erroneous, apart from any considerations of convenience. Such a reduction must, therefore, not be used; but refraction corrections must be applied, and a four-constant solution made. After that has been done, it becomes merely a question of convenience or personal opinion whether results shall be retained in terms of the measured coordinates, or transformed into right-ascensions and declinations.

A THEOREM CONCERNING THE METHOD OF LEAST-SQUARES,

By HAROLD JACOBY.

Let there be given two series of observation equations as follows:

$$(1)\quad \begin{cases} a_1x + b_1y + c_1z + \ldots + n_1 = 0 \\ a_2x + b_2y + c_2z + \ldots + n_2 = 0 \\ \quad\vdots \qquad\qquad\quad \vdots \end{cases}$$

$$(2)\quad \begin{cases} a_1x + b_1y + c_1z + \ldots + p_1w + \ldots + n_1 = 0 \\ a_2x + b_2y + c_2z + \ldots + p_2w + \ldots + n_2 = 0 \\ \quad\vdots \qquad\qquad\qquad\qquad \vdots \end{cases}$$

the equations being identical in the two series except for the addition of one or more new unknowns $w, \ldots$ in (2). Let each of these series of equations be solved by the method of least-squares, and let

$[vv]_1$ be the sum of the squares of the residuals resulting from the solution of equations (1).

$[vv]_2$ be the sum of the squares of the residuals resulting from the solution of equations (2):

then, no matter what may be the law of the coefficients $p_1, p_2, \ldots$, and even if these coefficients are assigned at random, $[vv]_1$ is always larger than $[vv]_2$.

DEMONSTRATION. — Let

S_1 represent the series of values found for $x, y, z, \ldots$ in the least-squares solution of (1).

S_2 represent the series of values found for $x, y, z, \ldots$ in the solution of (2).

S_2' represent the series of values found for $w, \ldots$ in the solution of (2).

The values S_2 and S_2' will make $[vv]_2$ smaller than would the substitution of any other series of values in (2), since they make $[vv]_2$ a minimum. One other possible series of values would be S_1 instead of S_2, and zeros instead of S_2'. But this possible series of values would make every residual in (2) exactly equal to the corresponding residual in (1), and therefore the sum of the squares of the residuals equal to $[vv]_1$. But as we have just seen, this squared residual-sum would be larger than $[vv]_2$. Therefore,

$$[vv]_1 > [vv]_2 \qquad \text{Q.E.D.}$$

CONCLUSION. — The method of least-squares is used ordinarily to adjust series of observation-equations so as to obtain the most probable values of the unknowns. But there is a subtler and perhaps more important use of the method: when it is employed to decide which of two hypothetical theories has the greater probability of really being a law of Nature; or to decide between two methods of reducing observations. Cases abound in astronomy where the method of least-squares is used for this purpose. It has been so employed, for instance, to decide whether stellar parallax observations should be reduced with equations involving terms depending on atmospheric dispersion, and terms depending on the hour-angle, to ascertain whether portable transit observations should be reduced on the supposition of a change of azimuth on reversal of the instrument, etc.

In such cases, astronomers not infrequently give preference to the solution which brings out the smallest value of $[vv]$, the sum of the squared residuals. But in the light of the above theorem, it becomes clear that the mere diminution of $[vv]$ alone is insufficient to decide between two solutions, when one involves more unknowns than the other. To give preference to the second solution it is necessary that the diminution of $[vv]$ be quite large, and that the additional unknowns possess a decided *a priori* probability of having a real existence.

Columbia University, New York, 1901 December.

NOTICE.

For the convenience of astronomers engaged upon the observation of variable stars, and of others who may be interested, separate reprints of the list prepared by the Committee of the *Astronomische Gesellschaft*, and printed in this number, will be sent on application to any desired address.

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THE EFFECT OF METEORIC DEPOSITS ON THE LENGTH OF THE TERRESTRIAL DAY.

By G. JOHNSTONE STONEY, M.A., D.Sc., F.R.S.

In No. 502 of the *Astronomical Journal*, published on the 23d of last July, there is a remarkable paper by Professor R. S. WOODWARD, in which, after a much needed criticism, correction and extension of the investigations by LAPLACE and his successors into the amount by which the sidereal day has been shortened by the secular cooling of the earth, Professor WOODWARD deals in the last section of his paper with the analogous problem of the effect of meteoric deposits upon the length of the day, so far as the effect is due to an alteration of the moment of inertia of the earth.*

The earth's moment of inertia is affected in two ways by the gradual accumulation of meteorites and meteoric dust over its surface. It is increased by an amount equal to the moment of inertia of the shell of added matter; and it is decreased in consequence of the rest of the earth being compressed by the weight of this load. As this second factor is not referred to by Professor WOODWARD the present note is written to call attention to it, since it has the effect of reducing the change of the earth's moment of inertia to a small fraction of the already small amount that is obtained when it is left out of account.

*Meteors that impinge upon the earth may conceivably affect the earth's rotation in three ways:

1. By having had a preponderance of moment of momentum in one direction or the other round the earth's axis, when they reached the earth's atmosphere;

2. By parts of some of those which have entered the earth's atmosphere nearly tangentially escaping outwards with a preponderance of moment of momentum in one direction or the other;

3. By becoming part of the earth and changing its moment of inertia.

If any such excess of moment of momentum, as is contemplated in case (1) has existed, it is to be transferred to the earth;

An excess such as that contemplated in case (2) is to be compensated for by applying an equal and opposite moment of momentum to the earth;

Case (3) is dealt with in the present communication.

We may proceed upon the assumption made by Professor WOODWARD that the meteoric deposit is spread uniformly over the earth. In this case the increment of I, the earth's moment of inertia, includes two terms,

$$\delta I = \delta_1 I - \delta_2 I \qquad (1)$$

where $\delta_1 I$ is the moment of inertia of a uniform spherical shell of meteoric deposit consisting of meteorites and meteoric dust, and $\delta_2 I$ is the decrement of the moment of inertia of the rest of the earth, due to its being compressed.

The moment of inertia of a thin spherical shell of thickness θ, density ρ, and of which the radius is a (a being here the radius of the earth) can be directly computed, and is

$$\delta_1 I = \tfrac{8}{3}\pi\, a^4 \theta\rho \qquad (2)$$

Again, the moment of inertia of the earth is known to be, nearly enough for the present investigation,

$$I = \tfrac{1}{3}\, M a^2 \qquad (3)$$

where M is the earth's mass. This equation though only approximate holds good both before and after the compression, inasmuch as when the earth is compressed all depths seem to be nearly equally affected. This is evidenced by the fact that the mean compressibility of the earth as a whole, as indicated by geology, is found to be not far from the known compressibility of its superficial strata.† It is accordingly legitimate to differentiate equation (3), and we thus get approximately

$$\delta_2 I = \tfrac{2}{3}\, Ma\, \delta a \qquad (4)$$

where δa is the compression. Now $\delta a/a$ (which is the earth's compressibility when loaded uniformly over its whole surface) is directly proportional to the load per unit of surface. That is,

† See the papers referred to in the last paragraph of the present paper.

$$(5) \qquad \frac{\delta a}{a} = K\theta\rho g$$

where g is gravity, and K the coefficient of compressibility.

Let ρ' be that density of deposit which, if spread uniformly over the whole earth, would depress the former surface of the ground by an amount equal to the thickness of the layer of matter placed upon it, then we find, as a particular case of equation (5),

$$(6) \qquad \frac{\theta}{a} = K\theta\rho' g$$

Eliminating K between (5) and (6) we get

$$(7) \qquad \frac{\delta a}{\theta} = \frac{\rho}{\rho'}$$

Again M, the mass of the earth,

$$(8) \qquad = \tfrac{4}{3}\pi a^3 \rho_0$$

where ρ_0 is the mean density of the earth. By these equations equation (4) becomes

$$(9) \qquad \delta_2 I = \tfrac{8}{9}\pi a^4 \theta\rho \frac{\rho_0}{\rho'}$$

Finally, putting the values (2) and (9) into equation (1) we obtain

$$(10) \qquad \delta I = \tfrac{8}{3}\pi a^4 \theta\rho \left(1 - \frac{\rho_0}{3\rho'}\right)$$

If we write 1 instead of $\left(1 - \frac{\rho_0}{3\rho'}\right)$ into this value of δI, it becomes the value assigned to δI by Professor WOODWARD.

It is known that ρ_0, the mean density of the earth, is slightly more than $5\frac{1}{2}$ times the density of water, and ρ' seems, from the geological evidence, to be somewhat less than twice the density of water. Hence $\left(1 - \frac{\rho_0}{3\rho'}\right)$ is a small fraction. We must, accordingly, substitute for Professor WOODWARD's estimate of the effect on the length of the day, one that is a small fraction of that value; instead of the day being lengthened by $\frac{1}{4}$ of a second in a million million years, the change in that time due to the effect of the meteoric deposit upon the earth's moment of inertia, is probably better represented by something like $\frac{1}{50}$ or $\frac{1}{100}$ of a second; and moreover the geological evidence that is available seems to leave it doubtful whether this excessively minute alteration is positive or negative.

Thus the effect on the length of the sidereal day brought about upon the earth by the alteration of its moment of inertia, is very small. And this is the whole effect produced by the accession of meteors to the earth, if, as is probable, they on the average strike equally all parts of the target which the earth presents to them; since in this case the positive and negative moments of momentum of the meteors immediately before they strike the earth balance, so that the matter reaching the earth has reached it without any moment of momentum of its own.

The foregoing analysis seems to be a sufficient investigation of the effect on the rotation of the earth of the meteoric matter that accumulates on its surface. But there is a small residuum of effect produced by meteors in another way. Some meteors that strike the atmosphere very obliquely, or rather the portions of them that have not been dissipated during their transit, escape from the atmosphere after having pursued a nearly horizontal flight through its upper regions. These meteors pass away from the earth with diminished mass, in a somewhat altered direction and usually with reduced relative speed. To estimate the effect upon the rotation of the earth of these escaping meteors, it would be necessary to search for such data as would make known the aggregate moment of momentum round the earth's axis, of these bodies when on the point of emerging. We have only to apply an equal and opposite compensating moment of momentum to the earth, to find their effect upon the earth's rotation. Adequately to pursue this branch of the subject would, however, require another paper quite as long as the present one, and the resultant effect obtained would prove to be excessively small.

It is of interest to point out that the earth seems to be the only one of the planets for which the two terms of equation (1) are nearly equal to one another. In passing from one planet to another ρ' varies inversely as a, g and K: from which and from the known values of a, g and ρ_0 for the several planets, it follows that if K, the coefficient of compressibility, be either nearly the same as, or less than, that of the earth, as is probably true of *Mercury*, *Venus* and *Mars*, then on these planets $\delta_1 I$ preponderates largely over $\delta_2 I$, and *a fortiori* on the moon and the smaller bodies of the solar system. To all of these our method of investigation may legitimately be applied, and on all of these meteors tend to lengthen the period of rotation.

As to *Jupiter*, *Saturn*, *Uranus* and *Neptune*, it is not likely that there is solid ground upon any of these planets to arrest the subsidence of bodies falling through their atmospheres. Unless there is such a support, the whole method of investigation would require to be altered. If, however, falling meteorites do reach solid ground under their atmospheres or oceans, then, even if the coefficients of compressibility of these planets were as small as that of the earth, $\delta_2 I$ on them would be much larger than $\delta_1 I$ on account of their great sizes; and the preponderance of $\delta_2 I$ becomes larger if, as is probable, these planets consist of more compressible materials than the earth. Hence, even if any of these great planets had a solid crust, the meteors that reach it would diminish their moments of inertia, which is the reverse of the effect which takes place upon the planets that are smaller than the earth.

It is also of interest to observe that inasmuch as meteorites and meteoric dust have a density higher than that which the facts of geology permit us to assign to ρ', it follows that the accession of these foreign bodies to the earth diminishes instead of increasing the size of the earth, contrary to what is sometimes stated in text books on Astronomy; in fact, if any extensive terrestrial deposit have the density of meteorites, its weight compresses the earth beneath by an amount greater than the thickness of the added matter.

In the treatment of the problem which has been pursued in the present paper, we have had to rely on geological evidence to fix the amount of the compressibility of the earth. In regard to this evidence, reference may be made to two papers on "Denudation and Deposition" in the numbers of the "London, Edinburgh and Dublin Philosophical Magazine" for April and June, 1899, pp. 372 and 557.*

30 *Ledbury Road, London W.*, 1901 *November*.

* Readers of the papers here referred to are requested to insert on p. 374, in the paragraph referring to Brazil, the following words, viz.: —*by the great fluviatile deposits carried down from the Andes and* — before the words "by the luxuriant tropical vegetation."

The cause of the small elevation of China over the level of the sea, seems to be substantially of the same kind as in the case of Brazil. In both, presumably, borings would reveal strata of vegetable mould alternating with fluviatile deposits to a great depth. All such strata were once at the surface, but have since subsided because they rest on the up-turned base of an immense inverted cone extending from the surface of the earth to its centre, and therefore nearly 4000 miles long, which shortens with every additional load placed upon it. The underground water which must permeate these strata, and perhaps others below them, has also to be taken into account. It is fed by rain and oozes into the ocean as hard water, and is therefore an agent which causes a certain amount of subterraneous denudation; thus tending, so far as this factor is concerned, to produce elevation.

THE VARIABLE 6458 *U DRACONIS*.

By ZACCHEUS DANIEL.

The variability of this star was discovered by ANDERSON, and announced by him in *A.N.* 3618. The position and definitive notation are given in *A.J.* 514 and *A.N.* 3752.

All my observations of this star were made with the 10-inch Clark refractor of the Bucknell Observatory, the method of observing being ARGELANDER'S.

I began to observe this star on 1900 June 20, and have followed it continuously since. At the first observation, the star was at 11^M.1, but on 1900 July 23 it had risen to 9^M.85, and the variability was fully confirmed. At the last observation, on 1901 Dec. 30, the star was at 11^M.0, and steadily rising. The reduction of the observations, thirty-nine in number, yields the following results:

OBSERVED MAXIMA AND MINIMA.

Max. 1900 July 27,	J.D. 2415228	9.8	8	obs.
Min. 1900 Dec. 1,	5355	13.9	12	"
Max. 1901 May 5,	J.D. 2415510	9.3	7	obs.
Min. 1901 Sept. 17,	5645	14.1	10	"

All observations above 11^M.7, the middle point in the light range, are assigned to the maxima; those below, to the minima.

The dates are all well determined, but the time of the first maximum is in doubt by a few days because the observations near maximum are not numerous enough to locate the curve definitely. The magnitude is also somewhat uncertain.

The period from the maxima is 282 days, from the minima 290 days. The mean period from fourteen similar points on the descending branch of each maximum is 283 days. Adopting the latter period as the most probable, and using the second maximum as the basis, we obtain the following provisional elements:

$$\text{Max. J.D. } 2415227 + 283\,E; \quad M - m = 155$$

From these elements the following ephemeris is computed:

EPHEMERIS FOR 1902.

		M
Max. 1902	Feb. 12, 5793	9.3
Min.	June 20, 5921	14.0
Max.	Nov. 23, 6076	9.3

The light-curve of this star is fairly regular. The rise is steady. Between 11^M.5 and 9^M.5 it takes place at the rate of 0^M.046 a day. The fall is equally steady and rapid. The maxima are sharp and well defined. The minima are fairly well defined, but the curve is almost twice as flat as it is at maximum. At the last maximum, the star was above 11^M.7 for 126 days, or 0.445 of the period.

I used fifteen comparison-stars from 9^M.0 to 14^M.0. The following are eight of them:

COMPARISON-STARS FOR *U Draconis*.

Star	Desig.	B.D.	H.	D.	Grades
		M	M	M	
m	DM. +54°1931	9.0	9.21	9.05	43.5
g	+55°2003	9.0	9.46	9.62	38.5
p	+55°2005	9.5	. .	9.99	35.2
b	. .	. .	. .	10.57	30.1
a	. .	. .	. .	10.63	29.6
e	11^s *f* 3′ *s U*	. .	. .	10.82	27.9
c	. .	. .	. .	11.05	25.9
d	. .	. .	. .	11.10	25.4

The stars *a*, *b*, *c*, and *d* are the ones given by ANDERSON in *A.N.* 3618.

The light-scale is composed of ninety-five separate comparisons. The magnitudes are based on the photometric magnitudes of *m* and *g* (given in column H.) found in *H.C.O. Annals*, Vol. 24, and the assumed magnitude of 11.2 as the limit of the 10-inch telescope in fair seeing. In this discussion the value of one grade is 0^M.111.

Bucknell University, Lewisburg, Penn., 1902 *January* 4.

OBSERVATIONS OF VARIABLE STARS, 1900–1901,

BY PAUL S. YENDELL.

103. *T Andromedae*.

Fourteen observations of *T Andromedae*, from 1901 Sept. 30 to Dec. 12, indicate that a maximum of $8^{M}.3$ was passed on Nov. 16.5.

At its first appearance the star was estimated as of 11^{M}, and when last observed had decreased to $8^{M}.9$.

7085a. *SU Cygni*.

I observed *SU Cygni* sixty-three times in 1900, and twenty-four times in 1901. From these observations I have deduced twenty maxima and twenty-five minima, using a mean light-curve formed on the elements of LUIZET (*A.N.* 3570). The time given in all the dates in this paper is Cambridge M.T.

Maxima	Wt.	Minima	Wt.
1900 May 20.34	1	1900 April 25.11	1
June 4.57	1	May 11.45	1½
23.72	1	30.27	½
27.16	½	June 11.56	1
July 1.40	½	18.40	½
17.18	½	30.65	½
21.11	½	July 4.38	½
28.37	½	27.06	½
Aug. 4.95	½	31.01	1½
16.86	1	Aug. 19.17	1
31.97	½	Sept. 19.10	1
Sept. 12.32	½	26.72	1
21.47	½	Oct. 15.71	1
Oct. 24.71	½	19.99	1
1901 June 8.38	½	23.37	1½
Oct. 6.29	1½	Nov. 11.74	½
9.89	½	1901 Sept. 30.86	½
Nov. 21.11	½	Oct. 4.98	1
Dec. 6.25	1	12.11	½
10.45	1½	16.76	1
		Nov. 4.73	½
		19.70	½
		Dec. 1.63	½
		5.76	½
		13.23	1

An examination of the residuals found by a comparison of these observed times of maxima indicated no significant correction to LUIZET's elements.

6733. *R Scuti*.

During the season of 1900 I observed *R Scuti* forty-three times, from May 20 to Nov. 14. An ill-defined maximum of $5^{M}.8$ was shown on May 30, followed by a much more definitely indicated minimum of $6^{M}.8$ on July 1. Another sharply-marked minimum of $7^{M}.0$ occurred on July 30, and a maximum of $4^{M}.9$ on August 22. A third minimum of $6^{M}.8$ took place on Sept. 26, and a maximum of $4^{M}.9$ on October 21. At the last observation, the star's light had fallen to $5^{M}.4$. A short series of observations from 1901 Oct. 1 to Nov. 10 shows a single maximum of $4^{M}.6$ about Oct. 16. The minima in 1900 were much more sharply indicated than the maxima.

7378. *SZ Cygni*.

This star I have observed sixteen times in 1900, and nine times in 1901. Maxima are indicated about 1900 October 16, and on 1901 October 15.5 and Nov. 1. These seem confirmatory of HARTWIG's period of 15.2 days.

7437. *X Cygni*.

I have sixty observations of *X Cygni* from 1900 April 21 to Nov. 23, and twenty-three from 1901 Oct. 1 to Dec. 16. These indicate eight maxima and eight minima, as follows:

Maxima	Wt.	Minima	Wt.
1900 June 16.8	1	1900 June 10.8	1
July 2.8	1	24.4	1
20.7	½	July 10.4	½
Aug. 5.7	1	30.0	1
Sept. 23.3	1	Aug. 15.3	1
Nov. 12.7	½	Sept. 18.5	1
1901 Oct. 5.7	1	Oct. 21.2	1
Dec. 10.3	½	1901 Oct. 16.4	½

7793. *SS Cygni*.

I have twenty-three observations of this star in 1900, and eighteen in 1901. Among these are observations of parts of five maxima, as follows:

1900 July 2.435	$7^{M}.68$	1901 Oct. 15.323	$8^{M}.32$
4.389	8.06	16.313	8.19
10.382	8.30	18.437	8.93
Oct. 24.367	10.13	21.306	9.47
Nov. 10.333	8.32	Dec. 10.299	8.36
12.319	8.70	11.309	8.35
14.309	8.82	12.306	8.29
21.315	10.24	15.344	8.86
23.309	11.02	16.306	9.51
Dec. 27.312	9.31		

Dorchester, Mass., 1901 Dec. 21.

VARIATION OF LATITUDE FROM BESSEL'S AND STRUVE'S OBSERVATIONS.

By S. C. CHANDLER.

The following article presents the determination of the fourteen-months' term of the latitude-variation from observations with the Königsberg and Dorpat Reichenbach & Ertel circles.

1. *Königsberg*, 1820–27. The material used is the polar-point determinations in *Abth.* X–XVI. *Königsberger Beobachtungen*. These were found by Bessel in two ways: first, by upper and lower culminations of *Polaris* (also of δ *Ursae minoris* to some extent); secondly, by fundamental stars culminating south of the zenith. This second series, which he employed for the calculation of the declinations of the sun, was made for the purpose of controlling the doubts that had been raised with regard to the constancy of the flexure by the places of the pole on the instrument as found by *Polaris* between 1820 and 1824. While it is by no means entitled to the weight of the *Polaris*-series, for obvious reasons, I have thought it proper and interesting to use both, as well for the purpose of comparison as for the value of their evidence, which is largely independent although of unequal weight.

If we denote by p and p' the place of the pole on the circle when it is West or East, respectively, we have the instrumental latitude by

$$\varphi = 90^\circ - \frac{p-p'}{2}$$

Table I gives the latitudes thus found by combining Bessel's values of p and p', using the mean of two adjacent values in one position of the circle in conjunction with the intervening value in the other position; of course excluding combinations where a manifest change of adjustment, either intentional or accidental, had occurred. A seven-years' series is taken because thus the results for the fourteen-months' term would be reasonably free from the effects of the annual term without any arbitrary correction for the latter, which we should have to introduce from extraneous sources since this series alone is not competent for an independent determination of both terms. Of course a fourteen-years' series would have been used had the material available allowed it.

To find the constants of the fourteen-months' term the 52 values of the latitude, for each series independently, were brought into order according to their place in that period by adding or subtracting multiples of 429 days so as to bring the modified dates within the compass of the interval 2387199–7596. Means were then taken in 13 groups of 4 values each. Table II gives these means in the 2d and 3d columns, and the deviations from their means in the 4th and 5th, which represent the variations of latitude due to the 14-mos. term on the assumption that the annual term has been eliminated by the method of grouping.

A solution by least-squares gives the following constants and their probable errors:

Polaris	$-0''.191 \cos (t-2386946)\, 0.84$;	$\pm 0''.039$ and $\pm 14^d$
Fund. Stars	$-0.094 \cos (t-2386959)\, 0.84$;	± 0.033 and ± 21

the Julian dates being the observed dates of minimum latitude at Königsberg. The reduction to the Greenwich meridian is -24 days.

The above determinations rest on the assumption that the uniformity of distribution of the observed data over the different times of the year, for the seven years as a whole, is sufficient to secure a satisfactory elimination of the annual term. This is approximately the case; still I think that we can get an undoubted improvement by applying corrections for this term derived from an extraneous source without injuring the independent character of the results. From the investigation in *A.J.* 489 we assume for this term, $-0''.125 \cos (\odot - 15^\circ)$, whence the last column of Table I. Subtracting from the values of φ, and classifying as before, we get the corrected values of $\varphi - \varphi_0$ in the 6th and 7th columns of Table II. The solution gives,

Polaris	$-0''.230 \cos (t-2386950)\, 0^\circ.84$;	$\pm 0''.034$ and $\pm 9^d$
Fund. stars	$-0.134 \cos (t-2386948)\, 0.84$;	± 0.046 and ± 24

The epochs are not materially different from those of the uncorrected data, but the semi-amplitudes are decidedly larger.

On general grounds the result from the pole star should probably have at least double the weight of that from the southern stars, and this is also in accord with the assigned probable errors; so that we have finally,

$$-0''.200 \cos (t-2386949)\, 0^\circ.84 ; \quad \pm 0''.021 \text{ and } \pm 9^d$$

Referred to Greenwich the epoch is 2386925.

The coefficients for this series used in *A.J.* 491, p. 109, were 0″.171 and 2386920 (Gr.). I am inclined to think the present values are preferable. In the last column of Table II are given the computed values by this formula.

2. *Dorpat*, 1822–26. The material used is the polar-point readings of the circle given in the *Positiones Mediae*, Introd. pp. XIV–XV. Struve has there used them for the purpose of ascertaining the possible effect of temperature upon the place of the pole on the circle. He supposed that he succeeded in establishing such a relation, and in consequence he applies a correction to the results of his observations. But I think it is now demonstrable that this assumed relation is purely factitious, and that the outstanding residuals which he endeavored to satisfy by this

hypotheses were merely exhibitions of the effects of the polar motion with which we are now familiar. From the nature of the case the seasonal distribution of his data would naturally lend color, in the absence of knowledge as to the real cause, to the erroneous supposition of an instrumental origin of the deviations.

TABLE I. KÖNIGSBERG.

t 2380000+	φ = 54° 42′+ Pol.	F. St.	Annual Term	t 2380000+	φ = 54° 42′+ Pol.	F. St.	Annual Term	t 2380000+	φ = 54° 42′+ Pol.	F. St.	Annual Term
5904	50.91	50.75	0.12	6345	50.64	50.73	−0.01	7468	50.00	50.51	+0.04
5915	.83	.47	.12	6424	50.20	50.32	+ .12	7518	.08	.71	+ .11
5931	.99	.48	.11	6504	49.80	49.89	+ .07	7584	.09	.09	+ .10
5972	.79	.72	.04	6589	50.54	50.05	— .10	7675	50.01	.28	— .09
5990	.59	.64	.00	6652	.70	.59	— .12	7787	49.82	.28	— .06
6006	50.22	.30	+ .02	6702	.44	.72	— .04	7878	50.22	50.58	+ .11
6018	49.94	.29	+ .05	6848	.59	.67	+ .11	7897	.63	51.12	+ .12
6028	50.25	.71	+ .08	6994	50.31	.37	— .12	7926	.12	51.30	+ .12
6043	.21	.86	+ .10	7084	49.95	.55	— .01	8036	50.20	51.21	— .07
6056	.24	50.77	+ .12	7125	50.06	.92	+ .08	8106	49.57	50.54	— .12
6105	.79	51.04	+ .11	7159	.15	.77	+ .12	8150	49.93	.83	— .07
6148	.02	49.88	+ .05	7234	.20	.65	+ .08	8221	49.37	.79	+ .08
6218	.64	50.73	— .08	7284	.01	.42	— .03	8298	48.97	.26	+ .11
6236	50.13	50.24	.11	7303	.03	.40	— .06	8419	50.40	50.73	— .10
6259	51.28	51.06	.12	7323	.02	.45	— .10	8490	49.40	51.21	— .10
6287	50.70	50.20	— .11	7356	.62	.56	— .12	8624	49.63	50.74	+0.12
6300	.55	.22	— .10	7394	.33	.02	— .10				
6316	50.58	50.46	−0.08	7407	50.31	50.75	−0.08				

TABLE III. DORPAT.

t 2380000+	φ	Annual Term	t 2380000+	φ	Annual Term	t 2380000+	φ	Annual Term	t 2380000+	φ	Annual Term
6841	48.04	+0.11	7159	48.19	+0.12	7345	47.90	−0.12	7667	48.17	−0.07
6859	47.86	+ .08	7166	48.17	+ .12	7360	47.85	— .12	7696	48.28	— .11
6896	48.35	+ .02	7170	48.09	.12	7371	47.72	— .12	7704	48.34	— .12
6958	47.74	— .10	7177	47.97	+ .12	7381	47.70	— .11	7722	48.29	— .12
6969	47.94	— .11	7192	48.24	.12	7392	47.77	— .10	7751	48.61	— .11
6983	48.00	.12	7213	48.51	.10	7414	47.80	— .07	7777	48.40	— .08
7002	47.83	— .12	7232	48.45	.07	7508	48.01	+ .10	7897	48.25	+ .12
7020	47.55	.11	7272	48.16	.00	7520	48.03	+ .12	7926	48.12	+ .12
7035	47.46	— .10	7301	47.73	— .06	7521	48.05	+ .12	7963	48.30	+ .07
7049	47.64	— .08	7330	47.73	— .11	7531	48.15	+ .12	8030	48.10	−0.06
7148	48.19	+0.11	7341	47.87	−0.12	7601	48.22	+0.07			

TABLE II. KÖNIGSBERG.

t	φ = 54° 42′+ Pol.	F. St.	$\Delta\varphi$ uncorr'd Pol.	F. St.	$\Delta\varphi$ corr'd Pol.	F. St.	Comp'd
7199	50.44	50.79	+0.19	+0.22	+0.27	+0.30	+0.17
7236	.20	.49	.05	.08	.00	.03	+ .10
7273	.54	.59	+ .29	+ .02	+ .23	— .01	.00
7293	.05	.49	.20	— .08	— .18	.06	— .06
7317	.11	.58	.14	+ .01	.18	.03	.12
7349	.08	.58	.17	+ .01	.19	— .01	— .18
7377	50.07	.44	.18	.13	.23	.18	.20
7426	49.90	.31	.35	.26	.35	.26	.16
7458	50.35	.56	+ .10	— .01	+ .05	.06	.08
7506	.35	.79	+ .10	+ .22	+ .11	+ .23	— .06
7535	.39	.73	+ .14	+ .16	+ .14	+ .16	+ .13
7569	.41	.43	+ .16	.14	+ .19	— .11	— .19
7596	50.37	50.64	+0.12	+0.07	+0.14	+0.09	0.20
Mean	50.25	50.57					

TABLE IV. DORPAT.

t	Wt.	$\Delta\varphi$ Obs'd	Comp'd
238 7271	1	+0.23	+0.26
7306	5	+ .31	+ .20
7345	5	+ .06	+ .09
7385	5	— .12	— .04
7426	1	— .12	— .13
7481	5	— .14	— .16
7530	1	+ .02	— .07
7589	1	+ .04	+ .11
7608	1	+ .07	+ .16
238 7656	3	+0.34	+0.26

From the given series of values of the polar-point in the two positions of the circle I have deduced the instrumental latitudes in a precisely analogous way to that employed for Königsberg as above described. They are given in Table III. To get the constants of the 14-mos. term in this case it is essential to apply a correction for the effects of the annual term, since the observed data cover only about half of a seven-year cycle. This correction is given in the last column. Subtracting it from the observed latitudes and classifying in precisely the same way as for Königsberg, and assuming the provisional mean latitude 58° 22′ 48″.00, we have the values of $\Delta\varphi$ in Table IV, with their weights. A solution gives the correction to the assumed mean latitude, +0″.051, and the constants of the 14-mos. term given below. The probable errors have, naturally, merely a nominal significance.

$$-0''.219 \cos(t-2387463)\,0^\circ.84\,; \quad \pm 0''.027 \text{ and } \pm 9^d$$

The corresponding computed values will be found in the last column of Table IV. The reduction of the above epoch to the Greenwich phase gives 2387436 (Gr.), which differs 2 days from the date used in *A.J.* 494, p. 109. The radius of the motion is considerably larger than the value there given, but I think the present revised value deserves decided preference.

3. *Comparison of results.* To compare the above results with those derived in *A.J.* XIV, p. 19, from the nearly contemporaneous but far more determinate series of POND, we reduce the observed dates of minimum latitude to a common phase, that of Greenwich, obtaining the values T_1 below. Referring these to a common epoch by the period 428^d.5 we have the last column. The weights assigned represent fairly, I think, the relative strength of the three determinations.

		T_1	Wt.	Red. to Common Epoch
Bessel	1820–27	2386923	1	2388637
Struve	1822–26	7436	½	8722
Mean, Struve and Bessel			1½	2388665
Pond	1825–36	2389521	5	2388664

The mutual corroboration of the three series for this epoch thus afforded is of great importance in establishing, by comparison with other observations, the length of the mean period of the 14-mos. term. I think this falls between 428.5 and 428.6 by any reasonable estimate of it. Nearer than this we cannot come until we know more exactly the law of the deviations from uniformity to which it is undoubtedly subject as demonstrated in *A.J.* 494. Assuming that the above epoch falls on the same phase of these deviations as the mean epoch +18 (*loc. cit.*) we have the interval 2413090 − 2388664 = 24426, corresponding to 57 periods of 428.52 days. Similarly, assuming for BRADLEY'S epoch (*A.J.* 514) 2352190 ± 30, and that this falls on the same phase as the epoch zero, we have the interval 2405331 − 2352190 = 53141, corresponding to 124 periods of 428.56 days. Taking into account the probable uncertainty of the various data the best guess I can now make as to the average period is 428^d.55 ± 0^d.15, where the second decimal is purely nominal.

ABERRATION-CONSTANT FROM POND'S OBSERVATIONS, 1825–36.

BY S. C. CHANDLER.

The determinations of the variation of latitude, the mean declinations for 1830 and the nutation-constant from this series have been published in *A.J.* 313, 315 and 361. In addition to these the discussion of the aberration-constant has long been completed, but it has not yet been printed. Awaiting a favorable opportunity for doing so with the requisite detail a brief summary of the principal results is here communicated.

The most suitable, and indeed the only satisfactory material for this purpose in POND'S work is comprised in the following eight stars. The other stars having sufficiently large aberration-factors in declination were either not sufficiently observed, or the distribution was defective, being confined to a few months of the year.

The first two columns of the table give the name and total number of observations. The next four columns show the distribution, approximately, as regards the maximum and minimum times of aberration in declination. Column f is the ratio of the maximum aberration in declination to the total aberration. Column r is the probable error of a single observation. Then follow the observed values of the aberration-constant deduced for the several stars, after eliminating the effects of the latitude-variation, with their probable errors and the weights used in combining them. Finally, in the last column I have placed the values which would result if we should ignore the annual term of this variation.

The value of the annual term comes out by the present calculation,

$$-0''.176 \cos(\odot - 21^\circ)$$

which is not sensibly different from that already derived

[illegible] different process, (A. J. XIV, p. 19), which [illegible]

$0''.173$ [illegible] 15 [illegible]

[illegible] this investigation of the aberration- [illegible]

$20''.512 \pm 0''.019$

[illegible] $0''.12$ larger than that found if we neglect [illegible]. Of course the fourteen-months term was [illegible] taking the solutions.

In order to make sure that no material was neglected which could be of any service for the purpose, solutions were made for many other stars, but they all proved, in accordance with what might have been anticipated from the small theoretical aberration-factors and imperfect distribution of the data, to be indeterminate in character; and I am convinced that they would only tend to vitiate the above result, if included. The above value may be regarded as definitive from POND's series.

Star	Obsns.	Observations near Max.	Zero	Min.	Zero	f	r	Obs'd Aberration		Wt.	Aberration Ignoring Annual Term
							$''$	$''$	$''$		
Polaris U. C.	317	62	115	108	32	0.99	±0.17	20.53	±0.04	8	20.51
Polaris L. C.	257	64	16	55	122	0.99	.67	20.41	.06	3	20.42
α Cygni	135	74	168	157	39	0.90	.53	20.52	.05	5	20.32
β Cephei	203	2	105	92	4	1.00	.34	20.52	.06	3	20.37
α Lyrae	292	33	126	97	36	0.88	.61	20.37	.07	2	20.17
β Lyrae	258	15	75	168		0.83	.52	20.52	.08	2	20.31
δ Draconis	180	15	88	77		1.00	.44	20.63	.08	2	20.46
α Cephei	206	5	23	165	13	0.98	.60	20.75	.11	1	20.59
α Aquilae	481	54	209	205	13	0.51	±0.59	20.11	±0.11	1	20.08
	2629							20.512	±0.019		20.393

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR.

BY A. W. QUIMBY.

1901	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1901	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1901	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
July 22	5	1	1	1	2	fair	Oct. 30	8	–	1	5		fair	Nov. 16	3	–	1	1	–	poor
Aug. 8	10				1	fair	31	9		1	3		fair	17	3	–	1	1	–	poor
Oct. 10	9	1	1	6		fair	Nov. 1	10		1	1		poor	19	8	–	1	1	–	fair
11	10		1	1		poor	13	8	1	1	1		fair	20	8	–	1	1	–	fair
28	3	1	1	5	1	fair	14	9	–	1	3	1	fair	21	8	–	1	1	–	fair
29	8		1	5	1	fair	15	8		1	1	1	poor	22	8		1	1	–	fair

DR. QUIMBY's most valuable series of observations, continued uninterruptedly for so long a series of years, was communicated by [illegible] in his usual form. But on account of the remarkable paucity of sunspots during the last half of 1901 it consisted largely of blanks, so that in this instance it has seemed desirable to print only those dates on which spots or faculae were visible, together with the following statement. Observations were made on every day from July 1 to Dec. 31, inclusive, except July 12, 13, 26, Aug. 6, Sept. 18, Nov. 18, Dec. 3, 10, 14, 29; but no spots or faculae were visible except on the dates given in the table.

ED.

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ILLUSTRATIONS OF PERIODIC SOLUTIONS IN THE PROBLEM OF THREE BODIES,

By G. W. HILL.

(FIRST ARTICLE.)

Whenever a system of moving bodies retakes after a certain lapse of time the same position relatively to the bodies themselves, the solution of the differential equations to which this case corresponds is called a periodic solution. Periodic solutions may be broadly divided into two classes. The first class contains those cases in which there has taken place a rotation of the whole system in the interval between the mentioned positions. The second those in which no such rotation has occurred, but the longitudes of the bodies, as well as their distances, have returned to the same values. In the illustrations to be given here we will confine ourselves to the latter class.

For expressing the differential equations of motion of two planets about their central body we will employ variables similar to those of DELAUNAY in the Lunar Theory, but (with POINCARÉ) we will denote the energy of the system by F. Then these equations may be given the form

$$\frac{dL}{dt} = \frac{\partial F}{\partial l}, \quad \frac{dG}{dt} = \frac{\partial F}{\partial g}, \quad \frac{dH}{dt} = \frac{\partial F}{\partial h}$$
$$\frac{dl}{dt} = -\frac{\partial F}{\partial L}, \quad \frac{dg}{dt} = -\frac{\partial F}{\partial G}, \quad \frac{dh}{dt} = -\frac{\partial F}{\partial H}$$

$$\frac{dL'}{dt} = \frac{\partial F}{\partial l'}, \quad \frac{dG'}{dt} = \frac{\partial F}{\partial g'}, \quad \frac{dH'}{dt} = \frac{\partial F}{\partial h'}$$
$$\frac{dl'}{dt} = -\frac{\partial F}{\partial L'}, \quad \frac{dg'}{dt} = -\frac{\partial F}{\partial G'}, \quad \frac{dh'}{dt} = -\frac{\partial F}{\partial H'}$$

In these formulae l and l' may be regarded as similar in signification to the two mean anomalies of the planets, while g and g' are the angular distances of their perihelia from their nodes, and h and h' are the longitudes of the latter. For L, G, H, L', G', H' we have the following equivalents:

$$L = M\sqrt{a}\,, \quad G = M\sqrt{a}\,.\sqrt{1-e^2}\,, \quad H = M\sqrt{a}\sqrt{1-e^2}\cos i$$
$$L' = M'\sqrt{a'}\,, \quad G' = M'\sqrt{a'}\sqrt{1-e'^2}\,, \quad H' = M'\sqrt{a'}\sqrt{1-e'^2}\cos i'$$

where M and M' are certain functions of the masses and the other letters have the customary signification.

Ω being the potential function

$$F = \frac{\mu}{L^2} + \frac{\mu'}{L'^2} + \Omega$$

where again μ and μ' are functions of the masses. Ω is developable in an infinite periodic series of cosines whose general argument is a linear function of the angular elements, l, g, h, l', g', h'; that is, we have

$$\Omega = \Sigma\,.\,C\cos(il + i'g + i''h + i'''l' + i^{\text{iv}}g' + i^{\text{v}}h')$$

where the i are positive or negative integers, and C involves only L, G, H, L', G', H'.

Now we may imagine that by a series of operations of DELAUNAY we may remove from F all the sensible terms whose arguments have a sensible motion. The forms of F and the differential equations are not changed by this procedure. It is evident that, at this stage of the investigations, we may suppose our work of integration finished, provided that the conditions, to be mentioned shortly, are fulfilled. For the sake of distinction, let us say that F has now become F_0, and suppose that L, G, H, L', G', H' are now constants. Then the six equations

$$\frac{\partial F_0}{\partial l} = 0\,, \quad \frac{\partial F_0}{\partial g} = 0\,, \quad \frac{\partial F_0}{\partial h} = 0\,,$$
$$\frac{\partial F_0}{\partial l'} = 0\,, \quad \frac{\partial F_0}{\partial g'} = 0\,, \quad \frac{\partial F_0}{\partial h'} = 0$$

ought to be satisfied. Let us suppose, in addition, that the perihelia and nodes are stationary; this gives the four equations

$$\frac{\partial F_0}{\partial G} = 0\,, \quad \frac{\partial F_0}{\partial H} = 0\,, \quad \frac{\partial F_0}{\partial G'} = 0\,, \quad \frac{\partial F_0}{\partial H'} = 0$$

The two remaining equations may be written

$$\frac{\partial F_0}{\partial L} = -n\,, \quad \frac{\partial F_0}{\partial L'} = -n'$$

where n and n', being constants, are the mean motions of

the planets. Let us next suppose that n and n' are in the proportion of the integers k and k'. Then, as an eleventh equation, we may write

$$k'\frac{\partial F_0}{\partial L} - k\frac{\partial F_0}{\partial L'} = 0$$

The last supposition is evidently necessary in order that a periodic solution may exist. Although k and k' may be taken at will, all cases will be arrived at by adopting numbers prime to each other. Thus we have eleven equations to determine twelve arbitrary constants; thus one is left indeterminate. This is as it should be, since the origin from which longitudes are measured may be taken at will.

For the group of six equations, first given, we may substitute others which are simpler. j being a positive integer, it is evident the form of F_0 is

$$F_0 = \Sigma\,.\,C\cos[j(k'l - kl') + i'g + i''h + i^{iv}g' + i^{v}h']$$

When this expression is partially differentiated with respect to any one of the elements l, g, h, l', g', h', we have a series of sines. The six equations are then satisfied if we make

$$g_0 = 0^\circ \text{ or } 180^\circ,\ h_0 = 0 \text{ or } 180^\circ,\ g_0' = 0^\circ \text{ or } 180^\circ$$
$$h_0' = 0 \text{ or } 180^\circ,\ k'l_0 - kl_0' = 0^\circ \text{ or } 180^\circ$$

These substitutions can be made in F_0, by which it is reduced to a function of L, G, H, L', G', H'. Using the same symbol to denote the function after this change, the equations, which remain to be satisfied, are

$$k'\frac{\partial F_0}{\partial L} - k\frac{\partial F_0}{\partial L'} = 0\,,\quad \frac{\partial F_0}{\partial G} = 0\,,$$
$$\frac{\partial F_0}{\partial H} = 0\,,\quad \frac{\partial F_0}{\partial G'} = 0\,,\quad \frac{\partial F_0}{\partial H'} = 0$$

As these equations are five in number, they leave one of the six elements L, G, H, L', G', H' undetermined. Then we can take either L or L' at will, and the rest follow. Hence, provided the equations last given have suitable roots, the differential equations, we stopped with, will be satisfied.

The previous reasoning has been conducted on the supposition that we are dealing with three bodies; but it is plain that, however many bodies there may be in the system, similar propositions are true. The whole gist of the matter being that the initial values of the elements must be so adjusted that the secular perturbations vanish.

In the preceding we have argued as if the l, g, h, l', g', h' of DELAUNAY were six independent angular elements in the Problem of Three Bodies. But, as there are only four, always two less than three times the number of planets, it is necessary to modify the reasoning. Let us denote the independent angular elements by l_1, l_2, l_3, l_4, of which the last may be the single element having a finite motion. Also let their conjugate linear elements be severally L_1, L_2, L_3, L_4. Then

$$F = \Sigma\,.\,C\cos[i_1 l_1 + i_2 l_2 + i_3 l_3 + i_4 l_4]$$

From this, by a series of DELAUNAY transformations, will be removed all the terms involving l_4. Then we shall have

$$F_0 = \Sigma\,.\,C\cos[i_1 l_1 + i_2 l_2 + i_3 l_3]$$

After this L_4 will be a constant; so that the coefficients C can be regarded as involving only the three variables L_1, L_2, L_3. Then, to bring about a periodic solution, it is necessary, in the first place, to make

$$l_1 = 0^\circ \text{ or } 180^\circ\,,\quad l_2 = 0^\circ \text{ or } 180^\circ\,,\quad l_3 = 0^\circ \text{ or } 180^\circ$$

These substitutions made in F_0 reduce it to a function of the three quantities L_1, L_2, L_3, still regarded as variable. But, in the second place, the equations

$$\frac{\partial F_0}{\partial L_1} = 0\,,\quad \frac{\partial F_0}{\partial L_2} = 0\,,\quad \frac{\partial F_0}{\partial L_3} = 0$$

must be satisfied. Thus, if we suppose that the constant element L_4 is selected at random, the values of the other linear elements become determinate; and the motion of the conjugate element l_4 is given by the equation

$$\frac{dl_4}{dt} = -\frac{\partial F_0}{\partial L_4} = \text{a constant}$$

It will be noticed that the six conditions, just set down, are precisely those required to make the function F_0 at a standstill in reference to the six variables L_1, L_2, L_3, l_1, l_2, l_3.

In appearance eight different periodic solutions ought to belong to each assigned value of L_4; but these are often not all distinct. Also we may be restricted to a limited range of values for L_4 if real solutions are to be obtained.

Our illustrations will be selected from the coplanar case of the Problem of Three Bodies; and thus one equation will drop out from each of the two groups of three conditions. In this case, if we extend somewhat the signification of the phase "line of apsides," a little consideration will show that the two remaining conditions, having reference to the angular elements, are tantamount to the statement that, in order for the existence of a periodic solution, the lines of apsides of the two planets must coincide, and of symmetrical conjunctions and oppositions there must be two upon this line in each period of the solution.

Moreover, we shall suppose that *Jupiter* is one of the planets and a minor planet the other. The latter being supposed to have an evanescent mass, the motion of *Jupiter* is Keplerian with elements not admitting adjustment. Hence one more of the second group of three conditions will fall out, and there is no opportunity for the selection of any linear element. Here all that distinguishes one periodic solution from another is the value of the rational ratio $\frac{k}{k'}$.

If we adopt the L and G of DELAUNAY for the minor planet, their values in this case must satisfy the equations

$$\frac{\partial F_0}{\partial G} = 0\,,\quad \frac{\partial F_0}{\partial L} = -n$$

where n is the mean motion of the minor planet, deduced at once from the ratio $\frac{k}{k'}$, and the known n' of *Jupiter*.

F, expanded in periodic series has the form

$$F = \Sigma C \cos[il + i'l' + j\gamma]$$

where i is a positive integer, but i' and j may be either positive or negative integers, and γ denotes the angular distance of the perihelion of the small planet from that of *Jupiter*. The coefficients C are of the form

$$C = A_0 e^{j} + A_1 e^{j+2} + A_2 e^{j+4} + \ldots$$

the A being constants. After all the terms having arguments with sensible motions have been removed from F it is plain we may write

$$F_0 = \Sigma C_{i,j} \cos[(i+j) l_1 + j\gamma]$$

where $C_{i,j}$ has the same quality as C before, and l_1 denotes the argument whose motion vanishes to make the periodic solution. Then there appear to be four different solutions for each selection of the argument l_1 as follow:

$$\text{Solution I} \quad \gamma = 0^\circ \;,\; l_1 = 0^\circ \;,\; k'\frac{\partial \Sigma C_{i,j}}{\partial L} + kn' = 0 \;,\; \frac{\partial \Sigma C_{i,j}}{\partial G} = 0$$

$$\text{Solution II} \quad \gamma = 180^\circ,\; l_1 = 0^\circ \;,\; k'\frac{\partial \Sigma (-1)^j C_{i,j}}{\partial L} + kn' = 0 \;,\; \frac{\partial \Sigma (-1)^j C_{i,j}}{\partial G} = 0$$

$$\text{Solution III} \quad \gamma = 0^\circ \;,\; l_1 = 180^\circ,\; k'\frac{\partial \Sigma (-1)^{i+j} C_{i,j}}{\partial L} + kn' = 0 \;,\; \frac{\partial \Sigma (-1)^{i+j} C_{i,j}}{\partial G} = 0$$

$$\text{Solution IV} \quad \gamma = 180^\circ,\; l_1 = 180^\circ,\; k'\frac{\partial \Sigma (-1)^i C_{i,j}}{\partial L} + kn' = 0 \;,\; \frac{\partial \Sigma (-1)^i C_{i,j}}{\partial G} = 0$$

But if we put

$$\Sigma\,.\,C_{i,j}\,(i \text{ and } j \text{ both even}) = D_1 \quad , \quad \Sigma\,.\,C_{i,j}\,(i \text{ even, } j \text{ odd}) = D_2$$
$$\Sigma\,.\,C_{i,j}\,(i \text{ odd, } j \text{ even}) = D_3 \quad , \quad \Sigma\,.\,C_{i,j}\,(i \text{ and } j \text{ both odd}) = D_4$$

the third and fourth equations of each Solution become

$$\text{Solution I} \quad k'\frac{\partial(D_1 + D_2 + D_3 + D_4)}{\partial L} + kn' = 0 \;,\; \frac{\partial(D_1 + D_2 + D_3 + D_4)}{\partial G} = 0$$

$$\text{Solution II} \quad k'\frac{\partial(D_1 - D_2 + D_3 - D_4)}{\partial L} + kn' = 0 \;,\; \frac{\partial(D_1 - D_2 + D_3 - D_4)}{\partial G} = 0$$

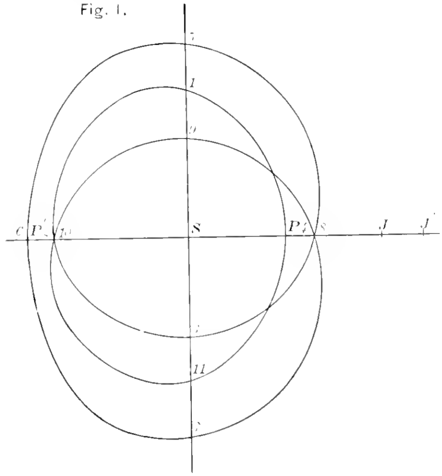
$$\text{Solution III} \quad k'\frac{\partial(D_1 - D_2 - D_3 + D_4)}{\partial L} + kn' = 0 \;,\; \frac{\partial(D_1 - D_2 - D_3 + D_4)}{\partial G} = 0$$

$$\text{Solution IV} \quad k'\frac{\partial(D_1 + D_2 - D_3 - D_4)}{\partial L} + kn' = 0 \;,\; \frac{\partial(D_1 + D_2 - D_3 - D_4)}{\partial G} = 0$$

When i is even $C_{i,j}$ involves only even powers of e. But when i is odd $C_{i,j}$ involves only odd powers of the same. Hence D_1 and D_2 contain only even powers of e, while D_3 and D_4 contain only odd powers of the same. Consequently Solution IV is not distinct from I, nor III from II. There is then no need to set $l_1 = 180^\circ$ as all distinct solutions may be got from the assumption $l_1 = 0^\circ$. As there are no arbitrary constants which may be taken at random in this matter, all that distinguishes one periodic solution from another are the values assigned to k and k', and the two values 0° or 180° which may be assigned to γ.

The elucidation of this subject in the fashion of synthetical geometry is worthy of attention. In the diagram let S denote the position of the *Sun* adopted as the origin of a system of rectangular coördinates, J the position of *Jupiter*, and let SJ the radius of *Jupiter* be the axis of x. Give the system of axes a variable rotation in the plane of *Jupiter's* orbit, so that the axis of x may continuously pass through the position of *Jupiter*. Then this planet will appear to oscillate on the axis of x between J, which we take to be the position of the planet at perihelion, and J' its position at aphelion. Suppose that, when *Jupiter* is at J, the minor planet crosses orthogonally the axis of x at the point P, or, in other words, that there is then a sym-

Fig. 1.



metrical conjunction. Now let an odd number of semi-revolutions of *Jupiter* elapse, *Jupiter* will then be at J' in aphelion. Next, let the radius SP of the minor planet and its velocity at the point P relative to the moving system of axes be so adjusted that the planet accomplishes an odd number of synodic semi-revolutions, and crosses othogonally the axis of x at a point P' on the farther side of S, or, in other words, that there is now symmetrical opposition. Again, let the same odd number of semi-revolutions of *Jupiter* elapse. This planet is then at J, or, again in perihelion. On account of the symmetry of the motion of *Jupiter* in these two equal intervals of time, it is plain the minor planet will, from the point of symmetrical opposition, repeat its path, but in reverse order with respect to time. Thus it will arrive again at the point P, and cross orthogonally the axis of x, and a complete period of its motion will be accomplished. The axis of x will divide what may be called the synodic orbit into symmetrical halves.

It is plain that we may, in the foregoing, interchange the points J and J'; that is, make the symmetrical conjunction occur when *Jupiter* is in aphelion, and the symmetrical opposition when it is in perihelion; thus the number of periodic solutions obtained will be doubled. If we have elaborated the first group of solutions in leaving e' the eccentricity of *Jupiter's* orbit indeterminate, it is plain the second group will result by simply changing the sign of e'.

The adjustment, spoken of above, is not proved to be possible for all values of the odd number of semi-revolutions mentioned; that is, in some cases we might find imaginary or unsuitable roots for the equations involved. But it is certain, however, that some periodic solutions of the character described have a real existence.

Let $2i+1$ denote the odd number of revolutions of *Jupiter* in the period of the solution, and $2j+1$ the odd number of synodic revolutions of the minor planet in the same time; then

$$\frac{n}{n'} = \frac{2i+2j+2}{2i+1}$$

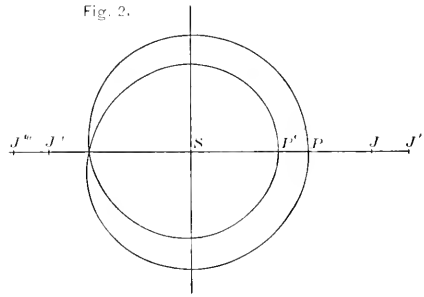
It is of sufficient interest to give a table to double entry for the values of this ratio corresponding to the smaller values of i and j.

i	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
0	2	4	6	8	10	12
1	$\frac{4}{3}$	2	$\frac{8}{3}$	$\frac{10}{3}$	4	$\frac{14}{3}$
2	$\frac{6}{5}$	$\frac{8}{5}$	2	$\frac{12}{5}$	$\frac{14}{5}$	$\frac{16}{5}$
3	$\frac{8}{7}$	$\frac{10}{7}$	$\frac{12}{7}$	2	$\frac{16}{7}$	$\frac{18}{7}$
4	$\frac{10}{9}$	$\frac{4}{3}$	$\frac{14}{9}$	$\frac{16}{9}$	2	$\frac{20}{9}$
5	$\frac{12}{11}$	$\frac{14}{11}$	$\frac{16}{11}$	$\frac{18}{11}$	$\frac{20}{11}$	2

The diagram is meant to illustrate the case where $j=1$, that is, where there are three synodic revolutions of the minor planet in the period of the solution. It will be seen that the synodic orbit of the planet has, in this case, two multiple points situated on the line of symmetrical conjunctions and oppositions; the first (marked 2) lies between S and P', the second (marked 4) lies beyond the point P. As the curve has not been plotted from calculation, being intended to illustrate only the general appearance of things, this order may need to be reversed. The order of motion of the minor planet is readily followed in the diagram by the numerals attached at the completion of each quadrant of synodic movement, viz., P, 1, 2, 3, 4 . . . . 10, 11, P.

It will be noticed that the table just given does not contain all possible rational quantities, and thus the field for periodic solutions is not yet exhausted. This is because we have limited the exposition to the case where the two orthogonal crossings of the line of syzygies lie on opposite sides of S, or where one is a conjunction and the other an opposition. But both may lie on the same side of S. Fig. 2 is designed to exemplify the later case.

Fig. 2.



Suppose that when *Jupiter* is in perhelion at J the minor planet is in symmetrical conjunction at P. After an odd number of semi-revolutions of *Jupiter* this planet will be in aphelion at J'. The radius SP of the minor planet and its velocity at P relative to the moving system of axes may be so adjusted that the planet accomplishes in the interval an integral number of synodic revolutions, and crosses orthogonally the axis of x at a point P' on the same side of S as P, and there is again symmetrical conjunction. Next, let the same odd number of semi-revolutions of *Jupiter* elapse. The planet is then at J and in perihelion. On account of the symmetry of the motion of *Jupiter* in these equal intervals, it is plain the minor planet will, from the second symmetrical conjunction, repeat its path, but in reverse order with regard to time. Thus it will arrive again at P, and intersect the axis of x orthogonally, and a complete period of its motion will have been gone through.

The figure represents the case where there are two synodic revolutions in the period of the solution. There is but one multiple point. If the points J and J' are rotated through a semi-circle, so that they fall into the positions

J'' and J''', the figure will represent the case where there are two symmetrical oppositions.

Let $2i+1$ denote the odd number of revolutions of *Jupiter* in the period of the solution, and $2j$ the even number of synodic revolutions of the minor planet in the same time. Then we shall have $\frac{n}{n'} = \frac{2i+2j+1}{2i+1}$ and the following table:

i	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
0	1	3	5	7	9	11
1	1	$\frac{5}{3}$	$\frac{7}{3}$	3	$\frac{11}{3}$	$\frac{13}{3}$
2	1	$\frac{7}{5}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{13}{5}$	3
3	1	$\frac{9}{7}$	$\frac{11}{7}$	$\frac{13}{7}$	$\frac{15}{7}$	$\frac{17}{7}$
4	1	$\frac{11}{9}$	$\frac{13}{9}$	$\frac{5}{3}$	$\frac{17}{9}$	$\frac{19}{9}$
5	1	$\frac{13}{11}$	$\frac{15}{11}$	$\frac{17}{11}$	$\frac{19}{11}$	$\frac{21}{11}$

Let us grant that in illustration we limit ourselves to cases where the two planets do not approach each other very nearly. Then at least a rough idea may be obtained of the course of the synodic orbit in neglecting the square of the disturbing force. Thus no DELAUNAY transformations are necessary.

From the point of view of calculation all periodic solutions may be divided into two classes; first, those where $k-k'$ is a large integer (say 8 or larger); second, those where the same difference is a small integer (ranging from 1 to 7). Dealing with the first class we may throw out from F all the periodic terms, since in F_0 they would be factored by powers of e higher than e^7, and retain only the so-called secular terms. With the second class, however, it will be necessary to retain, in addition, all the sensible terms involving the general argument $i(k'l-kl')+j\gamma$ whose motion vanishes for the periodic solution treated; for here these terms are secular to the same title as those mentioned for the first class.

With the limitations we have imposed, the construction of a periodic solution of the first class is a quite simple affair.

Let a' denote the semi-axis major of *Jupiter*, Δ the distance between the two planets, ω and ω' the longitudes of their perihelia, and R the secular portion of the periodic development of $\frac{a'}{\Delta}$. Neglecting the simple powers of e' because they disappear in the partial differentiation with respect to e, and carrying the approximation to the degree of neglecting terms of the 8th order with reference to the eccentricities, it is well known that

$$\begin{aligned} R = {} & A_1e^2 + A_2e^4 + A_3e'^2e^2 + A_4e^6 + A_5e'^2e^4 + A_6e'^4e^2 \\ & -(A_7e'e + A_8e'e^3 + A_9e'^3e + A_{10}e'e^5 + A_{11}e'^3e^3 + A_{12}e'^5e) \\ & \qquad \cos(\omega-\omega') \\ & +(A_{13}e'^2e^2 + A_{14}e'^2e^4 + A_{15}e'^4e^2)\cos 2(\omega-\omega') \\ & -A_{16}e'^3e^3\cos 3(\omega-\omega') \end{aligned}$$

where the A are positive constants and functions of α the ratio of the mean distances. Make in this expression $\omega-\omega' = 0°$ or $180°$, and take the partial derivative with reference to e. Thus

$$\begin{aligned} \frac{\partial R}{\partial e} = {} & \mp(A_7e' + A_9e'^3 + A_{12}e'^5) + 2[A_1 + (A_3+A_{13})e'^2 + (A_6+A_{15})e'^4]e \\ & \mp 3[A_8e' + (A_{11}+A_{16})e'^3]e^2 + 4[A_2 + (A_5+A_{14})e'^2]e^3 \\ & \pm 5A_{10}e'e^4 + 6A_4e^5 \end{aligned}$$

where the upper sign belongs to the value 0°, and the lower sign to the value 180° for $\omega-\omega'$. This expression must vanish in order that the line of apsides of the minor planet may have no secular motion. This affords the condition necessary to the determination of e. It is seen at once that if e is to have a positive value, the upper of the ambiguous signs must be taken. The assumption of the lower sign leads to a value of e the negative of the former; which would mean that the ellipse must be revolved through a semicircle. Thus the latter assumption is not really distinct from the former.

Making $e' = 0.04825336$, the equation in e has been computed for every 0.01 in the value of α from $\alpha = 0.01$ up to $\alpha = 0.70$, and the value of e obtained therefrom. The following table contains these values, but only for every 0.02 in the value of α.

α	e	α	e	α	e	α	e	α	e
0.02	0.0012091	0.16	0.0096411	0.30	0.0179216	0.44	0.0258982	0.58	0.0333726
0.04	0.0024178	0.18	0.0108366	0.32	0.0190838	0.46	0.0270022	0.60	0.0343837
0.06	0.0036258	0.20	0.0120286	0.34	0.0202392	0.48	0.0280955	0.62	0.0353776
0.08	0.0048326	0.22	0.0132167	0.36	0.0213875	0.50	0.0291772	0.64	0.0363529
0.10	0.0060379	0.24	0.0144005	0.38	0.0225281	0.52	0.0302466	0.66	0.0373080
0.12	0.0072414	0.26	0.0155796	0.40	0.0236605	0.54	0.0313029	0.68	0.0382412
0.14	0.0084426	0.28	0.0167531	0.42	0.0247811	0.56	0.0323453	0.70	0.0391503

Choosing $\frac{k}{k'} = \frac{n}{n'}$, we obtain α from the equation

$$\log\alpha = 9.9998618 - \frac{2}{3}\log\frac{n}{n'}$$

and, entering the table with this argument, have the value which must be attributed to the eccentricity of the minor planet in order that the line of apsides may have no secular motion.

The treatment of the case where $k-k'$ is a small integer is reserved for a second article.

PERSONAL EQUATION RELATIVE TO STELLAR MAGNITUDE FOR ALBANY OBSERVATIONS,

BY LEWIS BOSS.

During the progress of the Albany Zone-Observations in the years 1879, 1880 and 1882, special observations were made to determine the effect upon personal equation in transit produced by an apparent diminution in the brightness of the star's image. This diminution was produced by means of screens held in front of the objective. The general results of this investigation are printed, in some detail, in the Introduction of the Albany Zone-Catalogue (p. 17). In the discussion of these special observations we may assume, either that the effect on personal equation is directly proportional to the apparent diminution of star-magnitude (hypothesis I), or that the ratio of this effect on personal equation varies with the magnitude, and that it becomes, perhaps, greater for the fainter stars (hypothesis II). If M denote the magnitude of the star, and $4^{M}.0$ be taken as a convenient and natural point where the equation is assumed to be zero, we may express the magnitude-equation thus:

$$\text{Hypothesis I:} \quad h\alpha = (M-4)\,x_1$$
$$\text{Hypothesis II:} \quad h\alpha = (M-4)\,x_2 + (M-4)^2\,y_2$$

The results of the screen-experiments at Albany in 1879–82 may then be expressed in the following summary:

Year	No. Det.	Hyp. I. x_1	Hyp. II. x_2	y_2
		s	s	s
1879	91	−0.0118	−0.0098	−0.00018
1880	97	−0.0130	−0.0056	−0.00157
1882	89	−0.0177	−0.0119	−0.00124
Means	277	−0.0142	−0.0087	−0.00116

I have preferred the results from hypothesis II, which, however, differs only slightly from the other for magnitudes between $3^{M}.0$ and $9^{M}.5$.

Though aware of the explanation of this phenomenon derived from physical experiment it seemed to me possible that a large part of this equation in my own case might be due to my habit of observing transits. I had fallen into the habit of trying to tap the registry-key in such a manner that the sound produced would seem to me simultaneous with the apparent bisection of the star by a transit-thread. This method I found very natural and very precise. But it seemed to me that the impulse to tap the key might be governed in a very large degree by the relation to the transit-thread of the preceding edge of the star-image; so that the tendency might be to anticipate when observing the bright stars. Accordingly, in 1884, I determined to alter my method of observing. I would accept the impulse to tap the key when the image appeared to be bisected by the thread. With close concentration of attention I persevered in this practice until it seemed natural and resulted in what now appears to be a settled habit.

In my article upon the "Positions of 465 Comparison-Stars determined at Albany" in 1886–7 (see *A.J.*, Vol. X, p. 75) I called attention to this change of habit and alluded to the reason for it in the following words: "Whether, in accordance with intention, this plan has caused the error of transit dependent on magnitude, which is exhibited in the observations for the Albany A.G. Zone, to disappear, is a problem which remains for future investigation." At that time the recently adopted method of observing was found still difficult and the transits suffered in accuracy. In the observations made in 1896–1900 at the new Dudley Observatory, where something like 30,000 transits have been registered by the writer, this habit of observing, as I have stated, has become fixed and the casual probable error of transit over a single thread is less than $\pm 0^{s}.04$.

Accordingly, in the year 1899, I investigated anew my personal equation for star-magnitude. The method adopted was practically identical with that employed in 1879–1882. But I used only one wire screen, — screen II, of wire gauze, as described in the introduction of the Albany A.G. Zone-Catalogue (p. 16). In all 219 separate determinations were made. The observations were so conducted as to eliminate any possible error in the adopted wire intervals. The field illumination during each evening remained constantly very bright and the same for faint as for bright stars. The precision attained seems to indicate that there would have been no material gain in the further multiplication of observations.

There is some doubt about the absorption of the screen, — a doubt which it might be difficult to resolve in a satisfactory manner. The adopted absorption is determined from the observations themselves. It was always my custom to estimate the magnitude of the image after its light had been reduced by the screen, but for magnitudes of stars much brighter than $7^{M}.0$, estimates made of the image formed by an objective so large as ours (eight inches) must necessarily be very uncertain both in the systematic and casual sense. For the image at undiminished brightness when the star is $6^{M}.0$, or brighter, I have adopted the magnitudes of the Harvard Photometry. My own scale of magnitudes depends upon the Bonn *Durchmusterung*, and for stars from $7^{M}.5$ to $9^{M}.0$ is well fixed. For the immediate purpose contemplated the possible lack of uniformity in the scale of magnitudes employed is not of much consequence, as will be perceived. The estimates of magnitude

for the reduced magnitudes were intended to be, and probably are, sensibly conformable to the estimates of magnitude of the corresponding classes in the observations for the catalogue throughout the years 1896–1900. But when the experiment under consideration is treated as an abstract investigation of a physical constant the case is different. At the same time, uncertainty as to the absorption of the screen is probably not sufficient to affect the general conclusions of this paper, and I have not considered it worth while to enter upon a detailed discussion of the question.

If we take 199 estimates of the absorption of the screen, it results that the use of it in front of the objective diminishes the brightness of the star-image by $2^M.52$, a result quite accordant with that which was found in 1879–1882. But if we consider the absorption in relation to the scale of magnitudes employed, the observations indicate that it varies. In the following table of observed absorptions the magnitude of the star at full brightness is the argument.

Mag.	Observed absorption	No. Det.
M	M	
3.3	3.13	24
4.0	2.83	15
5.1	2.79	34
6.1	2.51	90
6.8	2.19	36

The mean magnitude of the diminished images can be found by adding the first and second columns. From this table of observed absorptions it has been assumed that the computed effective diminution of magnitude due to the employment of the screen was $3^M.1$ for a star of the third magnitude and $2^M.4$ for a star of magnitude 6.8. The effect for intermediate magnitudes has been interpolated from these values.

The effect upon time of transit due to interposition of the screen in front of the object-glass was determined on ten days in 1899 as follows:

Date	No. Det.	Screen effect.
		s
April 5	20	−0.0272
10	18	−0.0512
June 26	13	−0.0302
29	23	−0.0337
30	11	−0.0318
July 10	19	−0.0262
12	27	−0.0456
14	25	−0.0514
18	41	−0.0271
Aug. 1	22	−0.0315

The general mean, whether by nights or by individual determinations, is $-0^s.0356$. There is some ground for supposing that for two, or three, of the nights the personal equation might possibly have been larger than on the others. Thus, on July 14, only five of the 25 separate determinations are below the general mean, $-0^s.036$, and only two of these ($+0^s.008$ and $+0^s.031$) materially so. Subtracting the general mean from each individual determination, the indiscriminate mean of the resulting residuals is $\pm 0^s.028$, corresponding to the probable error of a transit over one group of threads (usually eight) of $\pm 0^s.015$. Most of the observations were made between zenith-distances 60° and 70°.

From the foregoing it results that when the brightness of the telescopic image is reduced by one magnitude the transit is registered later by $0^s.0141$. In other words the relative correction to the transit of a star, one magnitude fainter than another, is $-0^s.0141$. The corresponding correction obtained in 1878–1882 was $-0^s.0142$ (Alb. Zone-Catal., Int., p. 17). This agreement would be remarkable, even were it merely a question of persistence under identical conditions as to method; but it seemed to me still more remarkable (as it certainly was unexpected) when the fact is considered that between the two series the habit of the observer had been totally altered, in the hope that his magnitude-equation might be reduced to an insignificant quantity.

For the practical application of the correction it is probably better to treat the absorption of the screen as variable. We then have the following schedule of means:

No. Det.	Absorption	Ft. Image	Effect for one mag.
	M	M	s
12	3.10	5.0	−0.014
32	3.06	6.2	−0.011
10	2.90	6.9	−0.008
6	2.81	7.3	−0.011
12	2.73	7.7	−0.020
22	2.64	8.1	−0.014
33	2.57	8.4	−0.015
57	2.51	8.7	−0.011
35	2.44	9.0	−0.015

Discussing this determination according to hypothesis II, as described in the foregoing, we have as the correction of an observed transit,

$$\Delta a = -0^s.0132\,(M-4) - 0^s.00019\,(M-4)^2$$

This again is a very good approximation to the similar result for 1879–1882. This is clearly illustrated in the following comparison of corrections:

Mag.	1881	1899
M	s	
4.0	0.000	0.000
5.0	−0.010	−0.013
6.0	−0.022	−0.027
7.0	−0.036	−0.041
8.0	−0.053	−0.056
9.0	−0.073	−0.071
10.0	−0.094	(−0.086)

The probable errors calculated from the 219 separate determinations are for x_2, $\pm 0^s.00039$, and for y_2, $\pm 0^s.00023$. The real probable errors are doubtless materially greater. Thus the value of y_2 is decidedly less than its probable error; and from this series alone it would be impossible to assert anything in favor of the hypothesis that the ratio

of the effect per magnitude increases with diminishing brightness of the star. The present experiment, however, is not well designed to exhibit this effect in relation to very faint stars since in only 12 instances was the brightness of the diminished image estimated to be fainter than $9^{M}.0$, and in none fainter than $9^{M}.3$. With better evidence on this point in 1882 the term depending on the square of the magnitude-interval was found to be $-0^{s}.00116$, six times as great as in the present series.

Considered from every point of view, the observations of 1899 leave no doubt in my mind as to the nature of this personal equation effect in my own case. I interpret it as meaning that no important part of the equation is due to the apparent size of the focal image in relation to the transit-thread, as I had formerly surmised might be the case. That observations by the eye-and-ear method may also, in some instances, be affected in the same way as for chronographic registry seems very probable.

Observers hitherto scarcely seem to have realized the importance of this magnitude-equation relative to other important sources of error. The art of observation has so progressed and the accumulation of observations has become so great, that elements of systematic uncertainty hitherto unconsidered are emerging to view with increasing relative importance. There is now no particular difficulty in determining the right-ascension of a star by one observation with an apparent probable error of $\pm 0^{s}.025$ in the strict differential sense. This degree of precision has already been surpassed by several observers. Through a somewhat extended investigation of my own I find it to be a rare thing that the deviation of the calculated meridian as described by any good modern instrument is as much as $0^{s}.02 \sec\delta$ distant at any point from that of the mean system of all. With only few exceptions, among the important star-catalogues, the apparent accuracy of modern meridian-observations, contrasted with the prevailing opinion about them, is very reassuring. If, then, the personal equation in relation to star-magnitude produces a common error in the great majority of modern observations of as much as $-0^{s}.01\ (M-4)$, then by far the chief uncertainty in the observed right-ascensions of stars fainter than the seventh magnitude arises from our want of knowledge as to this source of error.

Obviously, then, the determination of the magnitude-equation is quite as important as the observations themselves. Even if, in order to be satisfactory, such an investigation must consume as much labor as the position-observations themselves, it must be regarded as labor well and economically employed.

ON THE LIGHT-VARIATIONS OF 7085*a* *SU CYGNI*.

BY PAUL S. YENDELL.

In *A.J.* 515, p. 88, mention is made of a mean light-curve of *SU Cygni*, formed upon the elements of LUIZET.

This curve was made by me in February, 1901, from the observations of the two previous seasons, in preparation for a reduction and discussion of these observations, with a view to the examination and possible improvement of the star's elements of variation.

This work was for the time interrupted at the completion of the mean light-curve, and was not resumed until the end of the season of 1901.

At that time, on examining the results of the season's work, the curve was found to be so satisfactory that it seemed not worth while to go to the labor of repeating the work of its formation for the sake of incorporating with it the somewhat meagre list of observations obtained during the autumn of 1901.

The curve was plotted in the usual way, from normals of twenty-four groups of five observations each, and the mean departure of the twenty-four normals from the curve as finally plotted was $0^{M}.06$.

The range of variation indicated is about a tenth of a magnitude greater than that shown by the Potsdam-curve, and the duration of the increase about $0^{d}.20$ greater. This latter, however, is not an excessive disagreement, considering the flatness and uncertainty of the star's light-curve at this phase. The general character of the two curves is identical. The comparison-stars used were as follows:

COMPARISON-STARS FOR 7085*a* *SU Cygni*. (1855.)

	DM.	α	δ		Light
		h m s	° ′	M	
h =	+30 3706	19 37 28.1	+30 19.9	6.30	15.0
a =	28 3147	19 37 1.0	28 59.3	6.76	10.9
d =	28 3412	19 31 23.1	28 10.9	7.16	6.3
c =	29 3710	19 37 30.8	29 53.5	7.18	2.7
b =	+29 3702	19 36 25.8	29 9.2	7.56	1.7

READINGS FROM MEAN LIGHT-CURVE.

	Y	*M&K*	*Y–M&K*		*Y*	*M&K*	*Y–M&K*
d	M	M	M	d	M	M	M
0.0	6.60	6.57	+0.03	2.2	7.44	7.27	+0.17
0.1	6.625	. .	. .	2.4	7.47	7.31	+0.16
0.2	6.70	6.65	+0.05	2.6	7.48	7.31	+0.14
0.4	6.84	6.73	+0.11	2.7	7.49	. .	. .
0.6	6.95	6.83	+0.12	2.8	7.48	7.37	+0.11
0.8	7.04	6.89	+0.15	3.0	7.46	7.37	+0.09
1.0	7.13	6.97	+0.16	3.2	7.40	7.23	+0.17
1.2	7.20	7.03	+0.17	3.4	7.26	7.05	+0.21
1.4	7.26	7.08	+0.18	3.6	6.91	6.85	+0.06
1.6	7.31	7.14	+0.17	3.7	6.72	. .	. .
1.8	7.36	7.18	+0.18	3.8	6.62	6.63	–0.01
2.0	7.40	7.23	+0.17	. .	. .	. .	. .

Dorchester, Mass., 1902 *Jan.* 25.

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NOTE ON THE MAGNITUDE-EQUATION IN RIGHT-ASCENSION FOR THE CAMBRIDGE A.G. ZONE AND FOR THE RECENT BONN OBSERVATIONS.

By LEWIS BOSS.

Suppose we have for a given epoch a series of observed right-ascensions which can be regarded as free from personal error dependent on star-magnitude. This may be employed as a standard of comparison for the purpose of ascertaining the magnitude-equation of other observed series of right-ascensions. But it is extremely desirable, in the first place, that the general systematic difference between the catalogues compared should be known, or at least that those terms of systematic difference which are functions of α and δ should be known with accuracy.

In the second place, if the two catalogues compared are not closely contemporaneous, the neglect of proper motion, or even the use of systematically imperfect proper motions, is liable seriously to impair the value of the conclusions if, indeed, it does not destroy it. Even for differences of epoch between the catalogues compared of no more than ten or fifteen years, the proper motion should be determined for each star, and especial care should be exercised as to its accuracy in the systematic sense.

For the sake of illustration let us examine the results obtained by Professor TURNER (*M.N.* LX, p. 3) in his comparison of the Cambridge (Eng.) A.G. Zone with the Oxford photographs. Professor TURNER found the constants of his plates by reducing the Cambridge zone-positions to the epoch of the photographs. He says (p. 5): "Proper motions have been applied (taken from the Greenwich Catalogue 1880.0) for the interval between the Cambridge and Oxford observations, though in other groups P.M. has been treated as accidental error." From inspection of his table of observed differences, Oxford — Camb. (p. 4), it is evident that the star-positions from his plates depend essentially upon the computed Cambridge positions of stars between the eighth and tenth magnitudes, corresponding to a mean of about $9^{\rm M}.1$. To what extent large discrepancies were rejected in the derivation of plate constants does not appear, but if we assume that the larger ones were rejected then the right-ascensions of ninth magnitude stars from the photographs require a systematic correction something like that indicated in the following table. If the larger discrepancies were not rejected then the corrections might have been somewhat larger.

R.A.	Corr'n.	R.A.	Corr'n.
h	″	h	″
0	+0.003	12	−0.027
3	−0.003	15	−0.021
6	−0.013	18	−0.011
9	−0.024	21	0.000

These corrections arise from two sources. Professor TURNER employed STRUVE's precessions in reducing Cambridge positions from about 1880 to about 1895. STRUVE's precession constant is almost certainly too large. The correction to STRUVE's precession which I have derived in a discussion published in *A.J.* 501 is founded upon a discussion of the motions of about 6000 bright stars, distributed over both hemispheres, with due attention to the systematic errors of the catalogues. While this discussion is no more than tentative, I think that the resulting correction is as likely to be too small as it is to be too large. I have therefore assumed that for the interval of fifteen years in Professor TURNER's reductions the correction required for that zone on account of precession is,

$$-0^{\rm s}.012 \quad -0^{\rm s}.0026 \ \sin\alpha$$

At the very least this represents a probable uncertainty.

Alluding to his neglect of proper motions in reducing Cambridge A.G. (1880) to the epoch of the plates, 1895, Professor TURNER says (*M.N.* LX, p. 3-4): "These may be regarded as accidental errors in the present discussion, which will disappear in the mean of a number of stars." Unless this statement is surrounded by several qualifications it is by no means correct. In this *Journal* (No. 496) I have shown that after we discard the known proper motions greater than 0″.10, even for the faint stars around the ninth magnitude, a well marked parallactic motion can be recognized, — that is to say, apparent motion of the stars due to the *Sun's* real motion in space. The persistence and general uniformity of this motion, even among the very faint stars, does not seem to be generally recog-

nized as a factor in computation, though attention has been called to it by several astronomers. It may be estimated that the average motion of ninth magnitude stars is very nearly 0".01. The general conformity of these with the reflected solar motion is such that, if one takes a group of 100 faint stars not more than 20° from the equator, it will be found in almost every instance that the mean of all the motions is distinctly southward. With few exceptions even 20 stars, or less, are sufficient to show this effect. At the same time the average motion in right-ascension will be found to be *minus* near 12^h, and *plus* around 0^h. For his fainter stars, by his method of reduction, in the interval of about fifteen years, Professor TURNER has thus introduced an error which requires for its correction (see *A.J.* 501, p. 168) something like

$$+0^s.017 \ \sin(\alpha+85°)$$

If he has rejected no stars for discordance this correction might be appreciably larger.

These two sources of error combined must have introduced a systematic error into the Cambridge right-ascensions reduced to 1895 of an amount which has been approximated in the preceding table.*

But if we assume that the stars employed in Professor TURNER's table (p. 4) are in all classes evenly distributed over the limits of right-ascension employed (0^h to 24^h for bright stars and 0^h to 12^h for faint stars) then the mean parallactic effect for all the classes of stars, while varying from a maximum of *plus* values near 0^h to a maximum of *minus* values near 12^h, might be very nearly zero in the mean.

But, if it should happen that in Professor TURNER's means for faint stars (0^h to 12^h) the distribution of stars in any class is not very symmetrical as to 6^h of right-ascension, then the parallactic effect might become sensibly different in its application, and more important for the groups near $7^M.0$ than for those near $9^M.0$.

There is always the chance of encountering an extensive group of non-conformable proper motions (perhaps a community of motions) like the one in *Taurus*. Though, perhaps, the fear of a serious mean effect from this source is not very great, still an uncertainty remains which is calculated to impair confidence in conclusions drawn under such circumstances.

These facts are calculated to raise a very tangible doubt as to whether Professor TURNER's table (p. 4) may not still contain a material residuum of effect due to parallactic motion of the stars not eliminated, as well as to irregularities of the proper motions.

For some of the brighter stars Professor TURNER employed proper motions in reducing from 1880 to 1895. If his remark (p. 5) means that he applied all the proper motions that could have been taken from Greenwich 1880, then an anomaly has been introduced into all the means of the brighter stars down to $7^M.0$. The proper motions in question are those of AUWERS-BRADLEY. The correction required to reduce these to conformity with NEWCOMB's Equinox is about $+0^s.0008$. A motion of the equinox, like NEWCOMB's, founded on or representing all the best meridian determinations, is so obviously superior to one based on BRADLEY and Greenwich 1860 alone, that it appears to leave no excuse for wasting time and space upon a discussion of the question. NEWCOMB's Equinox should be the system of reference until a better may be provided.

The result then is that where Professor TURNER has applied proper motions, his right-ascensions for 1895 require a further correction of about $+0^s.012$. Thus to make his system of brighter stars reduced with proper motions conformable to his basic system of faint stars the former should receive corrections amounting to $-0^s.024$ in the mean, assuming uniform distribution in right-ascension. That is to say, to the foregoing correction of the right-ascensions of faint stars,

$$-0^s.012 \ -0^s.0026 \sin\alpha \ +0^s.017 \ \sin(\alpha+85°)$$

should be added $-0^s.012$, the whole to represent a correction for the systematic relative difference between the Cambridge right-ascensions reduced to 1895 with proper motions, and those of faint stars reduced without proper motions. For the first three groups of Professor TURNER's table this correction ($-0^s.024$) applies in full force. For the later groups I have assumed that it applies in a diminishing ratio according to the proportion of stars having adopted proper motion. Correcting and condensing into groups I find for

OXFORD—CAMBRIDGE A.G.

Magn.	**	Corr'n.
3.6	20	+0.120
5.9	156	+0.100
7.0	167	+0.085
8.0	1076	+0.060
9.0	3227	−0.015
9.7	438	−0.065

As a matter of curiosity I have roughly plotted the curve of correction resulting from the preceding table; and

* While it is not a part of the present purpose to follow out the effect of Professor TURNER's procedure in the declinations which form the basis of his plate-constants, it may be of interest to note that, reasoning in a manner entirely analogous to that which has been employed in reference to the right-ascensions, his computed declinations, and consequently the declinations derived from his plates, require, in addition to the systematic correction of Cambridge A.G. for 1880, a further correction for precession and parallactic motion of about

$$-0''.25 + 0''.11 \sin(\alpha+5°) - 0''.08 \cos\alpha$$

so that the total correction would range between $-0''.12$ at 8^h and $0''.38$ at 20^h. But corrections for individual plates founded on a small number of stars might vary materially from those given by the formula, owing to possible groups of stars having non-conformable proper motions. It will be necessary hereafter to determine the peculiar error of each plate by means of stars whose positions and motions can be well determined.

while there is much room for difference of judgement, so that no claim for exact numerical accuracy in behalf of the practical application of the assumptions which underlie this revision can be made, there cannot be much doubt as to whether this revision is a move in the right direction. The following table exhibits the readings from the curve of correction reduced to the standard of $4^{M}.0$.

Correction to Cambridge A.G., R.A.

Magn.	Corr'n.	Magn.	Corr'n.
	s		s
1	[+0.012]	6	−0.021
2	[+0.010]	7	−0.040
3	+0.006	8	−0.071
4	+0.000	9	−0.116
5	−0.009	10	−0.182

The Cambridge observations of transits were all made by the method of eye-and-ear, and while the corrections which have just been deduced for the Cambridge right-ascensions are still open to possible and material modifications which might make the variation of the corrections between the first and eighth magnitudes less than I have made it, or even a vanishing quantity, yet it seems to me that those right-ascensions are very probably subject to material corrections for personality dependent on magnitude of a nature and amount similar to that which is found in the chronographic registry of transits. It has long been suspected that, whatever may be the case as to the brighter stars, eye-and-ear observations of the transits of very faint stars are peculiarly liable to errors of personality of a serious amount, and Professor Turner's discussion must be welcomed as a very satisfactory contribution to our knowledge upon this branch of the subject.

It will be noticed that from magnitudes $3^{M}.6$ to $8^{M}.0$ the mean observed rate of change in the equation is about $-0^{s}.014$, approximately the same as that which has been found elsewhere in many of the experiments made through the use of screens. That the rate may be really quite different from this for some observers by the eye-and-ear method there is abundant evidence.

It may not be without interest to observe that the results of the present discussion indicate that right-ascensions derived from the Oxford plates require approximately the following corrections to reduce them to the system of fourth magnitude stars of Auwers and Newcomb respectively. In constructing this table I assume that the fundamental right-ascensions of Auwers are systematically smaller by $0^{s}.012$ in 1895 than in 1880.

Systematic Correction in R.A., Oxford.

R.A.	A − O	N_1 − O	R.A.	A − O	N_1 − O
h	s	s	h	s	s
0	−0.119	−0.092	12	−0.149	−0.122
3	−0.125	−0.098	15	−0.143	−0.116
6	−0.135	−0.108	18	−0.133	−0.106
9	−0.146	−0.119	21	−0.122	−0.095

But in order to make visual observations, near the epoch 1895, comparable with right-ascensions from the Oxford plates the former must first be corrected for magnitude-equation.

Referring now to Professor Turner's paper in the *Monthly Notices*, (Vol. LXII, p. 3, Nov., 1901), which presents the results of a comparison of the Oxford photographic right-ascension with Dr. Küstner's Bonn observations it is found from indirect comparison (367 stars in Bonn) that for the mean magnitude $8^{M}.8$, the difference, Oxford − Bonn, is $+0^{s}.136$. But this result is subject to many of the uncertainties which I have pointed out as affecting the derivation of the magnitude-equation of Cambridge, A.G., which need not be recapitulated.

Since Küstner's positions depend upon the Fundamental System of Auwers, we presumably have, A. − Bonn $= 0^{s}.000$; though it does not follow that such is exactly the case. As shown above we have A − Oxford $= -0.134 - f(\alpha)$. Therefore we have for the difference of *systems* of right-ascension (corresponding, say, to $4^{M}.0$),

$$\text{Oxford} - \text{Bonn} = +0^{s}.134 + f(\alpha)^{*}$$

Subtracting this from the difference which Professor Turner finds ($+0^{s}.136$) for mean magnitude $8^{M}.8$ we have

$$\text{Oxford (corr'd for syst. diff.)} - \text{Bonn} = +0^{s}.002$$

But if we include only stars down to the ninth magnitude we shall have

$$\text{Oxford} - \text{Bonn}\ (8^{M}.3) = +0^{s}.129$$
$$\text{and Oxford (corr'd)} - \text{Bonn}\ (8.3) = -0.005$$

The natural conclusion would be that Dr. Küstner's magnitude-equation is very near zero. In dealing with small systematic differences, however, we must always bear in mind that the errors in such calculations may be cumulative; and especially does it not rigorously follow for a zone of stars determined at a given observatory that the general system of the observed positions is identical with that of the fundamental catalogue; although when proper precautions have been taken, the difference between the assumed and observed systems should be unimportant. Remembering, also, the uncertainty of Professor Turner's indirect comparison, Oxford − Bonn, in view of all the facts I should incline to the opinion that the most probable value of Dr. Küstner's magnitude-equation is zero, but that it may be anywhere from $-0^{s}.007\,(M-4)$ to $+0^{s}.007\,(M-4)$ without being inconsistent with the known facts, and under the assumption that the right-ascensions from the Oxford photographs are free from error having stellar magnitude as the argument.

* In this connection it would be interesting to have a direct comparison, Oxford—Bonn, arranged in order of right-ascension. The number of stars would probably be great enough to decide whether there is an inequality in the order of right-ascension comparable with that which I have predicted in the preceding table.

OBSERVATIONS OF MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1901 Greenwich M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(337) *Devosa.*[1]						
Jan. 26 16h 1m 16s	1	10, 6	−3m 51.46s	− 8′ 41.6″	9h 15m 8.41s	+26° 57′ 33.0″	n9.331	0.401	+2.68s	−15.9″
29 15 13 8	2	10, 8†	−0 1.90	+10 17.0	9 11 11.38	+27 1 15.3	n9.152	0.439	+2.74	−15.6
Feb. 6 15 10 52	3	8, 8†	+0 25.33	+ 6 3.7	9 2 15.50	+27 5 36.5	n9.177	0.370	+2.85	−14.8
7 15 32 22	3	15, 10	−0 43.25	+ 5 39.7	9 1 6.92	+27 5 12.6	n9.197	0.371	+2.85	−11.7
8 12 58 12	3	10, 8	−1 44.13	+ 5 8.0	9 0 5.74	+27 1 41.0	n9.613	0.517	+2.85	−11.6
				(372) *Palma.*[2]						
Feb. 12 15 17 16	4	10, 8	+1 9.67	+ 4 57.2	11 11 52.20	− 3 53 26.3	n9.530	0.787	+2.51	−15.5
19 16 2 4	5	10, 6	−1 27.10	+ 2 11.1	11 1 53.25	− 4 16 25.3	n9.347	0.797	+2.63	−16.6
				(451) *Patientia.*[1]						
Feb. 12 16 24 58	6	10, 8	−1 23.58	+ 2 24.7	11 37 24.71	+24 39 57.9	n9.190	0.504	+2.40	−19.5
19 17 6 3	7	10, 6	+0 53.36	+ 5 7.2	11 33 0.50	+25 36 56.6	n9.247	0.417	+2.58	−19.4
22 18 3 52	8	10, 8	−1 26.20	+ 2 30.2	11 30 51.50	+26 0 3.6	n8.438	0.374	+2.61	−19.2
				(196) *Philomela.*[1]						
Apr. 12 14 55 10	9	7, 8†	+0 6.20	+ 8 28.6	12 29 27.82	+ 7 3 54.9	n9.110	0.702	+2.86	−17.1
17 15 12 6	10	10, 8	+1 56.91	− 9 12.4	12 26 2.71	+ 7 14 46.7	7.725	0.696	+2.83	−16.7
				(315) *Tercidina.*[1]						
Apr. 27 16 29 9	11	6, 8†	+0 8.99	+ 6 27.9	12 3 41.65	− 5 6 9.3	9.317	0.803	+2.84	−19.2
				(247) *Eukrate.*[1]						
Sept. 25 13 34 26	12	9, 9*	−0 23.92	+ 3 4.6	23 56 42.26	− 0 4 37.4	n9.496	0.768	+4.26	+27.8
Oct. 1 14 13 35	13	4, 4*	−3 10.57	− 8 11.3	23 48 21.78	+ 0 34 13.3	n9.309	0.663	+4.26	+28.0
3 16 0 2	14	9, 9*	−1 43.51	− 9 27.1	23 45 33.23	+ 0 47 50.7	8.161	0.760	+4.27	+28.3
7 16 33 35	15	11, 11*	−0 29.21	+ 6 50.3	23 40 18.63	+ 1 14 40.7	9.073	0.757	+4.27	+28.6
				(325) *Heidelberga.*[2]						
Oct. 7 17 37 39	16	4, 4	−0 1.03	− 4 43.6	1 3 0.70	+14 31 31.3	8.937	0.606	+4.58	+25.3
8 15 20 55	16	5, 8	−0 46.94	− 6 35.1	1 2 14.80	+14 29 39.9	n9.254	0.617	+4.59	+25.4
9 14 16 19	17	10, 6	+0 13.79	0 0.0	1 1 24.30	+14 27 33.6	n9.351	0.626	+4.58	+25.6
10 14 0 25	17	8, 8†	−0 3.65	− 2 2.2	1 0 36.86	+14 25 31.5	n9.466	0.641	+4.58	+25.7
15 13 37 36	18	10, 6	+0 51.14	− 5 24.7	0 56 22.55	+14 13 30.5	n9.463	0.646	+4.58	+26.3

* Comparisons made with Square-Bar Micrometer. † Δα measured directly.

[1] Observations by CAROLINE E. FURNESS. [2] By MARY W. WHITNEY.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	9h 18m 57.19s	+27° 6′ 30.5″	Camb. (Eng.) A.G. 4911	10	12h 24m 2.94s	+ 7° 21′ 45.8″	Leipzig II, A.G. 6138
2	9 11 10.54	+26 51 43.9	" " " 4892	11	12 3 29.82	− 5 12 18.0	½[Munich I + Munich II]
3	9 1 47.32	+26 59 47.6	" " " 4830	12	23 57 1.92	− 0 8 9.8	Nicolajew A.G. 5942
4	11 10 40.02	− 3 58 8.0	Weisse I 11h	13	23 51 28.09	+ 0 41 56.6	" " 5920
5	11 6 17.72	− 4 18 19.8	½[Weisse I + Munich I]	14	23 47 12.47	+ 0 56 49.5	" " 5901
6	11 38 45.89	+24 37 52.7	Berlin (B) A.G. 4346	15	23 40 43.57	+ 1 7 21.8	" " 5888
7	11 32 4.56	+25 32 8.8	Camb. (Eng.) A.G. 5816	16	1 2 57.15	+14 35 49.6	Leipzig I, A.G. 303
8	11 32 15.09	+25 57 52.6	" " " 5818	17	1 0 35.93	+14 27 8.0	" " 289
9	12 29 18.76	+ 6 55 43.4	Leipzig II, A.G. 6169	18	0 55 26.83	+14 18 28.9	" " 261

NOTE ON CHAUVENET'S THEORY OF SOLAR ECLIPSES.

By WILHELM AUHAGEN.

As modifications to well established formulas should be made wherever justified by the attainment of greater accuracy and facility of computation, I venture to call attention to the expression which CHAUVENET assigns to the quantity ζ_1 (see page 458 of Part I, CHAUVENET's "Spherical and Practical Astronomy"), a quantity which is of the utmost importance in CHAUVENET's method of treating solar eclipses. Now, in the first place, even adopting CHAUVENET's suggestion of neglecting quantities of the order of i^2, I fail to see why

$$\zeta_1 = \pm[\cos\beta - i\sin\beta\cos(Q-\gamma)]$$

whereas it should read

$$\zeta_1 = \pm\cos\beta - i\sin\beta\cos(Q-\gamma)$$

unless we must be forced to assume an expression such as CHAUVENET finally arrives at (505) p. 459, and which he again finds it necessary to improve if a higher degree of accuracy be required (509) p. 460; but in all his developments the quantity $i\zeta_1\cos Q$ has been substituted for the quantity $\frac{i\zeta_1\cos Q}{\rho_1}$ where $1:\rho_1$ in most cases is by no means = unity.

I propose now to show that with scarcely more numerical work the quantity ζ_1 may be computed directly and with absolute rigor.

The formulae (499) p. 458, give

$$\xi^2+\eta_1^2 = 1-\zeta_1^2 = [x-(l-i\zeta_1)\sin Q]^2 + \left[\frac{y}{\rho_1} - \frac{(l-i\zeta_1)}{\rho_1}\cos Q\right]^2$$

or by (502)

$$\zeta_1^2\left\{1+i^2\left[\sin^2 Q + \frac{\cos^2 Q}{\rho_1^2}\right]\right\} + 2i\beta\left[\sin\gamma\sin Q + \frac{\cos\gamma\cos Q}{\rho_1}\right]\cdot\zeta_1 = \cos^2\beta$$

(1) Put
$$\sin\gamma\sin Q + \frac{\cos\gamma\cos Q}{\rho_1} = \kappa$$

(2)
$$\sin^2 Q + \frac{\cos^2 Q}{\rho_1} = \lambda^2$$

then, if we put
$$\operatorname{cotg}\mu = \frac{i\kappa\operatorname{tg}\beta}{\sqrt{1+i^2\lambda^2}} \qquad (3)$$

it is easy to show that the two values of ζ_1 are

$$\zeta_1 = \frac{\cos\beta\operatorname{tg}\frac{1}{2}\mu}{\sqrt{1+i^2\lambda^2}}\quad,\quad\text{or}\quad \zeta_1 = -\frac{\cos\beta\operatorname{cotg}\frac{1}{2}\mu}{\sqrt{1+i^2\lambda^2}}$$

β may always be taken in the first quadrant, therefore confining μ to the first or second quadrant as $\kappa \gtrless 0$, we have μ an acute angle, and since ζ_1 must always be >0, we have for all cases

$$\zeta_1 = \frac{\cos\beta\operatorname{tg}\frac{1}{2}\mu}{\sqrt{1+i^2\lambda^2}} \qquad (4)$$

or, still simpler, since $\sqrt{1+i^2\lambda^2}$ differs from unity by two units in the 5th decimal at most,

$$\zeta_1 = \cos\beta\,.\operatorname{tg}\tfrac{1}{2}\mu,\quad\text{where}\quad \operatorname{cotg}\mu = i\,.\,\kappa\,.\operatorname{tg}\beta$$

so that λ^2 need not at all be computed. The quantity κ is readily computed by putting

$$\rho_1\operatorname{tg}\gamma = \operatorname{tg}\alpha \qquad (5)$$

therefore
$$\kappa = \frac{\cos\gamma\cos(Q-\alpha)}{\rho_1\cos\alpha} \qquad (6)$$

If we do not hesitate to put $\lambda^2 = 1$ and $i\zeta_1\cos Q$ instead of $\frac{i\zeta_1\cos Q}{\rho_1}$ it will readily be seen that we should put

$$\operatorname{cotg}\mu = \frac{i}{\sqrt{1+i^2}}\,.\cos(Q-\gamma)\,.\operatorname{tg}\beta$$
$$= \sin f\,.\cos(Q-\gamma)\,.\operatorname{tg}\beta$$

where f is the angle of the penumbral cone; and then finally

$$\zeta_1 = \cos f\,.\cos\beta\,.\operatorname{tg}\tfrac{1}{2}\mu$$

or
$$\zeta_1 = \cos\beta\,.\operatorname{tg}\tfrac{1}{2}\mu$$

since $\cos f$ differs imperceptibly from unity.

These formulas allow of easy numerical application wherever Q is directly given, and offer no great difficulties where, as in the case of the northern and southern limits, a value of Q must first be established by an indirect process.

Washington, D.C., 1902 February 1.

THE CONSTANT OF ABERRATION FROM THE SAN FRANCISCO AND WAIKIKI OBSERVATIONS OF 1891–92.

By S. C. CHANDLER.

In Bulletins 28 and 32 of the U.S. Coast Survey, Mr. E. D. PRESTON has given values of the aberration-constant, computed by methods proposed by Prof. NEWCOMB, from the observations in 1891–92 as follows:

San Francisco, DAVIDSON observer,	20″.482
Waikiki, PRESTON observer,	20.433

An examination of these discussions has led me reluctantly to the conclusion that the results given are unacceptable on account of defects in the processes by which they were found. My objections relate to the form of equation of condition employed in the solution and the methods of eliminating the variation of latitude, which were differ-

ent in the two cases. I will state them as briefly as possible.

For San Francisco the equation assumed was of the form

(A) $$d\delta + (\varphi - \varphi_0) - A\,dk = n$$

where $d\delta$ is the correction of the assumed declination of a pair including error of assumed mean latitude, $(\varphi - \varphi_0)$ is the variation of latitude, A the aberration-factor, dk the correction of the assumed aberration-constant, and n the excess of the observed latitude for each observation over an assumed mean latitude. The value of $(\varphi - \varphi_0)$ was assumed as known, having been found by Schorr from the same observations as follows:

(B) $$\varphi - \varphi_0 = -0''.172 \cos(t - 2411623)\,0^\circ.84 \\ -0''.074 \cos(\odot - 169^\circ.5)$$

This was subtracted from eq. (A); $d\delta$ and dk were then found from the equations for each pair, and the results of dk combined.

To show the incorrectness of this method it will be necessary to revert to the expression for aberration-reduction from mean to apparent declination, or

$$-k \cos\odot(\sin\epsilon \cos\delta - \cos\epsilon \sin\delta \sin\alpha) - k \sin\odot \sin\delta \cos\alpha$$

Put $\delta = \varphi - \zeta$, $\delta_0 = \frac{1}{2}(\delta_2 + \delta_1)$, $\alpha_0 = \frac{1}{2}(\alpha_2 + \alpha_1)$, $\tau = \frac{1}{2}(\alpha_2 - \alpha_1)$

where the subscripts 1 and 2 denote the south and north stars of a pair.

Also put $m = \sin\epsilon \cos\varphi$, $n = \cos\epsilon \sin\varphi$

Then we have, rigorously,

$$-k \cos\zeta[m \cos\odot - \cos\tau\,(n \sin\alpha_0 \cos\odot - \cos\alpha_0 \sin\odot \sin\varphi)]$$

For our present purpose $\cos\zeta$ and $\cos\tau$ can be taken $= 1$. Also we take $T = \alpha_0 - \odot$; then the above becomes

$$k[n \sin T - m \cos\odot + \\ \sin\varphi(\cos\epsilon - 1)(\tfrac{1}{2}\sin 2\odot \cos T - \sin^2\odot \sin T)]$$

Of the three terms the last is quite small relatively to the others, since $\sin\varphi(\cos\epsilon - 1)$ is not over -0.05 for usual latitudes. Hence for our present purpose we can write simply, with enough precision,

(C) $$k(n \sin T - m \cos\odot)$$

T being very nearly the mean or apparent solar time of culmination.

For San Francisco and Waikiki ($\varphi = 37^\circ 47'$ and $21^\circ 16'$) this gives, nearly enough for our demonstration,

(D) $$\begin{cases} \text{San Fr.:} & Ak = k(+0.562 \sin T - 0.315 \cos\odot) \\ \text{Waikiki:} & k(+0.333 \sin T - 0.371 \cos\odot) \end{cases}$$

Now the determination of (B) from the observations will necessarily be affected by error in the value of k used in the reductions. Even if the observations be symmetrically distributed with reference to midnight, so that the term in T of eq. (D) will not affect the observed constants of the variation of latitude, the term in $\odot$ will remain. Thus, for example, the true value of the annual term in (B) will be in this case,

$$-0''.074 \cos(\odot - 169^\circ.5) + 0.315\,dk \cos\odot$$

That is, if $dk = +0''.10$, for example, the annual term ought to be

$$-0''.105 \cos(\odot - 172^\circ.7)$$

Thus, by the use made of (B) in the method described, a portion of the aberration is really taken out of the absolute terms in eq. (A); and what remains consequently does not correspond to the aberration-factors, A, used in the equations of condition, even in the most distant way.

Any solution for aberration thus made, therefore, is incorrect. Under the actual condition of things I am of opinion that the deduced value in this case (20''.482) is spurious, and probably bears no relation to what a correct method would have given. I will only add that it is manifest merely from an inspection of the values given by the eight groups (see p. 119, Bull. 32), that there is present a systematic error dependent on right-ascension, or date of observation, very like what might be expected from the error of method just pointed out. The values in question range from 20''.42 to 20''.85. Their weights are very unequal. Any systematic error of this sort is more likely to be eliminated in the brute mean, which is 20''.58, than in the weighted mean, 20''.48 adopted by Mr. Preston; so that I am inclined to anticipate that a correct solution would materially increase the result for the aberration now assigned for this series.

For Waikiki the method used was somewhat different, but I am constrained to think it is equally at fault. The form of $(\varphi - \varphi_0)$ was in this case assumed as

$$\varphi - \varphi_0 = x \cos X + y \sin X \qquad \text{(E)}$$

and x and y were treated as unknown together with $d\delta$ and dk. But the harmonic curve of the form of (B) which correctly represents this function can be only approximately represented by the form (E) even for the short interval embraced by these observations; and the effect of the differences between the two will be distributed in the numerical solutions between the deduced values of $d\delta$ and dk, and indeed be magnified in their effect on dk by the smallness of the aberration-coefficients (shown in eq. (D)) for this series. It is not easy to specify in this case the character of the error thus spuriously imposed on the deduced aberration-constant, from lack of details regarding the distribution of the observations. But such tests as I have been able to apply convince me that it is probably as untrustworthy as that for the San Francisco series.

It seems to me that a redetermination of the aberration from both of these series is desirable, without making any assumption as to the latitude-variation; eliminating it by use of approximately simultaneous dates, as is usually done in such investigations.

A NEW ANNUAL TERM IN THE VARIATION OF LATITUDE, INDEPENDENT OF THE COMPONENTS OF THE POLE'S MOTION.

By H. KIMURA.

On reading Prof. ALBRECHT's report in *A.N.* 3734, I noticed that the residuals of observations at some stations have certain periodicities, and I suspected that this fact might be caused by something like the change of the direction of the plumb line, common to all the stations presently referred to. I therefore recalculated* the coordinates of the instantaneous pole, based upon the materials† given in his paper, pp. 211–212, with the assumption.

$$\varphi - \varphi_0 = \xi + x\cos\lambda + y\sin\lambda$$

where ξ is variable with respect to time, but at any particular instant has a constant value for all the stations referred to. The results of ξ, x, and y thus calculated are as follows:

	ξ	x	y	p_ξ	p_x	p_y
	″	″	″			
1899.8	−0.003	+0.024	+0.124	4.27	1.88	3.09
.9	+0.018	+0.042	+0.082			
1900.0	+0.029	+0.052	+0.038	5.81	2.53	3.79
.1	+0.023	+0.056	−0.005			
.2	+0.015	+0.039	−0.047			
.3	+0.002	+0.001	−0.079			
.4	−0.015	−0.034	−0.095			
.5	−0.030	−0.049	−0.082			
.6	−0.033	−0.063	−0.043			
.7	−0.031	−0.067	+0.007			
.8	−0.011	−0.056	+0.054			
.9	+0.011	−0.026	+0.084			
1901.0	+0.034	+0.038	+0.095			

The mean error of a single observation of $\varphi - \varphi_0$ decreases from $\pm 0''.034$ to $\pm 0''.025$, the mean of the mean errors of the coordinates of a point upon the orbit, from $\pm 0''.019$ to $\pm 0''.015$, and that of ξ is $\pm 0''.011$.

Having applied the above values of ξ, x, and y to all stations, we have the following residuals:

Mizusawa		Tschardjui		Carloforte		Gaithersburg		Cincinnati		Ukiah	
1899	″	1899	″	1899	″	1899	″	1899	″	1899	″
—	—	.78	+.01	.82	+.02	.80	−.04	.78	−.02	.82	−.03
—	—	.88	−.04	.89	+.01	.89	−.03	.88	−.03	.90	+.10
.99	+.02	.95	−.08	.98	−.02	.98	−.02	.97	−.03	.97	+.09
1900 .06	.00	1900 .05	+.03	1900 .05	+.02	1900 .05	.00	1900 .06	.00	1900 .05	−.01
.12	−.03	.11	+.02	.12	+.01	.12	−.01	.12	+.02	.12	+.03
.18	.00	.19	−.02	.19	−.02	.19	.00	.19	+.04	.19	−.01
.26	.00	.25	−.08	.25	.00	.26	−.01	.25	−.01	.25	+.01
.32	−.02	.32	−.07	.32	−.01	.32	+.01	.33	.00	.32	−.01
.41	+.01	.40	−.02	.40	−.05	.40	+.05	.40	+.02	.40	−.04
.48	−.01	.48	+.02	.48	−.01	.48	+.03	.49	+.01	.48	−.03
.56	.00	.57	+.05	.57	.00	.57	+.02	.56	+.02	.57	−.02
.67	−.01	.67	+.01	.68	.00	.68	.00	.66	−.02	.68	.00
.78	.00	.78	+.02	.78	.00	.79	−.01	.78	+.02	.78	−.02
.88	+.05	.90	+.05	.89	−.01	.88	.00	.88	+.06	.90	−.01
.98	−.04	.97	+.10	.97	+.02	.97	+.02	.97	−.02	.98	−.04

Tōkyō		Kasan		Leiden		Philadelphia	
1899	″	1899	″	1899	″	1899	″
.80	.00	.87	−.01	.80	+.05	.79	+.05
.87	−.03	.97	+.08	.98	+.03	.85	−.01
.98	−.04	1900 .04	+.09	1900 .10	−.02	.89	+.05
1900 .06	.00	.12	+.02	.17	−.07	.97	+.07
.13	−.05	.22	+.01	.26	−.01	1900 .04	.00
.21	−.01	.29	+.01	.32	.00	.09	−.04
.29	+.06	.37	+.10	.39	−.01	.16	+.07
.36	+.04	.46	+.01	.48	−.02	.37	−.06
.45	.00	.51	−.02	.55	+.02	.43	−.09
.56	.00	.62	−.05	.63	+.04	.48	−.01
.62	−.05	.74	−.11	.71	−.02	.53	−.01
.71	.00	.80	−.06	.78	+.05	.58	+.01
.79	+.09	.86	+.02	.89	−.01	.62	.00
.87	+.01	1901 .03	−.11	.96	.00	.70	.00
.94	−.02	—	—	—	—	.79	.00
—	—	—	—	—	—	.83	+.04
—	—	—	—	—	—	.88	−.05
—	—	—	—	—	—	.96	+.05

From these the following mean deviations of observations were formed, and for comparison those of Prof. ALBRECHT (without ξ) are arranged on the right.

MEAN DEVIATIONS OF OBSERVATIONS.

	With ξ	Without ξ
	″	″
Mizusawa	±0.022	±0.041
Tschardjui	±0.050	±0.054
Carloforte	±0.018	±0.028
Gaithersburg	±0.026	±0.024
Cincinnati	±0.026	±0.028
Ukiah	±0.041	±0.054
Tōkyō	±0.038	±0.042
Kasan	±0.063	±0.074
Leiden	±0.033	±0.031
Philadelphia	±0.045	±0.050

The decreases of all the mean errors and the general improvement in the agreement of the observed values with the calculated show the possible existence of a new term ξ. On looking back to the tabular values of ξ, it is found to have an annual period, its maximum amplitude being nearly $0''.03$.

The orbit of the pole's motion according to my result being drawn, the lengths of its principal axes are found to remain nearly the same as that of Prof. ALBRECHT, but the direction of the apses, changes from $\lambda = -120°$ and $+60°$ to $\lambda = -110°$ and $70°$ respectively.

APPENDIX.

For the farther evidence of the existence of ξ, I have tried to deduce it from the observed values‡ of $\varphi - \varphi_0$, made in the preceding ten years at several observatories. But

* With the same weights of observations as he has taken.

† Beside them, $\varphi - \varphi_0$ of Tōkyō (from 1900.4 to 1910.0), and those of Philadelphia (from 1900.8 to 1901.0) in *A.J.* 509 are included. After the first approximation, φ_0 for Ukiah has been corrected by $-0''.01$, and that for Leiden, by $+0''.01$.

‡ *Bericht über den Stand der Erforschung der Breitenvariation von* TH. ALBRECHT 1898 and 1900.

on account of unfavorable distribution of the observatories and discontinuity of their observations, it is extremely difficult to determine ξ separately from x and y by taking all the materials together. I have therefore used for this computation only those simultaneous observations, for which $[p\cos\lambda]$ and $[p\sin\lambda]$ become numerically a minimum at the same time. This gives the following results:

ξ

Epoch	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
'91[1]						−39	28	+2	+12	+5
2[1]	+0.008	+18	+33	+22	−11					
5[2]							40	−35	−32	−5
6[2]	+0.009	+26	+22	+8	2	−7	+17	+16		
6									+8	+18
7[3]	+0.017	+15	−11	+3	−18	−41	40	+10		
8[3]									−11	+11
9[3]	+0.015	+19	+5	−6	17	47	−67	−62	−26	
	+0.021	+20	+17	+6	−12	−33	−32	−15	−12	+16

[1] Mean error, ±0″.032: Stations combined, Pulkowa, Berlin, San Francisco, Honolulu.

[2] Mean error, ±0″.027: Stations combined, Tokyō, Taschkent, Kasan, Potsdam, New York, ½ Washington.

[3] Mean error, ±0″.027: Stations combined, Tokyo, Kasan, Potsdam, New York, Philadelphia.

From these mean values of ξ for each tenth of a year, we see that each of them coincides very closely with that given in the preceding page for the international coöperation. It seems, therefore, to be an undeniable fact, that the variation of latitude contains a new element ξ, besides the components of the mere motion of the pole hitherto considered.

In above both computations, ξ has been regarded to be constant for all the latitudes referred to, but such cannot naturally be the case. Now since the materials are given by those observatories lying between $\varphi = 60°$N and $\varphi = 21°$N, the values of ξ thus obtained will correspond approximately to a mean latitude 42° N. If there is a sufficient number of series of observations in far different parallels, each of which enables us to determine ξ independent of x and y, we might find what function of latitude ξ may be.

The existence of ξ being thus affirmed, it is necessary to see how much influence the negligence of ξ will exert upon the values of x, y, and $\varphi - \varphi_0$. Let (x', y') and (x, y) be the coordinates, with and without the consideration of ξ respectively, and $(\varphi - \varphi_0)'$ and $(\varphi - \varphi_0)$ be the corresponding reductions for each station; then

$$x' = x - a\xi$$
$$y' = y - b\xi$$

where
$$a = \frac{[p\cos\lambda][p\sin^2\lambda] - [p\sin\lambda][p\sin\lambda\cos\lambda]}{[p\sin^2\lambda][p\cos^2\lambda] - [p\sin\lambda\cos\lambda]^2}$$

$$b = \frac{[p\sin\lambda][p\cos^2\lambda] - [p\cos\lambda][p\sin\lambda\cos\lambda]}{[p\sin^2\lambda][p\cos^2\lambda] - [p\sin\lambda\cos\lambda]^2}$$

and
$$(\varphi - \varphi_0)' = (\varphi - \varphi_0) + \xi(1 - a\cos\lambda - b\sin\lambda)$$

These equations show that, if $[p\cos\lambda]$ and $[p\sin\lambda]$ be zero at the same time, a and b will vanish, and ξ will affect $(\varphi - \varphi_0)$ in all stations by its whole amount, but not x and y at all. Till the present time, however, a majority of the stations in Europe and western coast of America, gave positively large values of $[p\cos\lambda]$ and, consequently, a is always positive and pretty large, while b is, in most cases, positively small on account of the smallness of the numerical value of $[p\sin\lambda]$; in consequence of such conditions in the values of a and b, the influence of the neglect of ξ on $(\varphi - \varphi_0)$ is greatest in Japan, next considerable in eastern America and western Russia, and least in Europe and western America.

Lastly, taking the mean of the two series of values of ξ above given for each tenth of a year, with weights according to their mean errors, I get

.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
+0″.029	+22	+16	+3	−14	−31	−33	−25	−9	+14

From a curve formed from these values, ξ is seen to possess an annual period, and is zero at 0.33 and 0.85, maximum at 0.0 and minimum at 0.57. It is very noteworthy that the zeroes lie near the equinoxes, and the maximum and the minimum, near the solstices.

The farther investigation respecting the nature and causes of ξ must be postponed to the future. I have stated in this paper only the empirical results according to my assumption.

In conclusion, I wish to return best thanks to Dr. Nakano, who has assisted me very kindly in carrying my double and independent calculations throughout this work.

Mizusawa, International Latitude Station, 1902 January 6.

ELEMENTS OF (6927) — *SAGITTAE*.

$\alpha = 19^h\ 14^m\ 26^s$, $\delta = +19°\ 25'.7$ (1900).

A cable dispatch from Dr. Kreutz (Feb. 2) gives the elements of Schwab's *Algol*-type variable

Min. 1902 Feb. 3.89 (Gr. M. T.) + $3^d.38\ E$

after correcting the word *achballada* to *acaballado*. The former is not in the code and the period, 5.70 days, which it would require, is discordant with the observed approximate minimum of 1901 Nov. 1.27 given in *A.N.* 3748. The latter accords within $0^d.02$, and is undoubtedly the correct interpretation, or period of $3^d.38$.

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ABERRATION CONSTANT FROM ZENITH DISTANCES OF *POLARIS*,

MEASURED WITH THE MERIDIAN CIRCLE OF THE DETROIT OBSERVATORY, ANN ARBOR,

BY A. HALL, JR.

The Ann Arbor Meridian Circle is by Pistor and Martins, of the old fashioned, heavy type, and unsymmetrical with respect to the cube, the object-glass end being the longer. The circles are approximately 37 inches in diameter. They are made rather light, so that they are distorted somewhat by gravity. The fine circle, which is the one read, is divided to 2′. The accidental division errors are small. The microscopes magnify about sixteen times.

The object-glass is 6.3 inches in diameter. It is rather poor, containing tree-like formations, though the images are round all the way across the field. But the rays of light are not brought sharply to a focus. The eye-piece employed has a power of 160.

The instrument is mounted between sandstone piers which are not covered with felt or wood. The observing room is joined as an east wing to the main part of the building. The walls are of heavy masonry. The room is small.

When the daytime observations of *Polaris* were made, the north slit was covered as far as possible with canvas, and a large piece of canvas was drawn up over most of the roof, being from one to four feet above it. All pointings were made with the tangent screw, the star being placed between two horizontal wires about 5″ apart. The field was bright.

It was not possible always to obtain successive culminations of *Polaris*, or to take both reflected and direct observations at the same culmination without working too near the edge of the field. The pointings were made symmetrically with respect to the middle vertical wire, seven being usually taken at each culmination, though sometimes five or nine.

The circle divisions read by the microscopes were not changed, so that no consideration is necessary of division errors. The probable error of a single pointing on *Polaris* is $\pm 0''.33$, and that of a nadir determination, the mean of four pointings, $\pm 0''.28$. This last was computed under the assumption that the nadirs at the beginning and end of a set of pointings differ only by accidental errors, so that $\pm 0''.28$ is somewhat too large.

Using the zenith-distances as observed, without applying flexure or division errors, observation-equations were written of the form

$$\Delta\delta + b \sin(\odot + B) \,.\, \Delta k + b \cos(\odot + B) \,.\, \pi + \delta_0 - \delta' = v$$

δ' is the observed declination, computed from the observed zenith-distance with the latitude $+42° \; 16' \; 48''.0$, δ_0 the declination of the *Berliner Jahrbuch*, Δk the correction to the aberration constant, $20''.445$, and π the parallax. With the term $\Delta\delta$ would be included all constant corrections. The n's as first computed were diminished in magnitude so as to make them conveniently small, and the quantities thus added restored to $\Delta\delta$ after solution. Thus, to the original n's was added in the first set of equations $-4''$, in the second set $+4''$, then $-1''$, $0''$, $+1''$, $-1''$, $-1''$, $+1''$.

Images and steadiness are marked on a scale 1–5, 1 being perfect. The dates refer to Ann Arbor mean time.

1. *Polaris* ABOVE POLE, CLAMP EAST.

No.	Date 1898	I	S	b sin $(\odot+B)$	b cos $(\odot+B)$	n' ″	v ″	No.	Date 1898	I	S	b sin $(\odot+B)$	b cos $(\odot+B)$	n' ″	v ″
1	Apr. 25.9	3	3	−0.31	−0.94	+0.36	+0.04	7	May 25.9	2–3	2–3	−0.72	−0.67	+0.36	+0.01
2	26.9	3	3	−0.32	−0.93	+0.29	−0.03	8	26.9	3	3	−0.73	−0.66	+0.18	−0.17
3	May 5.9	3	3	−0.46	−0.87	+0.57	+0.23	9	30.8	3	3	−0.77	−0.61	+0.83	+0.48
4	8.9	3–4	3	−0.50	−0.85	+0.81	+0.46	10	June 2.8	3	3	−0.80	−0.57	+0.09	−0.25
5	16.9	3	3	−0.61	−0.78	+0.72	+0.37	11	6.8	3	3	−0.84	−0.52	−0.10	−0.44
6	24.9	3	3	−0.71	−0.69	+0.49	+0.14	12	16.8	3	3–4	−0.91	−0.37	−0.33	−0.65

No.	Date	I	S	b sin (☉+B)	b cos (☉+B)	n′ ″	v ″
13	1898 June 19.8	3	3	−0.93	−0.33	+0.67	+0.36
14	20.8	3	3	0.94	−0.34	+1.24	+0.90
15	22.8	3	3	−0.95	−0.28	+0.26	0.04
16	July 5.8	2-3	3	−0.98	−0.07	−0.10	−0.36
17	7.8	3	3	−0.99	−0.04	+0.48	+0.23
18	10.7	3-2	3-2	−0.99	+0.01	+1.51	+1.27
19	13.7	3	3	−0.98	+0.06	0.06	−0.28
20	20.7	3	3	−0.97	+0.18	−0.19	−0.38
21	21.7	3	3	−0.97	+0.19	+0.98	+0.79
22	22.7	3	3	0.96	+0.21	+0.15	−0.03
23	25.7	3-4	3	−0.95	+0.26	+0.37	+0.20
24	26.7			−0.95	+0.27	+0.21	+0.05
25	Aug. 1.7	3-4	3	−0.90	+0.44	+0.12	+0.00
26	9.7	4	3	−0.86	+0.48	−1.28	−1.37
27	13.7	3	3	−0.83	+0.54	+0.61	+0.51
28	19.7	3	3-2	−0.77	+0.62	−0.74	−0.77
29	25.6	3	3	−0.70	+0.69	+0.01	+0.04
30	26.6	3	3-2	−0.69	+0.71	+0.44	+0.45
31	Sept. 21.5	3	2	−0.28	+0.95	−0.08	+0.09
32	26.5	–		−0.24	+0.96	−1.15	−0.97
33	28.5	3	2	−0.24	+0.96	−1.15	−0.96
34	Oct. 8.5	3	3	−0.04	+0.99	+0.75	+0.98
35	11.5	3	3	+0.01	+0.99	−0.83	−0.58
36	27.4	3	3	+0.28	+0.95	−0.32	−0.02
37	31.4	3	3	+0.34	+0.93	+0.14	+0.45
38	Nov. 7.4	3	3	+0.45	+0.88	−0.73	−0.40
39	12.4	3	3	+0.53	+0.83	−0.44	−0.11
40	13.4	3	3	+0.54	+0.82	−0.57	−0.24
41	26.3		–	+0.72	+0.68	+0.04	+0.38
42	Dec. 10.3	3	3	+0.86	+0.48	−0.91	−0.62
43	13.3	3	3	+0.88	+0.44	−0.16	+0.15
44	15.3	2-3	2-3	+0.90	+0.41	−1.14	−0.83
45	17.3	3	2-3	+0.91	+0.38	−0.79	−0.48
46	31.3	3-4	4-3	+0.98	+0.14	+0.27	+0.53
47	1899 Jan. 2.3	2-3	2-3	+0.98	+0.10	−0.64	−0.39
48	9.2	3	2	+0.99	−0.02	−0.34	−0.12
49	11.2	3	2-3	+0.98	−0.11	+0.23	+0.43
50	1899 Jan. 21.2	2-3	2-3	+0.96	−0.23	−0.43	−0.27
51	24.2	3	3	+0.95	−0.28	+0.15	+0.60
52	31.2	3	3	+0.91	−0.39	+0.11	+0.52
53	Feb. 8.2	3	3	+0.84	−0.54	+0.80	+0.86
54	13.2	3	3	+0.79	−0.59	−0.01	+0.02
55	16.2		–	+0.76	−0.63	−0.63	−0.62
56	24.1	3	3	+0.67	−0.72	+0.19	+0.16
57	Mar. 1.1	4-3	4-3	+0.60	−0.78	+0.25	+0.18
58	Apr. 1.0	3	3	+0.06	−0.98	+0.21	−0.04
59	9.0	3	3	+0.02	−0.99	+0.55	+0.29
60	18.9	3-4	4	−0.19	−0.97	+0.05	−0.25
61	20.9	–	–	−0.22	−0.96	+0.15	−0.16
62	23.9	3	3	−0.26	−0.95	−0.04	−0.35
63	25.9	3-4	3	−0.30	−0.94	+1.13	+0.81
64	27.9	3	3	−0.33	−0.93	−0.56	−0.88
65	28.9	2	3	−0.35	−0.92	−0.14	−0.47
66	May 8.9	3	3	−0.50	−0.85	+0.49	+0.15
67	10.9	4	3	−0.53	−0.84	+0.40	+0.05
68	23.8	3	3	−0.69	−0.70	−0.05	−0.10
69	24.8	3	3	−0.71	−0.69	+0.18	−0.17
70	June 1.8	2-3	2-3	−0.79	−0.59	−0.03	−0.38
71	3.8	3	3	−0.81	−0.56	+0.14	−0.20
72	4.8	3	3	−0.82	−0.55	−0.52	−0.86
73	29.7	3-4	3	−0.97	−0.17	+0.29	+0.01
74	July 8.7	4	3	−0.99	−0.03	+0.68	+0.43
75	10.7	2	2	−0.99	+0.01	+0.05	−0.19
76	18.7	3	3	−0.98	+0.14	−0.02	−0.22
77	22.7	3	3	−0.97	+0.20	−0.48	−0.66
78	Aug. 10.7	3-4	3	−0.86	+0.49	−0.17	−0.26
79	12.7	4-3	3	−0.84	+0.52	+0.71	+0.64
80	13.7	3-4	3	−0.83	+0.54	+1.13	+1.06
81	Sept. 1.6	3	2	−0.62	+0.77	+0.18	+0.22
82	11.6	3-4	3	−0.48	+0.86	−0.60	−0.50
83	13.5	4	3	−0.45	+0.88	+0.53	+0.64
84	26.5	4	3	−0.25	+0.96	+0.46	+0.64
85	30.5	4	3-4	−0.18	+0.97	+0.45	+0.65
86	Oct. 2.5	2-3	2-3	−0.15	+0.98	−0.32	−0.11

2. *Polaris* below Pole, Clamp East.

No.	Date	I	S	b sin (☉+B)	b cos (☉+B)	n′ ″	v ″
1	1898 Apr. 26.4	–		−0.31	−0.94	+0.22	−0.33
2	27.4	2-3	3	−0.33	−0.93	−0.42	−0.97
3	May 5.4	3-4	2	−0.45	−0.88	+0.24	−0.34
4	7.4	3	3	−0.48	−0.86	+0.34	−0.24
5	9.4	2	2	−0.51	−0.84	−0.14	−0.69
6	23.4	2-3	2-3	−0.69	−0.70	+1.29	+0.76
7	25.4	2-3	2	−0.72	−0.68	+0.46	−0.06
8	30.3	3	3	−0.77	−0.62	+0.38	−0.13
9	June 1.3	2-3	2	−0.79	−0.59	+0.19	−0.34
10	3.3	2-3	2	−0.81	−0.57	+0.91	+0.42
11	14.3	2-3	3	−0.90	0.44	+0.00	−0.45
12	17.3	3	2	−0.92	0.36	+0.63	+0.19
13	21.3	3	2-3	0.94	−0.30	+0.47	+0.05
14	22.3	3	3	−0.94	−0.29	+1.23	+0.84
15	July 6.2	2	3	−0.98	−0.06	+0.54	+0.19
16	9.2	2-3	2-3	−0.99	−0.01	−0.12	−0.15
17	11.2	2-3	2-3	−0.99	+0.02	+0.57	+0.25
18	12.2	3-2	3-2	−0.99	+0.04	+0.87	+0.56
19	14.2	3	3	0.98	+0.07	−0.03	−0.33
20	1898 July 21.2	3	3	−0.97	+0.18	+0.28	+0.02
21	22.2	3	3	−0.97	+0.20	+0.20	−0.06
22	29.2	3	3	−0.94	+0.31	−0.34	−0.55
23	Aug. 2.2	3	3	−0.91	+0.37	−0.07	−0.26
24	4.2	3-9	3-9	−0.90	+0.10	−0.11	−0.29
25	10.2	3-4	3-4	−0.86	+0.49	−0.25	−0.39
26	25.1	3	3	−0.71	+0.69	−0.37	−0.42
27	26.1	2	2-3	−0.70	+0.70	−0.02	−0.07
28	27.1	2	3	−0.68	+0.71	+0.03	−0.01
29	Sept. 25.0	3	2	−0.27	+0.95	+0.56	+0.65
30	27.0		–	−0.23	+0.96	−0.07	+0.03
31	29.0		–	−0.20	+0.97	−0.03	+0.08
32	Oct. 9.0	3-4	3	−0.03	+0.99	−0.61	−0.17
33	14.9	3-4	3	+0.07	+0.98	−0.26	−0.11
34	21.9	3	3	+0.24	+0.96	+0.81	+0.98
35	26.9	3	3	+0.27	+0.95	−1.07	−0.90
36	Nov. 2.9	2-3	2-3	+0.38	+0.91	+0.65	+0.82
37	15.9	3	3	+0.58	+0.80	+0.18	+0.34
38	24.8	2-3	2-3	+0.70	+0.70	−0.24	−0.10

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1898					″	″
39	Dec. 12.8	3–4	3–4	+0.88	+0.44	−0.94	−0.86
40	13.8	3	3	+0.89	+0.43	+0.47	+0.54
41	15.8	3	3	+0.90	+0.40	+0.95	+1.01
42	28.8	3	3	+0.97	+0.18	+1.18	+1.18
	1899						
43	Jan. 6.8	–	–	+0.99	+0.03	−1.14	−1.19
44	9.7	3–4	3	+0.99	−0.03	+0.09	+0.02
45	14.7	2	2	+0.98	−0.11	+0.51	+0.41
46	26.7	4	3	+0.93	−0.32	−0.47	−0.64
47	31.7	4	3	+0.90	−0.40	+0.28	+0.07
48	Feb. 6.7	4–3	3	+0.86	−0.49	−0.74	−0.98
49	13.7	3–2	3	+0.79	−0.59	+0.23	−0.06
50	21.6	2–3	2–3	+0.66	−0.73	+0.44	+0.09
51	27.6	4	4	+0.62	−0.77	+0.92	+0.55
52	Mar. 23.5	3–4	3–4	+0.26	−0.95	+0.10	−0.38
53	Apr. 8.5	3	3	−0.01	−0.99	+0.61	+0.08
54	9.5	3	2	−0.03	−0.99	+0.92	+0.39
55	10.5	3	3	−0.04	−0.99	+0.36	−0.17
56	20.4	3–4	3	−0.21	−0.96	+1.02	+0.47
57	22.4	3	3	−0.24	−0.96	−0.02	−0.57
58	25.4	2–3	2	−0.29	−0.94	+0.64	+0.09
	1899						
59	May 3.1	2–3	2–3	−0.42	−0.89	+1.19	+0.64
60	5.4	3	2	−0.45	−0.88	+0.50	−0.05
61	8.4	2–3	2	−0.49	−0.86	+0.20	−0.35
62	21.4	–	–	−0.70	−0.70	+0.87	+0.35
63	28.3	3	3	−0.74	−0.65	+0.83	+0.32
64	June 2.3	3	3	−0.80	−0.58	+1.71	+1.21
65	July 6.2	2	2–3	−0.98	−0.07	+0.72	+0.37
66	11.2	2	2–3	−0.99	+0.02	+1.04	+0.72
67	12.2	2–3	2–3	−0.99	+0.03	+1.14	+0.82
68	22.2	2	3	−0.97	+0.19	−0.04	−0.30
69	Aug. 2.2	3–4	3–4	−0.92	+0.37	−0.36	−0.55
70	7.2	2–3	2–3	−0.88	+0.44	−0.05	−0.21
71	12.2	3	3	−0.84	+0.51	−0.06	−0.19
72	Sept. 3.1	3–4	3	−0.60	+0.78	−0.28	−0.29
73	4.1	3	3	−0.59	+0.79	−0.49	−0.49
74	9.1	3	3	−0.52	+0.84	−0.10	−0.08
75	11.1	3	3	−0.49	+0.86	−0.24	−0.21
76	13.1	3	3–4	−0.46	+0.87	−0.82	−0.78
77	16.1	4	3	−0.41	+0.90	+0.78	+0.84

3. *Polaris* above Pole, R, Clamp East.

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1898					″	″
1	Aug. 1.7	3	3	−0.92	+0.36	+0.36	+0.33
2	14.7	2–3	3	−0.83	+0.54	−0.34	−0.45
3	20.6	3	3	−0.76	+0.63	−0.40	−0.55
4	Oct. 15.5	3	3	+0.08	+0.98	+0.76	+0.62
5	Nov. 2.4	–	–	+0.38	+0.91	−0.11	−0.13
6	3.4	–	–	+0.39	+0.91	−0.16	−0.18
7	19.4	3	3	+0.63	+0.76	+0.86	+0.99
8	27.3	3	3	+0.73	+0.67	−0.29	−0.08
9	Dec. 18.3	2–3	2–3	+0.92	+0.36	−0.88	−0.43
10	24.3	2–3	2–3	+0.95	+0.26	−0.34	+0.17
11	27.3	3–4	3–4	+0.96	+0.21	−0.76	−0.21
	1899						
12	Jan. 1.3	–	–	+0.98	+0.12	−1.40	−0.80
13	7.2	3	3	+0.99	+0.02	−0.25	+0.41
14	Jan. 10.2	3–2	3	+0.99	−0.04	−0.72	−0.02
15	19.2	3	3	+0.97	−0.19	−1.11	−0.33
16	29.2	3	3	+0.92	−0.36	−0.39	+0.48
17	July 17.7	3	3	−0.98	+0.13	+0.50	+0.59
18	21.7	4	3	−0.97	+0.19	−0.26	−0.20
19	31.7	4	3–4	−0.92	+0.34	+0.91	+0.89
20	Aug. 1.7	3	2	−0.92	+0.36	−0.23	−0.26
21	5.7	3–4	3	−0.89	+0.42	−0.51	−0.57
22	6.7	4	3	−0.88	+0.43	+0.24	+0.18
23	Sept. 2.6	3	2	−0.61	+0.77	−0.86	−1.06
24	12.6	–	–	−0.47	+0.87	+0.94	+0.72
25	14.5	3	3	−0.44	+0.88	+0.10	−0.12

4. *Polaris* below Pole R, Clamp East.

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1898					″	″
1	May 13.4	2–3	2–3	−0.56	−0.81	+0.62	+0.64
2	17.4	3	3	−0.62	−0.77	−0.20	−0.20
3	Dec. 27.8	–	–	+0.97	+0.20	−0.13	−0.39
4	31.8	3	3–2	+0.98	+0.13	+0.13	−0.11
	1899						
5	Jan. 2.8	3	3	+0.98	+0.10	+0.55	+0.32
6	15.7	3	3	+0.98	−0.13	+0.21	+0.05
7	18.7	3	3	+0.97	−0.18	−0.09	−0.23
8	21.7	3	3	+0.96	−0.24	−0.05	−0.17
9	Feb. 16.7	3	3	+0.76	−0.64	+0.90	+0.89
10	28.6	3	3	+0.61	−0.78	−0.27	−0.24
11	Apr. 3.5	2	2	+0.07	−0.98	+0.30	+0.38
12	4.5	2	2	+0.06	−0.99	+0.18	+0.26
13	Apr. 13.5	3	3	−0.10	−0.98	−0.13	−0.05
14	14.5	3	3	−0.11	−0.98	−0.73	−0.65
15	17.4	3	3	−0.16	−0.97	−0.45	−0.37
16	26.4	–	–	−0.31	−0.94	−0.27	−0.21
17	28.4	3	3	−0.34	−0.93	+0.12	+0.18
18	May 8.4	2–3	2	−0.49	−0.86	+0.10	+0.13
19	28.3	3	3	−0.74	−0.65	−0.68	−0.72
20	June 26.3	2	2	−0.96	−0.24	+0.06	−0.11
21	27.3	2–3	2	−0.96	−0.29	+0.32	+0.16
22	29.3	2–3	2–3	−0.97	−0.18	−0.31	−0.50
23	30.3	–	–	−0.97	−0.16	+0.20	+0.00
24	July 1.3	–	–	−0.98	−0.15	+1.11	+0.91

5. *Polaris* above Pole, Clamp West.

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1899					″	″
1	June 10.8	2–3	2–3	−0.87	−0.47	−0.38	−0.52
2	15.8	3	3	−0.91	−0.39	+0.46	+0.30
3	16.8	–	–	−0.91	−0.38	−0.11	−0.27
4	Oct. 5.5	4	3	−0.10	+0.98	+0.28	+0.06
5	9.5	3	3	−0.03	+0.99	+0.15	−0.06
6	22.4	3–4	3	+0.19	+0.97	−0.19	−0.37

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1899					″	″
7	Oct. 23.4	3	3	+0.20	+0.96	+0.02	−0.15
8	Nov. 26.3	3	2	+0.74	+0.68	+0.26	+0.20
9	Dec. 17.3	2	2	+0.91	+0.38	+0.06	+0.07
10	19.3	3	3	+0.92	+0.35	+1.03	+1.05
11	26.3	2–3	2–3	+0.96	+0.23	+1.02	+1.06
	1900						
12	Jan. 21.3	2–3	2–3	+0.96	−0.22	−0.13	−0.03
13	22.2	2–3	2	+0.96	−0.24	+0.08	+0.18
14	24.2	3–4	3	+0.95	−0.27	+0.01	+0.12
15	29.2	2–3	2–3	+0.92	−0.35	−0.83	−0.72
16	Feb. 1.2	2	2–3	+0.90	−0.40	−0.24	−0.12
17	9.2	1	1	+0.84	−0.52	+0.03	+0.15
18	10.2	3	3	+0.83	0.51	−0.56	−0.44
19	11.2	2	2	+0.82	−0.56	−0.72	−0.60
20	19.1	4	3–4	+0.73	−0.66	+0.06	+0.18
21	Mar. 2.1	3	3	+0.59	−0.79	−0.31	−0.19
22	3.1	3	3	+0.58	−0.80	+0.40	+0.52
23	23.0	3–4	3–4	+0.27	−0.95	+0.67	+0.76
24	24.0	1	4	+0.25	−0.95	+0.30	+0.39
25	31.0	3–4	3	+0.14	−0.98	−0.05	+0.03
26	Apr. 1.0	3	3	+0.12	−0.98	+0.04	+0.12
27	4.0	3–4	3–4	+0.07	−0.98	+0.52	+0.59
28	7.0	3–4	3–4	+0.02	−0.99	−0.49	−0.43
29	18.9	4	4	−0.18	−0.97	+0.26	+0.29
30	25.9	2–4	3	−0.30	−0.94	−0.57	−0.56
31	28.9	3–2	3–2	−0.34	−0.93	−1.04	−1.04
32	30.9	3	2–3	−0.37	−0.91	−0.01	−0.02
33	May 4.9	2	2	−0.43	−0.89	+0.15	+0.13
34	20.8		–	−0.65	−0.71	+0.35	+0.27
35	June 12.8	3	3	−0.88	−0.44	−0.78	−0.93
36	June 14.8	3	3	−0.90	−0.41	−0.38	−0.54
37	15.8	2	2–3	−0.90	−0.40	−0.09	−0.25
38	17.8	2	2–3	−0.92	−0.37	0.00	−0.16
39	19.8	2–3	2–3	−0.93	−0.34	−0.47	−0.64
40	July 11.7	3	3	−0.99	+0.02	+0.73	+0.50
41	12.7	3	3	−0.99	+0.03	+1.02	+0.79
42	15.7	3	3	−0.98	+0.08	+0.76	+0.52
43	21.7	3–4	3–4	−0.97	+0.18	+0.57	+0.32
44	25.7	3	3	−0.96	+0.24	+1.48	+1.23
45	Aug. 3.7	3	3	−0.91	+0.38	+0.42	+0.16
46	5.7	3	1–2	−0.90	+0.42	−0.18	−0.15
47	6.7	3	2	−0.89	+0.43	−0.08	−0.35
48	14.7	4	1	−0.82	+0.54	+0.69	+0.42
49	15.7	4	4	−0.81	+0.56	+0.51	+0.24
50	18.6	–	–	−0.79	+0.60	+0.10	−0.18
51	31.6	3–4	3	−0.64	+0.75	+1.58	+1.31
52	Sept. 21.6	1	3	−0.34	+0.93	+0.68	+0.43
53	27.5	4	3	−0.24	+0.96	+0.74	+0.50
54	29.5	3	3	−0.20	+0.96	−0.45	−0.69
55	Oct. 3.5	2–3	2	−0.14	+0.98	−0.13	−0.36
56	9.5	3	3	−0.04	+0.99	−1.30	−1.51
57	12.5	2–3	2–3	+0.01	+0.99	−0.04	−0.25
58	19.4	2–3	2–3	+0.13	+0.98	−0.35	−0.54
59	Nov. 26.4	2–3	2–3	+0.71	+0.69	−0.06	−0.12
60	Dec. 2.3	2	2	+0.78	+0.61	−0.33	−0.37
61	13.3	3	3	+0.88	+0.45	+0.11	+0.11
	1901						
62	Jan. 20.2	3	3	+0.97	−0.20	−0.65	−0.55
63	30.2	3	3	+0.92	−0.36	−0.02	+0.09
64	Feb. 7.2	2–3	2–3	+0.86	−0.49	+0.19	+0.31

6. *Polaris* below Pole, Clamp West.

No.	Date	I	S	b sin (⊙+B)	b cos (⊙+B)	n′	v
	1899					″	″
1	June 12.3	2	2	−0.88	−0.44	+1.25	+0.69
2	16.3	2–3	2–3	−0.91	−0.38	+1.13	+0.55
3	23.3	2–3	2	−0.95	−0.28	+0.42	−0.19
4	Oct. 11.9	3	3	+0.01	+0.99	+0.36	+0.01
5	17.9	3–4	3–4	+0.11	+0.98	−0.50	−0.80
6	20.9	2–3	2–3	+0.16	+0.97	−0.24	−0.52
7	23.9	3	3	+0.21	+0.96	+0.77	+0.51
8	24.9	3	3	+0.23	+0.96	+0.52	+0.27
9	Nov. 29.8	3	3	+0.75	+0.64	+0.06	+0.07
10	Dec. 5.8	2	2	+0.82	+0.56	−0.37	−0.32
11	7.8			+0.84	+0.52	−0.24	−0.18
12	19.8	3–2	3–2	+0.93	+0.34	−0.66	−0.54
13	20.8	3	3	+0.93	+0.32	−0.65	−0.53
	1900						
14	Jan. 21.7	3	3	+0.96	−0.23	−0.02	+0.18
15	22.7	3–4	3	+0.96	−0.25	+0.35	+0.55
16	27.7	3	3	+0.93	−0.33	0.59	−0.39
17	28.7	3–4	3–4	+0.92	−0.35	−0.66	−0.46
18	30.7	3–4	3–4	+0.91	−0.38	−0.53	−0.34
19	Feb. 9.6			+0.83	−0.53	−0.59	−0.41
20	10.6	3	3	+0.82	−0.55	+0.76	+0.94
21	13.6	4	4	+0.79	−0.59	+0.02	+0.19
22	17.6	3–4	3–4	+0.75	−0.64	+0.25	+0.40
23	Mar. 1.6	4	4	+0.60	−0.79	0.11	0.00
24	2.6			+0.58	−0.80	+0.35	+0.45
25	19.6	3	2–3	+0.33	−0.93	+0.38	+0.39
26	23.5	2–3	2–3	+0.26	−0.95	+0.51	+0.49
27	30.5			+0.15	−0.98	−0.07	−0.13
28	31.5	3	3	+0.13	0.98	+0.11	+0.04
29	Apr. 3.5	3	3	+0.08	−0.98	+0.54	+0.42
30	9.5	3	3–4	−0.02	−0.99	−0.13	−0.56
31	10.5	1	1	−0.04	−0.99	−0.25	−0.39
32	25.4	2–3	2–3	−0.29	−0.94	−0.34	−0.59
33	Apr. 27.4	–	–	−0.32	−0.93	+0.50	+0.23
34	30.4	–	–	−0.36	−0.92	+0.16	−0.13
35	May 6.4	2–3	2–3	−0.46	−0.88	+0.04	−0.29
36	26.3	2–3	2	−0.72	−0.68	+0.18	−0.29
37	June 12.3	2–3	2–3	−0.88	−0.45	+0.45	−0.11
38	18.3	2–3	2–3	−0.92	−0.36	+0.76	+0.17
39	28.3	–	–	−0.97	−0.20	−0.02	−0.65
40	July 4.2	2–3	2–3	−0.98	−0.10	+0.37	−0.28
41	5.2	2–3	2–3	−0.98	−0.09	+1.07	+0.12
42	9.2	2	2	−0.99	−0.02	+0.16	−0.50
43	13.2	–	–	−0.99	+0.04	+1.04	+0.38
44	21.2	3	2–3	−0.97	+0.17	+0.23	−0.44
45	28.2	2	2	−0.95	+0.28	+1.30	+0.62
46	Aug. 7.2	–	–	−0.88	+0.44	+0.78	+0.11
47	10.2	3	3	−0.86	+0.48	+0.55	−0.11
48	24.1	3	3	−0.73	+0.67	+0.77	+0.14
49	29.1	–	–	−0.67	+0.73	+0.86	+0.25
50	Sept. 6.1	3	3	−0.56	+0.81	+1.18	+0.61
51	12.1	3	3	−0.48	+0.86	−0.38	−0.92
52	14.1	2–3	2–3	−0.45	+0.88	+0.51	−0.02
53	30.0	3	3	−0.20	+0.97	+0.48	+0.04
54	Oct. 14.0	3	3	+0.04	+0.99	+0.97	+0.63
55	16.9	3–4	3–4	+0.09	+0.98	−0.22	−0.53
56	27.9	2–3	2–3	+0.28	+0.95	−0.07	−0.30
57	30.9	2–3	2–3	+0.32	+0.93	+1.02	+0.81
58	Nov. 3.9	2–3	2–3	+0.39	+0.91	+0.14	−0.04
59	Dec. 1.8	2–3	2–3	+0.77	+0.62	−0.09	−0.07
60	2.8	2	2	+0.78	+0.60	+0.71	+0.74
61	21.8	2–3	2–3	+0.94	+0.31	+0.35	+0.48
	1901						
62	Feb. 10.6	3	3	+0.82	−0.54	−0.93	−0.76
63	11.6	3–4	2	+0.79	−0.60	−0.10	+0.07

7. *Polaris* above Pole, R, Clamp West.

No.	Date	I	S	$b \sin (\odot + B)$	$b \cos (\odot + B)$	n' ″	v ″
1	1899 June 18.8	2–3	2–3	−0.92	−0.35	−0.44	−0.25
2	20.8	3	2–3	−0.94	−0.32	+0.03	+0.21
3	21.8	3	3	−0.95	−0.25	+0.35	+0.50
4	Oct. 12.5	2	2	+0.02	+0.99	−0.64	−0.67
5	15.5	3–4	3–4	+0.07	+0.98	−0.62	−0.63
6	31.4	2–3	2–3	+0.33	+0.93	−0.08	−0.01
7	Dec. 1.3	3	2–3	+0.77	+0.62	+0.15	+0.43
8	4.3	2	2	+0.80	+0.58	−0.23	+0.07
9	21.3	2–3	3	+0.94	+0.31	−0.37	+0.05
10	1900 June 22.8	2	2	−0.94	−0.29	−0.34	−0.18
11	25.8	2–3	2–3	−0.96	−0.21	−1.04	−0.89
12	27.8	2	2	−0.96	−0.21	−0.64	−0.51
13	July 22.7	3	3	−0.97	+0.20	−0.07	−0.07
14	1900 Aug. 25.6	4	4	−0.71	+0.69	+0.04	−0.06
15	27.6	2–3	2–3	−0.69	+0.71	+0.33	+0.22
16	30.6	2–3	2–3	−0.65	+0.71	+0.63	+0.52
17	Sept. 3.6	3	3	−0.60	+0.79	+1.19	+1.08
18	25.5	3	2	−0.27	+0.95	−0.54	−0.62
19	Oct. 5.5	3	3	−0.11	+0.98	+0.99	+0.93
20	14.5	3	3	+0.05	+0.99	+0.25	+0.23
21	17.5	3	2–3	+0.10	+0.98	+0.03	+0.02
22	20.4	2–3	2–3	+0.15	+0.98	+0.22	+0.23
23	Dec. 10.3	2	2	+0.85	+0.19	−1.49	−1.15
24	15.3	2–3	2–3	+0.90	+0.42	−1.05	−0.67
25	1901 Jan. 22.2	3	3	+0.96	−0.23	−0.12	+0.19
26	Feb. 10.1	2	2	+0.83	−0.51	+0.34	+1.02

8. *Polaris* below Pole. R, Clamp West.

No.	Date	I	S	$b \sin (\odot + B)$	$b \cos (\odot + B)$	n' ″	v ″
1	1899 June 11.3	3	3	−0.87	−0.46	+0.26	−0.14
2	13.3	–	–	−0.89	−0.43	+0.37	−0.05
3	17.3	2	2	−0.92	−0.37	+0.39	−0.06
4	Dec. 21.8	2–3	2–3	+0.94	+0.30	+0.31	+0.72
5	1900 Feb. 2.6	2–3	2	+0.89	−0.42	+0.40	+0.98
6	15.6	3	3	+0.77	−0.62	−0.09	+0.48
7	25.6	3	3	+0.65	−0.71	−1.05	−0.52
8	Mar. 11.6	2–3	2–3	+0.45	−0.88	−0.61	−0.15
9	26.5	2	2	+0.21	−0.96	−0.31	+0.03
10	Apr. 4.5	–	–	+0.06	−0.98	+0.05	+0.32
11	9.5	3	3–4	−0.02	−0.99	+0.24	+0.46
12	14.5	–	–	−0.11	−0.98	+0.59	+0.76
13	22.4	2–3	2–3	−0.24	−0.96	−0.40	−0.31
14	May 4.4	2–3	2–3	−0.13	−0.89	−1.12	−1.16
15	1900 May 30.3	2	1–2	−0.76	−0.63	+0.98	+0.68
16	June 3.3	2–3	2–3	−0.80	−0.58	−0.12	−0.45
17	12.3	2–3	2–3	−0.88	−0.45	+0.40	−0.01
18	17.3	2–3	2–3	−0.91	−0.37	+0.19	−0.26
19	18.3	2–3	2–3	−0.92	−0.36	+0.13	−0.33
20	19.3	2	2	−0.92	−0.34	+0.14	−0.02
21	25.3	2	2	−0.95	−0.25	+1.30	+0.79
22	July 18.3	2–3	2–3	−0.98	+0.12	+0.77	+0.15
23	25.2	2	2–3	−0.96	+0.24	+0.81	+0.20
24	26.2	–	–	−0.95	+0.25	+0.48	−0.16
25	Dec. 11.8	2–3	2–3	+0.87	+0.47	−0.90	−0.58
26	15.8	2–3	2–3	+0.90	+0.41	−0.52	−0.16
27	1901 Jan. 30.7	3	3	+0.91	−0.37	−1.06	−0.48
28	Feb. 7.6	3	3	+0.85	−0.50	−1.28	−0.70

From the foregoing observation-equations normals were formed, each set being taken by itself. The solution of the respective systems gives the following values for the unknown quantities and probable errors. The probable error of one equation may be taken as $\pm 0''.35$.

No.	$\Delta\delta$ ″	″	Δk ″	″	π ″	″
1	−4.01	±0.04	+0.24	±0.06	+0.26	±0.06
2	+3.81	±0.05	+0.14	±0.06	+0.34	±0.06
3	−0.58	±0.12	+0.26	±0.10	−0.59	±0.22
4	−0.22	±0.10	+0.03	±0.09	−0.31	±0.16
5	+0.92	±0.05	+0.15	±0.06	−0.14	±0.07
6	−1.24	±0.04	+0.43	±0.06	−0.12	±0.06
7	−0.70	±0.10	+0.24	±0.11	−0.33	±0.15
8	+0.96	±0.09	+0.56	±0.09	−0.27	±0.15

In the case of the direct observations the variation of latitude is eliminated by combining results on both sides of the pole. It may fairly be assumed also that temperature disturbances and other changes of like nature will disappear in the same way. For the reflected observations the dates are such that this elimination can hardly be assumed. Therefore, if the mean is taken of the results corresponding to solutions 1, 2, 5, 6, the correction to the aberration constant is

$$\Delta k = +0''.24 \quad , \quad \text{or } k = +20''.68$$

Provisionally the latitude of this observatory can be taken as

$$+42^\circ\ 16'\ 48''.8$$

The latitude as given in the ephemerides, 48″.0, was probably only an approximate determination. As to the negative parallaxes resulting from the reflected observations, the probable errors are large, so that such values might be looked for. The observations agree among themselves quite as well as those taken direct. The value $+0''.24$ of Δk is no doubt too great. If its probable error is assumed to be $\pm 0''.03$, it may, however, differ from the truth no more than might be reasonably expected.

REMARKS ON CERTAIN DETERMINATIONS OF THE CONSTANT OF ABERRATION BY THE U. S. COAST AND GEODETIC SURVEY.

BY SIMON NEWCOMB.

In No. 517 of this *Journal* Mr. CHANDLER objects to two values of the constant of aberration published some years since in Bulletins 28 and 32 of the U.S. Coast and Geodetic Survey. As the author, not unjustly, throws on me the principal responsibility for the alleged errors of method, it appears appropriate to reply to the strictures.

I may remark, at the outset, that the methods were suggested at a time when I was completing my work on the astronomical constants, and when any results to be incorporated must be speedily obtained. The series of observations made by DAVIDSON at San Francisco, seemed to me of special value for the end in view. But their complete discussion would have taken so long a time that the results would not be available for my work. I therefore suggested a more speedy process. Hence the adoption of an abbreviated method, which I shall set forth in such a manner as to make Mr. CHANDLER's objections as clear and forcible as possible. To condense the matter within the smallest limits I accept the problem in the form in which Mr. CHANDLER substantially gives it, replacing a pair of stars by a single star passing the meridian near the zenith, of which the zenith-distance is measured at transit at various seasons of the year. Granting that there is no variation of latitude, a series of observations of such a star lead to equations of condition for determining the aberration of the form:

$$z - A\delta k = n \tag{1}$$

Here z is the combined correction to the latitude and the star's declination, which is to be eliminated from all the observations of the same star. A is the aberration-factor in the expression for the star's apparent declination, and δk is the correction required to the aberration-constant k.

Were the latitude invariable, the problem would be simply that of eliminating z from the system of equations given by each star, thus forming a final normal equation in which δk would be the only unknown quantity. But, as a matter of fact, the value of the latitude is a variable quantity. Hence if we put $\delta\varphi$ for the deviation of the latitude from any mean value, our equations will be of the form

$$z + \delta\varphi - A\delta k = n \tag{2}$$

Now, if $\delta\varphi$ is a known quantity, determined from other observations, we have only to substitute its numerical value in each of the original equations of condition, (2), and thus reduce the latter to the form (1).

If I correctly understand Mr. CHANDLER's objections, they do not apply to this case, but to that in which the variations of the latitude have to be determined from the same observations as the aberration. In this case the latitude-variation may be written in the form

$$\delta\varphi = x\cos N + y\sin N + u\cos\odot + v\sin\odot \tag{3}$$

where N is any angle increasing at the rate of 360° in 428 days.

Substituting this value of $\delta\varphi$ in (2) we have equations of condition in the form

$$z - A\delta k + x\cos N + y\sin N + u\cos\odot + y\sin\odot = n \tag{4}$$

After eliminating z we shall have a normal equation for δk in the form

$$[aa]\delta k + [ab]x + [ac]y + [ad]u + [ae]v = [an] \tag{5}$$

together with normal equations in x, y, u and v which we need not write. From these equations the values of x, y, u and v are to be derived, and substituted in (5), which will then assume the form

$$[aa]\delta k = [an] - [ab]x - [ac]y - [ad]u - [ae]v \tag{6}$$

The gravamen of the objection to the DAVIDSON results seems to be that, instead of using this method strictly, the adopted values of x, y, u and v were derived assuming $\delta k = 0$, and then used in (6) to determine δk.

It is assumed that these quantities were derived from the observations themselves in this way, but as I shall presently show, it is practically indifferent whether they were so derived or obtained from other sources. In order, however, to get at the pith of the question, I shall provisionally accept the correctness of the former assumption, and trace its consequences, following as well as I can CHANDLER's line of thought. He throws the coefficient A into the form

$$A = n\sin T - m\cos\odot$$

where T is the mean time. The equations (4), writing only the necessary terms, may then be thrown into the form

$$-n\sin T\delta k + (m\delta k + u)\cos\odot + \text{etc.} = n$$

Then, putting $\delta k = 0$ in the first term, the solution will give us, instead of u, the value of

$$m\delta k + u = u - 0.315\,\delta k$$

Taking, with CHANDLER, as an extreme value,

$$\delta k = +0''.10$$

the value of u will be in error by 0″.0315. Then, by substitution in (5) the resulting error introduced into δk will be

$$0''.0315\,\frac{[ad]}{[aa]}$$

Since the coefficients a and d, from which $[ad]$ is formed, are of different signs in different equations, it is likely that $[ad]$ is much smaller than $[aa]$, and hence that the error is much less than $0''.03$. More than this cannot be said without a re-reduction of the observations, which seems to me very desirable, as I believe them to be of good quality, besides having the advantage of extending through the greater part of the night.

In such a re-reduction the value of δq, instead of being derived solely from the observations themselves, should be derived from all available data. The error introduced into the solution for δk is not simply that pointed out, but is the result of the total error of the adopted δq. This is why I have said the question whether δq was derived from the observations themselves or not is now indifferent.

The above criticism does not apply to the case of PRESTON'S Waikiki observations, which I shall hold to be reduced on correct principles until Mr. CHANDLER makes his objections clearer than he does. As, owing to the low latitude of the station, the weight of the resulting aberration must be small in any case, it does not seem worth while to occupy space in defending the method adopted in their discussion, except to justify and make clear the principles on which it is based.

In deriving the aberration-constant from the equations (2) it is only necessary that the values of δq finally used in the equations shall be numerically correct through the period of the observations. If such is the case, it matters not how widely the general formulas by which they are derived and expressed may deviate from the truth at other times. What PRESTON did, with my concurrence, was to assume that through the period of his observation, nearly thirteen months, the two terms in δq could be merged into a single one having a period of 384 days. The only way to determine the actual magnitude of the error thus introduced is, (1) to find the actual values of δq from all sources, especially CHANDLER'S latest formulas, from month to month during the period of observations; (2) to compute δq for the same dates by the values of x and y derived by PRESTON; (3) to determine the change in the aberration produced by substituting the corrected values of δq.

I now see that it would have been better to have formed and solved the equations in the form (4), notwithstanding the indetermination in the values of x, y, u and v then found in the solution, because the large errors to which these quantities were then liable would have compensated each other in forming the equation (5).

This does more than answer CHANDLER'S criticism of the method, since it corrects the result by introducing ulterior data which PRESTON did not have at command. To justify the method it is only necessary to show that the general form of δq supposed by PRESTON may be made to express δq during his observations.

I may be allowed to express my dissent from the opinion that, in such observations as these, it is best, in determining the aberration, to eliminate the variations of latitude by use of approximately simultaneous dates. I think the probable error of the result will be much larger by this method than by accepting the variations of latitude as known quantities. The demonstration of this view belongs rather to a treatise on Least-Squares than to the pages of this *Journal*.

THE SECULAR PERTURBATIONS OF THE *EARTH* BY THE ACTION OF *MARS*,

BY ERIC DOOLITTLE.

In the "*New Theory of Jupiter and Saturn*," Dr. G. W. HILL pointed out that in effecting the computation by means of series of the action of *Mars* on the *Earth* the terms of the fifth order with respect to the inclinations and eccentricities amount to about one per cent. of those of the first order, thus showing a lack of rapid convergence. It hence appeared desirable that the results should be verified by the method of GAUSS, and this was done by Dr. ASAPH HALL, Jr., (*A.J.*, No. 244), and by Mr. INNES, (*M.N.*, Vol. LII, Nos. 2 and 7). Dr. HALL divides the orbit of the *Earth* into twelve parts with regard to the eccentric anomaly, and Mr. INNES into sixteen; in the former computation, Dr. HILL'S first modification of GAUSS'S method is employed, and in the latter, Mr. INNES has employed the second modification, and has used in evaluating the elliptic integrals the tables prepared by himself.

In both of these applications of the method of GAUSS, the original elements of LEVERRIER were used; in the following computation, the elements were adopted from Dr. G. W. HILL'S "*New Theory of Jupiter and Saturn*," pages 192 and 554.

The Earth.		*Mars.*	
$\pi =$	$100^\circ\ 21'\ 39''.73$	$\pi' =$	$333^\circ\ 17'\ 51''.74$
$i =$	$0\ \ 0\ \ 0.00$	$i' =$	$1\ \ 51\ \ 2.24$
$\Omega =$	— — —	$\Omega' =$	$48\ \ 23\ \ 51.59$
$e =$	0.01677111	$e' =$	0.09326803
$n =$	$1295977''.416$	$n' =$	$689050''.784$
$\log a =$	0.0000000	$\log a' =$	0.1828971
$m =$	$\frac{1}{327000}$	$m' =$	$\frac{1}{3093500}$

Epoch 1850.0 G.M.T.

The orbit of the *Earth* was divided into sixteen parts with regard to the eccentric anomaly; the sums of the

functions corresponding respectively to the odd and even points of division were in substantial agreement, and the usual formulas of verification were found to be satisfied. The equation, $\sin q \,.\, \tfrac{1}{2} A_2^S + \cos q \,.\, B_2^C = 0$, gave the residual, $+0.000\,000\,088$.

If m' is left indefinite, the resulting values of the differential coefficients are as follows:

		log coeff.
$\left[\frac{de}{dt}\right]_{00}$	$= -\ \ 18626.511''$	$n4.686\,8731$
$\left[\frac{d\pi}{dt}\right]_{00}$	$= +3017241.7$	$p6.479\,6101$
$\left[\frac{dp}{dt}\right]_{00}$	$= +\ \ 19587.444$	$p4.291\,9711$
$\left[\frac{dq}{dt}\right]_{00}$	$= -\ \ 22233.621$	$n4.347\,0102$
$\left[\frac{dL}{dt}\right]_{00}$	$= -\ 725652.67$	$n5.860\,7288$

If the above value of m' be adopted, there finally results:

$$\left[\frac{de}{dt}\right]_{00} = -\ 0.015\,71893''$$
$$\left[\frac{d\pi}{dt}\right]_{00} = +\ 0.975\,31889$$
$$\left[\frac{dp}{dt}\right]_{00} = +\ 0.006\,331\,7101$$
$$\left[\frac{dq}{dt}\right]_{00} = -\ 0.007\,187\,2066$$
$$\left[\frac{dL}{dt}\right]_{00} = -\ 0.234\,57335$$

The results published by Mr. INNES are somewhat incomplete owing to uncorrected errors in $\left[\frac{dp}{dt}\right]_{00}$ and $\left[\frac{dq}{dt}\right]_{00}$ which arose from a misprint in Dr. HILL's original paper. I have hence recomputed the values of these quantities, and of J_3 and H_0, making use of the elements and auxiliaries employed by Mr. INNES. The results, together with those of HALL, NEWCOMB, HILL and LEVERRIER, upon being reduced to the above value of m', are found to compare with those here determined as follows:

	LEVERRIER	INNES	HALL	HILL	NEWCOMB	New Values
$\left[\frac{de}{dt}\right]_{00}$	$-0.01573''$	$-0.015722''$	$-0.0157232''$	. . . . .	$-0.01572''$	$-0.0157189''$
$\left[\frac{d\pi}{dt}\right]_{00}$	$+0.9754$	$+0.975224$	$+0.9754387$	. . . . .	$+0.9755$	$+0.9753189$
$\left[\frac{dp}{dt}\right]_{00}$	$+0.00635$	$+0.0063401$	$+0.0063414$	$+0.0063362$	$+0.00634$	$+0.006331\,710$
$\left[\frac{dq}{dt}\right]_{00}$	-0.00721	-0.0071898	-0.0071952	-0.0072112	-0.00719	$-0.007\,187\,207$
$\left[\frac{dL}{dt}\right]_{00}$	-0.2337	-0.23469	-0.2342416	. . . . .	. . . .	$-0.234\,57335$

The Flower Observatory, 1902 *Feb.* 17.

ON THE NEBULA SURROUNDING *NOVA PERSEI.*

A dispatch from Professor HUSSEY of the Lick Observatory, received through the Harvard College Observatory, states: "From recent Crossley photograph PERRINE finds no evidence of polarization in condensations A and D in nebula surrounding *Nova Persei.*"

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ILLUSTRATIONS OF PERIODIC SOLUTIONS IN THE PROBLEM OF THREE BODIES,

By G. W. Hill.

(Second Article.)

The examples we now propose to treat differ from those of the first article in that, $k-k'$ being a small integer, it is necessary to annex to the usually considered non-periodic portion of the perturbative function the terms which become non-periodic through the commensurability of the mean motions of the planets.

As a first example take the case where the mean motion of the minor planet is three times that of *Jupiter*, that is, make $k = 3$, $k' = 1$. In the development of the perturbative function we propose to stop with terms of the order of the fourth power of eccentricities. Then, making use of Leverrier's elaboration with his notation (χ denotes half an eccentricity)*, the new terms we have need of (they are sums of coefficients only) are

$$R = (172)^3\chi^2 + (173)^3\chi^4 + (174)^3\chi'^2\chi^2 + (182)^2\chi'\chi + (183)^2\chi'\chi^3 + (184)^2\chi'^3\chi$$
$$+ (193)^1\chi'^2\chi^2 + (206)^0\chi'^3\chi + (336)^6\chi^4 + (340)^5\chi'\chi^3 + (344)^4\chi'^2\chi^2 + (348)^3\chi'^3\chi$$

After substituting for the symbolic coefficients their expressions in terms of Leverrier's $A_j^{(i)}$, the preceding expression becomes

$$\begin{aligned} R = {} & [\tfrac{21}{2}+5+1]^3\chi^2 + [-31-\tfrac{61}{3}+5+12+4]^3\chi^4 + [-378-149+31+48+12]^3\chi'^2\chi^2 \\ & -[20+10+2]^2\chi'\chi + [70+29-37-42-12]^2\chi'\chi^3 + [262+83-59-54-12]^2\chi'^3\chi \\ & +[-34+7+59+48+12]^1\chi'^2\chi^2 - [0+13+27+18+4]^0\chi'^3\chi + [157+\tfrac{277}{3}+37+9+1]^6\chi^4 \\ & -[\tfrac{2090}{3}+\tfrac{1234}{3}+162+38+4]^5\chi'\chi^3 + [1144+683+265+60+6]^4\chi'^2\chi^2 \\ & -[816+500+192+42+4]^8\chi'^3\chi \end{aligned}$$

where the coefficients of the A are written alone, j being always 0 for the first term within each pair of brackets and augmenting by a unit in each step to the right, and the common value of i for all the terms being written above and to the right of each pair.

The argument for the A is $\alpha = 0.4805969$ and, with the assistance of Runkle's tables, the needed values have been obtained and their logarithms inserted in the following table (it should be noted that, on account of the reaction term of the perturbative function, α has been subtracted from $A_0^{(1)}$ and $A_1^{(1)}$):

Logarithms of $A^{(i)}$.

i	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
0	0.3290769	9.4911841	9.4124837	9.1660038	9.0446585
1	8.6890960	9.2152952	9.3334831	9.2075472	9.0337108
2	9.2854829	9.6363530	9.5165204	9.1946468	9.0600964
3	8.8905316	9.4032790	9.4925919	9.3071685	9.0701913
4	8.515813	9.1456982	9.3790588	9.3422364	9.1445298
5	8.152810	8.8746742	9.2172230	9.3020362	9.1988251
6	7.79756	8.595126	9.0253814	9.2090472	9.2024474

Employing the same value, as before, for the eccentricity of *Jupiter*, we obtain, by the help of the preceding data, for the second class of considered terms, the expression

$$\frac{\partial R}{\partial e} = \mp 0.1082128 + 1.250172\,e \mp 0.630954\,e^2 + 1.765393\,e^3$$

the upper sign belonging to the case where the perihelia coincide, the lower to the case where they are opposed.

From the part of the perturbative function which is usually denominated secular, we have

$$\frac{\partial R}{\partial e} = \mp 0.0080600 + 0.287698\,e \mp 0.046723\,e^2 + 0.202990\,e^3$$

The sum of the two expressions is

$$\frac{\partial R}{\partial e} = \mp 0.1162728 + 1.537870\,e \mp 0.677677\,e^2 + 1.968383\,e^3$$

The right member of this, equated to zero (the upper sign being taken), gives $e = 0.077565$.

But, on account of the neglect of higher powers than the fourth of the eccentricities, this result is likely to be error to a considerable amount. Therefore a second de-

* *Annales de l'Observatoire de Paris*, Tom. I, pp. 284–291.

termination of the amount of motion of the line of apsides of the minor planet has been made by mechanical quadratures for two different values of e. The following formulas may be used:

$$\epsilon - e \sin\epsilon = M \qquad\qquad \epsilon' - e' \sin\epsilon' = l'$$
$$r\cos f = a(\cos\epsilon - e) \qquad\qquad r'\cos f'' = a'(\cos\epsilon' - e')$$
$$r\sin f = a\sqrt{1-e^2}\sin\epsilon \qquad\qquad r'\sin f'' = a'\sqrt{1-e'^2}\sin\epsilon'$$
$$\Delta^2 = [r - r'\cos(f-f'')]^2 + [r'\sin(f-f'')]^2$$
$$R = a'^2\left[\frac{1}{\Delta^3} - \frac{1}{r'^3}\right] r'\cos(f-f'') - \frac{a'^2 r}{\Delta^3}$$
$$S = -a'^2\left[\frac{1}{\Delta^3} - \frac{1}{r'^3}\right] r'\sin(f-f'')$$

Then it is necessary to compute the definite integral

$$\frac{1}{\pi}\int_0^\pi \left[-R\cos f + S\left(1+\frac{r}{p}\right)\sin f\right] dl'$$

This we do for the two values $e = 0.077565$ and $e = 0.078565$. The values of the quantity under the sign of integration for every 5° of the mean anomaly of *Jupiter* are given in the following table:

l'	$e = 0.077565$	$e = 0.078565$	l'	$e = 0.077565$	$e = 0.078565$
0	−2.764126	−2.757124	95	−0.156316	−0.157956
5	2.835108	2.828391	100	−0.053211	−0.054595
10	2.825159	2.817919	105	+0.018231	+0.017387
15	2.493183	2.481721	110	+0.032612	+0.032315
20	1.904867	1.895961	115	−0.015514	−0.015502
25	1.217910	1.239685	120	0.102780	0.102618
30	0.665311	0.658612	125	0.178702	0.178315
35	−0.227037	−0.222175	130	0.180609	0.179565
40	+0.050129	+0.053255	135	−0.054306	−0.052616
45	0.180203	0.181698	140	+0.225216	+0.229136
50	0.192504	0.193372	145	0.652027	0.657593
55	0.122146	0.122512	150	1.183608	1.190146
60	+0.005293	+0.004863	155	1.749316	1.756153
65	−0.126795	−0.126801	160	2.253331	2.260756
70	0.214125	0.214309	165	2.598633	2.605567
75	0.325167	0.325641	170	2.726679	2.733158
80	0.355710	0.356586	175	2.689120	2.695806
85	0.331231	0.332528	180	+2.644931	+2.651387
90	−0.258528	−0.260128			

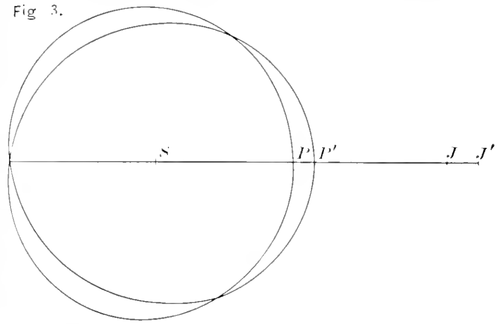
The sums of the numbers respectively in the second and third columns (the values appertaining to 0° and 180° receiving only half weight) are

$$+0.037466 \text{ and } +0.146495$$

Interpolating, we find that the value of e, proper to make the definite integral vanish, is $e = 0.07722136$.

We now construct to scale and by points, Fig. 3, exhibiting the synodic orbit of the minor planet; where, in explanation, it is necessary only to say that when *Jupiter* is severally at J and J' the planet is severally at P and P'. On comparison of Figs. 2 and 3, it will be noticed that, in the latter, P' lying between P and J instead of between P and S, two additional multiple points lying off the line of syzygies are necessitated. The latter figure probably shows what usually occurs in periodic solutions of this type.

Fig. 3.



Our second example will be the minor planet of the *Hecuba* type, where $k = 2$ and $k' = 1$. Then, with similar conditions and notation as in the first example, the terms of the perturbative function, made secular by the commensurability of the mean motions, are

$$R = (11)^2\chi + (12)^2\chi^3 + (13)^2\chi'^2\chi + (42)^1\chi'\chi^2 + (89)^0\chi'^2\chi$$
$$+ (21)^4\chi^2 + (22)^4\chi^4 + (23)^4\chi'^2\chi^2 + (51)^3\chi'\chi + (52)^3\chi'\chi^3$$
$$+ (53)^3\chi'^3\chi + (81)^2\chi'^2\chi^2 + (118)^1\chi'^3\chi + (27)^6\chi^3 + (60)^5\chi'\chi^2$$
$$+ (89)^4\chi'^2\chi + (33)^8\chi^4 + (69)^7\chi'\chi^3 + (98)^6\chi'^2\chi^2 + (118)^5\chi'^3\chi$$

Or, in the more explicit form,

$$R = -[4+1]\chi + [11+\tfrac{3}{2}-6-3]\chi^3 + [64+6-16-6]^2\chi'^2\chi + [-12+4+14+6]^1\chi'\chi^2$$
$$-[0+5+8+3]\chi'^2\chi + [22+7+1]^4\chi^2 + [-\tfrac{536}{3}-\tfrac{212}{3}+8+16+1]^4\chi^4$$
$$+[-1108-390+12+60+12]^4\chi'^2\chi^2 - [42+11+2]^3\chi'\chi$$
$$+[162+138-51-51-12]^5\chi'\chi^4 + [1071+270-78-66-12]^3\chi'^3\chi$$
$$+[-301-60+84+60+12]^4\chi'^2\chi^2 + [\tfrac{142}{3}-\tfrac{14}{3}-38-22-1]^1\chi'^3\chi$$
$$-[134+\tfrac{5}{2}+10+1]^6\chi^3 + [\tfrac{85}{2}+\tfrac{221}{2}+31+3]^5\chi'\chi^2 - [416+151+32+3]^4\chi'^2\chi$$
$$+[\tfrac{2576}{3}+\tfrac{9}{3}+80+13+1]^8\chi^4 - [3610+\tfrac{1992}{3}+310+51+4]^7\chi'\chi^3$$
$$+[5757+2131+511+84+6]^6\chi'^2\chi^2 - [\tfrac{12910}{3}+\tfrac{4534}{3}+382+58+4]^5\chi'^3\chi$$

In the *Hecuba* type of minor planet $\alpha = 0.6297651$. With this argument we take from RUNKLE's tables the values of LEVERRIER's A_{i}, and their logarithms are given in the following table (α has been subtracted in the cases of $A_0^{(1)}$ and $A_1^{(1)}$):

Logarithms of $A_j^{(i)}$.

i	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$
0	. . . .	9.8426431	9.9305853	9.9823155	0.4024322
1	9.1028882	9.6774459	9.9034748	9.9959539	0.1028525
2	9.5623275	9.9632340	9.9976231	0.0010594	0.1129766
3	9.2871510	9.8370606	0.0184658	0.0512301	0.1246387
4	9.0313180	9.6909111	9.9878184	0.0925462	0.1555446
5	8.786771	9.5334010	9.9236912	0.1062615	0.1934997
6	8.51962	9.368508	9.8366581	0.0921560	0.2229760
7	8.34774	9.198419	9.7331207	0.0544375	0.2359934
8	8.08977	9.02446	9.6171825	9.9974755	0.2305597

With the same value of *Jupiter's* eccentricity as before, we get, for the case where the perihelia coincide,

$$\frac{\partial R}{\partial e} = -1.482354 + 5.143972\,e - 13.889955\,e^2 + 17.1062\,e^3$$

and for the case where they are opposed,

$$\frac{\partial R}{\partial e} = -0.989656 + 2.193098\,e - 5.076561\,e^2 + 17.1062\,e^3$$

For the part of the perturbative function, usually denominated secular, we have, in the first case,

$$\frac{\partial R}{\partial e} = -0.028184 + 0.795798\,e - 0.34150\,e^2 + 1.8647\,e^3$$

and, in the second case,

$$\frac{\partial R}{\partial e} = +0.028184 + 0.795798\,e + 0.34150\,e^2 + 1.8647\,e^3$$

By the addition of the two portions, severally for each case, we obtain

$$\frac{\partial R}{\partial e} = -1.510538 + 5.939770\,e - 14.22245\,e^2 + 18.9709\,e^3 = 0$$

$$\frac{\partial R}{\partial e} = -0.961472 + 2.988896\,e - 4.73506\,e^2 + 18.9709\,e^3 = 0$$

The solution of these equations gives the value of e, for which, in each case, the secular motion of the line of apsides vanishes. The roots are, severally, $e = 0.45$ and $e = 0.30$. But a comparison of the values of the last with the first terms shows that these values are probably very wide of the mark. We must resort to mechanical quadratures. The formulas to be employed here are the same as in the first example, except that we substitute

$$\epsilon - e \sin \epsilon = 2l'$$

So little is known of the secular motion of the line of apsides in the difficult case of the planet of the *Hecuba* type that I feel justified in giving some details of the calculations I have made. Limiting ourselves at first to the case where the perihelia coincide, and where the two planets are in symmetrical conjunction when they occupy these positions, the quantity under the integral sign has the following values for the specified values of e:

l'	$e=0$	$e=0.12$	$e=0.14$	$e=0.2056$	$e=0.28$	$e=0.7$
0	−8.530590	−5.217868	−4.777629	−4.006547	−2.923007	−0.614550
10	8.461098	5.215575	4.957969	3.891521	2.927314	+0.229027
20	7.634481	4.496858	4.116617	3.049454	2.085997	1.089245
30	6.004867	3.231570	2.889654	1.927968	1.074403	1.286962
40	4.214893	2.062405	1.792204	1.062899	0.429075	1.207302
50	2.675585	1.221548	1.043364	0.552893	0.134531	0.976214
60	1.546403	0.712261	0.609033	0.323279	0.077717	0.653932
70	0.843500	0.468127	0.418578	0.337319	0.155461	+0.283321
80	0.513848	0.410583	0.395853	0.351480	0.306481	−0.098206
90	0.463977	0.460630	0.478429	0.481446	0.482434	0.455054
100	0.582306	0.612484	0.617623	0.634250	0.653194	0.751549
110	0.760216	0.771618	0.773134	0.784790	0.800732	0.949993
120	0.905745	0.911425	0.914456	0.913984	0.945747	1.010152
130	0.967521	1.003429	1.004982	1.002797	0.992655	0.896834
140	0.937333	1.008472	1.015565	1.022202	1.021184	0.611639
150	0.832348	0.916754	0.928795	0.959964	0.976675	0.272769
160	0.700922	0.752902	0.753064	0.793750	0.826882	0.138557
170	0.595117	0.589793	0.589561	0.590287	0.595183	0.354465
180	−0.554843	−0.520554	−0.514359	−0.620608	−0.466615	−0.256730

It thus appears that the line of apsides continually retrogrades for all values of e below 0.28, and it is not until $e = 0.7$ that the advancing seriously begins to counterbalance the retrogradation. For $e = 0$ the value of the definite integral is −2.287926, while, for $e = 0.7$, the value is −0.013825. However, much precision cannot be attributed to the latter, as 18 points on the semi-circumference are insufficient for anything but a rude approximation.

It seemed likely that the definite integral would vanish for a value of e in the neighborhood of 0.72; hence another computation was made for $e = 0.72$, doubling the number of points on the semi-circumference, with the following result:

l'	$e = 0.72$	l'	$e = 0.72$	l'	$e = 0.72$	l'	$e = 0.72$
0	−0.559176	50	+1.012882	95	−0.614318	140	−0.573594
5	0.103687	55	0.856245	100	0.757217	145	0.370910
10	+0.379255	60	0.682514	105	0.874217	150	0.179530
15	0.899073	65	0.496646	110	0.961737	155	0.044604
20	1.175027	70	0.303366	115	1.014920	160	0.010630
25	1.305755	75	+0.107266	120	1.019747	165	0.098793
30	1.345358	80	−0.087135	125	0.981329	170	0.274107
35	1.322269	85	0.276233	130	0.892546	175	0.379638
40	1.252687	90	0.453425	135	−0.753961	180	−0.243223
45	+1.147068						

These numbers make the value of the definite integral positive, viz., +0.024026. By interpolation between the results for $e = 0.7$ and $e = 0.72$, it is concluded that $e = 0.7073$ would make the definite integral vanish; thus we adopt $e = \sin 45°$, and with this Fig. 4, exhibiting the synodic orbit, has been constructed. But to establish the matter more firmly, the value of the definite integral was computed for $e = \sin 45°$, this time making ϵ the independent variable instead of l'. In this method it is only necessary to multiple the former expression for the quantity under the integral sign by $\frac{r}{a}$. This method undoubtedly has advantages over the method with l' as independent variable. Details of the result are

ϵ	$e = \sin 45°$	ϵ	$e = \sin 45°$	ϵ	$e = \sin 45°$	ϵ	$e = \sin 45°$
0	−0.17489	100°	+1.46880	190°	−1.21180	280°	−0.08244
10	0.19121	110	1.59050	200	1.50660	290	0.11638
20	0.21189	120	1.59338	210	1.61982	300	0.16077
30	0.18993	130	1.45842	220	1.53837	310	0.18694
40	−0.07980	140	1.18491	230	1.28591	320	0.18122
50	+0.11845	150	0.78321	240	0.92743	330	0.15205
60	0.37672	160	+0.28601	250	0.55782	340	0.11487
70	0.68259	170	−0.25141	260	0.27157	350	0.08509
80	0.98628	180	−0.77550	270	−0.11727	360	−0.07512
90	+1.25652						

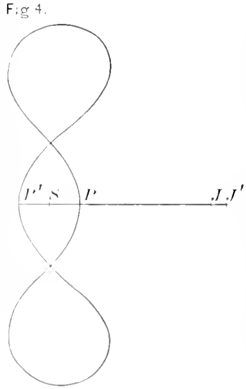
These numbers make the value of the definite integral −0.004481, which seems to show that the desired value of e somewhat exceeds sin 45°.

To see whether the other three arrangements of the elements could bring about periodic solutions in the case of the *Hecuba* type of minor planet, the value of the definite integral has been computed, in the first instance, for $e = 0.7$, but with *Jupiter* in aphelion instead of perihelion at the time of symmetrical conjunction, and, in the second instance, for $e = 0.2056$, but with the aphelion of the minor planet in conjunction with the perihelion of *Jupiter*, or, which amounts to the same thing, the computation is made with $e = -0.2056$. The details of these calculations are thus shown:

l'	$e = 0.7$	$e = -0.2056$	l'	$e = 0.7$	$e = -0.2056$
0	−0.114167	+25.81939	100	−0.592627	+0.54894
10	+0.210407	24.77975	110	0.923640	0.73260
20	0.906778	21.62186	120	1.168929	0.81629
30	1.106777	16.83959	130	1.263650	0.80290
40	1.096926	11.92495	140	1.143922	0.71732
50	0.967684	7.57606	150	0.817554	0.68782
60	0.752428	4.33151	160	0.511273	0.63966
70	0.469796	2.00845	170	0.549153	0.61324
80	+0.136597	0.78334	180	−0.335407	+0.60394
90	−0.226700	+0.41958			

The value of the definite integral, in the first instance, is −0.107769, which, as it is more decidedly negative than when *Jupiter* was in perihelion, leads us to think that a larger value of e than sin 45° is necessary for a periodic solution in this arrangement than in the case we have worked out, and, perhaps, it may not exist. In the second instance, the line of apsides continually advances, and plainly it does so all the way from $e = 0$ up to the point where the minor planet passes through the perihelion position of *Jupiter*, when the motion becomes infinite: hence no periodic solution is to be looked for in this direction.

Fig 4.



The figure shows that, although the minor planet crosses the orbit of *Jupiter* four times every synodic revolution, it keeps well out of the way of the latter planet. Thus the periodic perturbations must be quite small, probably no coefficient of any periodic term in the longitude exceeding 200″, with correspondingly small inequalities in the radius. The circumstance that the two planets change the order of their distance from the Sun is no bar to the representation of the

coordinates of the minor planet by periodic series. The elaboration of the latter for the periodic solution, just treated, is far easier than in the general case involving LINDSTEDT's series. But, as Nature affords us no example of this periodic solution, perhaps, the labor would be unwarranted. However, hints might be suggested in its course, profitable for the more complicated case.

NOTE ON THE COMPARISON OF PHOTOGRAPHS WITH MERIDIAN CATALOGUES. A REPLY TO REMARKS BY PROFESSOR LEWIS BOSS IN *ASTRONOMICAL JOURNAL*, NO. 517,

By H. H. TURNER.

It was with considerable surprise that I read the remarks of Professor LEWIS BOSS in his "Note on the Magnitude-Equation in R.A. for the Cambridge A.G. Zone and for the recent Bonn Observations." I can only think that his great experience in the comparison of one meridian catalogue with another has led him to overlook the essential difference which arises when we are dealing with a photographic catalogue derived from a series of plates each of which covers a comparatively small region of the sky, and is treated separately. Speaking generally, the question of *precession* does not arise in such a case, and accordingly the principal set of corrections which Professor BOSS proposes to apply are quite irrelevant. Parallactic motion might have been applied, but its amount in the mean of a number of plates extending over 12^h, even admitting the existing irregularity of stellar distribution, is negligible. Professor BOSS's remarks upon proper motions are conditional upon the interpretation of a certain paragraph in my paper which he finds, to my regret, of ambiguous meaning; when it is read as I meant it to be read his criticism no longer applies.

Perhaps I may take the last point first, as it involves a simple matter of fact. The paragraph from my paper which is the subject of criticism stands as follows (*M.N.*, LX, p. 4):

"In column III is given the mean photographic correction to the Cambridge R.A. For the first three groups the number of stars is so small that the results must not be treated too seriously. Proper motions have been applied (taken from the Greenwich Catalogue, 1880.0) for the interval between the Cambridge and Oxford observations, though in the other groups P.M. has been treated as accidental error."

The meaning these words were intended to have was that proper motions have been applied to stars in the first three groups, though in the other groups proper motion has been treated as accidental error. I am sorry this was not clear, though but for Professor BOSS's finding a difficulty I confess I should have thought it clear enough; especially when the first three groups are marked in a different way in the diagram; and when they are so conspicuously different in number of stars — 4, 7, 9 as against 38 for the next group. Surely Professor BOSS might have given me the benefit of the doubt, if there was one.

But now that the doubt is removed, his remarks about a correction of $0^s.012$ (see the middle of the second column of p. 102, *A.J.*, No. 517) only apply to the first three groups, which are by declaration "not to be treated too seriously;" the other groups do not require any correction.

Next as regards parallactic motion; taking Professor BOSS's expression

$$+0^s.017 \sin(\alpha+85^\circ)$$

we have to consider what the mean value from 0^h–12^h of this is likely to be when the stars are irregularly distributed. Immediately following my paper, is one by Mr. F. A. BELLAMY, showing the kind of distribution of the stars in right-ascension. The stars we are considering (viz.: those observed at Cambridge on the meridian) are approximately those in the DM., and we may take from the diagram 3 on plate 2 (opposite p. 16, *M.N.* LX) the following approximate numbers representing irregularity of distribution about zone $+26^\circ$:

α = R.A.	No. of Stars	Value of $\sin(\alpha+85^\circ)$	Product
0^h– 1^h	11	+1.00	+11
1– 2	11	+0.95	+10
2– 3	12	+0.84	+10
3– 4	15	+0.68	+10
4– 5	19	+0.46	+ 9
5– 6	23	+0.22	+ 4
6– 7	25	−0.04	− 1
7– 8	18	−0.30	− 5
8– 9	14	−0.54	− 8
9–10	10	−0.74	− 7
10–11	9	−0.89	− 8
11–12	8	−0.98	− 8
Total	175		+17

Thus the mean value of $+0^s.017 \sin(\alpha+85^\circ)$ with the distribution above assumed is less than $0^s.002$. To prevent misconception, I should say that this result is only illustrative; it does not actually apply, because, first, the DM. contains many stars not observed in Cambridge; and secondly, the correction for parallactic motion depends on the distribution of bright stars and faint stars separately, while we have taken all together. But it shows the *kind* of diminution of the coefficient $+0^s.017$ which we may expect on the average of a series extending over 12^h in R.A.

a diminution of 1 to 10, sufficient to render this correction, already a small one, of dimensions which I at any rate meant deliberately to neglect in the provisional discussion.

Coming now to precession, there is a vital difference between Professor Boss and myself. Let me state the case thus. About 1880 some meridian observations were made at Cambridge, to determine the places of certain stars of different magnitudes. Meridian observations are made by using the rotation of the Earth on its axis, and the resulting star places are thus referred to the position of the Earth's axis in 1880, or thereabouts. Subsequently we are going to photograph some or all of these stars; but in taking a photograph, the rotation of the Earth, far from being a help, is a positive nuisance. It has to be compensated by delicate machinery, and it leads to confusion in the calculations. After securing our meridian observations in 1880, let us then abolish the Earth altogether and substitute a body fixed in space. As we have disposed of proper motions (including that of the Sun) independently, we can suppose them all zero; *i.e.*, we can imagine the stars all absolutely fixed, and that we are henceforth to photograph them from a standpoint fixed in space. Hence, whenever we take a photograph we get the same picture, whether 10, 100 or 1000, or any number of years after 1880. We have no reference points or lines on the photograph beyond the stars themselves; there is no north or south on it, and certainly no information about its right-ascension. It is merely a picture, of which the orientation and even the scale are unknown. But when we compare it with the positions of the stars as measured at Cambridge in 1880, we can assign a north and south as they were in 1880; we can say what was the right-ascension and declination of each star in 1880; we can assign these exactly if the Cambridge observations are perfect; and if they are not perfect, we can do it approximately, fitting measures made on the photograph to the Cambridge places by a least-square solution, the residuals of which will give us the systematic errors of Cambridge. Let us call the place of any star at Cambridge in 1880.0 (ξ, η): where (ξ, η) correspond nearly to right-ascension and declination (1880.0), but are transformed in the well known manner so as to represent rectangular coordinates. Let us call corresponding measures on the picture, taken any number of years afterwards, (x, y); and suppose that by a least-square solution we have found axes of x and y to correspond to the Cambridge (1880.0) axes. Then we have

$$x = \xi + r \quad , \quad y = \eta + s$$

where r and s represent residuals for individual stars, which are all small, and have mean value zero. The discussion of r and s according to magnitude would tell us about the Cambridge "magnitude-equation:" and whenever the photograph was taken, r and s would be just the same. The Earth and its precession have been abolished, and cannot possibly affect the discussion.

Next, let us suppose that, before comparing the Cambridge places with the photograph, we made a transformation of axes, so that the coordinates of the star are now represented by $(\xi' \eta')$ where

$$\xi' = a + \xi \cos\theta - \eta \sin\theta \quad , \quad \eta' = b + \xi \sin\theta + \eta \cos\theta$$

(a, b) representing an arbitrary change of origin, and θ an arbitrary change of orientation. If we select axes of $(x' y')$ as before to fit $(\xi' \eta')$ by a least-square solution it can easily be shown that we shall reproduce exactly the same changes in (xy) as we have made in $(\xi\eta)$, *i.e.*, we shall have

$$x' = a + x \cos\theta - y \sin\theta \quad , \quad y' = b + x \sin\theta + y \cos\theta$$

and thus $\quad x' = \xi' + r' \quad , \quad y' = \eta' + s'$

where $\quad r' = r \cos\theta - s \sin\theta \quad , \quad s' = r \sin\theta + s \cos\theta$

If θ is a small angle, so that $r \sin\theta$ and $s \sin\theta$ may be neglected, these last become

$$r' = r \quad , \quad s' = s$$

That is, for *any* change of origin, and for any *moderate* change of orientation, we get the same residuals as before; and hence the same "magnitude-equation," etc.

Among such general changes of axes we may include any desired change connected with methods in use before the Earth was abolished. If, for instance, it was considered desirable to refer the Cambridge meridian observations to axes favored by Mr. Smith or Mr. Jones, however erroneous, provided only that the change of orientation proposed by either of these gentlemen were not too large, we could do this without in any way altering our residuals, or the deduced magnitude-equation. And now I remark that the change of orientation produced by precession in twenty years at declination +30° is about $\frac{1}{450}$ in circular measure, which is sufficiently small to allow us to neglect $r\theta$ and $s\theta$. For instance, a residual even of 10″ in one coordinate would only be introduced into the other as $\frac{10}{450}''$ or 0″.02. [It may be remarked, in passing, that *in high declinations*, we must keep a watch on this intermixture of errors of right-ascension and declination, not only in applying precession, but originally in forming (ξ, η), which no longer correspond closely to right-ascension and declination.]

Hence, we may make that particular change of axes which corresponds to bringing up the Cambridge meridian places for 1880.0 to the epoch 1900.0, using any constant of precession we please, provided it is of the same order of magnitude as 50″ (say, for instance, 55″ or 60″, or even 400″), and we shall not in any way interfere with our discussion of magnitude-equation.

Again, the epoch when the photograph is taken has *nothing whatever* to do with the application of precession.

Lest I should seem to be stating too obvious a fact, let me quote Professor Boss's words (*A.J.*, No. 517, p. 101, col. 2):

"Professor Turner employed Struve's precession in reducing Cambridge positions from about 1880 to *about 1895*."

The italics are mine. I take it that Professor Boss means by "about 1895" the epoch when the photographs were taken, as he uses the words lower down in the same column in connection with proper motions. In this latter connection the epoch would of course have a significance; but in connection with precession it has none whatever, as above stated; and as a matter of fact precession was applied to reduce the Cambridge places, not to "about 1895," but to 1900.0 definitely merely as a matter of convenience; and the same thing might have been done without any real inconsistency if the photographs had been taken in 1995.

In short, I cannot accept any of the conclusions of Professor Boss's paper as they stand. Is it that he is under some misconception as to the manner in which photographs are taken or treated?

ON THE APPROXIMATE MEAN PARALLAX OF A GROUP OF 405 STARS,

By J. G. Porter.

In studying the directions of the motions of a large number of stars, one will naturally expect to discover a preponderating tendency to move away from the apex of the solar motion. A large percentage, moreover, of the motion of stars which fall in this class is certainly parallactic. If we can tell what proportion of the total movement is parallactic, we can, of course, determine the actual mean parallax of the group under consideration as a whole, assuming the rate of the sun's motion to be known.

Out of 2400 stars I find 405, the direction of whose motion differs by less than 15° from that towards the antiapex. This is more than twice the number that should lie within these limits were the directions equally distributed. These 405 stars were divided into two groups according to magnitude. Group I including stars of seventh magnitude and brighter, and Group II stars fainter than seventh. The following table shows the distribution.

Table I.

Group	No. Stars	Mean Mag.	Mean Motion ″
I	206	5.7	0.310
II	199	8.1	0.304

The mean motions in the table are the actual proper motions divided by the sine of the apical distance, and no stars are included nearer the apex than 30°. Evidently the amount of motion, and presumably the distance of these stars, is virtually independent of the magnitude. We may therefore treat the two groups together.

In order to form some idea of what proportion of this motion may be considered parallactic I have proceeded in the following manner. Of the 405 stars there are 59 with motion less than 0″.125, 133 stars with motions between 0″.125 and 0″.200, 85 stars with motions between 0″.200 and 0″.300, and the remaining 128 have motions exceeding 0″.300. The average motions for each class are given in Table II. It is safe to assume that in the first class the actual motions to and from the apex are nearly equal, and practically the whole resulting movement is parallactic. For the fourth class I assume the mean parallactic movement to be one-half the observed movement. For the other classes, of course, the parallactic motion will fall between these values. I have roughly estimated it as in the table.

Table II.

Class	No. Stars	Aver. Motion ″	Assumed Par. Motion ″
1	59	0.10	0.10
2	133	0.16	0.13
3	85	0.25	0.17
4	128	0.58	0.29

We thus find for the average parallactic motion of all the stars 0″.185. This value in my opinion is more likely to be too small than too large. The resulting parallax, taking the sun's yearly motion as four radii of the earth's orbit, is 0″.046, a quantity considerably greater, according to Kapteyn's formula, than the average parallax of second magnitude stars, the mean magnitude of these stars being 6.9. Even should we reduce our estimate of the parallactic motion to one-half the total motion for all the stars, we still get an average parallax of nearly 0″.04.

In this investigation the apex was taken at $18^h 35^m$, +15°, but I do not anticipate that a shifting of its position further to the south would appreciably alter the results. While it has long been recognized that stars with considerable proper motion are, on the whole, nearest to our system, still I venture to think that this approximation to the average parallax of so large a group will not be without interest.

Cincinnati Observatory, 1902 *March* 12.

THE ABERRATION-CONSTANT FROM DAVIDSON'S SAN FRANCISCO OBSERVATIONS,

By S. C. CHANDLER.

As the best way to settle the point in discussion in *A.J.* 517 and 518 I have redetermined the aberration from Davidson's observations by what seems to me an unobjectionable method, and find

$$20''.555 \pm 0''.021 \text{ (p.e.)}$$

The details will be printed as soon as is consistent with the precedence that must be given to articles by other contributors.

The above value seems to confirm my surmise that a correct solution would considerably increase the result ($20''.182 \pm 0''.033$) of the Coast Survey discussion. I heartily concur with Prof. Newcomb that it is very desirable that that discussion should be revised with the amendment of his method he now proposes, to see if it verifies the present calculation.

With regard to my criticism of the method used for Waikiki, which Prof. Newcomb finds obscure, I regret that he asks something beyond my ability in requiring that I shall make it clearer. I have tried hard to do so but fail to compass a more limpid statement. However, fortunately the matter is not important, since here also I heartily concur with him that the aberration is practically indeterminate from this series. It is too ill-conditioned for the purpose, but by the same token absolute correctness of process would be essential in any attempt of the sort; which is exactly my point.

The concluding paragraph of Prof. Newcomb's article involves a question far more important than the mere numerical results of these two series of observations. Its extreme interest lies in the fact that a score of aberration-determinations have been published — to which I can add half a dozen of my own computation not yet printed — conducted on principles which he thinks can be supplanted by better ones. Any value of the aberration that may be finally agreed upon generally by astronomers for future conventional use — and we are rapidly nearing the point when the balance can be struck and this decision can be made — must rest in very large part on the testimony of these and similar undertakings now under way. The best mode of utilizing to this end this vast body of observations is a paramount element in the settlement of this burning question. Therefore, if the demonstration of which Prof. Newcomb speaks will lead to improvement of the present method of dealing with this material, he will confer a great service on astronomy by elucidating it.

EPHEMERIS OF MINIMA OF THE *ALGOL*-TYPE VARIABLE (6927) — *SAGITTAE*.

$\alpha = 19^h\,14^m\,26^s$, $\delta = +19°\,25'.7$ (1900).

E	Hel. Gr. M.T. (d h)	E	Hel. Gr. M.T. (d h)	E	Hel. Gr. M.T. (d h)	E	Hel. Gr. M.T. (d h)
45	1902 Apr. 2 9.7	60	1902 May 23 2.8	75	1902 July 12 19.8	90	1902 Sept. 1 12.9
46	5 18.9	61	26 11.9	76	16 5.0	91	4 22.0
47	9 4.0	62	29 21.1	77	19 14.1	92	8 7.2
48	12 13.2	63	June 2 6.2	78	22 23.3	93	11 16.3
49	15 22.3	64	5 15.4	79	26 8.4	94	15 1.5
50	19 7.4	65	9 0.5	80	29 17.5	95	18 10.6
51	22 16.5	66	12 9.6	81	Aug. 2 2.7	96	21 19.7
52	26 1.7	67	15 18.8	82	5 11.8	97	25 4.9
53	29 10.8	68	19 3.9	83	8 21.0	98	28 14.0
54	May 2 20.0	69	22 13.1	84	12 6.1	99	Oct. 1 23.2
55	6 5.1	70	25 22.2	85	15 15.2	100	5 8.2
56	9 14.2	71	29 7.3	86	18 21.3	101	8 17.4
57	12 23.3	72	July 2 16.5	87	22 9.5	102	12 2.5
58	16 8.5	73	6 1.6	88	25 18.6	103	15 11.7
59	19 17.7	74	9 10.8	89	29 3.8	104	18 20.8

From Ebell's elements. Min. 1901 Nov. $1^d\,6^h\,37^m$, Greenwich M.T. $+ 3^d\,9^h\,8^m\,10^s.2\ E$

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ON THE PARALLAX AND PROPER MOTION OF *NOVA PERSEI.*

By F. L. CHASE.

The distance of *Nova Persei* has an added importance since the discovered changes in the nebula surrounding it, as furnishing evidence as to the true explanation of these changes. Immediately after the discovery of the *Nova* I began a series of observations with the Yale heliometer for the purpose of determining its parallax.

A provisional solution was made from the first six months' observations, and the results presented at the Denver meeting of the A.A.A.S. in August last. Further observations have now been made, extending the period to an entire year, and I present here a more complete determination.

The comparison-stars selected were DM. +43°720 (a) and DM. +43°766 (b), the distances from the *Nova* being approximately 2900″ and 2700″, and the position-angles about 252° and 94° respectively, thus constituting a fairly symmetrical pair. The magnitudes of these stars are 7.4 and 8.0 according to the *Bonn Durchmusterung*, or 7.9 and 8.5 according to the *A.G. Catalogue*. The *Nova* was screened down to about these magnitudes during the earlier observations. In the later series no screen was necessary.

The observations are brought together in the following table, in which are given also the sums of the distances and the distance $a\,b$, the use of which distance will be explained later:

	Date	Sid. Time	Def.	Temp.	Dist. from a	Dist. from b	Sum	Dist. $a\,b$
		h m		°	R	R	R	R
1	1901 Feb. 24	8 52	2–3	15	231.348	212.672	444.020	. . .
2	25	8 48	2–3	26.5	.383	.705	.088	. . .
3	26	8 57	3	23	.349	.681	.030	436.399
4	27	8 45	2	17	.399	.710	.109	.409
5	28	8 40	2–3	17.5	.366	.683	.049	.388
6	Mar. 7	8 41	2	25	.366	.694	.060	.397
7	18	8 56	3	36	.374	.729	.103	.438
8	1901 July 19	22 46	3	66	231.386	212.714	444.100	436.415
9	21	22 39	2	74	.366	.700	.066	.407
10	23	22 19	1–2	66	.376	.709	.085	.407
11	27	23 1	2–3	61.5	.375	.706	.081	.396
12	30	22 52	1–2	73	.380	.696	.076	.418
13	Aug. 1	23 3	3	61	.378	.705	.083	.400
14	1	23 5	3	63.5	.358	.701	.059	.404
15	1901 Dec. 16	6 57	3	13	231.350	212.666	444.016	436.381
16	18	8 5	3	8.5	.378	.693	.071	.367
17	19	8 35	3	11.5	.349	.699	.048	.367
18	20	8 32	3	12	.380	.700	.080	.358
19	1902 Feb. 6	8 3	2	16	.375	.697	.072	.377
20	7	7 56	2	19.5	.356	.700	.056	.365
21	9	8 11	2–3	24	231.365	212.692	444.057	436.368
						Means	444.067	436.393

After observing the star the first two nights it was thought it might be a good plan to observe each night also the distance $a\,b$, which would furnish an independent scale value, and it was thought that we might thus make an in-

dependent solution of the parallax from each star separately, and thus see if there is any marked difference in the parallaxes of the comparison-stars. A closer investigation, however, by Dr. ELKIN later, showed that this method would involve the parallaxes of the comparison-stars in essentially the same manner as the usual method. Hence, although practically nothing is to be gained in this way by the use of the distance ab, I nevertheless continued to observe it each night throughout the work in order to make all of the observations as exactly comparable as possible; and as a matter of a little interest I have carried out a second solution, exactly like the first, except that instead of assuming the scale value for each night to be determined by the sum of the distances, I have in Sol. II assumed it to be determined by the value of ab for that night.

From the above observations I have formed the differences of the distance of the *Nora* from each of the comparison-stars, the corrections to be applied to these differences each night for the change of scale value (assuming for Sol. II a vanishing correction for the first two nights), and the corrected differences. These are given in the following table:

	Difference	Scale Corr. I	Scale Corr. II	Corr. Diff. I	Corr. Diff. II
	R			R	R
1	18.676	+2	0	18.678	18.676
2	.678	−1	0	.677	.678
3	.668	+2	0	.670	.668
4	.689	−2	−1	.687	.688
5	.683	+1	0	.684	.683
6	.672	0	0	.672	.672
7	.645	−2	−2	.643	.643
8	18.672	−1	−1	18.671	18.671
9	.666	0	−1	.666	.665
10	.667	−1	−1	.666	.666
11	.669	−1	0	.668	.669
12	.684	0	−1	.684	.683
13	.673	−1	0	.672	.673
14	.657	0	0	.657	.657
15	18.684	+2	+1	18.686	18.685
16	.685	0	+1	.685	.686
17	.650	+1	+1	.651	.651
18	.680	−1	+1	.679	.681
19	.678	0	+1	.678	.679
20	.656	0	+1	.656	.657
21	18.673	0	+1	18.673	18.671
Assumed mean corr. diff.				18.672	18.672

These corrected differences give rise to the equations of condition given below, in which x is the correction to the assumed mean difference, and y and z the parallax and proper motion of the *Nora*, both relative to the comparison-stars. In the last column are given also the residuals derived from Sol. I, those from Sol. II never differing more than two units in the last decimal used.

EQUATIONS OF CONDITION.

				I	II	Residuals I
				R	R	R
1	$+1.00x$	$-1.89y$	$+0.15z=$	+0.006	+0.004	+0.004
2	1.00	1.88	0.15	+ .005	+ .006	+ .004
3	1.00	1.88	0.15	− .002	− .004	− .003
4	1.00	1.87	0.16	+ .015	+ .016	+ .014
5	1.00	1.86	0.16	+ .012	+ .011	+ .011
6	1.00	1.80	0.18	.000	.000	− .001
7	1.00	1.65	0.21	− .029	− .029	− .030
8	$+1.00x$	$+1.71y$	$+0.55z=$	−0.001	−0.001	+0.001
9	1.00	1.78	0.55	− .006	− .007	− .003
10	1.00	1.80	0.56	− .006	− .006	− .003
11	1.00	1.84	0.57	− .004	− .003	− .001
12	1.00	1.87	0.57	+ .012	+ .011	+ .015
13	1.00	1.89	0.58	.000	+ .001	+ .003
14	1.00	1.92	0.59	− .015	− .015	− .012
15	$+1.00x$	$-0.92y$	$+0.95z=$	+0.014	+0.013	+0.014
16	1.00	0.98	0.96	+ .013	+ .014	+ .013
17	1.00	1.01	0.96	− .021	− .021	− .020
18	1.00	1.03	0.97	+ .007	+ .009	+ .007
19	1.00	1.91	1.10	+ .006	+ .007	+ .006
20	1.00	1.92	1.10	− .016	− .015	− .016
21	1.00	1.93	1.11	+0.001	+0.002	+0.001

These twenty-one equations of condition reduce to the following three normals:

$$
\begin{array}{rrrlll}
 & & & & \text{I} & \text{II} \\
+21.00x & -\ 9.69y & +12.28z & = & -0^{R}.0090, \text{ or} & -0^{R}.0070 \\
 & +62.09y & -\ 4.94z & = & -0.0507, \text{ or} & -0.0531 \\
 & & +\ 9.79z & = & -0.0091, \text{ or} & -0.0044
\end{array}
$$

which, solved, give the following values:

$$
\begin{array}{llll}
x = -0^{R}.0001 \ \pm 0^{R}.0036 & \text{or} & -0^{R}.0009 \ \pm 0^{R}.0036 & \text{Wt. } 5.39 \\
y = -0.00093 \pm 0.0011 & \text{or} & -0.00098 \pm 0.0011 & \text{"} \ 57.42 \\
z = -0.0013 \ \pm 0.0052 & \text{or} & +0.0015 \ \pm 0.0052 & \text{"} \ 2.60
\end{array}
$$

(I) $[nn] = 0.002881$ $[vv] = 0.002813$

Prob. error 1 eq. $= \pm 0^{R}.0084$ or $\pm 0''.107$

(II) $[nn] = 0.002893$ $[vv] = 0.002836$

Prob. error 1 eq. $= \pm 0^{R}.0084$ or $\pm 0''.107$

The value of y from either solution transformed into arc gives,

$$\pi = -0''.012 \quad \pm 0''.014$$

This result, based upon one of the most accordant series of observations I have as yet been able to make, shows the *Nora* to be not less distant than the comparison-stars employed, viz.: stars of approximately the eighth magnitude; and my observations further indicate that there is no perceptible proper motion of the *Nora* unless we assume the same motion for the comparison-stars. I have been able to find only the first mentioned of the comparison-stars in any of the catalogues at my disposal other than the A.G. Catalogue for 1875, and this star appears to have no appreciable proper motion.

The average parallax for an eighth magnitude star is according to KAPTEYN $+0''.007$, so that the absolute parallax for *Nova Persei* would be

$$\pi = -0''.005$$

This result, when we consider that doubtless some small systematic error should be added to the probable error, $\pm 0''.014$, above found, is not incompatible with the value $\pi = +0''.012$, deduced by WOLF, on the theory that the apparent displacements of certain condensations of the nebula represent a velocity equal to that of light or electricity.

NOTES ON POGSON'S OBSERVATIONS OF *U GEMINORUM*, *T SCORPII*, AND *R LIBRAE*.

By JOSEPH BAXENDELL.

Upon the death in 1891 of Mr. N. R. POGSON, the Government Astronomer at Madras, I was entrusted (in accordance with a request he had expressed a short time previously), with the whole of his observations of Variable Stars. The reduction of these, on the lines he had decided upon, has now been practically completed, and the results are being communicated to Prof. G. MÜLLER of Potsdam.

In the course of the work, three matters of special interest have been met with.

2815. *U Geminorum*.

In Mr. POGSON's journal, under date 1856 March 26, appears the following very remarkable entry with reference to this singular star, — an entry, indeed, so remarkable that it is probably without a parallel in the history of Variable Star Astronomy. It could scarcely be received or accepted were it not for the high reputation enjoyed by its author as an able and reliable observer. It is *possible* that the note may have been published by POGSON at the time, but, if so, its repetition now will, I think, be forgiven. The entry (which is very clearly written) reads thus:

"The variable subject to strange fluctuations at intervals of six to fifteen seconds, quite to the extent of four magnitudes; when the adjacent stars were quite steady and not at all twitching, like the variable. At times it surpassed the [comparison] star 8.9, at others it quite vanished. A new phenomenon to me! Watched it for half an hour, with powers 54, 65, and 95, on the equatorial." [At Oxford, England.]

On the succeeding evening (1856 March 27), the following note was made:

"Light [of *U*] very unsteady, but not subject to such pulsations as were seen last night."

Various observations of unsteadiness were recorded during subsequent years; an entry under date 1857 April 15 may be quoted as being typical of, but rather fuller than, the remainder:

"Slightly flickering; certainly less steady than the neighboring stars, but not varying in the marked manner previously observed."

But on certain other occasions POGSON describes the light of the variable as being "quite steady."

U Geminorum has always been regarded by the older observers of variable stars as a kind of connecting link between the so-called new stars and the ordinary long-period variables. An instance may be cited of the rapidity of its rise to maximum (which is, I believe, a feature of all the maxima of this star, whether belonging to the short or the long-duration types, and whatever may be the interval since the previous one).

On 1866 April 16, *U* was of the 13th magnitude. The following day it had attained its maximum brightness (9^{M}–10^{M})!

It is greatly to be desired that some suitably equipped observatory should undertake careful spectroscopic observations of *U Geminorum during its rapid rise* and then more gradual decline.

5826. *T Scorpii*.

Upon happening to observe the cluster *Messier* 80 on 1860 May 28 POGSON found that it presented the startling appearance "of a nebulous star of 7.6 magnitude." It subsequently transpired that the outburst had been discovered by AUWERS, one week earlier. On May 9 POGSON had perceived nothing unusual about the cluster. By June 10 the new object had become considerably fainter. It is commonly believed that this star has never since been seen. POGSON's observations, however, appear to prove that either it, or some other star (or stars) in the cluster, exhibit irregular, but at times striking, variations.

On 1863 June 9, what we must at present term *T Scorpii* appeared = 11.0 magnitude; the following night it was scarcely noticeable. On 1864 January 30 nothing was seen of it, but on February 10 it was = 9.0 magnitude, and on March 5 = 9.3 magnitude. Only two days later, however, it had fallen below 11.5 magnitude. Many other, but probably less striking, variations in the appearance of *Messier* 80 are recorded by Mr. POGSON.

5688. *R Librae*.

It has been generally supposed (at any rate until quite

recently) that the period of this star is longer than those of all, or almost all, other tolerably regular known variables. It would, however, seem that nearly forty years ago, Pogson became aware that such was not the case. Instead of a period of two years, the star possesses one of very slightly under eight months. Mr. Pogson's complete series of observations appear to prove this incontrovertibly. The point tends to still further emphasize the extraordinary circumstance of the general shortness of the periods of all variable stars that are known to be periodic at all.

The Fernley Observatory, Southport, England, 1902 *March* 25.

ABERRATION-CONSTANT FROM KASAN, PRAGUE, POTSDAM AND SAN FRANCISCO OBSERVATIONS.

By S. C. Chandler.

This article gives the determination of the aberration-constant from various series of observations by Talcott's method which have not previously been utilized for this object; or from which, in the case of San Francisco, what seems to me an erroneous value has been derived.

Before giving the results it is desired to explain a departure from the customary mode of calculation of the cyclical sum, ΣA_0, of the aberration-factors used in the known equation

$$\Sigma A_0 \, \Delta k + \Sigma r = 0 \tag{1}$$

for finding the aberration-correction by Küstner's polygonal method. The process usually followed is a tedious one even where the reductions from true to apparent declination for the individual observations are at hand. But this computation of the factors for the final equations permits of extraordinary amelioration, and the abridgement has a degree of accuracy which I hold to be entirely sufficient for the purpose. To show this take the reduction from true to apparent declination, which in *A.J.* 517 I have thrown into the form

$$(2) \qquad k\cos\zeta\,[n\sin T - m\cos\odot + \sin q\,(\cos\epsilon - 1)\,(\tfrac{1}{2}\sin 2\odot\cos T - \sin^2\odot\,\sin T)]$$

where $T = \alpha - \odot$, $m = \sin\epsilon\cos q$, $n = \cos\epsilon\sin q$.

This is rigorous within its one-thousandth part for a pair of stars of any zenith-distance, since the only neglected element is the interval between the times of culmination of the stars of the pair. Denote by the subscripts 1 and 2, respectively, the average values of T and ζ for two groups of stars observed on the same date. Then the difference of the aberration-factors for the two groups will be approximately,

$$(3) \quad \left\{\begin{array}{ll} A_0 = n\,(\cos\zeta_1\sin T_1 - \cos\zeta_2\sin T_2) & (a) \\ \quad - m\,(\cos\zeta_1 - \cos\zeta_2)\cos\odot & (b) \\ \quad + \sin q\,(\cos\epsilon - 1)\,\tfrac{1}{2}\sin 2\odot\,(\cos\zeta_1\cos T_1 - \cos\zeta_2\cos T_2) & (c) \\ \quad - \sin q\,(\cos\epsilon - 1)\sin^2\odot\,(\cos\zeta_1\sin T_1 - \cos\zeta_2\sin T_2) & (d) \end{array}\right.$$

The cyclical sum will be

$$\Sigma A_0 = \Sigma(a) + \Sigma(b) + \Sigma(c) + \Sigma(d) \tag{4}$$

Now, in (b), (c) and (d) the products of the factors of $\cos\odot$, $\sin 2\odot$ and $\sin^2\odot$ are all very small, and have nearly the same constant values in each for all the combinations of groups I–II, II–III, etc. Also, for observations distributed uniformly through the year, $\Sigma\cos\odot = 0$, $\Sigma\sin 2\odot = 0$, $\Sigma\sin^2\odot = \tfrac{1}{2}$. Consequently, $\Sigma(b)$ and $\Sigma(c)$ are $= 0$, and we can write the sum of $\Sigma(a)$ and $\Sigma(d)$,

$$\Sigma A_0 = n\,\tfrac{1}{2}\,(1 + \sec\epsilon)\,\Sigma\,(\cos\zeta_1\sin T_1 - \cos\zeta_2\sin T_2) \tag{5}$$

We may safely simplify still further in most cases by using a common value for $\cos\zeta$ for all the groups, and since this is ordinarily not far from 0.97 while $\tfrac{1}{2}\,(1 + \sec\epsilon) = 1.04$, so that their product is unity, we have finally the very simple expression

$$\Sigma A_0 = n\,\Sigma\,(\sin T_1 - \sin T_2) \tag{6}$$

If we use the average right-ascension for each group of stars, and the longitude of the sun for the mean date of the comparisons of any two groups, eq. (6) will give the value of ΣA_0 with abundant precision, generally within, say, its fortieth part, by a computation requiring only a half an hour. Since the aberration-correction is in general under $0''.10$ the error in its determination by (6) introduced in (1) will be less than $0''.003$ in most cases. If greater precision is not deemed finical it may be had by subdividing the observations of each combination of groups and taking the mean of the parts. If the difference of $\cos\zeta$ for the various groups is considerable, eq. (5) may be used. Indeed, any needed modifications in the application of the method will suggest themselves to the computer in each case.

The practical process may be exemplified by Battermann's discussion of the aberration from the Berlin observations, 1892. For Berlin we have $n = 0.728$. From pp. 11–17 of the memoir we get α and $\cos\zeta$ below, and from p. 33 the average dates below, whence by eq. (6) $\Sigma A_0 = 4.22$; Battermann's rigorous value is 4.16, the error being thus only one and a half per cent.

Gr.	a °	cos ζ		Date 1892	⊙	T_1	T_2	A_0 Approx.	A_0 Precise
I	45	0.97	II–III	Feb. 23	335	122	173	0.52	0.502
II	97	.98	III–IV	Mar. 19	0	148	177	0.35	0.342
III	148	.96	IV–V	Apr. 13	24	153	186	0.41	0.364
IV	177	.98	V–VI	May 11	51	159	192	0.41	0.404
V	210	.99	VI–VII	May 28	68	175	202	0.33	0.344
VI	243	.98	VII–VIII	July 9	108	162	192	0.38	0.383
VII	270	.97	VIII–IX	July 27	125	175	207	0.39	0.410
VIII	300	.99	IX–I	Oct. 23	211	121	194	0.80	0.778
IX	332	0.99	I–II	Dec. 5	254	151	203	0.63	0.634
							$\Sigma A_0 =$	4.22	4.161

Similar illustrations, using eq. (5) or (6) are.

			ΣA_0 Approx.	ΣA_0 Precise
New York	1893–94	4 gr.	3.08	3.04
Philadelphia	1896–98	4 "	3.10	3.06
Leyden	1899–00	10 "	3.69	3.80

The errors in the deduced aberrations from using the approximate process here developed are, for Berlin 0″.000, for New York 0″.000, for Philadelphia 0″.002, for Leyden 0″.003. From these illustrations the labor involved in the process commonly used would appear to be inordinate and profitless.

By this compendious method I have found various values of the aberration-constant. Following is a description of how the material has been treated.

Kasan. The data are taken from the publications of 1894, 1896 and 1900. From pp. 11–14 of the first we find:

Group	a °	cos ζ	Group	a °	cos ζ
I	42	0.99	VI	241	0.98
II	97	1.00	VII	272	0.97
III	148	0.99	VIII	298	0.99
IV	177	0.99	IX	332	0.99
V	210	0.99			

For Kasan, $\cos\epsilon \sin q = 0.759$. For the observations of Kowalski, 1892.4–93.5, the values of v and their probable errors, given below, were obtained from the table on pp. 78–80. Those of

$$A = \cos\epsilon \sin q\,(\cos\zeta_1 \sin T_1 - \cos\zeta_2 \sin T_2)$$

by eq. (5), and their sum multiplied by $\frac{1}{2}(1+\sec\epsilon) = 1.042$ gives ΣA_0.

	A	v ″	″
V–VI	0.40	−0.345	±0.021
VI–VII	0.40	+0.879	0.024
VII–VIII	0.33	+0.049	0.040
VIII–IX	0.43	−0.026	0.025
IX–I	0.84	−1.034	0.052
I–II	0.56	+0.014	0.042
II–III	0.59	−0.238	0.026
III–IV	0.34	+0.200	0.020
IV–V	0.42	+0.078	±0.039
Sum	4.31	−0.423	±0.096

Hence we have

$$4.49\ \Delta k\ -0''.423 = 0\quad,\quad \text{whence}\ \Delta k = +0''.094$$

and $k = 20''.539\ \pm 0''.023$

Similarly, for the observations of Gratchew and Trozki, 1893.6–95.0, we find from the data on pp. 112–114 of Publ. II:

	A	v ″	″
VII–VIII	0.31	+0.050	±0.030
VIII–IX	0.43	−0.056	0.016
IX–I	0.80	−0.971	0.020
I–II	0.63	−0.020	0.027
II–III	0.56	−0.157	0.060
III–IV	0.35	+0.138	0.037
IV–V	0.42	−0.026	0.017
V–VI	0.10	−0.327	0.036
VI–VII	0.39	+1.023	±0.018
Sum	4.29	−0.346	±0.096

Whence $4.47\ \Delta k\ -0''.346 = 0\quad,\quad \Delta k = +0''.077$

and $k = 20''.522\ \pm 0''.022$

For the observations of Gratchew, 1895.1–97.5, we employ the data on pp. 137–141 of Publ. III.

	A	v ″	″
II–III	0.56	−0.194	±0.027
III–IV	0.28	+0.179	0.016
IV–V	0.40	+0.048	0.017
V–VI	0.42	−0.370	0.021
VI–VII	0.39	+0.934	0.014
VII–VIII	0.31	−0.077	0.036
VIII–IX	0.38	−0.022	0.018
IX–I	0.72	−1.024	0.026
I–II	0.48	−0.085	±0.039
Sum	3.94	−0.613	±0.076

Whence $4.11\ \Delta k\ -0''.613 = 0\quad,\quad \Delta k = +0''.149$

and $k = 20''.594\ \pm 0''.018$

Potsdam. The data used are the observations by Schnauder and Hecker, 1893.9–98.0. They have been treated in two ways. First, by employing the differences of the groups given on pp. 35–44 (Publ. of 1900) having the weight 3. From pp. 2–5 we find,

Group	a °	cos ζ	Group	a °	cos ζ
I	47	0.99	VI	243	0.98
II	91	0.98	VII	270	0.99
III	146	0.98	VIII	300	0.98
IV	179	0.98	IX	332	0.99
V	211	0.98	X	7	0.99

Thus, computing A and ΣA_0 in the same way as for Kasan, we have

	A	r	
I–II	0.39	−0.199	±0.024
II–III	0.55	+0.018	0.016
III–IV	0.37	+0.086	0.015
IV–V	0.38	+0.017	0.014
V–VI	0.39	0.035	0.013
VI–VII	0.33	+0.225	0.012
VII–VIII	0.37	−0.069	0.014
VIII–IX	0.37	+0.046	0.012
IX–X	0.33	−0.056	0.011
X–I	0.44	0.022	±0.020
Sum	3.92	+0.041	±0.050

$$4.09\ \Delta k + 0''.041 = 0 \quad , \quad \Delta k = -0''.010$$
$$k = 20''.491 \ \pm 0''.012$$

The second method, which appears to me to be more advantageous, was the following. Each of the groups contains six pairs. We may therefore make six entirely independent cyclical connections by taking the differences of the observed latitudes (pp. 8–18) on identical dates, combining the first pairs, the second pairs, and so on up to the sixth pairs, of the various groups. The values for the two observers were at first kept separate, but as there was no evidence of systematic personal difference, the mean value being

$$\text{Schnauder–Hecker} = +0''.022 \ \pm 0''.021$$

the combined results were employed, giving the following table:

Pairs	I–II		II–III		III–IV		IV–V		V–VI		VI–VII		VII–VIII		VIII–IX		IX–X		X–I	
	″						″		″		″		″		″		″		″	
1	−.022	25	+.178	19	−.109	27	−.266	27	−.260	34	+.618	34	+.165	39	−.420	36	+.051	30	−.124	23
2	−.107	23	−.584	18	+.528	27	+.134	31	−.075	36	+.291	37	−.204	35	−.154	34	+.034	31	+.343	23
3	−.335	27	+.094	20	+.214	23	+.419	28	−.218	22	+.370	38	−.378	40	−.155	34	+.340	31	+.014	25
4	.105	26	+.206	21	−.221	24	+.031	23	+.228	33	+.032	32	−.302	29	+.256	30	−.045	29	−.250	23
5	−.049	30	+.057	18	+.354	21	−.141	23	+.190	32	−.415	33	+.688	33	−.021	31	−.234	27	−.555	24
6	−.331	29	+.295	17	+.163	21	−.109	22	−.124	33	+.516	31	−.571	33	+.681	31	−.334	27	+.127	24

The results for IX–X and X–I were adjusted by least-squares, using the comparisons of the groups IX and I, as well as those of IX and X, and of X and I.

Taking the horizontal sums for each pair we find,

	Σr	Obs.
	″	
Pair 1 of each group	−0.189	294
" 2 " "	−0.094	295
" 3 " "	+0.332	298
" 4 " "	−0.170	260
" 5 " "	−0.126	272
" 6 " "	+0.310	269
Mean	+0.012	±0″.068

Each of these six cyclical sums is free from variation of latitude and from errors of declinations of the stars and of the assumed micrometer revolution, and involves merely the error in the assumed aberration-constant, which in this case was 20″.501. Therefore we have the equation

$$4.09\ \Delta k + 0''.012 = 0 \quad , \quad \Delta k = -0''.003$$
$$k = 20''.498 \ \pm 0''.016$$

a result which is I think to be preferred to the one given above.

Prague. The data are taken from LIEBLEIN's reduction of the observations of WEINEK and GRUSS, 1889.2–92.4. Employing only those observations (pp. 7–11, Publ. of 1897) in which at least six of the pairs of each group are measured, in order to be practically free from the errors of the reductions to the mean of the groups, we have the values of r below, and their probable errors. The values of the aberration-factors A were taken from the data on p. 14.

	A	r	
		″	″
VI–VII	0.37	+0.500	±0.024
VII–VIII	0.32	−0.221	0.032
VIII–IX	0.42	+0.093	0.037
IX–I	0.71	−0.133	0.033
I–II	0.41	−0.705	0.039
II–III	0.55	+0.414	0.034
III–IV	0.32	+0.219	0.025
IV–V	0.31	−0.106	0.046
V–VI	0.30	−0.280	±0.031
Sum	3.71	−0.219	±0.102

Whence $\quad 3.71\ k - 0''.219 = 0 \quad , \quad k = +0''.059$
$$k = 20''.504 \ \pm 0''.027$$

San Francisco. The details of this series of observations by DAVIDSON are given very fully in the *Report of the Coast and Geodetic Survey*, 1893, Part II. From the data, pp. 454–490, we find,

Group	a	$\cos\zeta$	Group	a	$\cos\zeta$
I	233	0.92	V	52	0.94
II	279	0.97	VI	99	0.93
III	324	0.93	VII	144	0.95
IV	9	0.91	VIII	190	0.86

The distribution of the pairs in this case being unusually wide in zenith-distance the aberration-factors were found by eq. (1). The values of r were derived from the tables on pp. 495–498.

	A	r	
		″	″
I–II	0.45	−0.022	±0.017
II–III	0.43	+0.043	0.015
III–IV	0.40	−0.258	0.015
IV–V	0.39	−0.002	0.023
V–VI	0.43	−0.022	0.017
VI–VII	0.42	−0.162	0.021
VII–VIII	0.34	+0.026	0.027
VIII–I	0.38	+0.039	±0.016
Sum	3.24	−0.356	±0.068

Whence $3.24\ \Delta k - 0''.356 = 0$, $\Delta k = +0''.110$
$k = 20''.555 \pm 0''.021$

This result (20″.555) seems to confirm my surmise in *A.J.* 517 that a correct solution would materially increase the result (20″.482) obtained in the Coast Survey discussion of this series.

Summarizing the previous results we have

Observatory	Date	Aberr'n	Prob. Error
		″	″
Kasan	1892.4–93.5	20.539	±0.023
"	1893.6–95.0	20.522	0.022
"	1895.1–97.5	20.594	0.018
Potsdam	1893.9–98.0	20.498	0.016
Prague	1889.2–92.4	20.504	0.027
San Francisco	1891.4–92.6	20.555	0.021

The weighted mean of these six values is 20″.532. This is but a few thousandths in excess of the general mean flowing from all the determinations, twenty-six in number, by TALCOTT's method, now available; or 20″.525. Further, a critical discussion of all the extant values obtained from declinations with the vertical or meridian circle that are entitled to be considered gives their general mean 20″.515; and a similar review of the prime-vertical values available yields 20″.528. It can be shown also that the existing right-ascension determinations that are competent may be harmonized with a value in this neighborhood. From the present aspect of affairs, therefore, I am hopefully of the opinion that we shall soon be within reach of decision upon a conventional value, generally acceptable; with the probability that it will rest somewhere between 20″.52 and 20″.54; or possibly, if it can be so closely defined, between 20″.525 and 20″.530.

ABERRATION-CONSTANT FROM POND'S OBSERVATIONS OF *POLARIS*, 1812-19,

BY S. C. CHANDLER.

An examination of Dr. AUWERS's reduction of POND's observations with the Troughton Mural Circle, 1812–19, leads me to think that it gives material for an investigation of the aberration, which I accordingly present here. The discussion has the incidental interest also that it appears to explain the cause of the discordance found by Dr. AUWERS (p. 56) between the spring and fall observations of *Polaris*, for which he has applied empirical corrections.

The differences between the circle-readings for upper and lower culminations of *Polaris*, after reduction for refraction and to the epoch 1815.0, are given on pp. 54, 55. These differences, although of course unaffected by variation of latitude, necessarily involve twice the effect of the error of STRUVE's constant of aberration. To find the correction to this assumed constant, according to these observations, call $2P$ the observed values of double the polar distance, given on pp. 54, 55. Take an approximate value, $2P_0 = 3°\ 21'\ 24''.30$, assumed for convenience so as to make all the residuals positive, and put $n = 2P - 2P_0$. Solving the equations of condition of the form

$$(1)\qquad 2\,\Delta P + Y \sin\odot + Z\cos\odot = n$$

the correction to the aberration-constant, and the star's parallax, will be given by

$$(2)\qquad u = \frac{qY + pZ}{2(p^2+q^2)} \qquad \pi = \frac{qZ - pY}{2(p^2+q^2)}$$

where

$$p = \sin\epsilon\cos\delta - \cos\epsilon\sin\alpha\sin\delta \quad , \quad q = \cos\alpha\sin\delta$$

(see *A.J.*, Vol. XII, p. 177). For *Polaris* (1815) we have

$$(3)\qquad p = -0.209\ ,\ q = +0.970\ ,\ 2(p^2+q^2) = 1.969$$

The following table gives the values of n and their weights for the five sets of observations for which $2\,\Delta P$ in eq. (1) must be separately found

Circle 0° ; Six Microscopes.

Date	n	w
	″	
1812 Aug. 15	+0.90	2
Oct. 18	1.53	5
1813 June 12	1.33	11
Oct. 18	1.01	4
Nov. 4	1.43	4
Nov. 23	1.12	5
1819 Sept. 29	1.24	4
Oct. 18	2.47	1
Dec. 2	+0.65	3

Circle 20° ; Six Microscopes.

Date	n	w
	″	
1812 Dec. 8	+0.70	6
1816 Nov. 20	0.49	6
1818 Apr. 17	1.86	4
May 17	1.42	5
June 11	1.36	6
Nov. 19	+1.68	6

Circle 0° ; Microscopes AB.

Date	n	w
	″	
1812 Sept. 4	+0.75	1
Oct. 4	0.87	1
1814 Apr. 6	0.35	1
Apr. 30	1.04	5
May 30	0.77	8
Nov. 11	+0.08	5

Circle 10° ; Microscopes AB.

Date	n	w
	″	
1815 Apr. 17	+2.48	1
May 7	2.48	5
June 5	3.11	6
June 29	3.15	1
Sept. 19	4.16	1
Oct. 1	1.62	3
Oct. 23	2.45	5
Nov. 18	1.91	3
Dec. 12	+2.27	4

Circle 20° ; Microscopes AB.

Date	n	w
	″	
1816 Apr. 25	+2.00	3
May 16	1.65	2
June 10	1.52	4
July 24	1.61	1
Sept. 26	2.95	2
Oct. 20	1.92	3
Nov. 13	1.44	1
1817 Apr. 20	1.35	2
May 5	1.90	2
May 15	1.36	3
June 16	0.62	1
Nov. 12	1.07	1
Nov. 21	0.59	1
1818 Oct. 24	1.36	1
1819 May 6	1.52	8
June 10	1.58	6
Nov. 25	+0.60	1

NORMAL EQUATIONS. — ELIMINATION-EQUATIONS (2 IP eliminated).

Circle 0° : Six Microscopes.

$$\begin{array}{rrrrr} 39\,(2\,IP) & -\;3.07\,Y & -18.98\,Z & = & +18''.82 \\ & +22.10 & +10.28 & & -\;2.35 \\ & & +17.29 & & -24.15 \end{array} \qquad \begin{array}{rrrr} +21.86\,Y & +\;8.79\,Z & = & +1''.49 \\ & +\;8.05 & & -0.39 \end{array}$$

Circle 20° : Six Microscopes.

$$\begin{array}{rrrrr} 33\,(2\,IP) & -\;1.12\,Y & -\;0.05\,Z & = & +39.92 \\ & +24.29 & +11.55 & & +\;1.72 \\ & & +\;8.65 & & +\;1.15 \end{array} \qquad \begin{array}{rrrr} +23.70\,Y & +11.54\,Z & = & +7.07 \\ & +\;8.65 & & +4.51 \end{array}$$

Circle 0° : Microscopes AB

$$\begin{array}{rrrrr} 24\,(2\,IP) & +\;7.84\,Y & +\;5.66\,Z & = & +14.78 \\ & +12.15 & +\;8.59 & & +\;9.17 \\ & & +11.85 & & +\;5.89 \end{array} \qquad \begin{array}{rrrr} +\;9.59\,Y & +\;6.74\,Z & = & +1.34 \\ & +\;9.53 & & +2.10 \end{array}$$

Circle 10° : Microscopes AB.

$$\begin{array}{rrrrr} 29\,(2\,IP) & +\;1.12\,Y & -\;1.43\,Z & = & +73.77 \\ & +16.70 & +\;9.02 & & +\;9.83 \\ & & +12.30 & & -\;7.92 \end{array} \qquad \begin{array}{rrrr} +16.66\,Y & +\;9.19\,Z & = & +6.98 \\ & +11.62 & & +3.35 \end{array}$$

Circle 20° : Microscopes AB.

$$\begin{array}{rrrrr} 15\,(2\,IP) & +23.08\,Y & +\;8.58\,Z & = & +68.37 \\ & +27.12 & +14.75 & & +34.46 \\ & & +17.54 & & +11.51 \end{array} \qquad \begin{array}{rrrr} +15.59\,Y & +10.35\,Z & = & -0.60 \\ & +15.90 & & -1.53 \end{array}$$

Examination shows that the probable error for the unit of weight is sensibly the same for the sets with six and with two microscopes. From the sum of the elimination-equations we get

$$Y = +0''.256 \;\pm 0''.098 \quad , \quad Z = -0''.067 \;\pm 0''.112$$

whence by eq. (2) and (3)

$$\begin{array}{lcl} \text{Aberration} & = & 20''.578 \;\pm 0''.013 \\ \text{Parallax} & = & -0''.006 \;\pm 0''.066 \end{array}$$

Substituting these values in the normals in 2 IP, and adding the constant 3° 21′ 21″.30 we find

	$2P$ computed
Circle 0°; six microscopes;	3° 21′ 25″.50
" 20°; " "	25.54
" 0°; microscopes AB:	24.85
" 10°; " "	26.84
" 20°; " "	25.70

The values of aberration and parallax given by the independent solutions of the five sets are

	Aberration	Parallax
	″	″
Circle 0°; six microscopes;	20.51	−0.09
" 20°; " "	20.48	+0.18
" 0°; microscopes AB	20.72	−0.01
" 10°; " "	20.68	+0.01
" 20°; " "	20.48	−0.06

Thus all five of the sets are in accord in indicating a considerable positive correction to STRUVE's constant. The mean result of the general solution given above, 20″.578, is also in harmony, considering its probable error, with the general mean which I have obtained from an unpublished discussion of all the extant determinations of the aberration-constant suitable for use (more than forty in number) showing the true value of this constant to be very close to 20″.53.

If we subtract the above computed values of $2P$ from the observed individual values and take the weighted means by months we find the quantities in column O below. By their side I have placed the values C corresponding to the assumptions 20″.58 and 20″.53 for the aberration-constant; and in the last column the empirical corrections applied by AUWERS to adjust for this discordance.

	Weight	O	C (Ab.20″.58)	C (Ab.20″.53)	AUWERS'S Empir.Corr.
		″	″	″	″
April	19	+0.30	+0.06	+0.04	+0.20
May	33	+0.14	+0.18	+0.11	+0.20
June	38	+0.11	+0.26	+0.16	+0.20
July–Sept.	11	+0.42	+0.20	+0.12	0.00
October	23	+0.03	−0.05	−0.03	−0.20
November	33	−0.23	−0.25	−0.16	−0.20
December	13	−0.47	−0.26	−0.16	−0.20

From this it would appear that the source of the discordance in question is established, with a reasonable degree of probability, as the effect of the erroneous aberration used in the reductions.

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THE TERMS OF PRECESSION AND NUTATION.

By IRA STERNER.

I have completely investigated the subject of precession and nutation, first in the memoirs, and then independently of all previous developments of the subject. I thoroughly revised the memoirs of POISSON* and SERRET,† *Sur le Mouvement de la Terre autour de son centre de gravité*, and found that they satisfactorily obtain the fundamental differential equations of motion (E), on which are based the computations of PETERS‡ and OPPOLZER.§ The differential equations (E) involve a perturbative function V, one part of which contains the rotatory angle φ in every term: this part‖ does not lead to any sensible terms of nutation, and therefore can be omitted in the computations. All those terms in V which are independent of φ, will be called V_1-terms. POISSON and SERRET developed V_1 only partly; but PETERS and OPPOLZER developed V_1 in a fairly accurate manner.

I have independently developed all V_1-terms with entire accuracy. Substituting the accurate V_1-development in the differential equations (E), I put these equations in the following form:

$$\frac{1}{\kappa}\frac{d\psi}{dt} = \epsilon F_1(I,\ \omega_0+i,\ R'^3\cos\alpha,\ R'^3\cos(2\mathbf{v}'+\alpha)) + F_1(o,\ \omega_0+i,\ R^3\cos\alpha,\ R^3\cos(2\mathbf{v}+\alpha))$$

$$\frac{1}{\kappa}\frac{d\omega}{dt} = \epsilon F_2(I,\ \omega_0+i,\ R'^3\sin\alpha,\ R'^3\sin(2\mathbf{v}'+\alpha)) + F_2(o,\ \omega_0+i,\ R^3\sin\alpha,\ R^3\sin(2\mathbf{v}+\alpha))$$

The notation just used has the following meaning: $\kappa\epsilon$ and κ are fundamental constants to be computed for a particular epoch $t = o$; ψ and ω are respectively the precession of longitude and the obliquity; F_1 and F_2 are known functions of four expressions, as indicated; I is the actual inclination of the moon's orbit to the ecliptic at the time t: ω_0 is the obliquity of the ecliptic at the epoch $t = o$, and i is the secular inequality of ω; R' and R are respectively the ratios $\frac{\rho'_0}{\rho'}$ and $\frac{\rho_0}{\rho}$ where ρ_0 and ρ'_0 are the semi-major axes of the orbits of the sun and moon, and ρ and ρ' are the distances ES and EM respectively at the time t [E, S, M being respectively the centers of gravity of earth, sun and moon]; α denotes various angles, involving ψ and the longitude of the moon's ascending node: $\mathbf{v}$ and $\mathbf{v}'$ are respectively the actual longitudes of the sun and moon at the time t.

In case of the *sun's* action, we require merely the elliptic developments:

$$R^3 {}^{\cos}_{\sin}\,\alpha = f_1(A,\ e,\ \alpha,\ {}^{\cos}_{\sin}),$$
$$R^3 {}^{\cos}_{\sin}\,(2\mathbf{v}+\alpha) = f_2(A,\ e,\ 2\varpi+\alpha,\ {}^{\cos}_{\sin}),$$

where A, e, ϖ are respectively the *solar* mean anomaly, eccentricity and longitude of perihelion. I developed the functions f_1 and f_2 so as to include all terms of the first five orders in e, and substituted them in F_1 and F_2. Then the terms of precession due to the action of the *sun* are as follows:

$$\psi(S) = \kappa\int F_1\{o, \omega_0+i,\ f_1(A,e,\alpha,\cos),\ f_2(A,e,2\varpi+\alpha,\cos)\}dt$$
$$\omega(S) = \kappa\int F_2\{o, \omega_0+i,\ f_1(A,e,\alpha,\sin),\ f_2(A,e,2\varpi+\alpha,\sin)\}dt$$

In case of the *moon's* action, I developed the following series, for the actual motion of the moon:

$$I = f_3(A', A, D, J), \qquad R'^3 {}^{\cos}_{\sin}\,\alpha = f_4(A', A, D, J, \alpha, {}^{\cos}_{\sin}),$$
$$R'^3 {}^{\cos}_{\sin}\,(2\mathbf{v}'+\alpha) = f_5(A', A, D, J, 2\frac{1}{\varpi}+\alpha, {}^{\cos}_{\sin}),$$

where A', D, J, $\frac{1}{\varpi}$ are respectively the *moon's* mean anomaly, mean elongation, mean longitude *from* the ascending node,

* *Mémoires de l'Institut de la France*, T. 7 and 9 (1827–9).

† *Annales de l'Observatoire de Paris*, tome 5 (1859): *Mems. de l'Inst.*, T. 35.

‡ *Numerus Constans Nutationis* (1841); *Mém. de l'Acad. des Sciences de St. Petersbourg* (6, V, 1).

§ *Lehrbuch zur Bahnbestimmung*, 2d ed., Leipzig, 1882.

‖ *Tisserand, Mécanique Céleste*, tome 2, part 2, chapter on development of V.

and longitude of perigee. I computed, and verified twice, these trigonometrical series for f_2, f_4 and f_5, including all terms whose coefficients exceed ″.000001: for this purpose, I used the best available tables* of the moon's actual latitude, sine parallax and actual longitude. Then the terms of precession due to the *moon* are:

$$\psi(M) = \kappa\epsilon_1 \int F_1 \Big\{ f_2(A', A, D, \Gamma)\,\omega_0 + i_1 f_4(A', A, D, \Gamma, \alpha_1 \cos),\ f_5(A', A, D, \Gamma, 2\frac{1}{\varpi} + \alpha_1 \cos) \Big\}\, dt$$

$$\omega(M) = \kappa\epsilon_1 \int F \Big\{ f_3(A', A, D, \Gamma)\,\omega_0 + i_1 f_4(A', A, D, \Gamma, \alpha_1 \sin),\ f_5(A', A, D, \Gamma, 2\frac{1}{\varpi} + \alpha_1 \sin) \Big\}\, dt$$

In computing the fundamental constants $\kappa\epsilon$ and κ, I used NEWCOMB's constants of precession† and nutation‡ for the epoch 1900, and inserted literal correction-ratios for accuracy:

$$\sigma \equiv \frac{N(1900) - 9.214}{9.214} \qquad \eta \equiv \frac{P_1(1900) - 50.2564}{50.2564}$$

For the values of ω_0, i, A, e, ϖ, and L, I used NEWCOMB's secular inequalities of the sun for the epoch 1900.‡ For the values of A', L', ☊, and $\frac{1}{\varpi}$, I used LEVERRIER's§ secular inequalities of the moon, which are accurate enough for use here.

Completing the integrations, I obtained ψ and ω for the *epoch 1900*:

$$\psi = \psi(1900) + (50''.3639 + P_1\eta)\,t + ''.000\,108\,9\,(1+\eta)\,t^2 + \Psi(M)\,(1+\sigma) + \Psi(S)\,(1 + 3.158\,\eta - 2.175\,\bar{\sigma})$$

$$\omega = 23°\,27'\,8''.26 - ''.4685\,t - ''.000\,000\,59\,t^2 + \Omega(M)\,(1+\sigma) + \Omega(S)\,(1 + 3.158\,\eta - 2.175\,\bar{\sigma})$$

I computed all terms of nutation (Ψ and Ω) whose coefficients exceed ″.00004: the more important terms are given in the comparative table that follows. The second column contains the coefficients used by Professor NEWCOMB.‡ The third column contains my newly computed coefficients, which were verified twice. In the arguments, A' and A are the mean anomalies of the moon and sun, L' and L the mean longitudes, $D \equiv L' - L$, and ☊ is the longitude of the moon's ascending node.

NUTATION IN LONGITUDE (sines).

I Args.	II P. and O.	III Sterner	IV Diffs.
		″	″
☊	$-(17.234 + 0.0177T)$	$-(17.2398 + 0.0174\,T + 0.0083\,T)$	-0.006 $-0.008\,T$
2☊	$+0.209$ (elliptic)	$+(0.3109 - 0.0001\,T)$	$+0.102$
$2L'$	-0.204	$-(0.2050 - 0.0001\,T)$	-0.001
A'	$+0.067$	$+0.0679$	$+0.001$
$2L' - ☊$	-0.031	-0.0343	.
$2L' + A'$	-0.026	-0.0262	. .
$2D - A'$	$+0.015$	$+0.0150$	. .
$2L - A'$	$+0.012$	$+0.0115$	. .
$2L - ☊$	$+0.012$	$+0.0122$	. .
$2D$	$+0.006$	$+0.0061$	. .
$A' + ☊$	. . .	$+0.0058$	. .
$A' - ☊$	. . .	$+0.0057$	. .
$2D + 2L' - A'$	. . .	0.0052	. .
$2A' - 2D$	. . .	$+0.0045$	. .
$2L' + A' - ☊$	. . .	-0.0014	. .
$2L$	-1.257	$-(1.2571 + 0.0006\tfrac{1}{2}\,T)$	. . $-0.0006\,T$
A	$+0.127$	$+(0.1243 - 0.0003\,T)$	-0.003
$2L + A$	-0.019	$-(0.0192 - 0.0001\,T)$	. .
$2L - A$	$+0.021$	$+0.0211$	. .

NUTATION IN OBLIQUITY (cosines).

I Args.	II P. and O.	III Sterner	IV Diffs.
	″	″	″
☊	$+9.214$ (Newcomb)	$+(9.214(1+\bar{\sigma}) + 0.0018\,T)$	$9.214\,\sigma$ $+0.002\,T$
2☊	-0.090 (elliptic)	$-(0.1350 - 0.0001\,T)$	-0.045
$2L'$	$+0.088$	$+0.0890$	$+0.001$
$2L' - ☊$	$+0.018$	$+0.0183$	. .
$2L' + A'$	$+0.011$	$+0.0115$	. .
$2L' - A'$	-0.005	-0.0050	. .
$2L - ☊$	-0.007	-0.0066	. .
$A' + ☊$	. .	-0.0031	. .
$A' - ☊$	. .	$+0.0031$	. .
. .	. .	$+0.0023$	. .
. .	. .	$+0.0024$	. .
$2L$	$+0.546$	$+(0.5457 - 0.0006\,T)$	. . $+0.0006\,T$
$2L + A$	$+0.021$	$+(0.0213 - 0.0001\,T)$	. .
$2L - A$	-0.009	-0.0091	. .

* NEWCOMB and MEIER, Revision of HANSEN's Tables, *Astron. Papers, Amer. Ephem.*, Vol. I, pt. 2.

† The Precessional Motion, 1896: *Astron. Journal*, No. 405.

‡ Tables of the Sun, 1895, *Astron. Papers, Amer. Ephem.*, Vol. 6, pt. 1.

§ *Annales de l'Observatoire de Paris*, tome 2, 1856.

I carried all my series much farther than PETERS and OPPOLZER, and retained all terms of series with coefficients exceeding $''.000001$: this accounts for the differences in the fourth column, which I have verified twice. The values of the coefficients of $\sin 2\Omega$ and $\cos 2\Omega$ which PETERS and OPPOLZER had obtained, are precisely my *elliptic* values of those coefficients. These two coeffiients have large *oscillatory corrections*, caused by the moon's actual deviations from an ellipse. These two corrections result from integrating terms which are relatively small; but their integrating factor is much smaller than that of most other terms: hence the coefficients of these terms become relatively large after integration. Consequently the values of the coefficients of $\sin 2\Omega$ and $\cos 2\Omega$ obtained by PETERS and OPPOLZER, being merely elliptic values, are not near as large as the actual values which I have computed. I guarded against omissions of this kind, by computing all terms of nutation whose coefficients exceed $''.00004$.

Keller's Church, Pa., 1902 April 28.

CONCERNING THE MAGNITUDE-EQUATION FOR THE CAMBRIDGE ZONES (SEE *A. J.*, Nos. 517 AND 519),

BY LEWIS BOSS.

To my article (*A.J.* 517) upon magnitude-equations derived from comparisons with the Oxford photographic survey Professor TURNER offers objections (*A.J.* 519) which seem to call for further attention.

The point which seems to require particular explanation is that relative to the treatment of proper motions. This matter seems to me so simple and elementary that I should hesitate to enter upon a further explanation but for the fact that an astronomer of Professor TURNER's recognized acquirements has thrown doubts upon my conclusions. For a statement of our respective views I must refer to the articles already cited.

I readily accept Professor TURNER's suggestion that, in comparing his photographic plates, he can do this without adopting any special values of the precessional motion. It is a fact, that, in preparing my former article, I took this identical point of view; but in writing the explanation I fitted it to the processes which Professor TURNER actually employed, because I thought it would be more easily understood in that form.

We have a picture of a portion of the sky as it appeared in 1895. We wish to compare this with measured star-positions obtained at Cambridge in 1880. We have nothing but star-images on the plate. For the purposes of comparison we may either construct a picture from the Cambridge observations which can be compared with the Oxford picture; or we may measure the Oxford picture, referring each star to coordinate axes comparable with those of the Cambridge zones. Let us take the latter course, and let us suppose that the orientation of the plate can be determined on the assumption that the picture taken in 1895 is a sufficient approximation to the picture which might have been taken in 1880. This is by no means the case; but let us suppose, for the sake of argument, that no material error would be introduced by this treatment when the results are to be used for the purpose of ascertaining the magnitude-equation for Cambridge right-ascensions. Regarding our Oxford photographs, now, as sufficiently representative of the corresponding portion of the sky for 1880, we proceed to draw upon the plates meridians of right-ascension and parallels of declination by means of the Cambridge zone-positions reduced to their mean epoch, 1880. We may now determine, by measurements upon the plates, the right-ascension of each star. For stars very near the ninth magnitude we shall evidently get substantial agreement with the Cambridge right-ascensions, owing to the preponderating influence of the fainter stars in determining the axes of reference upon the plates. This is very clearly the actual case, as shown in Professor TURNER's table (p. 4, *M.N.*, LX). So far we have been quite indifferent as to what may be the true precessional motion.

But Professor TURNER seems to have decided that the positions of the brighter stars have been so influenced by proper motion in the interval, 1880–1895, that our Oxford picture no longer represents for these stars the state of the sky in 1880. He therefore corrects the stars of his first three groups (first to fifth magnitude) for the effect of proper motion.

So far there appears to be no vital divergence of view between Professor TURNER and me.

But in correcting these three groups of stars for proper motion, Professor TURNER employs the proper motions determined by Dr. AUWERS. Here is our point of divergence.

I assume that Professor NEWCOMB's values of annual variation are more nearly standard than are those of Dr. AUWERS. Consequently I correct the latter by $+0^s.0008$, annually, or by $+0^s.012$ to get the effect for 15 years. But in separating the proper motion from the annual variation it becomes evidently and vitally necessary to know the amount of precessional motion. If I assume that STRUVE's precessional motion for 15 years is too large by

$$+0^s.012 + 0^s.0026 \sin \alpha$$

(at δ, $+27^\circ.5$) then NEWCOMB's proper motions for that

interval are too small by this same amount. If this be granted then the corresponding proper motions for 15 years according to Dr. AUWERS are systematically too small, and they require the correction:

$$+0^s.012 + 0^s.012 + 0^s.0026 \sin \alpha$$

precisely the result arrived at in my former paper (*A.J.* 517). If my premises be granted I do not see how there can be any escape from this conclusion.

Therefore in the comparison, Oxford–Cambr., the right-ascensions of the Cambridge zones for bright stars are relatively too small by $0^s.024$ as compared with the corresponding right-ascensions of stars of the ninth magnitude. Hence the numbers in the first three groups of Professor TURNER's table (*M.N.*, LX, p. 4) must be corrected by $-0^s.024$ as in my former paper.

I owe Professor TURNER an apology for misapprehending the extent to which he had made use of proper motions in his reductions. My interpretation of his remark was evidently based on a false assumption. Accordingly, I have since corrected his groups for mean magnitudes, 5.2 and 5.7, for the proper motions of the Greenwich Catalogue for 1880, and then to what I conceive to be systematically nearly correct proper motions. Combining the two groups, $5^M.0$ to $5^M.9$, I find the following mean motions for 15 years:

MEAN P.M. FOR 15 YEARS ($5^M.0$ to $5^M.9$).

R.A.	**	Gr. p.m.	Corr'd p.m.
2^h	10	$-0^s.021$	$+0^s.004$
6	13	-0.030	-0.003
10	6	-0.054	-0.029
14	12	-0.021	$+0.002$
18	9	-0.015	$+0.006$
22	10	$+0.002$	$+0.025$
Means		-0.021	$+0.003$

The numbers in the last column of the foregoing table, resulting from corrected residual proper motions illustrate the necessity of the correction $+0^s.0016$ annually ($+0^s.024$ for 15 years) required in order to reduce the proper motions adopted by Dr. AUWERS to those which seem to me more probable. It will be observed that the mean is a vanishing quantity, and that the solar motion is very clearly indicated in the outstanding residuals.

Accordingly we have as the corrected result for Oxford–Cambridge:

**	Mean Magn.	Oxfd.–Camb.
20	$3^M.6$	$+0^s.120$
68	5.5	$+0.119$

Taking the separate groups of Professor TURNER I find:

**	Mean Magn.	Oxfd.–Camb.
4	$2^M.0$	$+0^s.094$
7	3.5	$+0.089$
9	4.5	$+0.156$
38	5.2	$+0.121$
30	5.7	$+0.117$

Thus from the first to the sixth magnitude there is no evidence of a material difference between the magnitude-equations of the Cambridge and Oxford right-ascensions.

If we take the 10 proper motions of the Greenwich Catalogue belonging to Professor TURNER's group, $6^M.0$ to $6^M.4$, we shall have as the correction to his, Oxford–Cambridge, for those stars, $+0^s.088$; likewise for the 11 stars of group, $6^M.5$ to $6^M.9$, a correction amounting to $+0^s.035$. If we suppose the residual effect of neglected proper motion for the remaining stars in each group to be zero, we shall have as corrections to Professor TURNER's group numbers respectively, $+0^s.010$ and $+0^s.003$. Then we shall have:

**	Mean Magn.	Oxford–Camb.
88	$6^M.0$ to $6^M.4$	$+0^s.114$
147	6.5 to 6.9	$+0.096$

This still further sustains the hypothesis that the magnitude effect for the Cambridge zones is almost wholly confined to the telescopic stars.

It should not be overlooked that, in order to reduce our points of difference, in the foregoing remarks I have waived special consideration of the fundamental uncertainties which underlie researches in which it is assumed that the residual effect of a large number of proper motions may be assumed to be zero in relation to a small quantity. For a brief discussion of this point I must refer to my former papers. A fairly complete catalogue of these futile works would, perhaps, surprise those who have not been observant in this matter.

It is well understood that the zone observations of LALANDE, BESSEL and ARGELANDER are now of very little value for any precise investigation, not so much on account of the errors of individual star-positions, as on account of the irregular nature of the systematic corrections required for the individual zones. Unless proper precautions are taken the work of some parts of the great photographic survey now going on will suffer for a like reason. Meridian-observers should see to it that the positions of several stars for each plate be determined at once. Furthermore, such meridian-observers should use special precautions to insure that the systematic corrections of their own observations

can be hereafter determined and applied. This can be done by observing large numbers of stars of the fourth to sixth magnitudes (say) in the vicinity of the zones in which they are working. Even with all precautions, since the plates will be in the tertiary scale at best, it is to be feared, from nearly every point of view as to accuracy, they will fall very materially below the best meridian observations.

OBSERVED MINIMA OF VARIABLE STARS OF THE *ALGOL*-TYPE, OCTOBER, 1901, TO APRIL, 1902,

By PAUL S. YENDELL.

Observations of the minimum phases of the *Algol*-type stars have been much hindered by unfavorable weather during the past six months.

Although I have availed myself of every opportunity that has offered from October, 1901 to April, 1902, during the hours when it is possible for me to observe, the following nine minima of three stars are all that I have succeeded in securing.

The reductions have been made in the same manner as those of other minima that I have published, excepting that in the case of the single minimum of *U Cephei*, as in those published in No. 512 of this *Journal*, and for the reason there stated, no mean curve reduction is given.

320 *U Cephei*.

One minimum.

1902 April 2, nineteen observations, from $7^h 33^m$ to $12^h 7^m$, Local Mean Time.

Time of minimum by single curve, $9^h 37^m$, wt. 1.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
8.6	7 42	11 24	9 33
8.8	8 2	10 59	9 30.5
9.0	8 20	10 52	9 36
		Mean	9 33.2

Least observed light, $9^M.22$.

1090 *Algol*.

Five minima, as follows:

1901 October 8, nineteen observations, from $6^h 37^m$ to $11^h 22^m$.

Time of minimum by single curve, $10^h 2^m$, wt. 3.

Time of minimum by mean curve, $10^h 15^m$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
3.4	9 37	11 25	10 31
3.6	9 45	10 35	10 10
		Mean	10 20.5

Least observed light, $3^M.73$.

1901 November 20, seventeen observations, from $6^h 15^m$ to $12^h 15^m$.

Time of minimum by single curve, $10^h 32^m$, wt. 4.

Time of minimum by mean curve, $10^h 22^m$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
3.4	9 40	11 15	10 27.5
3.6	10 5	10 56	10 30.5
		Mean	10 29.0

Least observed light, $3^M.69$.

1902 January 5, ten observations, from $6^h 2^m$ to $9^h 27^m$.

Time of minimum by single curve, $7^h 37^m$, wt. 5.

Time of minimum by mean curve, $7^h 33^m$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
3.2	6 6	9 22	7 44
3.4	6 12	8 15	7 43.5
3.6	6 59	8 20	7 39.5
3.8	7 22	7 52	7 37
		Mean	7 41.0

Least observed light, $3^M.96$.

1902 January 25, nineteen observations, from $6^h 55^m$ to $12^h 15^m$.

Time of minimum by single curve, $10^h 7^m$, wt. 5.

Time of minimum by mean curve, $9^h 43^m$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
2.8	7 12	12 15	9 43.5
3.0	7 50	11 25	9 37.5
3.2	8 22	11 13	9 17.5
3.4	9 8	10 51	9 29.5
3.6	9 34	10 37	10 5.5
		Mean	9 45.7

Least observed light, $3^M.71$.

1902 February 14, twelve observations, from $7^h 10^m$ to $11^h 58^m$.

Time of minimum by single curve, $10^h 42^m$, wt. 3.

Time of minimum by mean curve, $10^h 30^m$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
3.8	10 29	10 57	10 43

Least observed light, $3^M.88$.

2640 *R Canis Majoris.*

Three minima.

1902 January 13, nineteen observations, from $7^h\,50^m$ to $11^h\,40^m$.

Time of minimum by single curve, $10^h\,18^m$, wt. 1.

Time of minimum by mean curve, $10^h\,28^m$.

Time of minimum by equal brightness,

	Before	After	Mean
M	h m	h m	h m
6.3	9 18	11 23	10 20.5

Least observed light, $6^M.42$.

1902 January 30, sixteen observations, from $7^h\,25^m$ to $13^h\,0^m$.

Time of minimum by single curve, $11^h\,15^m$, wt. 5.

Time of minimum by mean curve, $11^h\,26^m$.

Time of minimum by equal brightness,

	Before	After	Mean
M	h m	h m	h m
6.4	10 9	12 47	11 28
6.6	10 25	12 18	11 21.5
6.8	10 50	12 3	11 26.5
7.0	10 58	11 30	11 14
		Mean	11 22.5

Least observed light, $7^M.13$.

1902 February 7, $7^h\,22^m$ to $11^h\,15^m$, sixteen observations.

Time of minimum by single curve, $10^h\,22^m$, wt. 5.

Time of minimum by mean curve, $10^h\,19^m$.

Time of minimum by equal brightness,

	Before	After	Mean
M	h m	h m	h m
6.3	8 19	11 39	10 14
6.4	9 0	11 23	10 11.5
6.5	9 29	11 2	10 15.5
6.6	9 45	10 39	10 12.0
6.7	9 59	10 31	10 16.5
		Mean	10 13.9

Least observed light, $6^M.76$.

The minimum of *R Canis Majoris* on January 13 is anomalous, the star decreasing only to $6^M.4$, at which brightness it remained nearly two hours.

This is not unexampled, as I have several other similar cases. In the present instance, every precaution was taken to make sure that the anomaly was not subjective, and I am fairly certain that such was not the case.

I made especial efforts to secure as many minima of the star as possible with a view to a recurrence of the anomaly, but so many of the available minima fell upon cloudy or moonlight evenings that only two later minima were secured, both of which, as will be seen, were perfectly normal.

Dorchester, Mass., 1902 *April* 26.

LETTER TO THE EDITOR REGARDING ASTRONOMICAL EPHEMERIDES,

BY A. HALL.

The managers of the *Connaissance des Temps* have taken a step which might well have been taken twenty years ago: that is, they will omit the lunar distances after 1905. With the great extension of accurate determinations of longitude, by means of telegraphic lines, the need of lunar distances has become small, and they are rarely used. If a case should occur it could be computed directly by some of the simple formulas such as BREMIKER'S. Such a change in the *American Ephemeris* would save yearly seventy-two pages of printed matter.

Again, what is the use of so much space being given to the small stars occulted by the moon. Probably one-fifth of this space is sufficient to give all the data really needed for the bright stars and the groups.

Although time will no doubt show defects in the theories of the planets, our present tables appear to be accurate and well established. This is not the case with the secondary systems. The twenty satellites of the outer planets need new orbits and tables, made with a uniform method and notation. It is time that the Saturnian system should be investigated as a whole, and not by parts. The great telescopes can now furnish good observations, and it is to be hoped that observers may unite on one plan. On account of the ring and the figure of the planet, and its brightness, *Titan* is a good center to which the observations can be referred.

Professor H. STRUVE has begun a good work on this grand system, and his investigations should be continued. The figure of the planet, the attraction of the ring, the action of the satellites on each other, the curious motions of *Hyperion*, and the action of the sun on the satellites, ought to be discussed in a general manner. This is a work that might well be undertaken by the Office of the *American Ephemeris*. The theories of *Jupiter's* satellites also need revision and new tables.

In the future, stellar astronomy will demand more attention. Besides the proper motions of the stars and the motion of the sun in space, which are being slowly disclosed, it would be well, I think, if the rapid double stars, the variable stars, and stars with peculiar and interesting motions could be noticed in a way that would direct the observers. They would need but a small space in the *Ephemeris*.

Goshen, Conn., 1902 *May* 7.

ON TWO THEOREMS CONCERNING THE METHOD OF LEAST-SQUARES,

By J. NIDZUHARA.

Mr. HAROLD JACOBY has demonstrated the following theorem in the *Astronomical Journal*, Vol. XXII, page 84:

"If there be given two series of observation-equations as follows:

$$(1)\quad \begin{cases} a_1x+b_1y+c_1z+\;.\;.\;.\;+n_1=0 \\ a_2x+b_2y+c_2z+\;.\;.\;.\;+n_2=0 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \end{cases}$$

$$(2)\quad \begin{cases} a_1x+b_1y+c_1z+\;.\;.\;.\;+p_1w+\;.\;.\;.\;+n_1=0 \\ a_2x+b_2y+c_2z+\;.\;.\;.\;+p_2w+\;.\;.\;.\;+n_2=0 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \end{cases}$$

then, no matter what may be the law of the coefficients $p_1, p_2, \ldots$ and even if these coefficients are assigned at random, $(vv)_1$, the sum of the squares of the residuals resulting from the least-square solution of (1) is always larger than $(vv)_2$, that from the solution of (2)."

By the light of this theorem we may also demonstrate the following theorem:

Let $px_1, py_1, pz_1, \ldots$ be the weights of the values of $x, y, z, \ldots$ found from the normal equations of (1) and $px_2, py_2, pz_2, \ldots$ be those from the normal equations of (2), then $px_1, py_1, pz_1, \ldots$ are always larger than $px_2, py_2, pz_2, \ldots$ respectively.

Demonstration. — Let there be given two series of observation-equations which are identical with those of the equations of (1) and (2), except that the former constant terms satisfy the following conditions:

$$(an') = -1$$
$$(bn') = (cn') = (dn') = \ldots = 0$$

then, substituting their numbers in the following general formula,

$$(vv) = (nn) + (an)x + (bn)y + (cn)z + \ldots$$

we have

$$(v'v')_1 = (n'n') - x_1' \quad , \quad (v'v')_2 = (n'n') - x_2'$$

But by above theorem of Mr. JACOBY we must have

$$(v'v')_1 > (v'v')_2$$

or

$$(v'v') - x_1' > (n'n') - x_2'$$

or

$$\frac{1}{x_1'} > \frac{1}{x_2'}$$

Now, it may be easily seen that the first member of the last inequality is equal to the weight of the value of x found from the normal equations of (1), and the second member of the same to that from the normal equations of (2), and therefore we must have

$$px_1 > px_2$$

By the same reasoning we may demonstrate the following inequalities:

$$py_1 > py_2$$
$$pz_1 > pz_2$$
$$\vdots$$

Mr. JACOBY says, as his conclusion of the above first theorem, that:

"In such cases, astronomers not infrequently give preference to the solution which brings out the smallest value of (vv) . . . . To give preference to the second solution it is necessary that the diminution of (vv) be quite large, and that the additional unknowns possess a decided *a priori* probability of having a real existence."

The first part of this conclusion perhaps depends on the author's misapprehension of the principle of probability. For I believe that to compare the probabilities of the two solutions we must necessarily take $\frac{(vv)_1}{m-n_1}$ and $\frac{(vv)_2}{m-n_2}$ where m expresses the number of observations, and n_1 and n_2 are the numbers of the unknown quantities in the first and second solutions respectively: if then, the last part of the above conclusion is a sufficient condition, but not necessary, for, it is only words that assign no weight to the result of the method of least-squares; therefore I shall only say, as to the conclusion of his theorem, that to give preference to the second solution it is necessary that the diminution of (vv) is larger than $(n_2-n_1)\;\frac{(vv)_1}{m-n_1}$.

If we give, however, preference to the solution which has least probable error of a certain proposed unknown quantity, such as time-correction-term in portable transit observations, it follows, by the light of the above second theorem, that if the probable errors of a single observation in the two solutions are equal we must give preference to the first solution.

Tokyo-Astronomical Observatory, 1902 *March* 6.

COMET a 1902 (BROOKS).

Mr. W. R. BROOKS announced his discovery of a comet on April 14; and telegraphed to the Harvard College Observatory its approximate position on April 15, 16h mean time, as $\alpha = 23^h 8^m 10^s$, $\delta = +27° 25'$. "Motion, east 12^m, south 2°; brightish, with tail." CAMPBELL, from Lick Observatory, telegraphed a position obtained by AITKEN, and Dr. KREUTZ one observed by HOSTELMANN at Konigsberg, as follows:

1902 Gr. M.T.	α	δ	
	h m s	° ′ ″	
April 16.5521	23 15 11.1	+26 6 35	Hostelmann
16.9916	23 21 7.7	+25 12 26	Aitken

Dr. KREUTZ also cabled the following elements and ephemeris:

$$T = 1902 \text{ May } 7.12 \text{ Gr.}$$

$$\left.\begin{aligned} \pi - \Omega &= 228° \ 23' \\ \Omega &= 52 \ 15 \\ i &= 66 \ 30 \end{aligned}\right\} \text{Ecliptic}$$

$$q = 0.1512$$

Gr. Midnight	α	δ	Br.
	h m s	° ′	
April 27	1 22 20	+1 30	1.11
May 1	1 57 24	−4 36	
5	2 25 8	−7 58	
9	2 46 36	−9 7?	0.71

Computed from observations of April 16, 17, 18.

Dr. LEUSCHNER (May 2) telegraphed elliptic elements and ephemeris to the Harvard College Observatory, as follows:

$$T = 1902 \text{ May } 28.39 \text{ Gr.}$$

$$\left.\begin{aligned} \pi - \Omega &= 271° \ 30' \\ \Omega &= 35 \ 3 \\ i &= 71 \ 50 \end{aligned}\right\} \text{Equator}$$

$$q = 0.5542$$
$$e = 0.3917$$
$$\text{Period} = 0.88 \text{ yr.}$$

Gr. Midnight	α	δ	Br.
	h m s	° ′	
May 1	1 54 8	−5 26	2.09
5	2 20 52	−9 58	
9	2 41 8	−12 30	
13	2 57 12	−13 25	1.07

Elements are stated to be rude, and as indicating possible identity with Comet 1748 II. Another solution gave a hyperbola. Computers, STEBBINS, CURTIS, WEYWOUTH and LEUSCHNER.

Manifestly these contradictory solutions are to be regarded as nominal results of computation, indicating merely the insufficiency of the data to establish any deviation from parabolic velocity.

Additional observations by HARTWIG and PECHULE (*A.N.* 3788) are as follows:

1902 Local M.T.		α	δ
h m s		h m s	° ′ ″
Apr. 16 15 30 10	Bamberg	23 16 27.57	+25 58 59.1
16 15 8 6	Copenhagen	23 16 12.81	+26 1 21.0

— ED.

ELEMENTS AND EPHEMERIS OF COMET a 1902,

By H. KREUTZ.

I send you for printing in the *Astronomical Journal* the following elements of the new Comet a 1902 (BROOKS), derived by Dr. E. STRÖMGREN and myself from observations at Konigsberg, April 16, and at Arcetri, April 17 and 18.

$$T = 1902 \text{ May } 7.519 \text{ Berlin M.T.}$$

$$\left.\begin{aligned} \omega &= 228° \ 22'.7 \\ \Omega &= 52 \ 15.4 \\ i &= 66 \ 30.1 \end{aligned}\right\} 1902.0$$

$$\log q = 9.65436$$

Ephemeris for 12h Berlin, computed by M. EBELL:

1902	α	δ	Br.
	h m s	° ′	
April 27	1 21 58	+1 34.1	1.41
May 1	1 57 7	−4 33.4	1.22
5	2 24 53	−7 56.7	0.97
9	2 46 25	−9 6.8	0.71
13	3 3 15	−8 47.5	0.48
17	3 16 45	−7 37.5	0.32
21	3 27 59	−6 3.8	0.22

The comet is lost for the northern hemisphere, but it can possibly be observed for some time at the observatories situated in higher southern latitudes.

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MICROMETRIC MEASURES OF DOUBLE STARS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By E. A. BOEGER.

The usual precautions were taken in measuring the position-angles. Each observation is the mean of four separate measures for angle, and four separate double distances. The positions are for 1880.

β 256.

$\alpha = 0^h\ 14^m.4\ ;\ \delta = -14^\circ\ 26'$

1900.82	264.5	2.66

$O\Sigma$ 179.

$\alpha = 7^h\ 37^m.2\ ;\ \delta = +24^\circ\ 41'$

1900.26	234.5	. .
.28	239.2	6.22
.32	238.7	. .
.36	244.6	6.66
1900.30	239.2	6.44

Σ 1273.

$\alpha = 8^h\ 40^m.4\ ;\ \delta = +6^\circ\ 52'$

1900.27	243.6	3.31
.27	239.3	3.04
1900.27	241.4	3.18

Σ 1280.

$\alpha = 8^h\ 44^m.4\ ;\ \delta = +71^\circ\ 16'$

1900.27	48.8	4.96
.27	43.7	4.89
1900.27	46.2	4.92

$O\Sigma$ 196.

$\alpha = 8^h\ 51^m.0\ ;\ \delta = +48^\circ\ 31'$

1900.27	355.3	8.75
.30	358.3	7.73
1900.28	356.8	8.24

Σ 1296.

$\alpha = 8^h\ 51^m.8\ ;\ \delta = +35^\circ\ 25'$

1900.36	95.1	3.89

Σ 1334.

$\alpha = 9^h\ 11^m.4\ ;\ \delta = +37^\circ\ 19'$

1900.26	240.5	2.93
.27	245.0	3.25
1900.27	242.8	3.09

Σ 1338.

$\alpha = 9^h\ 13^m.4\ ;\ \delta = +38^\circ\ 42'$

1900.36	165.0	2.31

$O\Sigma$ 200.

$\alpha = 9^h\ 16^m.6\ ;\ \delta = +52^\circ\ 5'$

1900.27	325.3	2.43
.28	329.7	1.74
1900.27	327.5	2.08

Σ 1374.

$\alpha = 9^h\ 34^m.0\ ;\ \delta = +39^\circ\ 30'$

1900.27	284.7	3.25
.28	282.6	3.25
1900.27	283.6	3.25

Σ 1386.

$\alpha = 9^h\ 45^m.6\ ;\ \delta = +69^\circ\ 28'$

1900.27	290.7	2.20
.27	296.5	2.06
1900.27	293.6	2.13

Σ 1428.

$\alpha = 10^h\ 18^m.4\ ;\ \delta = +53^\circ\ 14'$

1900.28	90.7	2.91
.29	89.1	2.61
1900.28	89.9	2.76

Σ 1439.

$\alpha = 10^h\ 23^m.5\ ;\ \delta = +21^\circ\ 25'$

1900.29	129.0	2.61

$O\Sigma$ 233.

$\alpha = 11^h\ 11^m.0\ ;\ \delta = +67^\circ\ 2'.1$

1900.30	331.8	1.58
.32	334.8	1.56
1900.31	333.3	1.57

Σ 1523.

$\alpha = 11^h\ 11^m.8\ ;\ \delta = +32^\circ\ 13'$

1900.30	147.3	2.47
.51	149.3	2.40
1900.42	148.3	2.44

Σ 1527.

$\alpha = 11^h\ 12^m.7\ ;\ \delta = +14^\circ\ 56'$

1900.30	15.6	2.93
.32	16.1	3.15
1900.31	15.8	3.04

Σ 1536.

$\alpha = 11^h\ 17^m.6\ ;\ \delta = +11^\circ\ 12'$

1900.30	50.5	2.70
.32	51.2	2.40
1900.31	50.8	2.55

Σ 1543.

$\alpha = 11^h\ 22^m.6\ ;\ \delta = +40^\circ\ 0'$

1900.30	359.5	5.05
.32	356.1	5.70
.54	358.1	5.95
1900.39	357.9	5.57

Σ 1553.

$\alpha = 11^h\ 30^m.0\ ;\ \delta = +56^\circ\ 48'$

1900.43	165.5	5.86
.48	171.8	5.86
1900.45	168.6	5.86

Σ 1596.

$\alpha = 11^h\ 58^m.0\ ;\ \delta = +22^\circ\ 8'$

1900.43	238.5	3.89
.50	240.3	3.64
1900.46	239.4	3.75

Σ 1687 AC.

$\alpha = 12^h\ 47^m.4\ ;\ \delta = +21^\circ\ 54'$

1900.49	123.9	28.78
.50	126.6	28.60
1900.49	125.2	28.69

$O\Sigma$ 257.

$\alpha = 12^h\ 51^m\ ;\ \delta = +46^\circ\ 16'$

1900.49	355.5	12.77
.50	355.3	13.27
.54	353.4	12.86
1900.51	354.7	12.97

Σ 1707.

$\alpha = 12^h\ 55^m.3\ ;\ \delta = +16^\circ\ 31'$

1900.36	36.5	9.59
.44	37.4	10.53
.54	36.6	10.02
1900.45	36.8	10.04

Σ 1722.

$\alpha = 13^h\ 2^m\ ;\ \delta = +16^\circ\ 8'$

1900.36	334.6	3.38
.48	336.8	3.31
1900.42	335.7	3.34

$O\Sigma$ 261.

$\alpha = 13^h\ 6^m.4\ ;\ \delta = +32^\circ\ 43'$

1900.53	343.2	1.88
.53	343.2	1.80
1900.53	343.2	1.84

Σ 1771.

$\alpha = 13^h\ 33^m.5\ ;\ \delta = +70^\circ\ 23'$

1900.53	72.0	2.11
.53	78.9	2.24
1900.53	75.4	2.18

Σ 1776.

$\alpha = 13^h\ 36^m.8\ ;\ \delta = +46^\circ\ 50'$

1900.53	199.2	7.16
.53	196.7	6.95
1900.53	198.0	7.06

Σ 1785.

$\alpha = 13^h\ 43^m.6\ ;\ \delta = +27^\circ\ 35'$

1900.53	277.1	1.94
.53	273.9	1.97
1900.53	275.5	1.96

Σ 1813.

$\alpha = 14^h\ 7^m.4\ ;\ \delta = +5^\circ\ 58'$

1900.50	186.1	5.35
.50	196.8	1.85
.51	195.7	1.49
1900.51	192.9	1.90

Σ 1816.

$\alpha = 14^h\ 8^m.0\ ;\ \delta = +29^\circ\ 40'$

1900.53	81.1	2.01
.54	80.6	1.71
1900.54	81.0	1.86

Σ 1820.

$\alpha = 14^h\ 9^m.1\ ;\ \delta = +55^\circ\ 58'$

1900.53	79.9	2.29
.54	81.8	2.52
.57	82.3	2.29
1900.55	81.3	2.37

Σ 1858.

$\alpha = 14^h\ 29^m.0\ ;\ \delta = +36^\circ\ 6'$

1900.50	41.2	3.57
.53	35.3	3.02
.57	39.1	2.40
1900.53	39.6	3.00

Σ 1864.

$\alpha = 14^h\ 35^m.1\ ;\ \delta = +16^\circ\ 56'$

1900.50	101.2	6.59
.53	104.0	6.57
1900.51	102.6	6.58

Σ 1871.

$\alpha = 14^h\ 37^m.5\ ;\ \delta = +57^\circ\ 55'$

1900.50	289.2	2.39
.53	294.6	2.06
1900.51	291.9	2.22

Σ 1877.

$\alpha = 14^h\ 39^m.7\ ;\ \delta = +27^\circ\ 35'$

1900.62	326.3	3.09
.65	331.3	2.97
.66	323.1	3.41
1900.64	326.9	3.16

OΣ 288.

$\alpha = 14^h\ 47^m.7\ ;\ \delta = +16^\circ\ 12'$

1900.53	189.1	2.22
.57	194.8	1.35
.61	194.2	1.69
1900.57	192.7	1.75

Σ 1909.

$\alpha = 14^h\ 59^m.8\ ;\ \delta = +48^\circ\ 7'$

1900.53	244.8	1.69
.53	236.9	1.94
.59	247.1	1.44
1900.55	242.9	1.69

Σ 1931.

$\alpha = 15^h\ 13^m.2\ ;\ \delta = +44^\circ\ 14'$

1900.53	29.7	6.80
.53	29.1	6.73
1900.53	29.4	6.76

OΣ 296.

$\alpha = 15^h\ 22^m.2\ ;\ \delta = +44^\circ\ 26'$

1900.53	305.7	2.11
.53	303.5	1.99
1900.53	304.6	2.05

Σ 1954.

$\alpha = 15^h\ 29^m.1\ ;\ \delta = +10^\circ\ 56'$

1900.53	185.0	3.71
.57	184.2	3.96
.59	184.6	3.52
1900.57	184.6	3.73

Σ 1956.

$\alpha = 15^h\ 29^m\ ;\ \delta = +42^\circ\ 13'$

1900.53	39.5	1.58
.53	42.4	1.97
1900.53	41.0	1.78

OΣ 298.

$\alpha = 15^h\ 31^m.7\ ;\ \delta = +40^\circ\ 13'$

1900.50	185.8	1.60
.53	179.1	1.58
1900.52	182.4	1.59

OΣ 299.

$\alpha = 15^h\ 32^m\ ;\ \delta = +64^\circ\ 15'$

1900.59	23.1	3.22
.59	26.8	3.20
1900.59	25.0	3.21

Σ 1965.

$\alpha = 15^h\ 34^m.9\ ;\ \delta = +37^\circ\ 1'$

1900.59	305.2	6.50
.59	304.3	6.39
.66	304.6	6.43
1900.61	304.7	6.44

Σ 1985.

$\alpha = 15^h\ 49^m.7\ ;\ \delta = -1^\circ\ 49'$

1900.59	338.3	5.49
.59	335.3	6.02
1900.59	336.8	5.76

Σ 2021.

$\alpha = 16^h\ 7^m.7\ ;\ \delta = +13^\circ\ 51'$

1900.59	157.2	3.94
.59	153.1	3.84
.66	153.8	3.41
1900.61	154.7	3.73

Σ 2032.

$\alpha = 16^h\ 10^m.2\ ;\ \delta = +34^\circ\ 10'$

1900.57	218.7	1.26
.59	216.6	1.40
1900.58	217.6	1.33

Σ 2041.

$\alpha = 16^h\ 20^m\ ;\ \delta = +37^\circ\ 19'$

1900.60	312.6	8.60
.61	314.0	8.93
1900.60	313.3	8.76

α Scorpii.

$\alpha = 16^h\ 22^m\ ;\ \delta = -26^\circ\ 10'$

1900.60	265.5	3.15
.61	268.3	3.43
1900.60	266.9	3.29

Σ 2054.

$\alpha = 16^h\ 22^m\ ;\ \delta = +61^\circ\ 58'$

1900.59	176.5	1.32
.59	180.2	1.28
1900.59	178.3	1.30

Σ 2052.

$\alpha = 16^h\ 23^m.6\ ;\ \delta = +18^\circ\ 40'$

1900.60	95.9	2.07
.61	97.1	2.19
1900.60	96.5	2.13

Σ 2055.

$\alpha = 16^h\ 24^m.9\ ;\ \delta = +2^\circ\ 15'$

1900.60	47.3	1.60
.61	60.2	1.92
.70	53.8	1.58
1900.61	53.8	1.70

Σ 2097.

$\alpha = 16^h\ 40^m.5\ ;\ \delta = +35^\circ\ 57'$

1900.60	85.3	2.01
.61	84.2	2.17
1900.60	84.8	2.09

OΣ 318.

$\alpha = 16^h\ 51^m\ ;\ \delta = +14^\circ\ 18'$

1900.70	241.9	3.06

Σ 2130.

$\alpha = 17^h\ 2^m.9\ ;\ \delta = +54^\circ\ 38'$

1900.66	144.0	3.25
.70	147.5	2.40
1900.68	145.8	2.82

Σ 2135.

$\alpha = 17^h\ 7^m\ ;\ \delta = +21^\circ\ 22'$

1900.66	178.0	6.59
.70	176.7	7.07
1900.68	177.4	6.83

Σ 2140.

$\alpha = 17^h\ 9^m.1\ ;\ \delta = +14^\circ\ 32'$

1900.62	115.7	4.58
.65	115.3	4.35
1900.63	115.5	4.46

Σ 2165.

$\alpha = 17^h\ 21^m.6\ ;\ \delta = +29^\circ\ 34'$.

1900.66	*23.8	8.10
.70	53.8	8.19
1900.68	53.8	8.14

* Probably read 30° wrong.

Σ 2190.

$\alpha = 17^h\ 30^m.9\ ;\ \delta = +21^\circ\ 4'$

1900.66	22.8	10.14
.70	27.7	10.18
1900.68	25.2	10.16

Σ 2192.

$\alpha = 17^h\ 35^m.4\ ;\ \delta = +29^\circ\ 18'$

1900.70	61.7	11.00

Σ 2205.

$\alpha = 17^h\ 40^m.4\ ;\ \delta = +17^\circ\ 46'$

1900.57	309.4	1.78
.60	308.5	2.57
1900.59	309.0	2.18

Σ 2218.

$\alpha = 17^h\ 39^m.5\ ;\ \delta = +63^\circ\ 44'$

1900.70	158.7	2.01
.70	158.5	2.24
1900.70	158.6	2.12

Σ 2262.

$\alpha = 17^h\ 56^m.5\ ;\ \delta = -8^\circ\ 11'$

1900.62	259.1	2.06
.65	255.6	2.08
1900.63	259.4	2.07

Σ 2271.

$\alpha = 17^h\ 57^m.7\ ;\ \delta = +52^\circ\ 51'$

1900.70	267.9	2.42

Σ 2267.

$\alpha = 17^h\ 57^m.8\ ;\ \delta = +40^\circ\ 11'$

1900.70	246.2	1.28

Σ 2272.

$\alpha = 17^h\ 59^m.4\ ;\ \delta = +2^\circ\ 33'$

1900.60	247.8	1.85
.61	256.4	1.88
.62	244.0	1.92
.65	247.9	1.67
.66	243.7	1.69
1900.63	248.0	1.80

Σ 2303.

$\alpha = 18^h\ 13^m.6\ ;\ \delta = -8^\circ\ 2'$

1900.65	229.2	2.45
.66	233.5	2.38
1900.65	231.4	2.41

Σ 2316.

$\alpha = 18^h\ 21^m\ ;\ \delta = +0^\circ\ 7'$

1900.57	312.9	4.26
.66	321.5	3.71
.68	317.5	3.87
1900.64	317.3	3.95

The following double stars were discovered by Prof. A. N. Skinner during his work upon the A.G. Zone −13° 50′ to −18° 10′. The positions are for 1890.0.

***DM. −15 3.**
$\alpha = 0^h\ 2^m.4$; $\delta = -14°\ 50'$

1900.82	107.2	9.93

*In angle and distance this agrees with Cin.$_5$1 identified by Stone as Weisse O 14, which was not noted as double during the zone observations here.

DM. −14 228 *K*.
$\alpha = 1^h\ 6^m.6$; $\delta = -14°\ 13'$

1900.82	250.7	*8.42
.82	252.6	9.61
1900.82	251.6	9.01

* Best.

DM. −15 1261.
$\alpha = 5^h\ 59^m.2$; $\delta = -15°\ 40'$

1900.83	169.1	4.82

DM. −17 1742.
$\alpha = 6^h\ 57^m.8$; $\delta = -17°\ 37'$

1900.83	274.3	5.17

DM. −16 1750.
$\alpha = 7^h\ 0^m.4$; $\delta = -16°\ 28'$

1900.83	338.1	4.37

DM. −16 1895 *A*
$\alpha = 7^h\ 14^m.4$; $\delta = -16°\ 36'$

1900.27	356.9*	6.59
.32	291.8	2.56

* Evidently two different stars.

DM. −16 2786.
$\alpha = 9^h\ 21^m.9$; $\delta = -16°\ 47'$

1900.32	357.2	6.04

DM. −14 3825 *B*
$\alpha = 13^h\ 48^m.2$; $\delta = -14°\ 35'$

1900.27	292.3	2.75
.30	297.1	2.13
1900.28	294.7	2.44

DM. −14 3891.
$\alpha = 14^h\ 4^m.8$; $\delta = -14°\ 17'$

1900.36	326.5	13.57
.43	324.6	12.77
1900.39	325.5	13.17

DM. −16 4169 *A*.
$\alpha = 15^h\ 46^m.1$; $\delta = -16°\ 54'$

1900.36	278.9	2.34
.48	281.0	2.61
.57	278.1	2.01
1900.47	279.3	2.32

DM. −17 4630 *A*.
$\alpha = 16^h\ 39^m.9$; $\delta = -17°\ 9'$

1900.36	270.2	3.87
.49	268.3	4.30
.57	269.1	3.52
1900.47	269.2	3.90

DM. −17 4821 *A*
$\alpha = 17^h\ 23^m.3$; $\delta = -17°\ 43'$

1900.57	269.2	2.24

DM. −15 4651 *B*.
$\alpha = 17^h\ 35^m.9$; $\delta = -15°\ 40'$

1900.57	286.4	4.23
.65	276.3	4.07
.66	279.4	4.31
1900.63	280.7	4.20

DM. −17 5949.
$\alpha = 20^h\ 14^m.8$; $\delta = -17°\ 2'$

1900.72	337.5	3.84
.80	336.9	3.29
.82	331.8	3.64
1900.78	335.4	3.59

DM. −17 6080 *A*
$\alpha = 20^h\ 40^m.3$; $\delta = -17°\ 6'$

1900.72	300.5	3.80
.82	298.7	3.57
.82	300.3	3.43
1900.79	299.8	3.60

DM. −16 6142 *A*.
$\alpha = 22^h\ 37^m.6$; $\delta = -16°\ 43'$

1900.82	285.4	1.92
.82	289.6	1.37
.82	289.8	1.67
1900.82	288.3	1.65

MAXIMA AND MINIMA OF VARIABLE STARS, OBSERVED BY THE LATE DAVID FLANERY, FROM 1895 TO 1900,

By PAUL S. YENDELL.

David Flanery, late of Memphis, Tennessee, was born in Limerick, Ireland, in the year 1828, and came to this country in 1847. At the breaking out of the Civil War, in 1861, he was residing in Jackson, Mississippi, as Superintendent of the Southwestern Telegraph Company.

He early entered the Confederate army, and remained in its telegraph service throughout the war. After the close of the conflict he went to Memphis; he at once became permanently identified with the telegraph business, and remained in connection with it, being at different times manager of the New Orleans and Richmond offices of the Western Union, until too frail for active duty.

Besides being a practical and theoretical electrician, Mr. Flanery was much interested in astronomy. It was in this connection that I first heard from him, early in the nineties, in a letter asking for information regarding certain variable stars. In this line of work he continued, with increasing interest, until his death in 1900.

Through the courtesy of his son, Mr. Charles M. Flanery, I had access to all his note-books for the years from 1895 to 1900, which contain practically all his work on the variables. They contain in all some four thousand observations on about eighty stars. In the first year, many of these are mere identifications, but about a dozen stars were carefully and persistently followed as long as he lived.

An examination of these notes shows that the observations were made with great care, and the notes are carefully and minutely kept. He was very solicitous in securing the best information accessible to him as to positions and magnitudes, and in taking every possible precaution against avoidable errors of observation.

Mr. Flanery was exceedingly modest as to the value of his astronomical work, and never seemed to think it good enough for publication. In spite of all discouragements and difficulties, and the occupations of a busy life, he secured valuable lines of observations of a number of variable stars, which I have the satisfaction of here making public.

806 *o Ceti.* (Six maxima.)

1895 Mar. 7.5	$3^{\rm M}.8$	34 obs.	1895 Jan. 16	to	Mar. 22	?
1896 Feb. 19	3.7	55 "	1895 Oct. 11	to	1896 Mar. 20	good
1896 Dec. 28	1.2	75 "	1896 Oct. 26	to	1897 Mar. 10	good
1897 Dec. 2	3.1	82 "	1897 Sept. 19	to	1898 Mar. 13	good
1898 Oct. 15	2.6	97 "	1898 Aug. 20	to	1899 Mar. 9	good
1899 Sept. 23*	1.0	80 "	1899 July 19	to	1900 Jan. 31	good

* A max. of $4^{\rm M}.3$ on Aug. 25; depression to $5^{\rm M}.0$ Sept. 13; Max. of $4^{\rm M}.0$ on Sept. 23.

2100 *U Orionis.* (Two maxima.)

1898 Mar. 23	$6^{\rm M}.3$	32 obs.	1898 Feb. 11	to	May 7	good
1899 Apr. 6	6.3	31 "	1899 Jan. 29	to	Apr. 30	good

3493 *R Leonis.* (Five maxima.)

1895 Feb. 26.5	$5^{\rm M}.0$	50 obs.	1895 Jan. 2	to	June 1	good
1896 Jan. 11.5	6.4	50 "	1895 Oct. 19	to	1896 Apr. 7	good
1896 Nov. 19	6.5	40 "	1896 Oct. 7	to	1897 Jan. 3	good
1899 June 2	6.2	71 "	1899 Feb. 19	to	July 7	good
1900 Apr. 13	5.6	29 "	1900 Apr. 4	to	May 17	uncertain

One minimum:

1898 Mar. 23	$<10^{\rm M}$	49 obs.	1898 Jan. 17	to	July 5	fair

4826 *R Hydrae.* (Two maxima.)

1897 May 5	$4^{\rm M}.0$	33 obs.	1897 Apr. 5	to	July 2	good
1898 July 4	4.9	38 "	1898 Apr. 28	to	July 29	not so certain

4817 *S Virginis.* (Three maxima.)

1896 May 30	$7^{\rm M}.4$	21 obs.	1896 Apr. 5	to	July 14	fair
1898 July 6	6.2	62 "	1898 Apr. 28	to	Aug. 22	good
1899 July 25	6.8	32 "	1899 May 29	to	Aug. 29	good

6733 *R Scuti.*

1895, 88 observations, from July 25 to Dec. 6.

Maxima			Minima		
1895 Sept. 22	$5^{\rm M}.3$	good	1895 Oct. 6.6	$7^{\rm M}.5$	good
Nov. 28	4.3	good			

1896, 90 observations, from Feb. 19 to Nov. 5.

1896 June 18	$6^{\rm M}.3$	good	1896 July 19	$7^{\rm M}.5$	good
Aug. 29	5.1	good	Sept. 10	6.1	(Sub-min.) good
Sept. 18	5.9	Sub-max. good	Oct. 2	6.5	good
Oct. 21	5.7	good			

1897, 137 observations, from Feb. 2 to Nov. 17.

1897 Feb. 26	$5^{\rm M}.1$	good	1897 July 2	$6^{\rm M}.6$	(Sub-min.) good
June 5	4.8	good	July 31	6.3	Sub-min. good
July 17	5.5	Sub-max. good	Sept. 21	8.0	good
Aug. 11	5.8	Sub-max. good			
Nov. 10	5.5	good			

1898, 78 observations, from June 6 to Nov. 18.

1898 July 31	$4^{\rm M}.9$	good	1898 June 27	$7^{\rm M}.5$	good
Sept. 18	5.4	good	Aug. 25	6.2	good
			Oct. 21	6.5	good

1899, 47 observations, from May 27 to Oct. 8.

Maxima			Minima		
1899 July 11	$5^{\rm M}.5$	good	1899 June 9	$6^{\rm M}.9$	good
Aug. 19	5.3	good	Aug. 5	6.5	good

7085$_a$ *SU Cygni.*

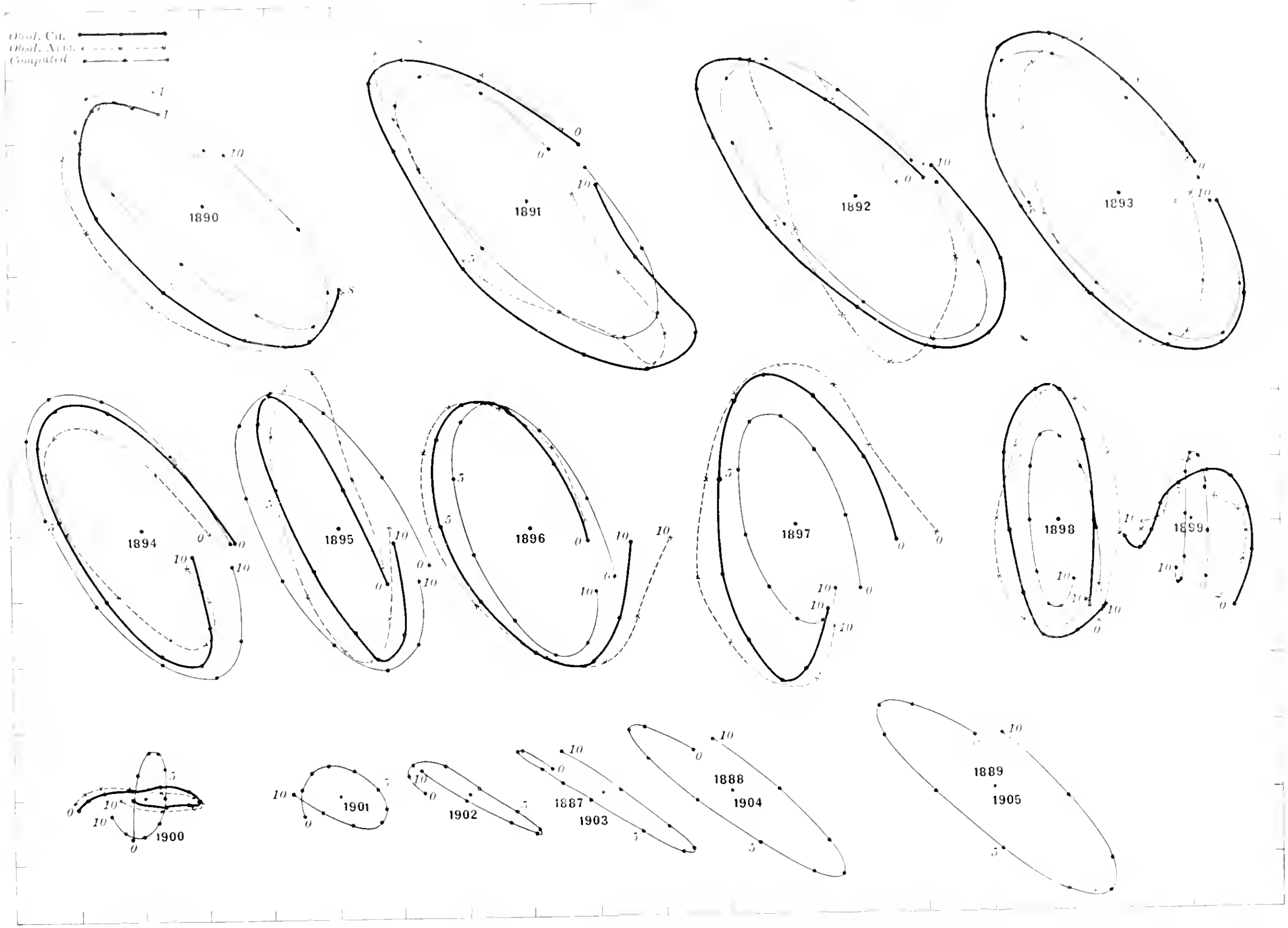
1899, 71 observations, from August 11 to Dec. 28.

Maxima		Minima	
1899 Aug. 26.88	2 obs.	1899 Aug. 30.98	2 obs.
31.73	2 "	Sept. 3.78	1 "
Sept. 4.02	2 "	11.08	2 "
12.38	1 "	15.12	2 "
27.38	1 "	30.24	3 "
Oct. 5.51	1 "	Oct. 15.23	2 "
9.22	1 "	23.08	2 "
24.62	2 "	27.28	1 "
28.18	2 "	30.88	2 "
Nov. 5.17	1 "	Nov. 3.98	1 "
28.38	1 "		

1900, 24 observations, from May 29 to July 14.

Maxima		Maxima	
1900 May 20.55	2 obs.	1900 June 21.00	2 obs.
24.17	1 "	27.59	1 "
31.89	1 "	July 5.11	3 "
June 16.16	1 "	9.58	1 "

RESULTANT OF THE TWELVE AND THIRTEEN MONTHS' MOTION OF THE POLE



NEW STUDY OF THE POLAR MOTION FOR THE INTERVAL 1890–1901.

BY S. C. CHANDLER.

The publication, since my last examination of the observations since 1890, of the results of latitude-observations for two or three additional years, has led to a new study of the whole series which seems to throw a more distinct light on the nature of the changes in the annual component of the polar motion. The conclusions from this discussion appear to be definite enough to warrant printing.

The basis of the investigation is the table of the observed coordinates of the pole on p. 147. Values of these coordinates have been determined from time to time during the past decade both by Dr. ALBRECHT and myself, substantially from the same body of observations and by independent processes. In view of the interesting conclusions that are to be here set forth, it has seemed to me advisable to include both of these series in the comparisons presently to be made, in order to demonstrate that the inferences are in no wise sophisticated by the particular values used. Both systems, therefore, are given in the table, under the designations "Ch." and "Albr." respectively. It was convenient and desirable, for the purpose of the investigation, to remove as far as possible merely local and accidental irregularities of individual points. This has been done in what seems to be an unobjectionable manner by comparison with a computed curve which so closely represents the track of the pole throughout the whole eleven years that the deviations during a short interval like three or four months can be regarded as linear, and their mean for a few adjacent values furnishes a normal value for the middle point. The values "Albr." given in the table were adjusted in this manner from the series given in his *Bericht* for 1898 for the interval 1890.0–95.0, and in those of 1900 and 1901 for 1895.1 onwards. The values "Ch." have been analogously adjusted from my independent determinations. They are expressed in the same system of coordinates that I have used from the beginning, $+x$ being directed to longitude 90° east of Greenwich (corresponding to ALBRECHT's $+y$), and $+y$ towards Greenwich (his $-x$).

The coordinates in the "Computed" columns of the table are the sums of those of the definitive fourteen-months' term and of the coordinates for the other terms as defined in this article. Incidentally it may be mentioned that the above table will be convenient for the purpose of computing variations of latitude. If λ is the longitude of the station (reckoned positive west of Greenwich) we have $\varphi-\varphi_0 = x\sin\lambda - y\cos\lambda$. Either set of values, "Ch.," "Albr.," or "Comp.," of x and y may be used.

From the observed coordinates of the total latitude-variation given in the table were subtracted those of the fourteen-months' term according to my definitive determination (see table III, *A. J.* 495, p. 119; also, *cf.* Nos. 490 and 494). To these residuals were added the following table of small corrections for the purpose of reducing the sums of the residuals for each calendar year to zero; *i.e.*, to correctly center the annual ellipses.

	Δx		Δy	
	Ch.	Albr.	Ch.	Albr.
	″	″	″	″
1890	+.006	−.004	−.040	−.035
91	+.010	+.015	+.020	+.015
92	−.022	−.006	+.005	+.005
93	+.003	+.009	+.006	+.005
94	+.003	+.002	+.016	+.019
95	+.001	+.005	−.011	−.015
96	−.001	.000	+.015	+.006
97	−.011	−.027	+.005	+.002
98	−.012	−.012	+.004	+.001
1899	−.010	−.005	−.008	−.003
1900	+.005	+.009	−.002	−.006

The results so obtained are drawn in the accompanying diagram, where the tenths of years are indicated by dots (or crosses), and the beginning, middle and end of the year by the numerals 0, 5 and 10, respectively. The heavy line indicates "Ch." and the broken line "Albr." For sake of completing the figure for 1891, the points for the first half of that year, when observations are lacking, are filled in as a mean of those for the same portions of 1890 and 1892. The faint computed curve will be explained later. The unit of the graduation on the margin is 0″.05.

An inspection of this diagram makes manifest three peculiarities. First, the direction of the major axis of the elliptical figures is subject to a continuous change, the upper end of it lying about 50° east of Greenwich in 1890, and directly towards Greenwich in 1898. Secondly, the calendar date on which the pole passes this apsis also gradually changes, from about the middle or end of March in 1890 to the middle of May in 1898. Thirdly, the size of the ellipse shows a notable and, in general, continuous diminution from a maximum in 1891 or 1892 until it nearly disappears in 1900. Only one year, 1897, shows a perceptible departure from this general decrease; but even there, in view of the uncertainty of such determinations, it is not significantly a contradiction of the general rule for the series.

Before we can accept as real any or all of the three peculiarities just pointed out, we must examine whether they may not have been factitiously introduced by the assumption of the particular constants of the fourteen-months' term by means of which the figures shown in the diagram have been derived. This I proceed to do by a demonstration that seems to leave no reasonable doubt.

First, suppose we make no assumption as to the fourteen-months' term and merely trust to its elimination by the employment of seven years' observation; an elimination that will be approximately effected, as is known, by the commensurability of the terms. Under the heading *A* below I have placed the mean value of the coordinates, found by merely taking the means of the total latitude-

variations in the table on p. 147, for each tenth of a year for the groups 1890.0–97.0 and 1894.0–1901.0. Again, I have placed under *B* the similarly found values assuming a constant mean value, 0″.14, for the radius of the fourteen-months' term. Finally, under *C* are given the values correspondingly found by the constants of the fourteen-months' term actually used in the present investigation, in which the radius is assumed to diminish from 0″.17 to 0″.09 between 1890 and 1901.

1890.0–1897.0.

	x_2 *A* ″	x_2 *B* ″	x_2 *C* ″	y_2 *A* ″	y_2 *B* ″	y_2 *C* ″
0.0	−.029	−.054	−.047	+.005	+.007	+.008
.1	+.018	.000	+.003	+.053	+.063	+.067
.2	+.073	+.050	+.050	+.085	+.103	+.103
.3	+.082	+.083	+.085	+.083	+.105	+.100
.4	+.072	+.085	+.080	+.045	+.063	+.057
.5	+.066	+.062	+.057	−.024	−.017	−.019
.6	+.006	+.006	+.004	−.086	−.080	−.077
.7	−.047	−.043	−.047	−.107	−.117	−.109
.8	−.086	−.080	−.079	−.106	−.106	−.097
.9	−.089	−.080	−.080	−.059	−.061	−.056

1894.0–1901.0.

	x_2 *A* ″	x_2 *B* ″	x_2 *C* ″	y_2 *A* ″	y_2 *B* ″	y_2 *C* ″
0.0	−.024	−.028	−.033	−.026	−.019	−.021
.1	−.007	−.007	−.014	+.017	+.024	+.024
.2	+.013	+.016	+.009	+.056	+.061	+.067
.3	+.023	+.033	+.024	+.067	+.070	+.076
.4	+.030	+.039	+.037	+.054	+.054	+.060
.5	+.029	+.034	+.037	+.019	+.017	+.021
.6	+.013	+.017	+.020	−.026	−.034	−.027
.7	.000	−.001	+.009	−.064	−.071	−.069
.8	−.012	−.017	−.006	−.069	−.077	−.076
.9	−.019	−.020	−.020	−.044	−.047	−.051

The differences between *A*, *B* and *C* are quite insignificant; and if the three sets of coordinates be plotted for the groups 1890–97 and 1894–01, it will be recognized that the resultant ellipses, which correspond to the mean dates 1893 and 1897, all manifest the same peculiarities of change, between the two epochs, that have been above spoken of. I think, therefore, that we may safely dismiss any anxiety on this score.

Reverting, then, to the curves of the diagram, it becomes an interesting question how they may best be explained. It is manifest that what we have hitherto treated as a single harmonic motion with an annual period is really the resultant of two or more motions of different periods. Still retaining the hypothesis that one of them is annual, we find that the results may be well represented for the whole interval 1890.0–1901.0 by the expressions

$$x_2 = 0''.016 \sin(\odot - 303^\circ) + 0''.054 \sin(t - 2411742)\ 0^\circ.924$$
$$y_2 = 0''.067 \cos(\odot - 11^\circ) + 0''.045 \cos(t - 2411763)\ 0^\circ.924$$

The first terms of the coordinates represent the annual term; the last terms correspond to an ellipse with a period of 390 days. The elements of these two ellipses are

	Annual Term	390-Day Term
T	April 8	2411816 +390 E
ω	34°	60°
a	0″.160	0″.115
b	0 .030	0 .080

where *T* is the date when the pole passes the major apsis which lies nearest to Greenwich; ω is the angle, reckoned to the east from the Greenwich meridian, of that end of the major axis; and *a* and *b* are the major and minor axes.

The resultant of these two motions is depicted on the diagram in the faint-line curves, so that a direct comparison with the observed curves can be made. I have also given at the bottom of the diagram the computed curves from 1901 onward, to illustrate the general nature of the motion. Since the period of 390 days is commensurable with the year, the curves repeat themselves every sixteen years; thus, that for 1889 is the same as that for 1905, for example; so that a whole cycle is shown on the chart.

To facilitate the comparison of the observed and computed results, I give in the following table the elements, read as closely as may be from the diagram, of the approximately elliptical figures for the various years. This can only be done approximately, since these resultant curves are not closed figures, but it is sufficient for the purpose of general comparison. It will be seen that the adopted hypothesis accounts satisfactorily for the observed changes in the size, position and dates of maximum departure, for the whole period under consideration, 1890–1901.

	T Observed Ch.	*T* Observed Albr.	*T* Comp.	ω Obs'd Ch. °	ω Obs'd Albr. °	ω Comp. °	*a* Observed Ch. ″	*a* Observed Albr. ″	*a* Comp. ″	*b* Observed Ch. ″	*b* Observed Albr. ″	*b* Comp. ″
1890	Mar. 15	Mar. 31	Mar. 17	43	51	46	0.25	0.24	0.25	*	*	0.08
1891	Apr. 7 :	Apr. 12 :	Mar. 28	44 :	48 :	43	.31 :	.32 :	.26	0.12 :	0.11 :	.09
1892	Apr. 19	Apr. 5	Apr. 15	47	27	44	.30	.27	.27	.11	.10	.10
1893	Apr. 2	Mar. 29	Apr. 17	33	20	38	.28	.25	.27	.13	.11	.11
1894	Apr. 18	Apr. 13	Apr. 19	29	32	31	.23	.20	.25	.10	.08	.11
1895	Apr. 20	Mar. 25	Apr. 24	22	15	24	.22	.22	.23	.05	.07	.11
1896	Apr. 16	Apr. 19	Apr. 26	25	29	17	.22	.23	.20	.10	.12	.09
1897	Apr. 23	Apr. 17	May 2	4	1	12	.23	.24	.16	.10	.13 :	.08
1898	May 7	May 11	May 9	2	0	3	.19	.18	.13	.06	.09	.05
1899	*	*	May 22	*	*	350	0.08 :	0.08 :	.10	†	†	.02 :
1900	*	*	May 15 :	*	*	335 :	†	†	0.07 :	†	†	0.02 :

* Indeterminate   † Small and indeterminate

Coordinates of the Pole, 1890–1902.

	x			y		
	Ch.	Albr.	Comp.	Ch.	Albr.	Comp.
	″	″	″	″	″	″
1890.0	−.050 :	−.076 :	+.001	+.234 :	+.277 :	+.209
1	+.108	+.110	.134	.260	.274	.228
2	.221	.227	.234	.198	.196	.168
3	.262	.276	.261	+.089	+.078	+.064
4	.221	.238	.213	−.043	−.061	−.065
5	+.103	+.123	+.095	.166	.182	.184
6	−.049	−.027	−.053	.223	.231	.244
7	.177	.158	.174	.202	.211	.232
8	.254	.237	.240	−.106	−.111	.150
9	−.260 :	−.273 :	.235	+.050 :	+.033 :	−.011
1891.0	–	–	.148	–	–	+.127
1	–	–	−.039	–	–	.225
2	–	–	+.102	–	–	.258
3	–	–	.189	–	–	.208
4	–	–	.214	–	–	+.092
5	+.190 :	+.171 :	.195	−.100 :	−.090 :	−.019
6	+.071	+.043	+.099	.230	.209	.175
7	−.035	−.046	−.009	.297	.286	.251
8	.161	.142	.121	.270	.266	.235
9	.186	.180	.182	.187	.193	.161
1892.0	.173	.172	.188	−.041	−.048	−.023
.1	.106	.130	.139	+.092	+.091	+.102
.2	−.005	−.060	−.046	.193	.203	.197
.3	+.095	+.040	+.052	.236	.245	.234
.4	.162	.101	.124	.191	.198	.192
.5	.189	.152	.154	+.088	+.087	+.094
.6	.164	.158	.138	−.038	−.046	−.030
.7	.107	.127	.086	.149	.160	.138
.8	+.032	+.057	+.019	.207	.212	.199
.9	−.044	−.027	−.050	.194	.191	.193
1893.0	.096	.097	.096	.122	.118	.128
.1	.111	.130	.111	−.017	−.017	−.026
.2	.094	.124	.093	+.087	+.084	+.078
.3	.050	.081	.050	.145	.135	.143
.4	−.004	−.037	−.007	.163	.151	.169
.5	+.045	+.023	+.042	.125	.122	.136
.6	.065	.061	.062	+.050	+.051	+.064
.7	.065	.080	.066	−.032	−.027	−.017
.8	.060	.080	.062	.096	.085	.084
.9	.036	.060	.041	.130	.119	.116
1894.0	+.012	.032	+.019	.134	.131	.117
.1	−.012	+.005	−.005	.097	.106	.075
.2	.029	−.021	.022	.048	.066	−.023
.3	.044	.044	.036	−.004	−.026	+.023
.4	.064	.067	.052	+.037	+.013	.059
.5	.067	.072	.054	.044	.031	.062
.6	.054	.062	.044	.044	.046	.058
.7	−.030	−.034	.028	.027	.036	.036
.8	+.005	+.003	−.004	+.007	.020	+.014
.9	.055	.054	+.032	−.011	+.006	−.005
1895.0	.089	.087	.059	.021	−.006	.018
.1	.106	.097	.078	.016	−.003	.017
.2	.092	.078	.076	.011	.000	.017
.3	+.054	+.036	.054	.011	−.004	.020
.4	−.003	−.017	+.012	.016	.023	.029
.5	.066	.073	−.041	.015	.029	.031
.6	.111	.110	.085	.022	.036	.039
.7	.123	.119	.106	−.001	−.011	−.021
.8	.102	.093	.099	+.026	+.029	+.006
1895.9	−.052	−.047	−.063	+.057	+.074	+.039

	x			y		
	Ch.	Albr.	Comp.	Ch.	Albr.	Comp.
	″	″	″	″	″	″
1896.0	+.019	+.021	−.002	+.081	+.105	+.066
.1	.091	.091	+.066	.091	.111	.081
.2	.140	.143	.112	.062	.073	.061
.3	.156	.164	.131	+.017	+.024	+.033
.4	.122	.132	.103	−.054	−.054	−.024
.5	+.063	+.072	+.052	.130	.121	.080
.6	−.028	−.027	−.025	.157	.152	.108
.7	.111	.120	.096	.150	.145	.111
.8	.158	.167	.131	.102	−.095	.083
.9	.157	.170	.132	−.012	.000	−.013
1897.0	.107	.121	.090	+.096	+.103	+.066
.1	−.028	−.017	−.025	.162	.173	.123
.2	+.072	+.073	+.061	.180	.193	.144
.3	.142	.147	.120	.149	.160	.123
.4	.164	.179	.140	+.070	+.078	+.061
.5	.148	.176	.123	−.046	−.041	−.033
.6	.107	.144	+.078	.147	.146	.112
.7	+.010	+.040	−.017	.198	.198	.151
.8	−.057	−.035	.080	.199	.200	.149
.9	.106	.097	.130	.141	.145	.100
1898.0	.118	.123	.136	−.038	−.046	−.018
.1	.094	.114	.106	+.078	+.070	+.076
.2	−.038	−.057	−.048	.161	.153	.142
.3	+.042	+.028	+.029	.199	.195	.168
.4	.108	.103	.092	.166	.167	.137
.5	.142	.144	.120	+.069	+.076	+.061
.6	.143	.153	.116	−.046	−.033	−.034
.7	.105	.121	.081	.133	.119	.113
.8	+.035	+.051	+.018	.186	.173	.161
.9	−.035	−.024	−.037	.178	.171	.155
1899.0	.103	.097	.091	.115	.115	.102
.1	.136	.133	.111	−.022	−.032	−.015
.2	.123	.126	.100	+.060	+.036	+.068
.3	.083	.093	.068	.121	.091	.128
.4	−.014	−.024	−.014	.145	.120	.150
.5	+.063	+.054	+.048	.125	.139	.122
.6	.112	.103	.083	.072	.063	+.056
.7	.132	.125	.097	+.010	+.012	−.021
.8	.128	.124	.089	−.049	−.039	.086
.9	.095	.092	.057	.090	.080	.125
1900.0	+.047	+.045	+.011	.097	.086	.120
.1	−.002	−.003	−.033	.078	.069	.084
.2	.051	.053	.073	−.043	−.036	−.031
.3	.087	.089	.095	+.003	+.007	+.029
.4	.096	.101	.089	.052	.049	.076
.5	.083	.090	.061	.084	.082	.098
.6	−.038	−.046	−.008	.085	.089	.088
.7	+.012	+.005	+.041	.067	.070	.051
.8	.057	.054	.079	+.028	+.030	+.004
.9	+.087	+.091	.097	−.007	−.003	−.037
1901.0	–	–	.093	–	–	.064
.1	–	–	.060	–	–	.072
.2	–	–	+.010	–	–	.062
.3	–	–	−.045	–	–	.040
.4	–	–	.089	–	–	−.003
.5	–	–	.111	–	–	+.023
.6	–	–	.098	–	–	.043
.7	–	–	−.058	–	–	.053
.8	–	–	+.007	–	–	.052
.9	–	–	.066	–	–	.043
1902.0	–	–	+.108	–	–	+.020

Doubtless a somewhat better approximation might be obtained for the constants of the computed coordinates x_2 and y_2 given on p. 146, but it is hardly worth while until one or two years more of observation have been secured. If these should substantiate the general nature of the motion required by this hypothesis, the elements may then be greatly improved. At present the length of the period, 390 days, is the most uncertain factor. It may be slightly longer, say 393 or 394 days. Nor can I now undertake to examine the question of the relation to the observed constants deduced from the best series of observations anterior to 1890.

If the conclusions from this investigation should be justified by future observation, we may sum up succinctly the present condition of the numerical theory as derived from observation as follows: The relative motion of the earth's axes of figure and rotation is the resultant of three elements; (1) a circular motion of fourteen months' period with variable radius and angular velocity; (2) an elliptical motion of thirteen months' period and of moderate eccentricity; (3) an elliptical motion of twelve months' period, closely approximating a harmonic rectilinear vibration.

It is noteworthy that there is a curious likeness between these results derived from observation and the theoretical conclusions of Prof. R. S. WOODWARD's remarkable paper on the "Mechanical Interpretation of the Variations of Latitudes" in *A.J.* 345. Treating the earth as a body of variable form, or subject to changes in the relative position of some portions of its mass, he arrives at general theoretical expressions for the coordinates of the pole, embodied in his equations (31). To quote his interpretation of them,—

"Thus, in this case, which appears to be the one with which astronomy is most concerned, the motion of the pole of rotation is defined as the resultant of three distinct types of motion, namely: First, a motion in a circle whose center is the pole of figure, with sensibly constant angular velocity λ; secondly, the sum of a series of elliptical motions dependent on the angular velocity μ; thirdly, the sum of a series of elliptical motions dependent on the angular velocity ν. Although these series of elliptical terms are infinite, it is practically certain that a small number of terms will suffice to express the observed motions of the pole of rotation."

The statement herein contained regarding the constancy of the angular velocity of the circular motion is subsequently qualified by observing that there are causes that might produce changes in both the velocity and the amplitude of this term.

The coincidence in every particular of the above conclusions arrived at from the diverse points of view of theory and observation, is not a little remarkable. The theory does not assign, *a priori*, either the velocities or the amplitudes of the terms in λ, μ, ν: these must be supplied by observation.

The apparent verification of WOODWARD's theory is a hopeful augury of progress. Should subsequent observation prove it to be real, a most important step will have been gained in our knowledge of this perplexing subject.

OBSERVATION OF SUNSPOTS AT BOSTON UNIVERSITY OBSERVATORY.

BY W. J. BANNAN AND W. H. STONE, STUDENTS IN ASTRONOMY.

The observations made during the college year 1901–2 when compared with those of the previous year (see *A.J.* 506), show a decline in solar activity, although the theoretical time of minimum was passed two years since. A publication of the detailed observations, as in former years, is scarcely warranted.

The following comparison of the results for the last two college years shows clearly the decline in activity. For each item the figures are for the two years in chronological order.

Number of different spots, 138, 24. Greatest number in any one group, 54, 7. Per cent. of days when no spots were found 77.6, 83.2.

There has been a slight increase in latitude, but scarcely enough to indicate compliance with SPOERER's law of sunspot latitudes when a minimum has been passed.

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ON THE FALLACY OF THE METHOD COMMONLY EMPLOYED IN FINDING THE PROBABLE ERROR OF A FUNCTION OF TWO OR MORE QUANTITIES WHOSE ADJUSTED VALUES HAVE BEEN DERIVED FROM THE SAME LEAST-SQUARE SOLUTION,

By HERBERT L. RICE.

It not unfrequently happens that an investigator has determined by the method of least-squares the most probable values of certain quantities, together with their probable errors, and requires, from the data thus furnished, the probable error of some function of two or more of the quantities in question. Thus, suppose that x, y and z denote the final values which result from a solution of three (simultaneous) normal equations in those quantities; p_x, p_y, p_z, their weights as found from the solution; and r_x, r_y, r_z, their probable errors. Then, if $u \equiv f(x, y)$ designate some function of x and y, it appears to be the practice among astronomers generally to compute the probable error of u from the simple formula

$$(1) \quad \left\{ \quad r_u = \sqrt{r_x^2\left(\frac{\partial u}{\partial x}\right)^2 + r_y^2\left(\frac{\partial u}{\partial y}\right)^2} \right.$$

The value of r_u thus found is, however, quite erroneous. Whence the object of this paper is to call attention to the fallacy involved, and to develop in general terms the proper formula to be employed in such cases.

If we suppose that x and y have been derived separately from two distinct and independent least-square solutions, each based upon its own set of observed quantities, then will the errors r_x and r_y be entirely independent of each other, and the value of r_u given by (1) is clearly correct under these conditions. Such, however, are not the conditions under consideration. In the case first supposed, x and y are inter-dependent, and *not* independent quantities; for it is clear that any change or error made in one of the observed values upon which the solution depends, will alter the values of x, y and z by amounts which must be mutually consistent.

In order to write down (for the present without proof) the proper formula for the computation of r_u, some consideration of the fundamental data is necessary. Let the normal equations determining x, y, z be —

$$\left.\begin{aligned}
[aa]x + [ab]y + [ac]z &= [am] \equiv K_1 \\
[ab]x + [bb]y + [bc]z &= [bm] \equiv K_2 \\
[ac]x + [bc]y + [cc]z &= [cm] \equiv K_3
\end{aligned}\right\} \qquad (2)$$

and let the solution of these in general terms take the form

$$\left.\begin{aligned}
x &= c_{11}K_1 + c_{12}K_2 + c_{13}K_3 \\
y &= c_{21}K_1 + c_{22}K_2 + c_{23}K_3 \\
z &= c_{31}K_1 + c_{32}K_2 + c_{33}K_3
\end{aligned}\right\} \qquad (3)$$

Then the correct value of r_u is given by the formula

$$r_u = r_0\sqrt{c_{11}\left(\frac{\partial u}{\partial x}\right)^2 + c_{22}\left(\frac{\partial u}{\partial y}\right)^2 + 2c_{12}\left(\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}\right)} \quad \Big\} \quad (4)$$

where r_0 is the probable error of an observation of weight unity.

The coefficients c in the second members of (3) are symmetrical with respect to the principal diagonal, $c_{11}K_1$, $c_{22}K_2$, $c_{33}K_3$; hence in (4) it is immaterial whether we write c_{12} or c_{21}.

Since $c_{11} = \dfrac{1}{p_x}$, $c_{22} = \dfrac{1}{p_y}$, $r_x = \dfrac{r_0}{\sqrt{p_x}}$, $r_y = \dfrac{r_0}{\sqrt{p_y}}$, it follows that the erroneous formula (1) gives identically the same result as (4) when the third term under the radical of the latter is omitted. It is proper to add that in general this term is by no means insignificant; there being no more reason for omitting it than for neglecting either of the other terms.

The problem, then, is fully indicated by the foregoing statements. I may here properly state that my own attention and interest were directed to it some time since by Professor W. S. HARSHMAN, U.S.N., now Director of the *Nautical Almanac*. About 1894 Prof. HARSHMAN (then assistant, *American Ephemeris*), presented as a Doctor's Thesis an investigation of the orbit of *Deimos*. Therein, following the usual method, he assumed circular elements, and thence formed equations of condition involving six unknowns, or corrections to the assumed elements. These were reduced in the usual manner to six normal equations, whence were derived the values of the six unknowns, together with their probable errors. Four of these quantities were immediately available as corrections to four of the elements, and with these we need not further concern ourselves. The remaining pair furnished by the solution were, in the usual notation, the quantities

$$\xi = e\cos\omega \quad , \quad \eta = e\sin\omega$$

where e and ω are the eccentricity and the angle determining the position of the line of apsides, respectively. Having found ξ and η, and their probable errors, the question which naturally presented itself to Prof. HARSHMAN was how to compute in a rigorous manner the probable errors of the functions

$$e = \sqrt{\xi^2+\eta^2} \quad , \quad \omega = \tan^{-1}\left(\frac{\eta}{\xi}\right)$$

An investigation of the problem was immediately made by the gentleman named, which resulted in a rigorous formula for computing r_e and r_ω; the solution taking substantially the form indicated in (4). As already stated, Prof. HARSHMAN has since kindly called my attention to the problem as one involving considerable interest, both from a theoretical and a practical point of view. Taking the matter up, I obtained independently the same result as did Prof. HARSHMAN. I have since been greatly surprised at finding a number of instances wherein astronomers of much prominence have fallen into the error of proceeding in accordance with formula (1); in fact, instances of the use of a correct formula are quite difficult to discover. I therefore believe that a rigorous investigation of the problem in general terms will not be out of place. This I proceed to develop along substantially the same lines as were followed in my original demonstration of the problem.

Let there be given n equations of condition, involving μ unknown quantities ($n > \mu$), as follows:

$$(5)\qquad \left\{\begin{array}{l} A_1'x_1 + A_2'x_2 + \;.\;.\;.\;.\;.\;.\;.\; + A_\mu'x_\mu = M_1 \\ A_1''x_1 + A_2''x_2 + \;.\;.\;.\;.\;.\;.\;.\; + A_\mu''x_\mu = M_2 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \\ A_1^{(n)}x_1 + A_2^{(n)}x_2 + \;.\;.\;.\;.\;.\;.\;.\; + A_\mu^{(n)}x_\mu = M_n \end{array}\right.$$

The absolute terms of these equations are independent quantities, resulting from n distinct observations. Let the weights assigned to these observations be denoted by $w_1, w_2, \ldots\ldots w_n$, respectively; and let us put, for brevity,

$$A_s^{(t)}\sqrt{w_t} = a_s^{(t)} \quad , \quad M_t\sqrt{w_t} = m_t \tag{6}$$

Accordingly, if each of the equations (5) be multiplied throughout by the square root of its assigned weight, thus reducing the group to a common standard of weight, we shall have the following set of uniform equations, in which the probable error of each absolute term is equal to that of an observation of weight unity:

$$\left.\begin{array}{l} a_1'x_1 + a_2'x_2 + \;.\;.\;.\;.\;.\;.\;.\; + a_\mu'x_\mu = m_1 \\ a_1''x_1 + a_2''x_2 + \;.\;.\;.\;.\;.\;.\;.\; + a_\mu''x_\mu = m_2 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \\ a_1^{(n)}x_1 + a_2^{(n)}x_2 + \;.\;.\;.\;.\;.\;.\;.\; + a_\mu^{(n)}x_\mu = m_n \end{array}\right\} \tag{7}$$

The normal equations are, therefore,

$$\left.\begin{array}{l} [a_1a_1]x_1 + [a_1a_2]x_2 + \;.\;.\;.\; + [a_1a_\mu]x_\mu = [a_1m] = K_1 \\ [a_2a_1]x_1 + [a_2a_2]x_2 + \;.\;.\;.\; + [a_2a_\mu]x_\mu = [a_2m] = K_2 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \\ [a_\mu a_1]x_1 + [a_\mu a_2]x_2 + \;.\;.\;.\; + [a_\mu a_\mu]x_\mu = [a_\mu m] = K_\mu \end{array}\right\} \tag{8}$$

where

$$\left.\begin{array}{l} [a_ra_s] = [a_sa_r] = a_r'a_s' + a_r''a_s'' + \;.\;.\;.\; + a_r^{(n)}a_s^{(n)} \\ [a_sa_s] = a_s'a_s' + a_s''a_s'' + \;.\;.\;.\; + a_s^{(n)}a_s^{(n)} \end{array}\right\} \tag{9}$$

and (10)

$$K_s = [a_sm] = a_s'm_1 + a_s''m_2 + \;.\;.\; + a_s^{(t)}m_t + \;.\;.\; + a_s^{(n)}m_n$$

Let the solution of the equations (8) be found in general terms, as follows:

$$\left.\begin{array}{l} x_1 = c_{11}K_1 + c_{12}K_2 + \;.\;.\;.\;.\; + c_{1\mu}K_\mu \\ x_2 = c_{21}K_1 + c_{22}K_2 + \;.\;.\;.\;.\; + c_{2\mu}K_\mu \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; \\ x_s = c_{s1}K_1 + c_{s2}K_2 + \;.\;.\;.\;.\; + c_{s\mu}K_\mu \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; \\ x_\mu = c_{\mu1}K_1 + c_{\mu2}K_2 + \;.\;.\;.\;.\; + c_{\mu\mu}K_\mu \end{array}\right\} \tag{11}$$

Having expressed the solution of our normal equations in this general form, we have only to substitute from (8) the numerical values of $K_1, K_2, \ldots\ldots K_\mu$, in order to obtain the required values of the μ unknowns. Moreover, it is well known that the weights $(p_1, p_2, \ldots\ldots, p_\mu)$ of $x_1, x_2, \ldots\ldots, x_\mu$ are the reciprocals of the coefficients $c_{11}, c_{22}, \ldots\ldots, c_{\mu\mu}$ respectively; so that we have, generally,

$$p_s = \text{weight of } x_s = \frac{1}{c_{ss}} \tag{12}$$

Now let

$$u = F(x_r, x_s, x_t, \ldots\ldots, x_v, x_w) \tag{13}$$

denote a function of any or all of the variables $x_1, x_2, \ldots, x_\mu$. Thus the subscripts $\alpha, \beta, \gamma, \ldots, \eta, \theta$ may designate any series of positive integral numbers — ranged in order of magnitude — within the limits 1 and μ.

Upon substituting in the second member of (13) the numerical values of $x_\alpha, x_\beta, \ldots, x_\theta$, found from (11), we have at once the most probable value of the function u. The problem now before us, therefore, is to find an expression for r_u, the probable error of u.

By (10) and (11), or by (8) alone, it is evident that u is ultimately a function of the n independent quantities, $m_1, m_2, \ldots, m_n$. Hence we have

$$\epsilon_u^2 = \epsilon_0^2\left[\left(\frac{\partial u}{\partial m_1}\right)^2 + \left(\frac{\partial u}{\partial m_2}\right)^2 + \ldots + \left(\frac{\partial u}{\partial m_n}\right)^2\right]$$

that is

$$\frac{\epsilon_u^2}{\epsilon_0^2} = \sum_1^n{}_i \left(\frac{\partial u}{\partial m_i}\right)^2 \tag{14}$$

ϵ_u being the mean error of u, and ϵ_0 that of an observation of weight unity. We shall considerably shorten our subsequent operations by adopting the following abridged notation. Let us put

$$\}\varphi(\alpha)\{^* \equiv \varphi(\alpha) + \varphi(\beta) + \ldots + \varphi(\eta) + \varphi(\theta)$$

and

$$\}\Psi(\alpha, \beta)\{^{**} \equiv \Psi(\alpha, \beta) + \Psi(\alpha, \gamma) + \ldots + \Psi(\eta, \theta)$$

The following are simple applications by way of illustration:

$$x_\alpha^* = x_\alpha + x_\beta + x_\gamma + \ldots + x_\eta + x_\theta$$
$$(x_\alpha^*)^2 = \}x_\alpha^2\{^* + \}2x_\alpha x_\beta\{^{**}$$

We now proceed to develop the right-hand member of (14). We have

$$\frac{\partial u}{\partial m_i} = \frac{\partial u}{\partial x_\alpha}\cdot\frac{\partial x_\alpha}{\partial m_i} + \frac{\partial u}{\partial x_\beta}\cdot\frac{\partial x_\beta}{\partial m_i} + \ldots + \frac{\partial u}{\partial x_\eta}\cdot\frac{\partial x_\eta}{\partial m_i} + \frac{\partial u}{\partial x_\theta}\cdot\frac{\partial x_\theta}{\partial m_i} = \left\}\frac{\partial u}{\partial x_\alpha}\cdot\frac{\partial x_\alpha}{\partial m_i}\right\{^*$$

and therefore

$$\left(\frac{\partial u}{\partial m_i}\right)^2 = \left\}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\cdot\left(\frac{\partial x_\alpha}{\partial m_i}\right)^2\right\{^* + \left\}2\frac{\partial u}{\partial x_\alpha}\cdot\frac{\partial u}{\partial x_\beta}\left(\frac{\partial x_\alpha}{\partial m_i}\cdot\frac{\partial x_\beta}{\partial m_i}\right)\right\{^{**}$$

Whence, by summation,

$$\sum_1^n{}_i\left(\frac{\partial u}{\partial m_i}\right)^2 = \left\}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\cdot\sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\right)^2\right\{^* + \left\}2\frac{\partial u}{\partial x_\alpha}\cdot\frac{\partial u}{\partial x_\beta}\cdot\sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\cdot\frac{\partial x_\beta}{\partial m_i}\right)\right\{^{**} \tag{15}$$

The function u is general; let us assume, in particular, that $u = x_\alpha$; then (14) becomes

$$\frac{\epsilon_\alpha^2}{\epsilon_0^2} = \sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\right)^2 \tag{16}$$

ϵ_α being the mean error of x_α. But since

$$\epsilon_\alpha^2 = \frac{\epsilon_0^2}{p_\alpha}$$

it follows from (12) that

$$\frac{\epsilon_\alpha^2}{\epsilon_0^2} = r_{\alpha\alpha}$$

and hence (16) becomes

$$\sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\right)^2 = r_{\alpha\alpha} \tag{17}$$

Upon substituting this expression in (15), and combining the result with (14), we obtain

$$\frac{\epsilon_u^2}{\epsilon_0^2} = \left\}r_{\alpha\alpha}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\right\{^* + \left\}2\frac{\partial u}{\partial x_\alpha}\cdot\frac{\partial u}{\partial x_\beta}\cdot\sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\cdot\frac{\partial x_\beta}{\partial m_i}\right)\right\{^{**} \tag{18}$$

It now remains to develop the terms of which

$$\sum_1^n{}_i\left(\frac{\partial x_\alpha}{\partial m_i}\cdot\frac{\partial x_\beta}{\partial m_i}\right)$$

is the type. For this purpose, let us denote by Δ the determinant whose elements are the coefficients of $x_1, x_2, \ldots, x_\mu$ in (8); that is, let us put

$$\Delta = \begin{vmatrix} [a_1a_1], [a_1a_2], \ldots [a_1a_\alpha] \ldots, [a_1a_\mu] \\ [a_2a_1], [a_2a_2], \ldots [a_2a_\alpha] \ldots [a_2a_\mu] \\ \ldots \\ \ldots \\ [a_\mu a_1], [a_\mu a_2], \ldots [a_\mu a_\alpha] \ldots [a_\mu a_\mu] \end{vmatrix} \tag{19}$$

and let the quantities

$$\left.\begin{matrix} A_{11} & A_{12} & \ldots & A_{1\alpha} & \ldots & A_{1\mu} \\ A_{21} & A_{22} & \ldots & A_{2\alpha} & \ldots & A_{2\mu} \\ \ldots \\ \ldots \\ A_{\mu 1} & A_{\mu 2} & \ldots & A_{\mu\alpha} & \ldots & A_{\mu\mu} \end{matrix}\right\} \tag{20}$$

designate the co-factors of the corresponding elements in Δ.

Expressing the value of x_α from (8) in determinant form, we have

$$x_\alpha = \frac{\Delta_\alpha}{\Delta}$$

where Δ_α is formed directly from Δ by replacing the column $[a_1a_\alpha], [a_2a_\alpha], \ldots [a_\mu a_\alpha]$ by a new column whose elements are $K_1, K_2, \ldots K_\mu$. Developing

Δ, in terms of the elements and corresponding co-factors of this particular column, we obtain

$$(21)\qquad \Delta x_\alpha = A_{1\alpha} K_1 + A_{2\alpha} K_2 + A_{3\alpha} K_3 + \ldots\ldots + A_{\mu\alpha} K_\mu$$

Comparing this expression with (11), we have, generally,

$$(22)\qquad c_{\alpha\beta} = \frac{A_{\beta\alpha}}{\Delta}$$

Differentiating (21) with respect to m_i, observing that Δ and the co-factors A do not involve the quantities m, we have

$$\Delta \frac{\partial x_\alpha}{\partial m_i} = A_{1\alpha} \frac{\partial K_1}{\partial m_i} + A_{2\alpha} \frac{\partial K_2}{\partial m_i} + \ldots + A_{\mu\alpha} \frac{\partial K_\mu}{\partial m_i}$$

From (10) we derive

$$\frac{\partial K_\beta}{\partial m_i} = a_\beta^{(i)}$$

and hence the preceding equation becomes

(23)

$$\Delta \frac{\partial x_\alpha}{\partial m_i} = a_1^{(i)} A_{1\alpha} + a_2^{(i)} A_{2\alpha} + \ldots + a_\beta^{(i)} A_{\beta\alpha} + \ldots + a_\mu^{(i)} A_{\mu\alpha}$$

By interchanging α and β, we have

(24)

$$\Delta \frac{\partial x_\beta}{\partial m_i} = a_1^{(i)} A_{1\beta} + a_2^{(i)} A_{2\beta} + \ldots + a_\alpha^{(i)} A_{\alpha\beta} + \ldots + a_\mu^{(i)} A_{\mu\beta}$$

Multiplying (23) and (24) together, member for member, we obtain

$$\begin{aligned}
\Delta^2 \frac{\partial x_\alpha}{\partial m_i} \cdot \frac{\partial x_\beta}{\partial m_i} = {} & A_{1\beta}\left(a_1^{(i)} a_1^{(i)} A_{1\alpha} + a_2^{(i)} a_1^{(i)} A_{2\alpha} + \ldots + a_\mu^{(i)} a_1^{(i)} A_{\mu\alpha}\right) \\
& + A_{2\beta}\left(a_1^{(i)} a_2^{(i)} A_{1\alpha} + a_2^{(i)} a_2^{(i)} A_{2\alpha} + \ldots + a_\mu^{(i)} a_2^{(i)} A_{\mu\alpha}\right) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots \\
& + A_{\alpha\beta}\left(a_1^{(i)} a_\alpha^{(i)} A_{1\alpha} + a_2^{(i)} a_\alpha^{(i)} A_{2\alpha} + \ldots + a_\mu^{(i)} a_\alpha^{(i)} A_{\mu\alpha}\right) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots \\
& + A_{\mu\beta}\left(a_1^{(i)} a_\mu^{(i)} A_{1\alpha} + a_2^{(i)} a_\mu^{(i)} A_{2\alpha} + \ldots + a_\mu^{(i)} a_\mu^{(i)} A_{\mu\alpha}\right)
\end{aligned}$$

Upon giving to the character i in this relation the successive values, $1, 2, 3, \ldots\ldots, n$, we obtain n such equations; and the sum of these, by virtue of (9), may be written as follows:

$$\begin{aligned}
\Delta^2 \sum_1^n{}_i \left(\frac{\partial x_\alpha}{\partial m_i} \cdot \frac{\partial x_\beta}{\partial m_i}\right) = {} & \\
(25)\qquad & A_{1\beta}\left([a_1 a_1] A_{1\alpha} + [a_2 a_1] A_{2\alpha} + \ldots + [a_\mu a_1] A_{\mu\alpha}\right) \\
& + A_{2\beta}\left([a_1 a_2] A_{1\alpha} + [a_2 a_2] A_{2\alpha} + \ldots + [a_\mu a_2] A_{\mu\alpha}\right) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots \\
& + A_{\alpha\beta}\left([a_1 a_\alpha] A_{1\alpha} + [a_2 a_\alpha] A_{2\alpha} + \ldots + [a_\mu a_\alpha] A_{\mu\alpha}\right) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots \\
& + A_{\mu\beta}\left([a_1 a_\mu] A_{1\alpha} + [a_2 a_\mu] A_{2\alpha} + \ldots + [a_\mu a_\mu] A_{\mu\alpha}\right)
\end{aligned}$$

Among the elementary theorems of determinants is the following:

If the elements of any column be multiplied in order by the co-factors of the corresponding elements of any *other* column, the sum of the products thus formed will be *zero*.

To apply this theorem, we observe that the μ expressions, or series, contained within parentheses in the second member of (25), are simply the μ results derived by summing the products formed by multiplying the quantities in column α of (20) (*i.e.*, the co-factors $A_{1\alpha}, A_{2\alpha}, \ldots, A_{\mu\alpha}$) by the corresponding elements of columns $1, 2, \ldots, \alpha, \ldots, \mu$, respectively, in (19). Hence, by the above theorem, each of the μ parentheses in (25) is equal to zero, except the one whose coefficient is $A_{\alpha\beta}$; and this parenthesis is merely an expansion of Δ. Thus the right-hand member of (25) reduces to the simple term $\Delta A_{\alpha\beta}$, and the equation becomes, therefore,

$$\sum_1^n{}_i \left(\frac{\partial x_\alpha}{\partial m_i} \cdot \frac{\partial x_\beta}{\partial m_i}\right) = \frac{A_{\alpha\beta}}{\Delta} \qquad (26)$$

Since the first member of this equation remains unaltered in value when α and β are interchanged, it follows that the second member is also unaffected by such interchange; thus we have

$$A_{\alpha\beta} = A_{\beta\alpha} \qquad (27)$$

and therefore, by (22),

$$c_{\alpha\beta} = \frac{A_{\beta\alpha}}{\Delta} = \frac{A_{\alpha\beta}}{\Delta} = c_{\beta\alpha} \qquad (28)$$

Hence the co-factors A in (20), and likewise the coefficients c in (11), are symmetrical with respect to their principal diagonals, $A_{11}, A_{22}, \ldots, A_{\mu\mu}$, and $c_{11}, c_{22}, \ldots, c_{\mu\mu}$, respectively. These relations result directly from the fact that a like symmetry exists among the coefficients of the normal equations (8).

From (26) and (28) we have

$$\sum_1^n{}_i \left(\frac{\partial x_\alpha}{\partial m_i} \cdot \frac{\partial x_\beta}{\partial m_i}\right) = c_{\alpha\beta} = c_{\beta\alpha} \qquad (29)$$

and hence (18) becomes [1]

$$\frac{\epsilon_u^2}{\epsilon_0^2} = \left\{ c_{\alpha\alpha} \left(\frac{\partial u}{\partial x_\alpha}\right)^2 \right\}^{*} + \left\{ 2 c_{\alpha\beta} \frac{\partial u}{\partial x_\alpha} \cdot \frac{\partial u}{\partial x_\beta} \right\}^{**} \qquad (30)$$

Since $r_u : r_0 = \epsilon_u : \epsilon_0$, the relation (30) at once furnishes the required value of r_u. In order to express the solution more completely, however, let us put

$[vv]$ = the sum of the squares of the residuals derived from (7); we then have, finally,

[1] In deriving the relation (17), in the earlier portion of this discussion, we naturally assumed the validity of the well known formula (12). A casual review of the steps in the development of (29) will clearly show, however, that we are justified in writing α for β in the latter expression. Doing this, we have (17) at once; and thus the truth of the relation (12) is independently demonstrated.

$$(31)\quad \begin{cases} r_0 = 0.6745\,\epsilon_0 = 0.6745\sqrt{\dfrac{[vv]}{n-\mu}} \\ \text{and} \\ r_u = r_0\sqrt{\left\{c_{\alpha\alpha}\left(\dfrac{\partial u}{\partial x_\alpha}\right)^2\right\}^* + \left\{2c_{\alpha\beta}\dfrac{\partial u}{\partial x_\alpha}\cdot\dfrac{\partial u}{\partial x_\beta}\right\}^{**}} \end{cases}$$

If, above the terms comprising the second members of (11), we write x_1 over $c_{11}K_1$, x_2 over $c_{12}K_2$, ... , and x_μ over $c_{1\mu}K_\mu$, then the coefficients c to be employed in (31) may be readily selected by observing the following precepts:

The coefficient $c_{\alpha\alpha}$ of the term $\left(\dfrac{\partial u}{\partial x_\alpha}\right)^2$ is found at the intersection of row x_α and column x_α.

The coefficient $c_{\alpha\beta}$ of $2\dfrac{\partial u}{\partial x_\alpha}\cdot\dfrac{\partial u}{\partial x_\beta}$ is found at the intersection of $\left\{\begin{matrix}\text{row}\\ \text{column}\end{matrix}\right\}$ x_α and $\left\{\begin{matrix}\text{column}\\ \text{row}\end{matrix}\right\}$ x_β.

If we neglect the second set of terms under the radical in (31), the value of r_u thus obtained is readily seen to be precisely that which results from the arbitrary and erroneous assumption that x_α, x_β, ... , x_θ may be treated as independent quantities. For upon making this assumption, we have directly, from (13),

$$\epsilon_u^2 = \left\{\epsilon_\alpha^2\left(\frac{\partial u}{\partial x_\alpha}\right)^2\right\}^* = \left\{\frac{\epsilon_0^2}{p_\alpha}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\right\}^* = \epsilon_0^2\left\{c_{\alpha\alpha}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\right\}^*$$

Whence

$$\frac{\epsilon_u^2}{\epsilon_0^2} = \left\{c_{\alpha\alpha}\left(\frac{\partial u}{\partial x_\alpha}\right)^2\right\}^*$$

a comparison of which with (30) proves the above statement.

To illustrate the foregoing principles, I have selected an example from Prof. HALL's investigation of the orbit of the satellite of *Neptune*, published as Appendix II of the Washington Observations for 1881. For brevity of reference, I shall designate any page of this appendix simply "HALL, page —."

The observations are divided into five distinct groups, each receiving a separate discussion. The example selected is taken from the solution of third group, viz., that of 1883–1884.

The observation equations involve the six unknowns, ΔN, ΔJ, Δu, ξ, η, and Δa (HALL, page 13): where $\xi = ae\cos\omega$, and $\eta = ae\sin\omega$. From the fifty-three equations of condition comprising the group in question (HALL, pages 17–18), the following normal equations are derived (HALL, page 22):

1883–84.

$$\left.\begin{array}{rrrrrrcrcl}
+11.0092\,\Delta N & -\ 0.7876\,\Delta J & -\ 5.2266\,\Delta u & -\ 5.2201\,\xi & -\ 2.6067\,\eta & -\ 5.7328\,\Delta a & = & +\ 1.6759 & = & K_1 \\
-\ 0.7876\,\Delta N & +12.6805\,\Delta J & -\ 5.1747\,\Delta u & -\ 2.9053\,\xi & +\ 2.9935\,\eta & -\ 3.1076\,\Delta a & = & -\ 9.6847 & = & K_2 \\
-\ 5.2266\,\Delta N & -\ 5.1747\,\Delta J & +15.4787\,\Delta u & +\ 9.6153\,\xi & +\ 5.6117\,\eta & +\ 2.2564\,\Delta a & = & +11.9035 & = & K_3 \\
-\ 5.2201\,\Delta N & -\ 2.9053\,\Delta J & +\ 9.6153\,\Delta u & +43.9472\,\xi & +32.1071\,\eta & -\ 8.2074\,\Delta a & = & +\ 3.3609 & = & K_4 \\
-\ 2.6067\,\Delta N & +\ 2.9935\,\Delta J & +\ 5.6117\,\Delta u & +32.1071\,\xi & +60.2877\,\eta & -\ 7.5748\,\Delta a & = & +\ 2.5668 & = & K_5 \\
-\ 5.7328\,\Delta N & -\ 3.1076\,\Delta J & +\ 2.2564\,\Delta u & -\ 8.2074\,\xi & -\ 7.5748\,\eta & +42.3308\,\Delta a & = & +\ 0.9216 & = & K_6
\end{array}\right\}\quad(32)$$

The solution of these equations in general terms gives the following values of ξ, η and Δa, the numbers within brackets being logarithms:

$$\begin{array}{llllllll}
 & \text{(I)} & \text{(II)} & \text{(III)} & \text{(IV)} & \text{(V)} & \text{(VI)} & \\
 & \dots & \dots & \dots & (\xi) & (\eta) & (\Delta a) & \\
\text{(IV)}\ \ \xi = & +[8.22850]\,K_1 & +[8.16179]\,K_2 & -[8.05456]\,K_3 & +[8.66538]\,K_4 & -[8.35141]\,K_5 & +[7.95016]\,K_6 & = -0.14106 \\
\text{(V)}\ \ \eta = & -[7.84010]\,K_1 & -[8.15030]\,K_2 & -[7.55019]\,K_3 & -[8.35141]\,K_4 & +[8.46487]\,K_5 & -[6.96440]\,K_6 & = +0.08157 \\
\text{(VI)}\,\Delta a = & +[8.30696]\,K_1 & +[8.04197]\,K_2 & +[7.05802]\,K_3 & +[7.95016]\,K_4 & -[6.96440]\,K_5 & +[8.45759]\,K_6 & = -0.00506
\end{array}\quad(33)$$

The assumed value of the mean distance being

$$a_0 = 16''.275$$

HALL, page 11), we find, therefore,

$$(34)\quad \begin{cases} a = a_0 + \Delta a = 16''.270 \\ \omega = \tan^{-1}\left(\dfrac{\eta}{\xi}\right) = 150^\circ.0 \\ e = \dfrac{\sqrt{\xi^2+\eta^2}}{a} = 0.010015 \end{cases}$$

These results agree substantially with those given by Prof. HALL. The latter, however, in computing e, takes $a = 16''.275$ $(= a_0)$ instead of $16''.270$, and hence gets a value of e which differs slightly from that recorded above (see HALL, page 24).

Let the coefficients of K_1, K_2, ... , K_6 in (33) be designated in accordance with the notation adopted in the general expressions (11). Thus, if in the latter we suppose $\mu = 6$, and take x_4, x_5, x_6 to represent the quantities ξ, η and Δa, we have from (33) —

$$(35)\quad \begin{cases} \log c_{44} = 8.66538 & \log c_{45} = \log c_{54} = 8.35111_n \\ \log c_{55} = 8.16487 & \log c_{46} = \log c_{64} = 7.95016 \\ \log c_{66} = 8.15759 & \log c_{56} = \log c_{65} = 6.96110_n \end{cases}$$

Again, since $\nu = 53$, $\mu = 6$ and $[vv] = 2.8777$ (HALL, page 23), we find by (31) —

$$(36)\qquad \log r_0 = 9.22245$$

Having tabulated the foregoing data, we proceed to determine r_ω, the probable error of ω. From (34) we derive

$$\frac{\partial\omega}{\partial\xi} = -\frac{\eta}{\xi^2+\eta^2} = -\frac{\eta}{a^2e^2}$$

$$\frac{\partial\omega}{\partial\eta} = +\frac{\xi}{\xi^2+\eta^2} = +\frac{\xi}{a^2e^2}$$

Whence, by (31), we have

$$(37)\qquad r_\omega = \frac{57.29\, r_0}{a^2e^2}\sqrt{c_{44}\eta^2 + c_{55}\xi^2 - 2c_{45}\xi\eta}$$

the numerical factor being introduced in order that r_ω may be expressed in degrees of arc. Employing the numerical data given in (33) — (36), and evaluating the second member of (37), we find

$$r_\omega = \pm 6^\circ.94$$

Omitting the third term under the radical in (37), we find $r_\omega = \pm 10^\circ.73$, which agrees with the (erroneous) value computed by Prof. HALL (see HALL, page 24).

Let us now find the probable error of the eccentricity. By (34) we observe that e is a function of the three quantities, ξ, η and a; hence we require the derivatives

$$\frac{\partial e}{\partial\xi} = \frac{\xi}{a^2e}\ ,\quad \frac{\partial e}{\partial\eta} = \frac{\eta}{a^2e}\ ,\quad \frac{\partial e}{\partial a} = -\frac{e}{a}$$

The expression for r_e is, therefore,

$$(38)\qquad r_e = \frac{r_0}{a^2e}\sqrt{c_{44}\xi^2 + c_{55}\eta^2 + c_{66}a^2e^4 + 2c_{45}\xi\eta - 2c_{46}\xi ae^2 - 2c_{56}\eta ae^2}$$

from which we find

$$r_e = \pm 0.002516$$

By omitting the three terms containing the factor 2, we get $r_e = \pm 0.002102$, which is substantially the result found by Prof. HALL (HALL, page 24). It is therefore plain that the latter has computed r_ω and r_e upon the erroneous assumption that ξ, η and a are strictly independent quantities.

I have already stated that instances of error of this kind are far from uncommon. A number of such have recently come to my notice, conspicuous among which are the following:

The probable errors of π and e given by Prof. S. J. BROWN in *A.J.*, No. 467 (middle of page 83) are erroneous. The results were evidently computed in the same manner as those found by Prof. HALL.

The same criticism applies to the probable errors of Q and e given by H. STRUVE at the foot of page 43 of his "*Beobachtungen des Neptunstrabanten am 30-zölligen Pulkowaer Refractor*" (*Mém. de l'Acad. de St.-Pétersbourg*, VII^e Série, Tome XLII, No. 4).

The four formulas for probable error given by Prof. S. C. CHANDLER in *A.J.*, No. 315 (foot of page 19) are likewise erroneous through the omission of necessary terms. Moreover, it will be observed that the last two of these formulas are otherwise incorrect; in the expression for the probable error of T_1, the quantities ϵ_1^2 and ϵ_2^2 are interchanged; in that for G, the quantities ϵ_3^2 and ϵ_4^2 should be transposed.

ON THE POSSIBLE EXISTENCE OF STILL ANOTHER TERM OF THE POLAR MOTION,

By S. C. CHANDLER.

One indication afforded by the investigation in *A.J.* 522 was passed in silence because it was desirable to separate what was fairly demonstrated by that scrutiny, as it seemed to me, from what was problematical. But, since it will not escape the attention of any one who undertakes to examine the subject critically that there still remain outstanding residuals, of small amount to be sure, but of a peculiarly marked character, between the observed and computed coordinates of the table on p. 147, there seems to be no harm in mentioning the strong indication of a minute motion of fifteen-months' period. I cannot commit myself to the assertion of its existence, for two reasons. First, it is not certain that these small systematic differences may not disappear by a future adjustment of the coefficients of the other terms, although I confess I do not see how this can be done. Secondly, the differences in question are of an order too small for us at present to speculate much about, their range being only about 0''.05; the coordinates being

$$x = 0''.025 \sin (t - 2411700)\ 0^\circ.789$$
$$y = 0.025 \cos (t - 2411700)\ 0.789$$

Nevertheless these departures so persistently and harmoniously pervade the whole series that I think they should not be lost sight of, although they may be merely nominal results of computation or a deceptive concurrence of constant errors in the observations, but that their reality should be rigorously tested by future observation. In dealing with a phenomenon so obviously complex as these motions of the earth's axis are, and until we are certain of the superior limit of precision in astronomical measurement, it would be unphilosophical to ignore without examination such indications as these.

MAXIMUM OF 2815 *U GEMINORUM.*

By J. A. PARKHURST.

The following observations of the last maximum were made with the 12 and 40-inch refractors of the Yerkes Observatory. With the two exceptions noted, they were made with the wedge photometer described in the *Astrophysical Journal*, XIII, 249, by the method of equalizing with an artificial star.

Gr. M.T.		Aperture	
1902 April 3 15.8	13.7	12	visual
4 16.3	13.38	40	
12 16.9	13.14	40	
13 14.4	10.45	12	
14 13.9	9.76	12	
19 14.9	11.67	12	
23 14.2	12.9	12	visual
May 2 14.8	13.26	40	
8 14.5	13.24	40	

The magnitudes of the comparison-stars were determined with the same photometer, using as standards the three stars of the Potsdam Photometric Durchmusterung.

	DM.	Mag.
A	+21°1714	7.02
D	+21°1724	7.56
B	+22°1803	7.30

Following is the list of magnitudes adopted, the notation being that used by KNOTT and BAXENDELL, with the addition of the two fainter stars, *n* and *l'*:

	M		M
a	8.84	*k*	12.42
b	9.34	*l*	12.94
f	11.13	*n*	13.83
g	11.46	*l'*	14.33
h	11.61	*l + l'*	12.75

The star *l'* is about 10″ north following *l*, and the two seem to have been glimpsed as one star by various observers with apertures of 6 and 7 inches.

The coordinates from the variable of the three nearest stars were measured with the micrometer on the 40-inch as follows:

	R.A.	Decl.
l	−0.9	+1 44
n	+1.8	−0 50
k	+6.1	−2 5

Yerkes Observatory, 1902 *June* 9.

OBSERVATIONS OF VARIABLE STARS OF SHORT PERIOD, 1900–1902.

By PAUL S. YENDELL.

2279 *T Monocerotis.*

Eleven observations of this star in 1900, and forty-six from 1901 Dec. 1 to 1902 April 24, indicate four maxima and five minima, as follows, all deduced from the single curves.

MAXIMA	Wt.	MINIMA	Wt.
1901 Dec. 21.3	4	1900 Mar. 29.5	3
1902 Jan. 17.3	5	1901 Dec. 12.0	4
Feb. 13.3	4	1902 Jan. 31.5	4
Mar. 10.5	3	Mar. 3.4	3
		Mar. 29.4	3

2335 *W Geminorum.*

I have fifty-one observations of *W Geminorum* from 1901 Nov. 20 to 1902 April 28. From these five maxima and one minimum are deduced by the single curves, as follows:

MAXIMA	Wt.	MINIMUM	Wt.
1901 Dec. 18.1	3	1902 Jan. 14.1	4
1902 Jan. 9.8	4		
Jan. 25.4	3		
Feb. 11.5	4		
Mar. 30.1	2		

2509 *ζ Geminorum.*

Thirty-two observations of *ζ Geminorum*, from 1901 Dec. 27 to 1902 May 2, indicate four maxima and three minima, as follows:

MAXIMA	Wt.	MINIMA	Wt.
1902 Jan 7.1	4	1902 Jan. 1.3	2
Jan. 16.4	3	Jan. 12.0	3
Jan. 27.5	3	Jan. 20.5	3
Feb. 6.7	2		

2676 *U Monocerotis.*

Twenty observations, 1902 Jan. 13 to April 1, show a minimum of $7^M.1$ on Jan. 30, and a maximum of $6^M.2$ on Feb. 12. After the end of February the observations were very scattered, and not sufficient to indicate any phases.

Dorchester, 1902 *May* 17.

CONCERNING THE MAGNITUDE EQUATION FOR THE CAMBRIDGE ZONES, (SEE *A.J.*, NOS. 517, 519, 521).

BY H. H. TURNER.

Professor BOSS and I seem to have been at cross purposes. In my original paper (*M.N.*, LX, p. 3) I neglected proper motions altogether, with a small exception which has been the origin of the misunderstanding. Professor BOSS thought, owing to my careless wording, that I had applied P.M.'s, and pointed out that in that case certain systematic corrections were necessary, arising from the way in which the adopted P.M.'s had been deduced. To get P.M.'s from meridian observations at different epochs, we must use precession; and if we use a wrong precession we get wrong P.M.'s; and any work in which these are used must be corrected accordingly. This we are quite agreed about. But, as already stated, P.M.'s were *not* applied; and thus the results given in *M.N.*, LX, p. 3, do not require corrections of the kind indicated. I think Professor BOSS allows this, with a possible small exception to which I will recur.

Unfortunately I did not read Professor BOSS's paper (see *A.J.* 517) in this sense. I thought he was applying corrections arising out of precession itself, and was bewildered accordingly. Reading his paper again now in the light of his explanation I can see how I went astray; I ought to have allowed the second paragraph of his paper to color the whole; it should especially govern the fourth paragraph, whereas I read the latter independently. In palliation I may perhaps add that others read it in the same way.

Returning now to the small point on which we apparently still differ, perhaps the shortest cut to an agreement would be to exclude the first three groups of stars *altogether*. I corrected them for proper motion, *not* because they were bright, but because they were *few*, and uncorrected results were hopelessly wild. Hence the following paragraph in Professor BOSS's last paper is rather misleading:

"But Professor TURNER seems to have decided that the positions of the brighter stars have been so influenced by proper motion in the interval, 1880–1895, that our Oxford picture no longer represents for these stars the state of the sky in 1880. He therefore corrects the stars of his first three groups (first to fifth magnitude) for the effect of proper motion."

This does not properly represent my reason, which is given better in my original paper (*M.N.*, LX, p. 4) in words already quoted once (*A.J.*, No. 519, p. 121); to which I will now add, in italics, a little fuller explanation:

"For the first three groups the number of stars is so small that the results must not be treated too seriously. Proper motions have been applied *to these three groups* (taken from the Greenwich Catalogue 1880.0) for the interval between the Cambridge and Oxford observations, *because owing to the small number of stars in each group the results would not otherwise have been even approximately in accordance with those from the other groups.* In the other groups P.M. has been treated as accidental error."

To make the point clear let me give in illustration the individual results for the first group. The mean of the results uncorrected for P.M. is clearly meaningless; but when P.M. is applied, the mean is sufficiently like that for other groups to set down for comparison with them, but "not to be taken too seriously." Here they are:

Star's Name	Oxford Epoch	Adopted P.M.	Oxford–Camb. Uncorr.	Oxford–Camb. Corr. for P.M.
		s	s	s
α Androm.	1895.0	+0.0095	+0.24	+0.05
β Tauri	1896.6	+0.0013	+0.19	+0.16
Pollux	1895.1	−0.0481	−0.75	+0.18
α Coronae	1895.4	+0.0085	+0.25	+0.08
		Mean	−0.018	+0.118

But it seems to me waste time to further correct the $+0^s.118$ for minute systematic errors when the results are as yet only a first approximation. The constants with which the plates have been reduced will probably be much improved on revision; and then we can consider minutiæ, which will affect individual places. On the other hand results depending on the mean of a number of stars will not be much altered; and I quite expect to find that the Cambridge magnitude equation, for magnitudes fainter than 5.0, is sensibly that indicated in *Mon. Not.*, LX, p. 3.

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THE MASS OF THE RINGS OF *SATURN*.

By A. HALL.

The determination of this mass by BESSEL in 1831 is one of the results drawn from his investigation of the orbit of *Titan*, the bright satellite discovered by HUYGENS in 1655. This value of the mass is too great, as BESSEL himself points out, since the actions of the other satellites, and the figure of the planet are neglected; but BESSEL considers his result as giving an approximate value of the mass. The recent investigations of the theory of the Ring, and of its stability, have led to the idea that the mass of the Ring must be very small. MAXWELL's theory is that the Ring is composed of a great number of very small, discrete bodies, moving independently around *Saturn* in circular orbits. This is a return to the old notion of CASSINI, but MAXWELL was led to it by theory. If p denote the number of bodies in the Ring, MAXWELL finds the condition,

$$\text{Mass of Ring} < \frac{2}{p^2}$$

Thus if $p = 10000$,

$$\text{Mass of Ring} < \frac{1}{50000000}$$

the mass of *Saturn* being taken for the unit of mass. BESSEL's value is

$$\text{Mass of Ring} = \frac{1}{118}$$

These are the extreme values.

Although MAXWELL's theory is obscure in some places his results agree with observations. Thus, when *Saturn* is seen through the thin part of the Ring there is no refraction of light, indicating that the particles are discrete. From a discussion of the variations of light of the Ring Professor SEELIGER comes to conclusions which support MAXWELL's theory.

From a discussion of the observations of *Titan*, BESSEL found the motion of the line apsides in a Julian year to be

$$+1744''.7$$

This result has been confirmed by recent observations, and the numerical value has been a little increased, to about

$$+1793''.0$$

This is a fact to be accounted for by theory. The orbits of the seven other satellites are not so well known as that of *Titan*.

In order to compute the part of this motion produced by the figure of the planet we are obliged to make an assumption on the density of the matter forming the equatorial protuberance. *Saturn* is a planet of 70,000 miles diameter; its mean density is only three-fourths that of water, and it revolves on its axis in $10\frac{1}{4}$ hours. Hence the matter of the equatorial protuberance may be of very small density. The potential of the ellipsoid, with respect to a satellite, may be written, with sufficient approximation,

$$V = \frac{1}{2a}\cdot\left(\frac{k}{a^2} + \frac{5}{2}\cdot\frac{l}{a^4} + \frac{5.7}{2.4}\cdot\frac{m}{a^6} + \ldots\right).e^2$$

where a is the semi-major axis of the satellite's orbit, e its eccentricity, and $k, l, m, \ldots$ are constants depending on the constitution of the ellipsoid, and must be found from observations. This series converges so rapidly that we need only the first two terms. Taking the derivative of V with respect to e, and calling n the mean motion of the satellite, we have for the secular motion of the perisaturnium,

$$\frac{d\pi}{dt} = n\cdot\left(\frac{k}{a^2} + \frac{5}{2}\,\frac{l}{a^4} + \ldots\right)$$

We may change to the equatorial radius of the planet, and if we call this radius a, and take $a = 8''.75$, we have for the case of *Titan*,

$$\frac{d\pi}{dt} = (4.86260)\cdot\frac{k}{a^2} + (2.65158)\cdot\frac{l}{a^4} \qquad (1)$$

These logarithmic coefficients are for seconds of arc.

BESSEL has given an elegant formula for the attraction of a plane circular Ring on a point in its own plane (*Astr. Nachr.*, Bd. 9). In the case of *Titan* he finds

$$\frac{d\pi}{dt} = \frac{2\pi\,.\,365.25}{T}\,.\,m\,.\left\{\frac{3}{2}\cdot\left(\frac{1}{2}\right)^2\frac{\rho'^4-\rho^4}{\rho'^2-\rho^2}\cdot\frac{1}{a^2(1-e^2)^2}+\frac{5}{3}\left(\frac{1.3}{2.4}\right)^2\cdot\frac{\rho'^6-\rho^6}{\rho'^2-\rho^2}\cdot\left(2+\frac{3}{2}e^2\right)\frac{1}{a^4(1-e^2)^4}\right.$$
$$\left.+\frac{7}{4}\cdot\left(\frac{1.3.5}{2.4.6}\right)^2\cdot\frac{\rho'^8-\rho^8}{\rho'^2-\rho^2}\cdot\left(3+\frac{15}{2}e^2+\frac{15}{8}e^4\right)\cdot\frac{1}{a^6(1-e^2)^6}+\text{\&c.}\ \ldots\right\}$$

T is the period of the satellite, m is the mass of the Ring, and ρ, ρ', are the interior and exterior radii of the Ring. BESSEL's result may be found by taking the potential in the form,

$$V=\iint\frac{r'\,dr'\,d\theta}{(r^2+r'^2-2rr'\cos\theta)^{\frac{1}{2}}}$$

expanding the denominator by means of the $b_{\frac{1}{2}}^{(i)}$ coefficients of LAPLACE, and integrating with respect to θ from 0 to 2π, the result is

$$V=\pi\,.\int b_{\frac{1}{2}}^{(0)}\,.\,\frac{r'dr'}{r}$$

r' is the distance from the center of the planet to a point in the Ring, and as $\alpha=\frac{r'}{r}$, in the value of $b_{\frac{1}{2}}^{(0)}$ (*Méc. Cél.*, Tome I, p. 271), we find, after integration between the limits ρ and ρ', and noticing that the mass of the Ring is proportional to $\pi\,(\rho'^2-\rho^2)$,

$$V = m\,.\left[\frac{1}{r}+\frac{1}{2}\left(\frac{1}{2}\right)^2\cdot\frac{\rho'^4-\rho^4}{\rho'^2-\rho^2}\cdot\frac{1}{r^3}+\frac{1}{3}\left(\frac{1.3}{2.4}\right)^2\cdot\frac{\rho'^6-\rho^6}{\rho'^2-\rho^2}\cdot\frac{1}{r^5}\right.$$
$$\left.+\frac{1}{4}\left(\frac{1.3.5}{2.4.6}\right)^2\cdot\frac{\rho'^8-\rho^8}{\rho'^2-\rho^2}\cdot\frac{1}{r^7}+\ \ldots\right]$$

If we take the derivative of V with respect to r we shall have the force along r, which is all we need, since in this case the force perpendicular to the radius is zero. We have

$$-\frac{\partial V}{\partial r}=\frac{m}{r^2}\left[1+\frac{3}{2}\left(\frac{1}{2}\right)^2\cdot\frac{\rho'^4-\rho^4}{\rho'^2-\rho^2}\cdot\frac{1}{r^2}\right.$$
$$\left.+\frac{5}{3}\left(\frac{1.3}{2.4}\right)^2\cdot\frac{\rho'^6-\rho^6}{\rho'^2-\rho^2}\cdot\frac{1}{r^4}+\frac{7}{4}\left(\frac{1.3.5}{2.4.6}\right)^2\cdot\frac{\rho'^8-\rho^8}{\rho'^2-\rho^2}\cdot\frac{1}{r^6}+\ldots\right]$$

In the formula for the perturbation of the perisaturnium change the independent variable from t to v, the true anomaly; write $\frac{1+e\cos v}{a(1-e^2)}$ for $\frac{1}{r}$, expand powers of $1+e\cos v$, and integrate from 0 to 2π. The terms in the odd powers of $\cos v$ will contain $\sin v$ as a factor, and will disappear. The terms in even powers of $\cos v$ will be of the form $f(\cos v)\times\sin v$, which will disappear, and will also contain a term in v. Hence BESSEL's form for the attraction of the Ring. Reducing to numbers we have

$$(2)\qquad\qquad \frac{d\pi}{dt}=(5.31319)\,.\,m$$

m being the mass of the Ring.

There remain the actions of the satellites on *Titan*. I have computed the secular perturbations produced by the three bright satellites interior to *Titan*: *Rhea*, *Dione*, and *Tethys*. These results are

$$\textit{Rhea},\qquad \frac{d\pi}{dt}=(6.78547)\,.\,m_1 \qquad (3)$$

$$\textit{Dione},\qquad \frac{d\pi}{dt}=(6.40903)\,.\,m_2 \qquad (4)$$

$$\textit{Tethys},\qquad \frac{d\pi}{dt}=(6.16084)\,.\,m_3 \qquad (5)$$

Collecting results we have for the motion of the line of apsides of *Titan*, in a Julian year,

$$\left.\begin{aligned}\frac{d\pi}{dt}=(4.86260)\,.\,\frac{k}{a^2}+(2.65158)\,.\,\frac{l}{a^4}+(5.31319)\,.\,m\\ +(6.78547)\,m_1+(6.40903)\,.\,m_2+(6.16084)\,.\,m_3\end{aligned}\right\}\quad(6)$$

Strictly there should be four more terms in this equation giving the perturbations produced by *Iapetus* and *Hyperion*, the two satellites outside of *Titan*, and by *Enceladus* and *Mimas*, the satellites near the Ring. These are omitted, because three of them are distant from *Titan*; the mass of *Hyperion* must be very small, and all the masses are uncertain. From what precedes we make a rough estimate of their resultant action. In equation (6) the numerical coefficients are known with a fair degree of accuracy, but the values of the masses are uncertain, and the quantities $\frac{k}{a^2}$ and $\frac{l}{a^4}$ must be found from assumptions on the constitution of the planet. The masses may be inferred from the magnitudes of the satellites, since the mass of *Titan* is known approximately from its action on the line of apsides of the orbit of *Hyperion*. Taking the mass of *Saturn* for unity, the mass of *Titan* is nearly $\frac{1}{4500}$. The magnitude of *Titan* at an opposition of *Saturn* is 8^M, and that of *Rhea* is $9^M.5$. From these data we can compute a mass of *Rhea*, and this method might be followed for the other satellites. But I prefer to adopt the values of these masses found by Professor H. STRUVE from his observations. It is worth while to notice, however, that these values are considerably smaller than those found by comparing magnitudes. STRUVE's mass of *Rhea* is $\frac{1}{250000}$, while from the magnitudes we would find $\frac{1}{63000}$. For the masses we adopt,

Rhea,	$m_1=\frac{1}{250000}$
Dione,	$m_2=\frac{1}{536000}$
Tethys,	$m_3=\frac{1}{907600}$

These values give by (3), (4), (5), the following results:

$$\begin{array}{lrl} \textit{Rhea.} & \dfrac{d\pi}{dt} = & +24''.41 \\ \textit{Dione.} & \dfrac{d\pi}{dt} = & +\ 4.78 \\ \textit{Tethys.} & \dfrac{d\pi}{dt} = & +\ 1.60 \\ \text{Sum} & = & +30.79 \end{array}$$

If we add 15″ for the actions of the four other satellites, we have 45″.8 for the action of all the satellites. STRUVE finds from the motions of *Mimas* and *Tethys*,

$$\frac{k}{a^2} = 0.0242 \quad ; \quad \frac{l}{a^4} = 0.00070$$

With these values the first two terms of equation (6) give 1764″.0; and adding the actions of the satellites the sum is 1809″.8; or greater than the observed value, 1793″.0. From the physical condition of the planet I reduce the value of $\frac{k}{a^2}$ to 0.0222, an older result. The terms of (6) give 1618″.2, and hence

$$\frac{d\pi}{dt} = 1664''.0 + (5.31319).m = 1793''.0$$

The mass of the Rings is

$$m = \tfrac{1}{7092}$$

It is probable, I think, that the satellites produce more motion than 45″.8, and this mass of the Rings may be too great. The values of eleven constants are needed, and the uncertainties in the theory of this system can be removed only by further observation and discussion of the motions of the satellites. There is not much theoretical difficulty to be encountered, except in the case of *Hyperion*, whose theory has not yet been worked out, and which may require much labor. After a few years *Saturn* will return to our northern skies, and the means of observing are now so good that we may hope for a complete theory of this interesting system.

Goshen, Conn., 1902 *June* 9.

THE RATE OF THE RIEFLER SIDEREAL CLOCK NO. 56,

BY CHARLES S. HOWE.

In order that a clock may run at its best rate, it must be under constant pressure and constant temperature, or there must be a perfect compensation for pressure and temperature. It is impossible to obtain a perfect compensation for either of these, and therefore it is desirable to maintain a constant pressure and temperature in the clock case. Constant pressure can only be obtained by enclosing the clock in an air-tight case, and partially exhausting the air. Then if the temperature is kept constant, the pressure will remain constant. The clock room at the Case Observatory is maintained at practically constant temperature by means of gas stoves outside of the room, and electric lights within the room, automatically controlled by thermostats composed of steel and hard rubber. A paper, giving an account of the method by which this is done, was read before the Astronomical and Astrophysical Society of America at its last meeting in Washington, and will shortly be published. It is sufficient to say here that the temperature of the clock room varies only a few tenths of a degree during the entire year. Such a clock room can be constructed at comparatively low cost, and seems to solve the question of constant temperature for the clock. So far as I am aware, the only clock-maker who has put upon the market a clock so arranged as to remain at a constant pressure, is RIEFLER of Munich. He encloses the clock in a glass case, which is made air-tight. There are three of these clocks in this country. One is at the Philadelphia Observatory, one at the Georgetown College Observatory, and one at the Case Observatory. The clock at the Case Observatory was received about a year ago, but was not permanently sealed up until September, 1901, on account of a broken barometer which had to be replaced. The clock is fastened to a wall of the constant temperature room by means of a bracket. The iron bracket is securely bolted to the wall by bolts which run clear through the wall, and are fastened upon the outer side. A long glass cylinder, closed at the bottom, and open at the top, is slipped through this bracket, and rests upon three screws which level and hold it in place. The clock movement is fastened by four screws to the top of this cylinder. The pendulum, of course, is lowered down into the open cylinder. Then a bell glass is put over the top of the movement, and the surfaces are rendered air-tight by means of vaseline. A small foot-pump is attached to a stop-cock at the bottom of the lower cylinder. The clock is wound by two dry cells about once in seven minutes. The weight, which is a small piece of brass weighing, perhaps, half an ounce, is raised by means of an electro-magnet operated by the battery. The weight is at the end of a lever, which is attached to the minute shaft. A barometer and thermometer are kept within the case, and can readily be seen through the glass cover. The pressure maintained has been 673 mm. There has been no leakage since last September. Not a single stroke of the pump has been necessary to maintain constant pressure, Before being finally closed the clock was regulated so as to lose about one and a half seconds per day. The air was

then exhausted until by trial the rate was very small. The final adjustment of the rate is always easily effected by the air-pump. If the clock loses, air is taken out; if it gains, air is let in. The present series of determinations was made to see what rate the clock would have under a pressure slightly lower than atmospheric pressure. It is the intention later on to test the rate in a vacuum, or at least under a pressure not more than two or three mm. A special copper cylinder has been provided, which will stand this pressure, as Dr. RIEFLER wrote that the glass case would probably not stand complete exhaustion. So far as is known no clock has ever been run in a vacuum, and the experiment will therefore be of interest, even if the results are no better than those of the present series.

The following table shows the clock rates from December 17 to March 24. Observations were made with the Case Almucantar, and the clock corrections depend upon four or six stars each night. There are not as many observations as might be desirable, but the weather in Cleveland during the winter is very bad, and good observing nights are few and far between. Every observation made has been placed in the table with the exception of January 15, when only one pair was observed, and February 28 and March 6, when the observing book indicated that the results were very poor. These results were omitted because the object of this investigation was to see what the clock was capable of doing, and its rate should rest upon the very best observations possible.

TABLE I.

Date 1902	Clock Time h m	Clock Error s	Daily Rate s	Hourly Rate s	Mean Daily Rate Minus Daily Rate s
Dec. 17	3 18	—13.58	+0.097	+0.0040	0.019
Dec. 20	2 53	—13.29	+ .112	+ .0047	.004
Dec. 21	0 29	—13.18	+ .094	+ .0039	.022
Jan. 17	2 20	—10.63	+ .103	+ .0043	.013
Jan. 19	5 36	—10.11	+ .134	+ .0056	.018
Jan. 25	2 53	— 9.62	+ .100	+ .0042	.016
Jan. 27	2 43	— 9.42	+ .100	+ .0042	.016
Jan. 28	5 0	— 9.31	+ .114	+ .0048	.002
Jan. 30	3 11	— 9.09	+ .135	+ .0056	.019
Feb. 1	4 12	— 8.41	+ .135	+ .0056	.019
Feb. 10	6 10	— 7.59	+ .137	+ .0057	.021
Feb. 15	8 52	— 6.89	+ .127	+ .0053	.011
Mar. 5	9 20	— 4.60	+ .117	+ .0049	.001
Mar. 10	7 55	— 4.02	+ .137	+ .0057	.021
Mar. 13	7 50	— 3.61	+0.098	+0.0041	0.018
Mar. 19	8 3	— 3.02			
	Mean Daily Rate,		+0.116	Mean Var.	0.015
				Max. Var.	0.022

It will be seen from this table that the maximum difference between the mean daily rate and the daily rate is 0.022 of a second, and the average difference is 0.015 of a second. I think no other published rates show as small differences as these. The mean daily rate of the clock for this period is 0.116 of a second. If the observations of December 17 and March 19 alone had been used, the daily rate for this whole period would have been 0.115 of a second, and the interpolated values of the clock rates would never have differed from the true values by more than 0.022 of a second. It seems to me that such rates are remarkable, and could not possibly be obtained from any clock unless it was running under conditions of constant pressure and temperature.

When all the observations are used the results are as follows:

TABLE II.

Date 1902	Clock Time h m	Clock Error s	Daily Rate s	Hourly Rate s	Mean Daily Rate Minus Daily Rate s
Dec. 17	3 18	—13.58	+0.097	+0.0040	0.019
Dec. 20	2 53	—13.29	+ .112	+ .0047	.004
Dec. 21	0 29	—13.18	+ .089	+ .0037	.027
Jan. 15	1 59	—10.95*	+ .159	+ .0066	.043
Jan. 17	2 20	—10.63	+ .103	+ .0043	.013
Jan. 19	5 36	—10.11	+ .134	+ .0056	.018
Jan. 25	2 53	— 9.62	+ .100	+ .0042	.016
Jan. 27	2 43	— 9.42	+ .100	+ .0042	.016
Jan. 28	5 0	— 9.31	+ .114	+ .0048	.002
Jan. 30	3 11	— 9.09	+ .135	+ .0056	.019
Feb. 1	4 12	— 8.41	+ .135	+ .0056	.019
Feb. 10	6 10	— 7.59	+ .137	+ .0057	.021
Feb. 15	8 52	— 6.89	+ .138	+ .0058	.022
Feb. 28	7 5	— 5.10†	+ .098	+ .0041	.018
Mar. 5	9 20	— 4.60	+ .087	+ .0036	.029
Mar. 6	7 18	— 4.52†	+ .124	+ .0052	.008
Mar. 10	7 55	— 4.02	+ .137	+ .0057	.021
Mar. 13	7 50	— 3.61	+0.098	+0.0041	0.018
Mar. 19	8 3	— 3.02			
	Mean Daily Rate,		+0.116	Mean Var.	0.018
				Max. Var.	0.043

* One pair. † Poor.

It will be seen that the mean daily rate is just the same when every observation is used as in the other table where the three observations are omitted. The mean variation from the mean daily rate is in this case 0.018 of a second per day, and the maximum variation is 0.043 of a second. This maximum variation occurs between January 15 and January 17, but on January 15 only one pair of stars was observed, and therefore the clock correction for that day is not as exact as it ought to be. The next largest variation is 0.029 of a second. With these poor observations included I believe the daily rates and mean variations from the mean daily rates are better than any other published observations with one exception — namely, that of the clock at the Berlin Observatory.

Case Observatory, Cleveland, Ohio.

BESSEL'S OBSERVATIONS FOR PARALLAX OF μ *CASSIOPEIAE*.

By HERMAN S. DAVIS.

In the *Abhandlungen von F. W. Bessel*, Bd. II, 215–6, are given the formulas and eighty equations from which BESSEL deduced

$$-0''.122 \quad \pm 0''.251$$

as the parallax of μ *Cassiopeiae* — derived from differences of right-ascension between μ and θ from 1814 Nov. 1 to 1816 May 20.

In March, 1893, I made a rediscussion of these eighty equations, by solving them anew after revising the differences of right-ascension in such a manner as would base them on the following values for 1815.0:

$$\begin{array}{lll} n = 20''.05911 & \alpha_\theta = \;\; 0^h\,59^m\,54^s.7 & \alpha_\mu = \;\; 0^h\,56^m\;\; 2^s.3 \\ k = 20\;.44511 & \delta_\theta = 54^\circ\;\; 9'\;11''.9 & \delta_\mu = 54^\circ\;\; 0'\;30''.8 \\ \epsilon = 23^\circ\,27'\,47''.2 & \mu_\theta = +0^s.0233 & \mu_\mu = +0^s.3860 \end{array}$$

These give

$$-0^s.32986\,(t-1815) + 0^s.0113 \sin(\Omega + 185^\circ\,22') + 0^s.0402 \sin(\odot + 178^\circ\,4'),$$

whereas BESSEL used

$$-0^s.3219\;(t-1815) + 0^s.0119 \sin(\Omega + 186^\circ\,19') + 0^s.0398 \sin(\odot + 178^\circ\,11')$$

In the new solution, BESSEL's coefficients were used exactly as printed in the equations of the *Abhandlungen*. From the revised differences of right-ascension, the results marked "Davis I" in the accompanying table were derived. These were communicated to the New York Academy of Sciences November 13, 1893, and a Note on their determination was printed in the *Transactions* of that Academy for the same date, Vol. XIII, page 75.

A table including this value of parallax, together with BESSEL's and eight others, can be found on page 15 of Dr. G. N. BAUER's thesis, "The Parallax of μ *Cassiopeiae*" (Contribution No. 18 from the Observatory of Columbia University, New York, 1901). In that table should also be inserted the value of $+0''.120 \pm 0''.044$ published by Mr. FLINT in *Science*, Vol. III, April 24, 1896.

In 1899, Dr. FRITZ COHN published his *Bearbeitung von Bessel's Beobachtungen am Dollond'schen Mittagsfernrohre in den Jahren*, 1813–1819 (Königsberg, 1899). In consequence of having already at hand the material with which to make easily another solution of BESSEL's equations for parallax by use of the definitive right-ascensions of Dr. COHN, I thought it might be worth while to do so, — mainly for the purpose of saving anyone else the trouble, since it seems to have been rightly concluded by BESSEL himself that the parallax quest from these observations is futile.

The results now derived are marked "Davis II" in the table herewith:

1815.0	Bessel	Davis I	Davis II
$\alpha_\mu - \alpha_\theta$	$-3^m\,52^s.0821 \;\; \pm$	$-3^m\,52^s.0390 \;\; \pm 0^s.0258$	$-3^m\,52^s.0358 \;\; \pm 0^s.0246$
$\pi \;\, - \pi_\theta$	$- \;\; 0''.1218 \;\; \pm 0''.2510$	$+ \;\; 0''.0210 \;\; \pm 0''.2115$	$- \;\; 0''.2106 \;\; \pm 0''.2295$
$\mu \;\, - \mu_\theta$	. . . .	$+ \;\; 0^s.2981 \;\; \pm 0^s.0290$	$+ \;\; 0^s.2974 \;\; \pm 0^s.0275$
$\Sigma\, v^2$	$2^s.192$	$2^s.415$	$2^s.100$

The diminution of the sum of the squares of the residuals indicates that each reduction has been an improvement on its predecessor. As was expected, the difference in right-ascension of the two stars, and the difference of their proper motions, remain almost unchanged in the two new solutions. On the other hand, the parallax is materially altered, though no more than the probable error renders permissible. It is curious that BESSEL's value happens to be the mean between the two new values.

It is to be noted, however, that for my second solution the eighty equations no longer remained unchanged except in their absolute term, but they differed in the following particulars:

(*a*) The equations of 1814 Dec. 22 and 1815 April 16 have been omitted, and a second equation for 1815 Jan. 4 has been inserted. Both omissions and insertion were due solely to absence or presence of the observations in Dr. COHN's publication.

(*b*) Only two decimals were retained in the coefficients of proper motion. Likewise from Dr. COHN's publication it was possible to express the absolute term also to only two decimals.

The magnitude of the probable error of the parallax — together with its attendant probability of great fluctuation in the value of the parallax itself, as here shown, by only slight changes in the absolute term, though having insignificant effect upon the other unknowns — confirms previous conclusions that it is a waste of time to make new solutions for the other stars observed by BESSEL in the same manner.

International Latitude Observatory, Gaithersburg, Maryland, 1902 *June* 28.

ON THE LIGHT-VARIATIONS OF *RX HERCULIS*.

BY PAUL S. YENDELL.

This star was announced by SAWYER, in 1898 (*A.J.*, XIX, p. 120), as a variable of the *Algol*-type.

I began to observe it in May of the current year, and my observations up to the present time show that it is not of the type of *Algol*, but distinctly of that of *β Lyrae*.

On 1902 May 8, I observed a minimum of the star, and found it brighter at that phase than I thought I had reason to expect. Another partially observed minimum on May 24 showed the same peculiarity, the light at minimum, as on the previous occasion, not going below 1.0 of my light-scale.

A day or two later, on making up my working list for June and July, I found a clerical error of ten hours in my ephemeris of *RX*. This threw these two minima into the middle of the star's period. Only two explanations of this could be admitted; either the observations themselves were in error, presumably through the expectation of finding the star faint, or the period of variation was subdivided. The former supposition I was by no means willing to admit, especially as the observations of May 24 had been made without recent reference to the observing list, and under the expectation and for the purpose of finding the star at its normal brightness. I had previously on several evenings observed the star for the same purpose, and found it scarcely constant at its normal light.

According to the corrected ephemeris the next available minimum came on May 29. I succeeded in observing the minimum light and a part of the increase, and found the former to be about one and a half steps brighter than the comparison-star *e* (DM. 12°3546), or 1.5 of my light-scale. On the following evening I again observed the star at its minimum light, which I found on that evening to be one step brighter than *e*, being 1.0 of my light-scale.

I had already concluded that I had no reason to reject the observations of May 8 and 24, and upon obtaining these later data, I at once recognized that the star was in all probability not of the type of *Algol*, but of that of *β Lyrae*, the minima determined by previous observers being principal ones, and my own of May 8 and 24 secondary.

I immediately put the star under close watch, observing at least twice or three times on every evening when observations were possible. I observed a principal minimum, the only really well-determined one I have so far been so fortunate as to obtain, on June 5, at $11^h 12^m$, Local M.T., and have observed well-determined secondary minima on the following dates:

1902 May	8	10^h	25^m	Local M.T.
June	9	12	25	
July	3	9	18	
	12	9	10	

Using LUIZET's epochs, we find from these observations the following intervals:

E 1475	May 8.434	
		$32^d.083$
1511	June 9.517	
		$23^d.975$
1538	July 3.192	
		$8^d.911$
1548	12.403	

From which I find values for the period respectively of $0^d.8913$, $0^d.8880$, and $0^d.8911$, the mean of all being $0^d.8899$, LUIZET's value being $0^d.8893$, and SAWYER's $0^d.8896$.

These correspondences make strongly for the reality of the phenomenon, for it is highly improbable that they can be accidental coincidences or the result of errors of observation.

My observations of the star from 1902 May 8 to and including July 12, number in all 122. Using the elements of LUIZET (*A.N.* 3764), I have formed from them a provisional mean light-curve, which completely confirms my conclusion above stated as to the character of the star's light-changes.

The readings from the curve are as follows, beginning at the principal minimum:

h	m		h	m		h	m		h	m	
0	0	1.7	6	0	7.7	12	0	5.3	19	0	7.5
0	30	2.0	7	0	7.6	13	0	5.9	19	30	6.3
1	0	3.2	8	0	6.8	14	0	7.1	20	0	4.6
2	0	5.4	9	0	5.8	15	0	7.8	20	30	3.0
3	0	6.4	10	0	5.2	16	0	8.3	21	0	2.0
4	0	7.2	10	30	5.0	17	0	8.5	21	20.5	1.7
5	0	7.6	11	0	5.0	18	0	8.3			

From 13^h to 19^h there is a dearth of observations, so that this part of the curve is less known than the rest, and it is not possible to state with certainty the time or brightness of the maximum following the secondary minimum, though the indications seem to be that it is somewhat brighter than the one at 6^h.

The important point, however, the secondary minimum, is determined from no fewer than 59 observations, being nearly one-half the whole number available. The entire character of the curve, as will be seen, is much like that of *β Lyrae*.

The comparison-stars and light-scale used are as follows:

		Light
a	DM. +13°3658	10.4
b	DM. +13°3677	6.5
c	DM. +12°3598	8.0
e	DM. +12°3546	0.0

Dorchester, 1902 *July* 15.

NOMENCLATURE OF NEWLY DISCOVERED VARIABLE STARS.

The following list contains a collection of stars which have been recognized as certainly variable within the last few months, and for which the notation, in the usual manner, can therefore be assigned. Photography takes the principal share in the new discoveries, and it is noteworthy that of the 24 objects no fewer than 4 are variables of the *Algol*-type (Ch. 3707, 6927, 7318, 7891), among them one with the strikingly long period of 31.3 days. The "chart-place," for stars between declination $-23°$ and the south pole, is for the equinox 1875; for the others, that of 1855. A *v* or *ph* in the last column indicates whether the maximum and minimum magnitudes are visual or photographic.

Continuous observation of the stars in the list is earnestly requested, in order that a certain determination of the elements of their variations may be made as soon as possible.

No. Ch.	Provis. Notation *A. N.*	Name	Position for 1900.0 R.A.	Position for 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-Place R.A.	Chart-Place Decl.	Magnitude Max.	Magnitude Min.	
			h m s	° ′	s	′	h m s	° ′			
885	. . .	*X Eridani*	2 27 26	−41 54.1	+2.35	+0.27	2 26 27	−42 0.8	9.10	<11	*ph.*
1232	. . .	*T Fornacis*	3 25 24	−28 44.8	+2.50	+0.21	3 24 22	−28 49.6	8.9	10	*v*
2096	8. 1902	*V Camelopardalis*	5 49 24	+74 29.9	+7.89	+0.02	5 43 29	+74 29.0	9	14	*ph.*
2709	27. 1900	*S Volantis*	7 31 29	−73 9.9	−1.00	−0.13	7 31 54	−73 6.6	9	<12	*v*
3394	. . .	*Y Velorum*	9 25 40	−51 44.6	+2.02	−0.26	9 24 49	−51 38.0	8.9	<12	*v*
3536	28. 1900	*Z Velorum*	9 49 25	−53 42.5	+2.09	−0.28	9 48 33	−53 35.5	9	<13	*v*
3707	91. 1901	*RR Velorum*	10 17 48	−41 51.3	+2.56	−0.30	10 16 44	−41 43.6	10	11	*v*
4055	. . .	*RY Carinae*	11 15 49	−61 19.0	+2.60	−0.33	11 14 44	−16 10.0	10	<12	*v*
4953	. . .	*RX Centauri*	13 45 33	−36 26.8	+3.51	−0.30	13 44 5	−36 19.3	9	<12	*v*
4957	29. 1900	*T Apodis*	13 46 6	−77 18.5	+5.72	−0.30	13 43 44	−77 11.0	8.9	<12	*v*
5928	77. 1901	*SS Herculis*	16 28 1	+ 7 3.0	+2.92	−0.13	16 25 50	+ 7 8.9	9	<12	*v*
6513	6. 1902	*W Draconis*	18 5 27	+65 56.5	+0.08	+0.01	18 5 23	+65 56.2	9	14	*ph.*
6521	7. 1902	*X Draconis*	18 6 47	+66 8.3	+0.05	+0.01	18 6 45	+66 7.9	9.10	14	*ph.*
6827	5. 1902	*RT Lyrae*	18 57 46	+37 22.4	+2.08	+0.08	18 56 12	+37 18.7	10.11	<12	*ph.*
6895	11. 1902	*RU Lyrae*	19 9 6	+41 8.1	+1.96	+0.10	19 7 37	+41 3.7	11	13	*ph.*
6927	93. 1901	*U Sagittae*	19 14 26	+19 25.7	+2.63	+0.11	19 12 27	+19 20.8	6.7	9	*v*
7318	78. 1901	*UW Cygni*	20 19 36	+42 54.9	+2.05	+0.19	20 18 4	+42 46.4	10.11	13	*v*
7360	. . .	*RU Capricorni*	20 26 44	−22 1.7	+3.51	+0.20	20 24 6	−22 10.7	9	<12	*v*
7505	96. 1901	*UX Cygni*	20 50 55	+30 2.0	+2.51	+0.23	20 49 2	+29 51.8	9.10	<12	*ph.*
7514	1. 1902	*UY Cygni*	20 52 16	+30 2.8	+2.51	+0.23	20 50 23	+29 52.6	9.10	10.11	*v*
7800	95. 1901	*RR Pegasi*	21 39 59	+24 32.9	+2.72	+0.27	21 37 56	+24 20.6	9	<12	*v*
7891	10. 1902	*UZ Cygni*	21 55 14	+43 51.9	+2.41	+0.29	21 53 26	+43 39.1	9	11.12	*ph.*
7964	12. 1902	*RS Pegasi*	22 7 24	+14 3.6	+2.91	+0.30	22 5 13	+13 50.4	8.9	<10	*v*
8182	2. 1902	*U Lacertae*	22 43 39	+54 38.0	+2.46	+0.32	22 41 49	+54 23.8	8	. .	*v*

Committee on Publication of a Catalogue of Variable Stars:
DUNÉR, HARTWIG, MÜLLER, OUDEMANS.

ON TWO CASES OF SUSPECTED VARIABILITY,

By PAUL S. YENDELL.

(7401) —*Delphini* = DM. +17°4370.

$\alpha = 20^h\,33^m\,21^s$; $\delta = +17°\,54'.8$ (1900).

This star is Birmingham 566 = Espin-Birmingham 679. It is given in the DM. as $7^M.0$. The Bonn Zones have estimates as follows: 1856 Nov. 9, $7^M.9$; Nov. 10, $6^M.8$.

The Potsdam DM. gives it as $6^M.04$, RG. The individual measures are: 1888 Sept. 17, $5^M.92$; 1892 Aug. 23, $6^M.17$. The former observation has the note, "*Die rothe Farbe stört.*"

My attention was called to the star by GORE in 1890. I have 42 observations of it during that year, which indicate irregular fluctuation from $6^M.3$ to $7^M.6$, but in view of the star's strong color, and the difficulty of observation due thereto, I did not consider the observations decisive.

FLANERY, in 1896 and 1897, made 150 observations of the star, which indicate variation of about a magnitude, from 6^M to 7^M.

The star appears to be a variable of the *R Scuti* type.

DM. +54°2863.

$\alpha = 22^h\,41^m.8$; $\delta = +54°\,23'.8$ (1855).

This star is given as $9^M.5$ in the DM. It does not appear in the Zones.

It is Espin-Birmingham 735. Espin, *A.N.* 3232, "Stars with Peculiar Spectra," says, "Var?" GRAFF calls attention to the star, *A.N.* 3774.

It has been on my observing list since 1894. I have observations as follows, the magnitudes being based on those of the DM.:

		M		M
1894 Sept. 24	8.25		1896 Sept. 11	8.10
Dec. 25	8.75		20	8.25
1895 Dec. 15	8.85		Oct. 8	8.15
1896 Aug. 19	8.30		31	8.3
Sept. 11	8.10			

The star is probably variable within the limits 8^M and 9^M, but does not seem to have been observed fainter than this, except at Bonn.

NOTE ON KIMURA'S SUGGESTION IN *A.J.* 517,

By S. C. CHANDLER.

It seems proper to remark with reference to Mr. KIMURA's interesting article in *A.J.* 517, that it is important, in an examination of such a minute and obscure effect as the one in question, to attend particularly to the correction of the various series employed to the true value of the aberration.

The methods by which the various series of latitude-variations that he has employed have been treated are quite heterogeneous in this particular. The constant adopted in the reduction of the observations has been generally STRUVE's. The results so obtained have been variously modified. In some cases the "closing error" has been divided equally between the various groups. The effect of this is to make a partial correction for the erroneous aberration. But there will still remain errors of two sorts due to the aberration error. First, the mean of the times of observation of the morning and evening groups varies in general with the season of the year; the effect of this is to introduce a systematic error in the latitude whose coefficient may amount to 0″.02. Secondly, there will also remain outstanding by this process an aberration error directly dependent on the sun's longitude of $\Delta k \sin\epsilon \cos\varphi \cos\odot$. If, as seems to me most probable, the true aberration-constant is about 20″.525, the periodical error in the latitudes so arising will have a coefficient of about 0″.025.

In the case of other series the latitudes have been corrected to a value of the aberration found from the series themselves, and the values so adopted have ranged all the way up to 20″.55.

The result of this heterogeneity of processes is that an effect of the same order as the phenomenon pointed out by Mr. KIMURA might have been introduced by this cause, which must be carefully eliminated. I by no means deny that such a phenomenon may have an objective existence, for we are by no means certain that the earth's center of gravity may not have an annual periodical oscillation, of say a few feet, in the line of the axis; indeed, I have for some years been watching current observations for evidence of its possible manifestation.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

By A. W. QUIMBY.

1902		Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1902		Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1902		Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan.	5	9	1	1	4	–	fair	Mar.	7	9	–	2	38	1	poor	May	25	7	–	1	2	1	fair
	6	10	–	1	8	–	fair		8	10	–	1	31	1	poor		26	2	–	1	2	1	fair
	7	10		1	10		poor		9	3	–	1	26	–	poor		27	11	–	1	2	–	poor
	9	9		1	15	–	fair		10	4	–	1	22	–	fair		28	8	–	1	2	–	poor
	10	8	–	1	4	–	poor		11	4	–	1	21	–	poor		29	4	–	1	2	–	fair
	11	4		1	8	–	poor		12	4	–	1	15	1	fair		30	8	–	1	1	–	fair
	12	9		1	10	–	poor		13	11	–	1	5	1	fair		31	8	–	1	1	–	fair
	13	9		1	5	–	fair		14	9	–	–	–	1	fair	June	1	9	–	1	4	–	fair
	14	8		1	3	1	poor		30	8	–	–	–	1	fair		2	7	–	1	1	–	fair
	15	9		1	1	1	fair		31	8	–	–	–	1	fair		3	8	–	1	1	1	fair
Mar.	2	9	1	1	5	–	poor	Apr.	1	4	–	–	–	1	good		4	8	–	1	1	1	fair
	3	10		1	6	–	fair		6	8	–	–	–	1	fair		5	7	–	–	–	1	fair
	4	8		1	11	–	fair	May	23	8	1	1	2	1	fair		23	8	–	–	–	1	fair
	6	8	1	2	66		good		24	6	–	1	2	1	poor		24	7	–	–	–	1	fair

Observations were made on 122 other days of the semester, beginning January 1, when neither spots nor faculae were seen. The sun was invisible on January 8, 21, 22, 26, 29, 31; Feb. 1, 17, 22, 25, 28; March 5, 16, 28; April 8, 29; May 3.

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ON THE STATISTICAL RELATIONS AMONG THE PARALLAXES AND THE PROPER MOTIONS OF THE STARS.

By SIMON NEWCOMB.

§ 1.

The idea of a parallactic survey of the heavens, with a view of detecting all the stars having a measurable parallax, is a familiar one in our practical astronomy, especially since the improvement of the photographic method has suggested the possibility of putting the idea into execution. A beginning was made a few years ago when plates of a small portion of the sky were taken with this special object at Helsingfors, and worked up by KAPTEYN. The results of this attempt appeared first in a brief summary published by KAPTEYN in the *Astronomische Nachrichten*, and later *in extenso* in a "Publication" of the Astronomical Laboratory at Groningen.

A study of these papers seemed to me to show a difficulty in reaching any well defined conclusions from such a survey, unless the statistical relations among the parallaxes and proper motions were better understood than they yet have been. The present paper is an attempt in this direction which will, I trust, at least pave the way for subsequent improvements of the results, and suggest the points to be considered in planning a general parallactic survey, and interpreting its results.

Interpretations of stellar statistics necessarily involve certain preliminary hypotheses, which form the basis for our conclusions. So far as the latter agree with observation we may assume the hypotheses to be confirmed, while systematic deviations from observations will call for farther investigation in order to determine their causes.

The two fundamental hypotheses have been enunciated by KAPTEYN in his recent remarkable paper "On the Distribution of Cosmic Velocities." They are:

1. That, at least within the limits of space containing the stars with a proper motion exceeding a certain limit, the stars are scattered through space with an approach to uniformity.

2. That, a few exceptional cases aside, the proper motions have no systematic tendency toward any particular direction.

It seems to me that the first hypothesis is fully justified by the remarkable equality in the thickness of the stars down to a certain magnitude in any two opposite regions of the heavens, and the uniformity of the law of increase in star-thickness from the galactic poles to the galaxy itself. The second hypothesis is, perhaps, less securely founded; but the chance character of the scattering, both of the observed proper motions on the celestial sphere, and of the radial motions, seem to show that it is at least worthy of acceptance as a preliminary hypothesis.

These hypotheses being accepted, a study of KAPTEYN's paper leads to the following interesting result.

If we have given the law of distribution in magnitude of the linear motions of the stars, that is to say, a table or expression showing for every degree of speed the probability that a star selected at random will move with that speed — if also, we know the thickness of the stars in space — then we can determine, with only accidental errors, the number of stars in the sky having apparent proper motions of any and every magnitude.

In the present investigation I make a preliminary attempt to do this.

§ 2.

To make clear the way of proceeding, we begin with some considerations on the problem of distribution of accidental linear motions.

First, as to the general problem of the distribution of velocities. We may represent the absolute speed of a star both in quantity and direction, by a vector drawn in space from a fixed origin in the direction of the motion. The position of the end of the vector, a point in space, will then completely represent the motion. The three rectangular coordinates of each point will then represent the three components of the speed of the star, resolved in the

direction of the axes. We may then consider the statistical distribution of

(α) The components of the speed as projected on any one axis;

(β) The projections of the velocities or vectors on a plane, or the two-coordinate distribution;

(γ) The lengths of the vectors themselves, or the three-coordinate distribution.

Motions in the line of sight and proper motions in one coordinate, reduced to linear measure, belong to class α; proper motions on a great circle, similarly reduced, to class β.

Fundamental properties of the distribution, based both on observation and the reason of the case, are,

1. The vector-points are equally thick in different directions at equal distances from the origin.

2. This thickness is a maximum at the origin, and varies continuously from the origin in every direction.

It follows that within a small sphere around the origin, the projections of class α will be equally distributed along any axis. The number of projections of class β lying between two such limits as r and $r+dr$ will be of the form Mr, M being a function which varies but slowly for small values of r. The number of projections of class γ between the same limits will be of the form Mr^2

The method of proceeding in defining the actual law of distribution is necessarily tentative. We have to assume a law, and then correct it by repeated approximations until we find an agreement with observation. The simplest assumption to start from is the usual exponential law of errors, according to which the number of projections on any one axis having a value between the limits x and $x+dx$ would be proportional to the quantity

$$e^{-\frac{x^2}{a^2}}dx$$

a being a constant modulus.

From this follows that if the motions are projected on a plane, the number having a value between x and $x+dx$ is proportional to

$$xe^{-\frac{x^2}{b^2}}dx$$

b being a simple function of a. The number of actual motions in space will be proportional to a quantity of the form

$$x^2e^{-\frac{x^2}{c^2}}dx$$

There can be no serious doubt that the slower motions follow this law of distribution. But it is readily shown that, assigning any admissible value to the constant b, such rapid motions as those of Groombridge 1830 and *Arcturus* exceed the limits of reasonable probability. Moreover, we shall see in the course of the present discussion that, if we ignore these rapid motions as exceptional, the number of large apparent proper motions given by our theory will fall far short of the actual number.

Were measures of motions in the line of sight sufficiently numerous — were they counted by thousands instead of by hundreds — our method of proceeding would be to start from the distribution α, given by these measures, and from it derive the distributions β and γ. In any case this would be the method to be followed in developing the logical theory; that is to say, the distribution α should be taken as the base of the theory, and from it the others should be derived. KAPTEYN has, however, taken the distribution β as the basic one, doubtless for the very good reason that he has shown how this can be derived at once from the observed parallactic angles of the proper motions, that is, the angle which the direction of the proper motion makes with the great circle through the star and the solar apex.

In the absence of sufficient data our method must be tentative. We have two quantities to adjust to the observed statistics of proper motions. One is the star-thickness in space, the other the number and magnitude of the exceptionally rapid motions.

The first quantity can best be determined from the measured parallaxes of the nearer stars. These, though probably incomplete, seem to indicate that the average volume of space containing a single star is not very different from that of a sphere concentric with the sun, the parallax of a point on whose surface would be 0″.5. The sphere in question may be somewhat smaller than this; but if we assign to its surface parallaxes of 0″.6, 0″.7, etc., we shall be led to numbers of undetected large parallaxes which seem quite inadmissible.

Deeming it unnecessary to set forth tentative processes, I now pass at once to the data which form the basis of the present investigation, and which may be continually improved as farther data become available.

§ 3.

Adopted units. I take as the unit of distance that of a star having a parallax of 1″.

As the unit of volume of space is taken the volume of a sphere of radius unity, that is, of a sphere concentric with the sun, the parallax of a point on whose surface is 1″.

As the unit of linear speed of a star is taken a speed which would carry it from the sun to the earth in one year, or about 4.75 km. per second. This is the most convenient unit in problems relating to the motions of the stars in space, and its general adoption seems to be desirable.

That the relations of these units do not conform to the conventional ones of geometry and physics is no drawback in the present case.

Notation and formulas. In notation I follow KAPTEYN, putting

μ, the apparent annual proper motion of a star on the celestial sphere, the effect of parallactic motion not being eliminated:

m, this motion reduced to linear measure by multiplication by the distance of the star;

$\rho, = \frac{1''}{\pi}$, the distance of the star (π being, as usual, the annual parallax).

We then have between m and μ the relation

$$m = \mu\rho = \frac{\mu}{\pi}$$

T is put for the star-thickness, that is, the quotient of the number of stars contained in a sphere concentric with the sun by the volume of this sphere. From what has been already remarked, we may assume with some probability,

$$\tfrac{1}{5} > T > \tfrac{1}{8}$$

Distribution of m. No attempt to construct a formula for this distribution would be of any use at present; but I have constructed a tabular one based on the following principles:

1. The distribution should follow the exponential law for moderate values, but assign a proportionate excess of large values continually increasing with large values of m.

2. The mean of all the m's should be equal to that estimated from observation. The best estimate I can make, based on KAPTEYN's investigations and CAMPBELL's measures of radial motions, is

$$\text{Mean} \quad m = 6.5$$

This is greater than the supposed absolute mean motion, projected on the sphere, in consequence of the solar motion, assumed to be about 4.0.

It is implied that m has only integral values. We lose nothing by replacing every value by the nearest integral value.

When we come to values of m so large that, occurring only once in thousands of times, we have no way of estimating their number except by the agreement of our results with statistics of the observed proper motions, it is not necessary even to indicate every integer. We may leave gaps increasing in breadth as the value of m increases.

By such tentative processes has been formed the following table.

In the first column is given a list of all the values which it is necessary to suppose that an m, selected at random, may possibly have.

In the second column p is the supposed probability that a star, selected at random from space, will have an m equal to that in the same horizontal line. For convenience the quantity tabulated is $1000\,p$, so that the table shows how many stars out of 1000 have each separate value of m.

The products of p by the three first powers of m, which occupy the last three columns, are of use in the investigation.

The dropping of useless decimals in these columns does not seem to need explanation.

It will be seen that the mean speed, Σmp, comes out a little larger than that assigned. This is legitimate, because, in the determination of this mean, it happened that no cases of exceptionally large motion were included.

TABLE OF THE ADOPTED DISTRIBUTION OF THE LINEAR SPEEDS OF STARS, RELATIVELY TO THE SUN, AND PROJECTED ON A PLANE.

m	$1000\,p$	mp	m^2p	m^3p
0	5	.00	.0	0
1	36	.04	.0	0
2	66	.13	.3	0
3	92	.28	.8	3
4	107	.43	1.7	7
5	114	.57	2.9	14
6	112	.67	4.0	24
7	103	.72	5.0	35
8	91	.73	5.8	16
9	75	.68	6.1	55
10	59	.59	5.9	59
11	44	.48	5.3	58
12	32	.38	4.6	55
13	22	.29	3.7	48
14	14	.20	2.7	38
15	9	.14	2.0	30
16	6	.10	1.5	24
17	3	.05	0.9	15
18	2	.04	0.6	11
19	1	.02	0.4	7
20	1	.02	0.4	8
22	1	.02	0.5	10
25	1	.02	0.6	16
30	1	.03	0.9	27
40	1	.04	1.6	64
50	1	.05	2.5	125
60	1	.06	3.6	216
Σ		6.78	61.3	995

§ 4.

We now proceed with the derivation of results, placing the questions in the form of problems.

PROBLEM I. *To find the total number of stars whose proper motions on the celestial sphere lie between the limits* μ *and* $\mu + d\mu$

We suppose the stars classified according to their values of m. In order that a star of motion m may have a proper motion lying between the assigned limits, it must lie between the spheres, concentric with the sun, whose radii are

$$\rho = \frac{m}{\mu}$$

and $$\rho - d\rho = \frac{m}{\mu} - \frac{m\,d\mu}{\mu^2}$$

The volume of space between these spherical surfaces is

$$V_m = 3\rho^2 d\rho = 3m^3\frac{d\mu}{\mu^4}$$

The entire number of stars of all classes contained in this space is

$$TV_m = 3\,Tm^3\frac{d\mu}{\mu^4}$$

The number of these of the class m is found by multiplying this total by the corresponding value of p, which we call p_m. We therefore have, for this number,

$$dN_{m,\mu} = 3\,Tm^3 p_m\frac{d\mu}{\mu^4}$$

The total of all the stars in question is found by summing this expression for all the values of m. We find from the table

$$\Sigma m^3 p_m = 995$$

The number of stars in question is therefore

$$dN_\mu = 2985\,T\frac{d\mu}{\mu^4}$$

From this differential expression we have to pass to the number whose proper motion lies between any assigned limits.

PROBLEM II. *To find the number of stars whose proper motion lies between the limits μ and μ'.*

This number is equal to the definite integral

$$\int_\mu^{\mu'} dN_\mu = 995\,T\left(\frac{1}{\mu^3} - \frac{1}{\mu'^3}\right)$$

By making μ' infinite we have the following table of the number of proper motions exceeding the limits set in the first column.

μ	Number	$T=\frac{1}{8}$	$T=\frac{1}{4}$
$<6''$	$5\,T$	1	1
5	8	1	2
4	15	2	4
3	37	5	9
2	124	15	31
1	995	124	249
0.5	7960	995	1990

In the last two columns are given the numbers for two values of T, of which it seems to me that the second, $\frac{1}{4}$, is inadmissibly large. Yet, I conceive that even these fall short of the actual number, though, except in the lower lines, they are not in excess of the known number. The following considerations are here to be adduced.

1. Although we suppose the actual star-thickness uniform, this cannot be true for the visible stars down to any given limit of magnitude, because stars of any given absolute luminosity will be omitted from our catalogues when they lie beyond a certain distance. The stars actually catalogued will therefore be thicker in our neighborhood, and thin out as we recede from the sun.

2. From this cause, as well as from the incompleteness of our knowledge of the fainter stars, the number of unknown proper motions must increase rapidly as they diminish in amount.

PROBLEM III. *To find the distribution of the proper motions of all the stars which have a given parallax, π, and the mean of all these proper motions.*

Since these stars all lie at the same distance their proper motions will be $\frac{m}{\rho}$, or $m\pi$. We shall therefore have, for any value of π

$$\text{Mean proper motion} = 6.78\,\pi$$

A table of the distribution for any given π is formed simply by multiplying the tabular values of m by the given π. For example, out of 1000 stars having a parallax 0″.10

5 will have a *p.m.* of 0″.0
36 " " " 0.1
66 " " " 0.2
. .
. .

In general, *only* 1 *star out of* 25 *has a proper motion less than* 1.5 *of its parallax.* Only 1 star out of 250 will, in the space of two years, be displaced by proper motion by a quantity less than its parallax.

A practical conclusion is:

In a parallactic survey of the heavens we may almost confine our attention to those stars which, on the plates, show a displacement through proper motion.

PROBLEM IV. *To find the mean parallax of all the stars having a given proper motion.*

We start from the expression found in treating Problem I for the total number of stars having a given μ and m, namely,

$$dN_{m,\mu} = 3\,Tm^3 p_m\frac{d\mu}{\mu^4}$$

These stars have the common parallax

$$\pi = \frac{\mu}{m}$$

the product of which into the number is

$$\pi dN_{m,\mu} = 3\,Tm^2 p_m\frac{d\mu}{\mu^3}$$

The sum of these products for all the values of m is

$$3\,T\frac{d\mu}{\mu^3}\Sigma m^2 p_m \qquad (a)$$

while the total number of the stars, as found in Problem I, is

$$(b) \qquad 3T\frac{d\mu}{\mu^4}\Sigma m^3 p_m$$

The mean parallax is the quotient of (a) by (b) or

$$\text{Mean } \pi = \mu\frac{\Sigma m^2 p_m}{\Sigma m^3 p_m} = 0.064\,\mu$$

That is, if we measure the parallaxes of all the stars having a given proper motion, we may expect the mean result to be about $\frac{1}{15}$ of the proper motion.

The preceding considerations suggest the limitation of any parallactic survey of the heavens to those stars having a discoverable proper motion. I now add another consideration bearing in the same direction, placing it in the form of an example, which the reader can readily modify to suit any case that may arise.

Let it be granted that we measure the parallaxes of 100,000 stars, each with a probable error of $\pm 0''.03$. Then, as a result of these accidental errors, we shall have, from the measures, several thousand parallaxes exceeding $0''.07$, and several hundred exceeding the limit $0''.10$ which have no reality, but are only errors of observation. These numbers will far exceed the probable number of stars within the corresponding parallactic limits, and, in consequence, it will be impossible to separate the actual parallaxes from the probable errors.

From a cosmological point of view perhaps the most important conclusion of the present paper is that which flows from the table of Problem II above, and may be stated in the following form. The reason why such great speeds as those of 1830 Groombridge and *Arcturus* have never been observed with the spectroscope is that they occur only once in, perhaps, a thousand times. Yet, this small probability will give 100 cases among the 100,000 stars which lie nearest to us, and these cases form an important fraction of the totality of stars having a considerable proper motion.

The writer may be allowed to add that, this investigation having been made where no astronomical literature is accessible, he is unable to compare his results with statistics, and, in some cases, cannot state exact numerical data.

Maloja, Engadine, 1902 *July* 15.

DEFINITIVE ORBIT OF COMET 1898 IX,

By HENRY A. PECK.

The ninth comet of 1898 was independently discovered by PERRINE at the Lick Observatory September 12, and by CHOFARDET in Besançon on September 14. The coma was round, with a diameter of five minutes of arc, and contained a small but definite nucleus. By the end of September it was bright enough to be faintly visible to the naked eye, and showed traces of a tail. The last observation was made at Mt. Hamilton, October 10, after which it vanished in the rays of the sun. Ten days later it passed perihelion, and should have been subsequently visible in the Southern hemisphere, but no observations have been reported. The heliocentric arc described during the observation period was forty-nine degrees.

The following discussion of the observations is based upon the elements published by BERBERICH in *A.N.* 3524. These elements are

$$T = 1898 \text{ Oct. } 20.57786 \text{ Berlin M.T.}$$

$$\left.\begin{aligned} \omega &= 162^\circ\ 20'\ 25''.5 \\ \Omega &= \ \ 34\ \ 53\ \ 31.6 \\ i &= \ \ 28\ \ 51\ \ \ \ 1.2 \end{aligned}\right\} 1898.0$$

$$\log q = 9.6237490$$

and to these elements correspond the equatorial coördinates,

$$\begin{aligned} x &= 9.9827922\,r\,.\,\sin(v + 283^\circ\ 45'\ 32''.03) \\ y &= 9.8466526\,r\,.\,\sin(v + 210\ \ 40\ \ 19.23) \\ z &= 9.8827098\,r\,.\,\sin(v + 179\ \ 41\ \ 34.11) \end{aligned}$$

$$\log m = 0.5245042$$

The star positions are mainly based upon the A.G. Catalogues of GRAHAM, AUWERS and BECKER. In combination with these the Paris Catalogue has been used wherever possible. Single positions have been modified by material drawn from the Armagh, Glasgow, Radcliff, Romberg, Schjellerup and Washington Catalogues. PORTER's Catalogues of proper motions have also been used. The adopted positions are as follows:

No.	α 1898.0	δ 1898.0	No.	α 1898.0	δ 1898.0	No.	α 1898.0	δ 1898.0
	h m s	° ′ ″		h m s	° ′ ″		h m s	° ′ ″
1	9 34 50.26	+31 12 13.5	8	9 55 26.51	+29 16 25.9	15	10 4 36.90	+28 41 35.4
2	9 36 45.64	30 34 33.6	9	9 55 48.27	29 0 39.7	16	10 5 47.66	28 20 47.0
3	9 47 27.81	30 6 42.6	10	9 59 49.31	29 13 8.4	17	10 7 22.91	27 55 37.2
4	9 48 7.98	29 41 37.4	11	9 59 52.14	29 7 59.9	18	10 10 26.98	27 58 28.3
5	9 51 34.39	29 39 25.3	12	10 1 9.36	28 59 25.6	19	10 12 20.37	28 2 3.9
6	9 53 56.69	29 48 20.3	13	10 1 44.77	28 32 50.3	20	10 12 25.78	27 55 28.7
7	9 55 23.18	+29 15 53.3	14	10 3 37.89	+28 28 56.4	21	10 13 2.83	+27 43 28.9

No.	α 1898.0	δ 1898.0	No.	α 1898.0	δ 1898.0	No.	α 1898.0	δ 1898.0
	h m s	° ′ ″		h m s	° ′ ″		h m s	° ′ ″
22	10 15 26.69	+27 12 40.7	35	10 39 53.21	+21 6 51.1	48	11 14 28.19	+19 38 27.2
23	10 17 43.66	27 0 33.4	36	10 40 6.58	25 1 10.4	49	11 15 22.86	19 50 39.2
24	10 18 24.59	26 51 59.7	37	10 40 53.39	25 6 48.4	50	11 18 3.07	19 35 38.6
25	10 23 26.47	26 25 9.9	38	10 45 5.16	23 56 44.2	51	11 19 5.56	17 54 12.5
26	10 23 57.45	26 42 15.9	39	10 49 4.04	23 29 48.0	52	11 22 20.51	18 28 30.3
27	10 27 38.86	25 39 1.7	40	10 49 59.52	23 42 31.0	53	11 25 9.76	18 58 17.4
28	10 28 32.84	25 42 41.5	41	10 54 34.26	22 34 21.4	54	11 40 38.84	15 35 19.4
29	10 29 4.94	26 11 14.7	42	10 56 30.11	22 33 47.3	55	11 50 12.59	14 11 6.4
30	10 31 46.96	25 6 16.2	43	11 2 4.30	21 42 8.8	56	11 56 0.51	12 56 43.9
31	10 32 24.86	26 0 37.2	44	11 3 59.08	21 33 56.1	57	11 57 40.61	13 22 32.5
32	10 35 32.34	24 43 27.0	45	11 8 20.14	20 41 16.2	58	12 4 4.70	11 51 37.5
33	10 36 7.43	24 51 15.0	46	11 9 0.37	20 35 10.4	59	12 29 9.57	+ 6 56 44.6
34	10 39 30.86	+24 49 19.0	47	11 12 3.65	+20 32 36.0			

In the following paragraph are contained all the observations I have been able to find, except one made at Vienna, Sept. 18, and two made at Bordeaux on the 20th and 24th of the same month. These three observations depend upon stars that are not given in any Catalogue that has been accessible. The observations did not seem of sufficient importance to warrant asking any observatory to locate the stars. The times of observation have been corrected for aberration. When better results could be obtained in both coördinates, I have not hesitated to alter the hour angle to allow for errors that experience has shown readily occur. When two or more observations were made at an observatory on the same day, they were compared separately with the ephemeris, but only the mean appears here. For such observations, therefore, the time of observation given does not strictly correspond with the coördinates.

Time	Place	α app.	π	Δα cos δ	δ app.	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
Sept. 13.06884	Mt. Hamilton	9 35 49.27	−0.34	+0.07	+31 4 31.0	+2.4	− 0.7	1
14.04280	Mt. Hamilton	41 43.84	.35	+0.09	30 35 19.2	2.8	− 1.5	2
14.68338	Besançon	45 38.79	.30	+0.29	15 15.4	3.6	− 2.0	3
15.00518	Mt. Hamilton	47 36.80	.35	−0.07	4 56.9	3.5	− 0.9	3
15.62650	Kopenhagen	51 25.72	.24	−0.36	29 41 30.7	4.5	+ 0.3	5
15.62668	Munich	26.24	.29	+0.01	30.0	4.1	− 0.3	4, 5
15.64545	Strassburg	33.27	.29	+0.07	43 53.2	4.2	+ 0.8	5, 6
15.65659	Vienna	37.33	.30	+0.01	33.3	3.8	+ 2.7	4
15.65870	Besançon	38.17	.30	+0.06	28.3	4.0	+ 2.0	6
15.67146	Arcetri	43.70	.32	+0.73	42 56.3	3.5	− 4.8	5, 6
15.68265	Pola	46.32	.31	−0.57	40.1	3.3	+ 1.4	5
15.68808	Marseilles	48.99	.33	−0.03	28.6	3.5	− 5.6	5
15.70513	Bordeaux	55.21	.32	−0.06	41 51.2	3.6	− 1.3	4
16.62877	Utrecht	57 38.16	.25	−0.15	10 0.2	4.7	− 7.1	8
16.63366	Hamburg	39.99	.25	−0.15	9 57.7	4.5	+ 0.8	8
16.63841	Kopenhagen	40.96	.24	−0.73	49.3	4.5	+ 1.7	10
16.63973	Toulouse	42.64	.31	+0.13	58.8	4.2	+14.2	7
16.64266	Pola	42.85	.31	−0.63	41.1	3.9	+ 2.3	8
16.64451	Strassburg	44.37	.29	+0.11	35.1	4.2	+ 0.4	8, 11
16.64885	Besançon	46.09	.30	+0.19	28.1	4.2	+ 3.0	10
16.66407	Kremsmünster	51.80	.30	+0.23	8 51.4	3.8	− 2.3	11
16.66756	Munich	53.01	.30	+0.15	49.3	3.9	− 1.9	9
16.66774	Arcetri	53.68	.32	+0.65	46.0	3.6	0.0	8
16.67478	Liverpool	55.60	.26	+0.08	30.2	4.5	− 0.1	11
16.68028	Kiel	57.50	.26	−0.03	20.2	4.1	+ 1.3	11
16.68685	Bamberg	59.52	.29	−0.43	2.4	3.9	− 2.9	8
16.68935	Teramo	58 0.78	.33	−0.19	7 55.1	3.0	− 5.9	10
16.69177	Paris	1.91	.29	+0.22	56.5	3.9	+ 0.4	11
16.69740	Marseilles	4.06	.33	+0.05	39.7	3.5	− 3.9	10
16.70064	Bordeaux	5.38	.32	+0.16	35.6	3.7	− 0.7	11
17.63982	Mt. Hamilton	10 0 11.73	.36	−0.10	28 55 31.4	3.1	− 0.9	12
17.63084	Utrecht	3 52.92	.25	−0.07	33 55.8	4.7	− 2.9	15
17.65526	Munich	1 2.20	.30	−0.01	5.5	4.1	+ 0.4	13, 14, 15
17.66402	Kremsmünster	4.65	.30	+0.24	32 53.6	3.9	+ 1.0	13
17.67475	Arcetri	10.15	.32	+0.53	20.6	3.6	− 1.8	14, 15
17.67487	Pola	10 1 9.16	−0.32	−0.12	+28 32 25.1	+3.5	+ 2.8	15

Time	Place	α app.	π	Δα cos δ	δ app.	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
Sept. 17.67524	Besançon	10 4 9.48	−0.30	−0.21	+28 32 23.9	+4.0	+ 3.1	14
17.68160	Padua	12.10	.31	0.00	8.9	3.6	+ 1.6	13, 14
17.68825	Jena	14.11	.28	−0.41	31 49.2	3.9	− 2.8	15
17.69069	Paris	15.45	.30	−0.04	44.5	4.0	− 2.0	14
17.71750	Bordeaux	25.60	.32	+0.19	46.5	3.5	− 0.8	14
17.93859	Washington Naval	5 48.49	.35	+0.14	22 38.7	3.0	+ 6.1	16
18.05268	Mt. Hamilton	6 31.43	.36	−0.11	18 16.3	3.0	+ 1.1	16
18.66935	Arcetri	10 24.58	.33	+0.61	27 54 31.9	3.7	− 6.6	19, 20
18.67016	Marseilles	24.10	.33	−0.04	32.1	3.9	− 4.2	20
18.67400	Kremsmünster	26.03	.30	+0.40	37.5	3.8	+10.0	20
18.67490	Padua	25.77	.32	−0.15	24.3	3.7	− 1.3	17, 20
18.68366	Teramo	29.37	.33	+0.11	3.7	3.4	− 1.7	18
18.72269	Bordeaux	44.03	.32	+0.05	52 30.0	3.5	− 4.2	20
18.92558	Washington Naval	12 0.60	.35	−0.02	44 42.2	3.2	+ 3.9	20
19.03947	Mt. Hamilton	43.56	.34	−0.11	40 6.1	3.3	− 2.7	21
19.67281	Padua	16 46.41	.32	−0.17	14 46.6	3.8	+ 2.4	22
19.68132	Toulouse	46.80	.31	−0.02	32.2	4.0	+ 9.0	22
19.68355	Besançon	47.72	.31	+0.04	18.7	4.0	+ 0.9	22
19.68604	Strassburg	48.58	.30	−0.02	13.2	4.0	+ 1.4	22
19.69347	Teramo	51.44	.33	−0.01	13 53.2	3.3	− 1.0	22
19.69534	Marseilles	51.98	.33	−0.16	50.1	3.7	+ 0.8	22
19.70381	Bordeaux	55.35	.32	−0.02	27.4	3.8	− 1.0	22
20.05973	Mt. Hamilton	19 8.83	.36	+0.12	26 59 2.3	3.1	− 2.0	23, 24
20.66413	Munich	23 0.77	.30	+0.13	33 28.3	4.1	− 0.5	25, 26
20.66904	Arcetri	3.05	.33	+0.47	13.9	3.8	− 2.7	25, 26
20.67906	Padua	6.15	.32	−0.18	32 47.3	3.7	− 3.8	29
20.68164	Besançon	7.41	.31	+0.08	43.4	4.1	− 0.7	25
21.04009	Mt. Hamilton	25 24.10	.36	−0.05	17 19.7	3.4	− 2.3	25
21.67717	Strassburg	29 27.91	.30	+0.12	25 49 24.0	4.2	− 1.4	27, 31
21.68754	Bescançon	31.98	.31	+0.27	48 59.8	4.1	+ 2.0	27
21.69092	Marseilles	32.79	.33	−0.21	52.1	3.9	+ 3.1	27
21.69943	Arcetri	36.63	.32	+0.32	24.5	3.6	− 1.2	27, 28
21.71122	Greenwich	40.48	.28	−0.21	57.8	4.3	+ 3.5	27, 31
21.71297	Liverpool	41.77	.27	+0.32	47 48.6	4.4	− 1.0	27
22.69070	Rome	35 56.77	.33	+0.16	3 21.9	3.6	− 2.5	37
22.69224	Besançon	57.12	.31	−0.03	19.7	4.1	0.0	33
22.69285	Teramo	57.51	.33	+0.09	19.0	3.4	+ 0.4	36
22.71323	Bordeaux	36 5.76	.32	+0.48	21.3	3.9	− 0.1	30
23.03778	Mt. Hamilton	38 13.79	.36	−0.03	14.2	3.5	− 1.4	32, 34
23.69304	Besançon	42 22.51	.31	+0.06	24 15 55.1	4.2	− 1.7	35
23.71616	Bordeaux	31.22	.32	−0.14	14 46.8	4.0	− 3.3	35
23.94293	Washington Naval	58.79	.35	−0.05	3 47.3	3.6	− 2.6	38
24.66847	Strassburg	48 39.12	.29	+0.11	23 27 50.4	4.5	− 4.8	39
24.66942	Arcetri	39.76	.33	+0.35	. . . . .	. .	. . .	39, 40
24.67344	Teramo	40.79	.33	−0.15	36.6	3.9	− 4.2	39
24.68439	Besançon	44.91	.30	−0.23	6.9	4.3	− 0.5	39
25.67202	Kremsmünster	55 7.72	.30	+0.28	22 36 37.3	4.4	+ 1.2	41
25.68271	Munich	11.67	.30	+0.11	0.7	4.3	− 2.0	41, 42
25.68284	Arcetri	12.64	.33	+0.95	35 58.9	4.0	− 3.5	41, 42
25.69045	Besançon	14.54	.31	−0.01	38.5	4.3	+ 0.1	41
25.70709	Liverpool	19.59	.26	+0.23	34 59.2	4.8	+ 0.4	41
26.67116	Kremsmünster	11 1 35.41	.30	+0.50	21 43 32.4	4.4	− 2.2	43
26.67257	Strassburg	35.39	.29	−0.01	27.3	4.6	− 2.7	43
26.69396	Munich	43.55	.30	−0.15	42 20.1	4.0	− 1.0	43
26.69668	Utrecht	44.85	.25	+0.04	12.2	5.1	+ 3.0	43
26.70669	Besançon	48.44	.30	−0.18	41 36.1	4.1	− 4.4	43
27.06355	Mt. Hamilton	4 7.23	.36	−0.03	21 10.8	3.5	− 4.0	44
27.69648	Rome	8 13.75	.33	+0.58	20 47 11.4	3.8	+ 0.3	45
27.93261	Washington Naval	9 45.98	.33	+0.06	33 39.2	3.8	−15.3	46
28.06352	Mt. Hamilton	10 35.86	.36	+0.02	26 27.2	3.5	− 3.3	47
28.70148	Teramo	11 14 44.00	−0.33	+0.04	+19 50 55.4	+3.9	− 2.2	48

Time	Place	α app.	π	Δα cos δ	δ app.	π	Δδ	*
		h m s	s	s	° ′ ″	″	″	
Sept. 28.71439	Bordeaux	11 14 18.11	−0.32	+0.31	+19 19 18.0	+4.3	−4.0	49
28.72497	Besançon	53.01	.30	−0.01	18 31.2	4.2	−3.1	49
28.94200	Washington Naval	16 17.17	.35	−0.31	35 58.2	3.7	+0.4	48
29.06182	Mt. Hamilton	17 5.25	.36	−0.08	28 41.1	3.6	−6.3	50
29.70876	Utrecht	21 16.06	.27	+0.11	18 50 33.1	4.8	−5.4	53
29.71993	Besançon	20.19	.30	−0.09	49 56.2	4.3	−2.8	53
29.92199	Washington Naval	22 11.25	.35	(+2.12)	. . . . .	. .	. . .	52
30.06800	Mt. Hamilton	23 35.69	.36	−0.17	29 3.5	4.6	+1.1	52
30.71090	Teramo	27 16.27	.33	+0.07	17 19 38.5	3.9	+2.8	51
Oct. 2.72010	Utrecht	40 48.98	.26	+0.29	15 42 6.6	4.6	−2.8	54
4.07193	Mt. Hamilton	49 35.47	.35	−0.08	14 12 5.9	3.8	−7.1	55
4.68713	Vienna	53 34.75	.29	+0.03	. . . . .	. .	. . .	57
5.07827	Mt. Hamilton	56 7.31	.35	+0.24	13 3 3.5	3.8	−8.7	56
6.07711	Mt. Hamilton	12 2 35.98	.35	+0.14	11 52 51.3	3.8	−9.2	58
10.08362	Mt. Hamilton	12 28 35.14	−0.35	+0.25	+6 55 25.4	+3.9	−12.2	59

In general the observations have been given the weight unity. In a number of cases, however, where the residual has departed too sharply from the mean of those in the immediate vicinity, the weight has been diminished. The observations, whose residuals are enclosed in parentheses, have been disregarded in forming the normal places. These normal places are

	α app.	Δα cos δ	δ app.	Δδ
	h m s	s	° ′ ″	″
Sept. 16.5	9 56 46.428	+0.22	+29 14 59.31	−0.20
20.5	10 21 54.199	+0.26	26 40 47.95	−0.50
27.5	11 6 53.018	+0.63	20 58 30.83	−2.13
Oct. 6.0	12 2 2.186	+1.01	+11 58 46.70	−8.00

While under the conditions of the present problem it might almost seem an over refinement, the perturbations by *Jupiter* and *Saturn* were computed, and the following corrections added to the normal places, the osculation being at perihelion:

Δα cos δ	Δδ
″	″
−0.05	+0.05
.04	.04
.02	.02
−0.01	+0.01

The coefficients of the Equations of Condition were formed by SCHÖNFELD's method, and are given by their logarithms. These equations are

								Weight
$+9.6571\,\partial k$	$-9.7676\,k\sqrt{2}\,\partial T$	$+0.0241\,\partial q$	$-8.3878\,\partial\lambda$	$-9.1535\,\partial\nu$	$-9.0349\,\frac{\partial e}{2}$	-9.2305	$=0$	36.0
9.6119	9.7873	9.9879	7.1243	9.4866	9.1065	9.3424		32.3
9.5185	9.8120	9.8956	+8.7579	9.4974	9.1405	9.7853		23.2
9.3933	9.8316	9.7082	9.1586	9.4071	9.0457	0.0128		6.0
−9.2007	+9.5396	−9.5862	−8.6489	−9.7146	+9.2644	+9.1761		38.8
9.2496	9.6153	9.6265	7.3129	9.6750	9.2652	9.6628		33.2
9.2987	9.7282	9.6437	+8.8386	9.5781	9.2411	0.3243		24.7
9.3209	9.8321	9.5507	9.1422	9.3908	9.1259	0.9025		5.0

To render the equations more homogeneous, the following substitutions were made:

$$\partial k = 9.5648\,u \qquad \partial\lambda = 0.4523\,x$$
$$k\sqrt{2}\,\partial T = 9.4543\,v \qquad \partial\nu = 9.4910\,y$$
$$\partial q = 9.1978\,w \qquad \frac{\partial e}{2} = 9.9412\,z$$

Log. of unit of residuals = 8.7480

The Normal Equations are

$+0.4070\,u$	$-0.5180\,v$	$+0.4091\,w$	$+8.5933\,x$	$-9.5847\,y$	$-0.4035\,z$	-9.8475	$=0$
−0.5180	+0.6619	−0.5180	−9.0871	−9.1083	+0.5742	+0.1298	
+0.4091	−0.5180	+0.4148	+8.9800	−9.5919	−0.4051	−9.8122	
+8.5933	−9.0871	+8.9800	+0.6148	−9.3625	−9.4714	+0.0639	
−9.5847	−9.1083	−9.5919	−9.3625	+0.4638	−0.1164	−9.7141	
−0.4035	+0.5742	−0.4051	−9.4714	−0.1164	+0.5585	+0.0417	

A preliminary solution of these equations as they stand produces a value of z indicating a hyperbola with eccentricity 1.00012. As might be expected from the brevity of the time of observation, the theoretical weight of this solution is so small that the probable error is several times the value of the variable. The solution also reveals the fact that the coefficient of w becomes very small in the reduced equations, thus introducing an element of considerable uncertainty in the value of the perihelion distance. An inspection of the normals shows the entire series of coefficients of u and w to be nearly commensurable. For these reasons it was thought best to express the remaining variables as functions of w and z, with the result that

$$\begin{array}{lcrrr}
\log u & = & -0.2954 & -0.0484\,w & +9.6000\,z \\
\log v & = & -0.2367 & -8.9315 & -9.7123 \\
\log x & = & -9.5105 & -8.2071 & +8.9027 \\
\log y & = & -9.2663 & -8.2643 & +9.6861
\end{array}$$

Substituting these values in the original equations of condition the residuals are

$$\begin{array}{rrr}
\overset{''}{+0.22} & -7.4287\,w & +7.9724\,z \\
-0.19 & +7.2828 & -7.9184 \\
-0.19 & +7.5400 & -8.0655 \\
+0.38 & -7.9764 & -7.8942 \\
-0.43 & +6.9615 & -7.1741 \\
+0.34 & -7.0247 & +7.4408 \\
+0.75 & -7.2481 & +7.5184 \\
-2.72 & +7.9247 & -8.2819
\end{array}$$

where the coefficients of w and z are given by their logarithms. If w and z are each made equal to zero, the probable error of an equation of condition whose weight is unity becomes $\pm 2''.75$, and the probable errors of the other variables are well within their limits.

Multiplying each of the above residuals by the square root of the weight previously assigned to the corresponding equation, and at the same time making the substitutions

$$w = 1.6345\,m \qquad z = 1.2495\,n$$

there results from a least-square solution

$$\log m = -0.5238 \;\; +9.8639\,n$$

and introducing the correction to the assumed parabolic eccentricity in the place of the variable n the residuals are

$$\begin{array}{rr}
\overset{''}{-0.17} & -7.4228\,\partial e \\
+0.08 & +7.4467 \\
+0.31 & +7.4964 \\
-0.98 & +8.1494 \\
-0.30 & -5.8724 \\
+0.19 & -6.7030 \\
+0.49 & -5.9578 \\
-1.51 & +7.3836
\end{array}$$

When $\partial e = 0$ the probable error of the unit equation becomes $\pm 1''.43$. The value of ∂e which most nearly satisfies the residuals is $+30''.2$, which corresponds to a hyperbola with eccentricity 1.00015. Substituting this value the final residuals are

$$\begin{array}{rr}
\overset{''}{-0.25} & \overset{''}{-0.30} \\
+0.17 & +0.17 \\
+0.41 & +0.49 \\
-0.56 & -1.43
\end{array}$$

On comparing these residuals with those obtained on the supposition that $\partial e = 0$ it is seen that no improvement has taken place if we except the final places in both coördinates. Even here the change in declination is almost insignificant. The probable error of a unit equation only drops from $\pm 1''.43$ to $\pm 1''.29$. We are therefore compelled to conclude that the observations do not show any definite departure of the orbit from the parabolic form.

The relations between the variables yield the following numerical values:

$$\begin{array}{lcr}
\log w & = & -2.1583 \\
\log u & = & +2.0992 \\
\log v & = & -1.2675 \\
\log x & = & -0.5401 \\
\log y & = & -9.8144
\end{array}$$

from which follows:

$$\begin{array}{lcr|lcr}
\partial i & = & -\overset{''}{2.79} & \partial T & = & -\overset{d}{0}.001050 \\
\partial \Omega & = & -19.53 & \partial \log q & = & -0.0001137 \\
\partial \omega & = & +63.25 & & &
\end{array}$$

Adding these corrections to the original elements, and computing the probable errors the definitive elements are

$$\begin{array}{lcll}
T & = & 1898 \text{ Oct. } 20.576810 & \pm 0.000262 \text{ B.M.T.} \\
\omega & = & 162^\circ\ 21'\ 28''.75 & \pm 13''.17 \\
\Omega & = & 34\ 53\ 12.07 & \pm\ 5.41 \quad 1898.0 \\
i & = & 28\ 50\ 58.41 & \pm\ 0.73 \\
\log q & = & 9.6236353 & \pm 0.0000305
\end{array}$$

To which corresponds the equatorial coördinates,

$$\begin{array}{lcl}
x & = & 9.9827979\,r\,.\sin(v+283^\circ\ 46'\ 17''.42) \\
y & = & 9.8466416\,r\,.\sin(v+210\ 40\ 56.91) \\
z & = & 9.8827103\,r\,.\sin(v+179\ 42\ 28.55) \\
 & & \log m = 0.5246747
\end{array}$$

On comparing the normal places with positions computed from these elements, the residuals are

$\Delta\alpha$	$\Delta\delta$
$-0''.14$	$-0''.36$
+0.12	+0.16
+0.33	+0.48
−1.05	−1.55

The close agreement with the results obtained by substituting in the equations of condition proves the numerical accuracy of the work.

Syracuse University, 1902 *Sept.* 25.

OBSERVATION OF COMET *b* 1902 (PERRINE),

BY E. E. BARNARD.

The comet has a fairly well defined nucleus of about the 11th magnitude. The head is large, 3′–4′ diameter, with a brushing out of the nebulosity in the direction of about 230°. The comet is about the 9th magnitude.

The observations were made through a very thick sky, between passing clouds, and the comet was faint, but the measures were satisfactory.

Measured $\Delta\alpha = 254''.92$ (10 obs.).

Two sets of $\Delta\delta$ were secured.

1st set	13h 13m 50s	−3′ 18″.91	(5 obs.)
2d set	13 32 19	−2 58.81	(6 obs.)

From these the $\Delta\delta$ at time of $\Delta\alpha$ is obtained.

OBSERVATION OF PERRINE'S COMET, *b* 1902.

	$6^h\,0^m\,0^s$ slow of G.M.T.	C	$\Delta\alpha$	$\Delta\delta$	α app.	δ app.
Sept. 2, 1902	13h 24m 9s	10–11	+0m 20s.85	−3′ 8″.0	3h 16m 14s.08	+35° 21′ 36″.1

Comparison-Star for 1902.0.

R.A.	Red. to app. place	Decl.	Red. to app. place	Authority
3h 15m 49s.21	+4s.02	+35° 24′ 42″.9	+1″.2	Lund A.G.C. 1731

The measures were made with the large telescope. The time is $6^h\,0^m\,0^s$ slow of G.M.T.

Yerkes Observatory, Williams Bay, Wis., 1902 Sept. 3.

EPHEMERIS OF COMET *b* 1902,

(From *A.N.* 3817, by NIJLAND, for Berlin midnight.)

1902	α	δ	log Δ	Br.	1902	α	δ	log Δ	Br.
	h m s	° ′				h m s	° ′		
Oct. 16	18 16 24	+16 30.5			Nov. 6	17 4 22	− 9 18.1		
17	9 55	14 9.1			7	17 2 0	9 54.5	0.0257	9.0
18	18 4 7	11 58.9	9.7294	13.0	8	16 59 36	10 29.7		
19	17 58 52	9 59.1			9	57 10	11 3.6		
20	54 6	8 8.9			10	54 41	11 36.6		
21	49 13	6 27.3			11	52 8	12 8.6	0.0643	9.6
22	45 41	4 53.6	9.8029	10.9	12	49 32	12 39.9		
23	41 56	3 26.9			13	46 53	13 10.5		
24	38 26	2 6.7			14	44 8	13 40.6		
25	35 11	+0 52.1			15	41 19	14 10.1	0.0963	10.5
26	32 7	−0 17.2	9.8698	9.6	16	38 25	14 39.2		
27	29 12	1 22.0			17	35 25	15 7.9		
28	26 25	2 22.6			18	32 20	15 36.3		
29	23 44	3 19.4			19	29 10	16 4.4	0.1210	11.4
30	21 10	4 12.9	9.9288	8.9	20	25 55	16 32.3		
31	18 39	5 3.5			21	22 37	17 0.1		
Nov. 1	16 13	5 51.3			22	19 14	17 27.7		
2	13 49	6 36.6			23	15 49	17 55.1	0.1373	11.5
3	11 26	7 19.8	9.9806	8.7	24	12 22	18 22.3		
4	9 5	8 1.0			25	16 8 53	−18 49.4		
5	17 6 44	−8 40.4							

Unit of brightness Sept. 6 (about $7^M.5$).

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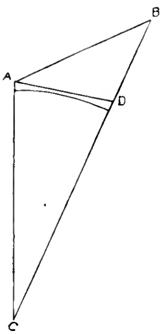
TERRESTRIAL REFRACTION AND THE TRIGONOMETRICAL MEASUREMENT OF HEIGHTS,

By WILLIAM HARKNESS.

Although no great change can be expected in our present theories respecting the trigonometrical measurement of heights, yet it is hoped that the following paper embodies some progress, especially in that part of the subject which relates to terrestrial refraction.

I. — Mathematical Theory of Trigonometrical Leveling.

Let A be the lower station, B the upper station, and C the intersection of their verticals near the center of the Earth. Strictly speaking, the verticals AC and BC do not generally lie in the same plane, and therefore cannot intersect at all, but for our present purpose it is usual to assume that they do, and the error introduced by that assumption is evanescent. Further, let ζ' be the observed zenith-distance of B as seen from A, ζ'' the observed zenith-distance of A as seen from B, s the length of the line joining A and B, measured at the sea level, R the radius of curvature of the Earth at the middle of that line, and h' and h'' the respective altitudes of A and B above the sea level. Then in the triangle ABC we have



$$AC = R+h' \qquad BC = R+h''$$
$$R+h' : R+h'' :: \sin B : \sin A$$

and by putting $h''-h' = \Delta h$

$$\frac{\sin A}{\sin B} = \frac{R+h''}{R+h'} = 1+\frac{\Delta h}{R+h'}$$

whence

$$(1) \qquad \Delta h = (R+h')\left\{\frac{\sin A}{\sin B}-1\right\}$$

in which R is given by formula (16), and

$$(2) \qquad \left\{\begin{array}{l} A = 180^\circ - \zeta' - k'C \\ B = 180^\circ - \zeta'' - k''C \end{array}\right.$$

$k'C$ and $k''C$ being respectively the terrestrial refractions at the stations A and B.

Formula (1) is the fundamental expression for the trigonometrical computation of heights, but unfortunately the base which it involves is so long that ten-figure logarithms are required to yield an accurate result. To avoid their use (1) is usually expanded into a series, of which there are many different forms; but a better result can be attained by a proper selection of the zero point of the angle A.

Referring to the figure, let the distance CA be set off from C to D upon the side CB. Then the triangle ACD will be isoceles, and after putting

$$<BAD = A' \quad , \quad <ADB = C'$$

we shall have

$$\left.\begin{array}{r} <ADC = DAC = \frac{1}{2}(180^\circ - C) \\ C = s \,/\, R \text{ arc } 1'' \\ A' = A+\frac{1}{2}C-90^\circ \\ C' = 90^\circ+\frac{1}{2}C \end{array}\right\} \qquad (3)$$

and the triangle ADB will give

$$DB = \Delta h = AD\frac{\sin A'}{\sin B} \qquad \Big\} \quad (4)$$

But AD is the chord of the arc AD, while the corresponding arc at sea level is s, and therefore

$$\text{Arc } AD : s :: \text{chord } AD : \text{chord } s :: R+h' : R$$

whence

$$\text{chord } AD = \text{chord } s\left(\frac{R+h'}{R}\right) \qquad \Big\} \quad (5)$$

Again, (6)

$$\text{chord } s = 2R\sin\tfrac{1}{2}C = 2R\left(\frac{s}{2R}-\frac{s^3}{48R^3}\right) = s\left(1-\frac{s^2}{24R^2}\right)$$

and by substituting in (4) the values of chord s and chord AD from (6) and (5), we obtain

$$\Delta h = \frac{R+h'}{R}\left\{s\frac{\sin A'}{\sin B}\left(1-\frac{s^2}{24R^2}\right)\right\} \qquad (7)$$

In computing Δh it will usually be more convenient to employ the observed value of ζ' or ζ'' rather than the derived angles A and B; and by substituting the values of the latter from (2) and (3), (7) becomes

$$(8)\qquad \Delta h = s\,\frac{\cos(\zeta'+k'C-\tfrac{1}{2}C)}{\sin(\zeta'+k'C-C)}\left\{1-\frac{s^2}{24R^2}+\frac{h'}{R}\right\}$$

or, if ζ'' is observed

$$(9)\qquad \Delta h = s\,\frac{\cos(\zeta''+k''C-\tfrac{1}{2}C)}{\sin(\zeta''+k''C)}\left\{1-\frac{s^2}{24R^2}+\frac{h'}{R}\right\}$$

The term h'/R represents the effect of the altitude of the lower station, and although it is usually small, it cannot be safely neglected in precise work. Conversely, the term multiplied by $s^2/24R^2$ is so insignificant that in it we may employ the mean value of R, viz. 3963.1 miles [see formula (18)], and take

$$(10)\qquad \log(1/24R^2) = [3.97816-20]\text{ ft.} = [5.01042-20]\text{meters}$$

For differences of altitude not exceeding 2500 feet a very simple and convenient formula for Δh can be derived by expanding (8), and rejecting all terms above the second order. Proceeding in that way

$$\frac{\cos[\zeta'-C(\tfrac{1}{2}-k')]}{\sin[\zeta'-C(1-k')]} = \frac{\cos\zeta'\cos[C(\tfrac{1}{2}-k')]+\sin\zeta'\sin[C(\tfrac{1}{2}-k')]}{\sin\zeta'\cos[C(1-k')]-\cos\zeta'\sin[C(1-k')]}$$

whence, with sufficient accuracy

$$(11)\qquad \frac{\sin A'}{\sin B} = \frac{\cos\zeta'}{\sin\zeta'}\times\frac{1+[C(\tfrac{1}{2}-k')]\tan\zeta'}{1-[C(1-k')]\cot\zeta'} = \cot\zeta'+C(\tfrac{1}{2}-k')$$

And as

$$(12)\qquad \text{arc } C = s/R$$

the substitution of (11) and (12) for the equivalent quantities in (7) gives

$$(13)\qquad \Delta h = s\cot\zeta+\frac{s^2}{R}(\tfrac{1}{2}-k)$$

where ζ indicates either ζ' or ζ'', k indicates either k' or k'', and the formula is accurate to about one part in 3,000. As the first term is either positive or negative according as ζ is less or greater than 90°, and the second term is always positive, careful attention must be given to the signs.

The factor

$$(14)\qquad (\tfrac{1}{2}-k)/R = Q$$

is frequently almost constant over large areas of country, and if these happen to contain many points whose zenith-distances have been measured, their reduction may be greatly facilitated by writing (13) in the form

$$(15)\qquad \Delta h = s\cot\zeta+Qs^2$$

To find the radius of curvature of the Earth at the stations, we have

$$R = a(1+\tfrac{1}{2}e^2\sin^2\varphi-e^2\cos^2\varphi\cos^2\alpha)\qquad(16)$$

where a is the equatorial semi-diameter of the Earth, e the eccentricity $=\sqrt{(a^2-b^2)}/a$, φ the mean latitude of the stations, and α the azimuth of the line joining the stations, counted from the south around by the west, as usual.

All the accuracy afforded by (16) is required when s is large, but when it is small we may frequently disregard α, and use the mean curvature of the Earth due to the latitude of the stations. In such cases $\frac{1}{2}$ must be substituted for $\cos^2\alpha$, and then (16) takes the very convenient form

$$R = a(1-\tfrac{1}{2}e^2)+ae^2\sin^2\varphi\qquad(17)$$

whence, with $a = 3963.1$ miles, and $e = 0.081\,553$

$$R = 3949.9+26.36\sin^2\varphi\text{ miles}\qquad(18)$$

II. — Terrestrial Refraction.

Owing to the confused distribution of heat in the lower strata of the Earth's atmosphere, it is difficult to determine the data necessary for the exact computation of the instantaneous coefficient of terrestrial refraction, and that constitutes the principal reason why trigonometrical measurements of heights are inferior to spirit levelings. The method ordinarily pursued is to make simultaneous reciprocal observations of the zenith-distances of the two stations — that is, B is observed from A at the same time that A is observed from B — and then to assume that the total refraction is kC at each station; k being the coefficient of terrestrial refraction. From as large a number of such observations as possible, the mean value of k is derived, and that is used throughout the region in which it was determined. The mathematical theory of the process is as follows:

$$\begin{aligned}
A &= 180°-\zeta'-kC &&(19)\\
B &= 180°-\zeta''-kC &&(20)\\
A+B &= 360°-(\zeta'+\zeta'')-2kC &&\\
A+B+C &= 180° &&\\
-C &= 180°-(\zeta'+\zeta'')-2kC &&\\
kC &= 90°-\tfrac{1}{2}(\zeta'+\zeta'')+\tfrac{1}{2}C &&(21)\\
k &= \tfrac{1}{2}+\frac{R\text{ arc }1''}{2s}[180°-(\zeta'+\zeta'')] &&(22)
\end{aligned}$$

In effect this amounts to determining the value of k for a mean state of the atmosphere, and as we know that k is a function of the density of the air, it is not clear why that fact should be ignored. Normally the refraction at the lower station exceeds that at the upper by a term of the second order, whose exact form has never been demonstrated; but a little study of the problem shows that the

mean value of the coefficient of terrestrial refraction can be satisfactorily represented by an expression of the form

$$k = E + Fd \qquad (23)$$

where E and F are coefficients to be determined from observation, and d is the density of the air. Let quantities pertaining to the lower and upper stations be distinguished respectively by one and two accents, while those without accent refer indifferently to either station. Then, by putting

$$k' = E + Fd' \quad , \quad k'' = E + Fd''$$

we get for our second order term

$$\tfrac{1}{2}(k' - k'') = \Delta k = \tfrac{1}{2}F(d' - d'') \qquad (24)$$

whence

$$\left.\begin{aligned} k' &= k + \Delta k = k + \tfrac{1}{2}F(d' - d'') \\ k'' &= k - \Delta k = k - \tfrac{1}{2}F(d' - d'') \end{aligned}\right\} \qquad (25)$$

where

$$d' = \frac{B'}{1 + a(t' - t_0)} \quad , \quad d'' = \frac{B''}{1 + a(t'' - t_0)} \qquad (26)$$

while B' and B'', and t' and t'' are respectively the readings of the barometer and thermometer at the two stations; $t_0 = 50°$ Fah., is the standard temperature at which all densities are measured, and $a = 0.002\,039$, is the coefficient, for one degree Fahrenheit, of the volume expansion of air under constant pressure. The substitution of the numerical values of a and t_0 in (26) gives, for the Fahrenheit thermometer and a barometer graduated to English inches,

$$d = \frac{B}{1 + 0.002\,039\,(t - t_0)} = \frac{B}{0.89805 + 0.002\,039\,t} \qquad (27)$$

and for the centigrade thermometer and a barometer graduated to millimeters

$$d = \frac{B}{24.468 + 0.093\,22\,t} \qquad (28)$$

We have next to derive the numerical values of E and F from observation, and for that purpose theory provides at least three different methods; as follows:

1st Method. — At any pair of stations whose difference of altitude has been determined by spirit leveling, the values of k' and k'' can be found independently of any theory of terrestrial refraction; and then we have from (23)

$$E = \frac{d'k'' - d''k'}{d' - d''} \quad , \quad F = \frac{k' - k''}{d' - d''} \qquad (29)$$

2d Method. — Let the true difference of level between any two stations be Δh, and upon the assumption that the refraction is the same at the upper and lower stations, let the computed difference of level between the same stations be $\Delta h'$. Then, from (25), we shall have with sufficient accuracy

$$\Delta h = \Delta h' - \tfrac{1}{2}F(d' - d'')\,Cs \text{ arc } 1'' \qquad (30)$$

Again, by putting $H_1 + x_1$ and $H_2 + x_2$ respectively for the true altitudes of the lower and upper stations, we have

$$\Delta h = (H_2 - H_1) + x_2 - x_1 \qquad (31)$$

and the subtraction of (30) from (31) gives

$$0 = (H_2 - H_1 - \Delta h') + x_2 - x_1 + \tfrac{1}{2}F(d' - d'')\,Cs \text{ arc } 1'' \qquad (32)$$

Then, in the adjustment of any net of trigonometrical levels, each pair of stations will furnish an observation-equation of the form (32), and the solution of these equations by the method of least-squares will give the value of F along with the corrections x_1, x_2, etc., to the assumed heights of the several stations.

3d Method. — In accordance with (23), every observed value of k will furnish an observation-equation of the form

$$0 = -k + E + Fd \qquad (33)$$

where d is the mean of the observed densities of the air at the upper and lower stations; and from a sufficient number of such equations, having an adequate range in altitude, the values of E and F can be found by the method of least-squares.

For the numerical determination of the values of the constants in the preceding formulas, the Annual Reports of the U.S. Coast and Geodetic Survey, and the magnificent volume entitled "The Trans-continental Triangulation and the American Arc of Parallel," published by the Coast and Geodetic Survey in 1900, are the only sources within my knowledge from which the requisite data can be obtained.

For the first method of determining E and F, by formula (29), no suitable data are available. The nearest approach to such data are found in the experimental observations on the lines Bodega Head to Ross Mountain, Cal., (Coast Survey Report, 1876, Appendix 16, pp. 342 and 346), and Martinez East to Mt. Diablo, Cal., (Coast Survey Report, 1883, Appendix 12, pp. 293–304), which give respectively the following results for the day minimum,

$$k = +0.1619 - 0.002\,435\,d \qquad (34)$$

$$k = -0.2416 + 0.011\,74\,d \qquad (35)$$

but these results are worthless, because the relative values of k' and k'' are falsified by the circumstance that one end of each of the lines is in a sea coast climate, while the other end is in a climate largely inland.

For the second method of determining F, by equation (32), the most suitable portion of the trans-continental triangulation is that across the Sacramento and San Joaquin valleys, Cal., where the heights of the stations range from

34 to 10,380 feet, and the lengths of the lines from 28 to 117 miles. There ten observation-equations were selected, involving five unknowns, and their solution by the method of least-squares gave

$$F = +0.001\,613 \pm 0.000\,115 \tag{36}$$

but that result is vitiated by the climate being partly sea coast and partly inland.

The next most available part of the triangulation is that in the Salt Lake base net, where nine observation-equations were selected, involving five unknowns, and their solution by the method of least-squares gave

$$F = +0.000\,915 \pm 0.000\,583 \tag{37}$$

but although the heights of the stations are great, ranging from 9571 to 12 100 feet, and the lines are long, ranging from 61 to 148 miles, the differences of level are comparatively small, and therefore the resulting value of F has little weight.

For the third method of determining E and F, by equation (33), the whole trans-continental triangulation is available, but to avoid excessive labor only part of it has been employed. The data actually used in the group of observation-equations (38) is as follows:

The first equation is from the mean of 21 lines in the primary triangulation of northern Georgia, for which the average value of $B = 715.6^{\mathrm{mm}} = 28.171^{\mathrm{in}}$ (Coast Survey Report, 1876, Appendix 18, p. 371).

The second equation is from observations of the day minimum at Ragged Mountain, Me., where $B = 28.668^{\mathrm{in}}$ and $t = 64.5°$ F. (Coast Survey Report, 1876, Appendix 17, p. 361).

The seven remaining equations are from 70 lines in the primary triangulation between Pike's Peak, Colo., and Round Top, Cal.; each equation being the mean of ten consecutive lines, arranged in the order of the density of the air ("Trans-continental Triangulation," pp. 329–332).

Observation-Equations.

				k, Comp.	r
(38)	$0 =$	$-0.0716 + 1E$	$+28.18F$	0.0728	$+0.0012$
		$-\;\;709 + 1$	$+27.85$	721	$+\;\;12$
		$-\;\;643 + 1$	$+23.01$	626	$-\;\;17$
		$-\;\;644 + 1$	$+22.03$	606	$-\;\;38$
		$-\;\;575 + 1$	$+20.51$	577	$+\;\;2$
		$-\;\;572 + 1$	$+20.19$	570	$-\;\;2$
		$-\;\;564 + 1$	$+19.76$	561	$-\;\;3$
		$-\;\;546 + 1$	$+19.17$	550	$+\;\;4$
		$-\;\;500 + 1$	$+18.35$	0.0534	$+0.0034$

While these equations are very strong for densities between 18.35 and 23.01 inches, they are weak for want of data between densities 23 and 30 inches. The resulting normal equations are,

$$\begin{aligned} 0 &= -\;\;0.5469 + \;\;9.00E \\ &\;\;\;\; -12.3070 + 199.11 \quad + 4510.09F \end{aligned} \tag{39}$$

whence

$$0 = -0.003\,58 + \;\;0.2098E$$
$$0 = -0.2077 \;\; + 105.12F$$

$$\log E = 8.23207 \qquad E = +0.017\,064 \pm 0.003\,154$$
$$\log F = 7.29567 \qquad F = +0.001\,975 \pm 0.000\,141 \tag{40}$$
$$k = +0.017\,06 + 0.001\,975\,d \pm 0.002\,564$$

The value of k given in (40) is the mean for the summer day-minimum in the neighborhood of latitude 39°, throughout the interior of the United States, and as it is believed to be the best result which can be obtained from the data now available, it is accepted provisionally. The probable error attributed to it is that which results from a comparison of the computed values of k with the observed values on the 70 lines of the trans-continental triangulation used in the observation-equations (38).

The investigations of the U.S. Coast and Geodetic Survey show ("Trans-continental Triangulation," pp. 254–256), that the daily course of the coefficient of refraction consists of a day-minimum, usually lasting with little change from about 10 A.M. until 4 P.M., a night-maximum lasting with more or less unsteadiness from 9 or 10 P.M. until sunrise, and the junction lines uniting the maximum and minimum. The latter are of little importance because observations are best made while the refraction is most steady, preferably during the day-minimum.

Having thus obtained a provisional formula for the day-minimum of the refraction, we have next to determine the corresponding night-maximum, for which unfortunately very little data exists. Indeed, the only available night observations seem to be those made at the Coast Survey experimental stations, Ragged Mountain, Me.; Martinez East to Mt Diablo, Cal.; and Jackson Butte to Round Top, Cal. (Coast Survey Reports, 1876, p. 361, and 1883, pp. 293–304, and "Trans-continental Triangulation," pp. 286–292).

Table (11).—Experimental Determination of the Ratio of the Night-Maximum of k to the Day-Minimum.

Stations	h	d	k Night	k Day	N/D
	m.	in.			
Martinez East	57	30.22	0.1391	0.1045	1.331
Ragged Mt.	397	28.12	.0995	.0720	1.375
Jackson Butte	714	26.98	.0789	.0655	1.205
Mt. Diablo	1173	26.58	.0884	.0653	1.354
Round Top	3171	20.97	0.0635	0.0523	1.215
				Mean =	1.296

From them we get the data exhibited in table (11), where the first and second columns give respectively the names of the stations and their heights in meters above the sea level; the third column gives the density of the air for the night observations, to which the day observations are reduced; the fourth and fifth columns give respectively the values of k

for the night-maximum and the day-minimum, and the sixth column gives the quotient of the night values divided by the day values.

According to these data, the mean ratio of the night-maximum to the day-minimum is 1.296, and by multiplying the value of k in (40) by that quantity, we get provisionally for the mean value of the night-maximum in the inland summer climate of the United States,

$$k = +0.022\,11 + 0.002\,560\,d \pm 0.003\,323 \tag{42}$$

We have next to investigate the peculiar sea-coast climate of California, which extends far beyond the limits of the triangulation under consideration; but unfortunately many of the coast stations north of San Francisco are lost to that work because no meteorological observations were made at them. Among the remaining stations only 16 lines are available, viz.: In series C, Ross Mt. to Tamalpais, Diablo to Yolo Base S.E., Mocho to Tamalpais, Helena to Diablo, Helena to Tamalpais, Diablo to Ross Mt.; in series E, Mocho to Diablo, Diablo to Sierra Morena, Tamalpais to S. Morena, Mocho to S. Morena, Diablo to Loma Prieta, Loma Prieta to S. Morena; in series F, Helena to Lola, Helena to Round Top, Diablo to Lola, and Diablo to Round Top ("Trans-continental Triangulation," pp. 276–277, 302 and 309). Dividing these lines into two groups, and prefixing the two groups of experimental observations made respectively at Bodega Head and Martinez East, Cal., we have the four observation-equations,

		k, Comp.	v
(43)	$0 = -0.0899 + 1E + 29.57F$	0.09643	+0.00653
	$- .1017 + 1 \quad + 29.28$	9538	− 632
	$- .0854 + 1 \quad + 26.20$	8420	− 120
	$-0.0762 + 1 \quad + 24.33$	0.07741	+0.00121

The resulting normal equations are

$$\begin{aligned} 0 &= -0.3532 + 4.00E \\ & -9.7275 + 109.38 \quad + 3010.09F \end{aligned} \tag{44}$$

whence

$$0 = +0.000\,275 + 0.025\,37E$$
$$0 = +0.002\,532 - 0.698F$$

$$\left\{ \begin{aligned} \log E &= 8.0350\,n & E &= -0.010\,840 \\ \log F &= 7.5595 & F &= +0.003\,627 \end{aligned} \right. \tag{45}$$

and for the day-minimum of the coast climate,

$$k = -0.010\,84 + 0.003\,627\,d \pm 0.002\,31 \tag{46}$$

The probable error here assigned to k has been found by comparing the computed values of the latter with the observed values in the 16 lines employed in the observation-equations (43), and it will not escape notice that although the weights of both E and F are small in the normal-equations, the probable error of k is nearly the same as in (40). However, that was to be expected, because both for (40) and (46) the probable error in question is simply the accidental variation between the mean value of k and the value actually existing at any given moment.

To obtain an approximation to the mean night-maximum of k for the sea-coast climate of California, we multiply (46) by the factor 1.296, from table (41), and find

$$k = -0.014\,05 + 0.004\,700\,d \pm 0.002\,99 \tag{47}$$

As we pass inward from the ocean, the sea-coast climate of California is gradually converted into the inland climate, but the transformation is not complete until the crest of the Sierra Nevada is reached. On the coast the climate is purely oceanic; at the Sierra Nevada it is purely inland; and if we assume that the change from one to the other is proportional to the distance from the ocean, then by putting k' and k'' respectively for the coast and inland values of the coefficient of refraction, and S and s respectively for the distance from the coast to the Sierra Nevada, and from the coast to the point where k is required, we shall have

$$k = \frac{k'(S-s)}{S} + \frac{k''s}{S} = k' - \frac{s}{S}(k' - k'') \tag{48}$$

All atmospheric refraction, whether astronomic or terrestrial, is due to the same cause and subject to a common theory. Its factors are four in number: viz.: the density of the air, the status of the successive layers of the atmosphere, which are normally concentric, and the elevation and length of the path of the ray. The density of the air is readily ascertained by means of the barometer and thermometer, and the action of the layers upon rays passing completely through the atmosphere at elevations exceeding ten degrees presents no serious difficulty. For all such cases the total refraction can therefore be computed with extreme accuracy; but below that altitude the problem rapidly increases in complexity. The status of the layers, which is involved in the quantity k, then becomes of prime importance, and at the same time very difficult to determine satisfactorily. As it is certainly variable in space as well as in time, it may not be the same at two trigonometrical stations some miles apart; and yet the only method of determining k heretofore employed rests upon the assumption that all the conditions are identical at such a pair of stations. The consequence of that faulty assumption is that the resulting value of the refraction does not apply strictly at either station, and the altitudes computed respectively from day and night observations do not agree. Fortunately, these difficulties are considerably ameliorated by the theory developed in the present paper, which takes strict account of the effect of changes of atmospheric density, but it does not seem possible to deal generally with the status of the layers otherwise than by average values.

Just at the outer limit of the Earth's atmosphere the terrestrial refraction must be identical with the astronomic

refraction, and consequently the one may be derived from the other when the instantaneous height of the atmosphere above the observing station is known. Designating the latter quantity by the symbol x, it may be derived with sufficient accuracy from the barometrical formula for the measurement of heights by writing

$$(49)\qquad x = \log B \times 60\,159 \text{ ft.} = \log B \times 11.3938 \text{ miles}$$

where B is the reading of the barometer in English inches at the observing station, at the instant for which x is required.

Next, in the right-angled triangle formed by the vertical line passing from the observer to the center of the Earth, the horizontal line passing from the observer to the outer limit of the atmosphere, and the line joining the latter point with the Earth's center, let R be the length of the first mentioned side, s the length of the second, and $R+x$ the length of the hypothenuse. Then we shall have

$$(50)\qquad s^2 = 2Rx + x^2$$

and for the angle at the center of curvature, after putting $\tan C'''/C''' = n$

$$(51)\qquad \tan C''' = nC''' = s/R$$

Next, with m for the numerical coefficient in (49), and the value of s from (50), we have with sufficient accuracy

$$\frac{1}{C'''} = \frac{nR}{s} = \frac{nR}{\sqrt{(2Rx+x^2)}} = \left(1-\frac{x}{4R}\right)\sqrt{\left(\frac{n^2R}{2x}\right)}$$

$$(52)\qquad \frac{1}{C'''} = \left(1-\frac{m\log B}{4R}\right)\sqrt{\left(\frac{n^2R}{2m\log B}\right)}$$

which, with $m = 11.3938$ miles, $R =$ its mean value, viz.: 3963.1 miles, and $\log\tan C''' = 8.965000$, whence $\log n = 4.686801-10$, gives

$$\frac{1}{C'''} = (1-[6.85657-10]\log B)\frac{[5.80697-10]}{\sqrt{\log B}}$$

$$(53)\qquad \frac{1}{C'''} = \frac{[5.80651+0.000\,005\,(30^{\text{in.}}-B)-10]}{\sqrt{\log B}}$$

where C''' is expressed in seconds of arc, and the quantity within the square brackets is the logarithm of the number which it represents.

Then with r_0 for the astronomical refraction at a zenith-distance of 90°, we have at the outer limit of the atmosphere

$$(54)\qquad r_0 = kC'''$$

whence, by substituting the value of $1/C'''$ from (53)

$$(55)\qquad k = \frac{[5.8065-10]\,r_0}{\sqrt{(\log B)}}$$

and thus we have found the coefficient of terrestrial refraction in terms of the astronomical refraction at the point where the observations are made. This ought to arrest attention, because it seems to afford a new means of eliminating systematic errors, and thereby increasing the accuracy of trigonometrical leveling. Heretofore the fundamental difficulty with that method has been that while k is in general different at the two stations, the value given by the observations was a mean which did not apply strictly at either station; but formula (55) suggests a way of avoiding that source of error completely by determining the refraction at the very spot where it is required. The procedure will be as follows:

Let the time of transit of some celestial body be observed across the horizontal wire of a carefully adjusted engineer's level, and at the same instant let the zenith-distance of the station whose Ih is required be measured with any suitable instrument. Then, from the latitude of the station, the observed time of transit, and the right-ascension and declination of the celestial body, the true zenith-distance of the latter at the instant of its observed transit can be found, and that zenith-distance, less 90°, will be the value of r_0; whence (55) immediately gives the exact value of k. The Sun will usually be the most convenient celestial body to observe, and probably observations made near sunrise will be preferable to those made near sunset.

It may be asked why the astronomical refraction is taken at 90° zenith-distance in (54) and (55). The answer is, because the lower strata of the atmosphere lie in contact with the Earth's surface, and consequently any line of sight joining two points on that surface must hold substantially the same relations to these strata as is held by a horizontal line passing to a celestial body. Possibly (49) may be so far inexact that the numerical coefficient of (55) can be improved by comparing values of k got from that expression with those given by simultaneous reciprocal zenith-distances.

If we substitute in (55) ARGELANDER's mean value of the astronomical refraction at 90° zenith-distance, together with his mean values of the pressure and temperature, viz.: $r_0 = 33'\,51'' = 2031''$, $B = 30.00^{\text{in}}$, $t = 50°$F., we shall get

$$k_0 = \frac{[5.8065-10]\,2031''}{\sqrt{(1.4771)}} = 0.1070 \qquad (56)$$

which slightly exceeds the night-maximum for temperate land climates given by (42), viz., 0.0989; but it cannot be affirmed that (56) is the absolute maximum, because, while most of the derangements to which the layers are subject tend to diminish k, it is possible to imagine some which will increase it. Experience shows that for an atmospheric density of 30 inches, at a temperature of 50° F., the usual values are, for the day-minimum in an inland temperate climate, about 0.0763; and for the night-maximum in the sea-coast climate of California, about 0.1270.

III. — Desiderata.

1. On the California coast the ocean climate penetrates far inland, but the observations at Ragged Mountain, Me., seem to indicate that such is not the case there. Further investigation is needed to show whether the conditions in Maine are exceptional, or whether the inland climate extends quite up to the sea shore all along the Atlantic coast.

2. More observations of the day-minimum of k, at atmospheric densities ranging from 23 to 30 inches, are much needed in the interior summer climate of the United States for combination with the observation-equations (38), in order to strengthen formula (40).

3. As the heights of mountains and headlands are frequently measured from ships at sea, a better knowledge of the mean value of k for lines passing wholly over the ocean is greatly needed. Suitable experimental lines, from the main land to outlying islands, could readily be selected both on the Atlantic and Pacific coasts.

4. An experimental investigation of the value of k over country covered by ice or snow would be extremely interesting from a scientific point of view, because of the light it would be likely to throw on the general theory of terrestrial refraction. During the New England winter the region surrounding Mount Washington, N.H., would afford a convenient field for such an investigation.

5. Five lines of the Trans-continental Triangulation, in the region Holcolm Hills, Holt, Square Bluffs, Cramer Gulch, Big Springs, Colo., pass very near the surface of the earth throughout large portions of their length, and give abnormally small values of k. The theoretical explanation seems to be that the temperature of the air in the neighborhood of the ground is nearly constant, and therefore the density of the air in the path of the ray remains almost uniform. Probably k is always diminished under such circumstances, and in extreme cases it may be reduced to as little as one-fifth of its normal value; but further investigation is needed before any final conclusion can be formulated.

6. Our present knowledge of the relation between the day-minimum and night-maximum of k is very unsatisfactory, and to improve it more hourly observations are needed. The latter should be made in series, each extending over a period of two or three weeks.

7. It is exceedingly desirable that the degree of accuracy with which k can be determined from (55) should be investigated at every available opportunity.

IV. — Practical Conclusions.

1. A comparison of formulas (40) and (46), or of (42) and (47), shows at once that the values of E and F are influenced by something else besides the density of the air, and there can be little doubt that that something is the status of the atmospheric layers through which the visual rays pass. It constitutes the great obstacle to the formation of a rigorous theory of terrestrial refraction, and there is apparently no general method of dealing with it, except that of averages. In extreme cases we may be warned of the existence of abnormal refraction by abnormal relations between the temperatures at the upper and lower stations, but even then we cannot predict its amount, and the difference between the mean and instantaneous values of the refraction remains in effect an accidental error. The mean value can be determined from readings of the barometer and thermometer, but the instantaneous value can be found only by actually observing its amount.

As the observation-equations (38) span the American continent, they show pretty clearly that ordinary variations in topography and surface cultivation hardly affect the mean value of k; whence it may be fairly inferred that the status of the layers is tolerably uniform over all land surfaces in temperate climates. Furthermore, if the conditions which determine the mean value of k are substantially uniform over the land, it would seem that they must be even more uniform over the sea; and thus we are led to the conclusion that probably only two great types of terrestrial refraction exist, viz.: a land type and a sea type. If so, the former must be represented by formulas (40) and (42), and the latter by formulas (46) and (47). Over land covered by ice and snow, the radiation must be so much reduced that the refraction probably approximates to the sea type.

2. The density of the air is just as important in the determination of terrestrial refraction as in that of astronomic refraction, and readings of the barometer and thermometer should never be omitted in measuring zenith-distances for the trigonometrical determination of heights.

3. It seems probable that simultaneous reciprocal zenith-distances, reduced by means of the formula (25), constitute the most accurate method of determining terrestrial refraction; and if so, eleven quantities are required for the best possible trigonometrical determination of Δh, viz.: the zenith-distance, readings of the barometer and thermometer, and latitude at each station, together with the distance between the stations, the azimuth from one to the other, and the altitude of one of them.

4. When it is impossible to observe simultaneous reciprocal zenith-distances, the method of formula (55) will probably be the next best way of determining the refraction. As yet we have no experience with it, but it seems to promise excellent results, particularly in the measurement of inaccessible mountain peaks, where present methods leave the refraction very uncertain.

5. On account of the enormous expense of occupying many trigonometrical stations at once, the method of non-simultaneous reciprocal zenith-distances will doubtless continue to be used in the future, as it has been in the past;

but the results will always be uncertain, and probably much inferior to those obtainable by the method of formula (55).

6. In the case of land observations, if no direct determination of the refraction has been made the best course will be to compute d from the readings of the barometer and thermometer when the zenith-distance was observed, and then to find the corresponding value of k from formula (40) = Table III. In such cases the reduction of the observations may very properly be effected by means of formula (15); the small terms neglected in it being of the same order as the uncertainty in the mean refraction.

7. Altitudes measured from ships at sea should be reduced by means of formula (15), with the mean refraction given by formula (46).

8. Tables I, II and III have been computed respectively by means of formulas (27), (28) and (40). The values of k given in Table III are for the summer day-minimum in the neighborhood of latitude 39° throughout the interior of the United States; but, for reasons already stated, it is highly probable that they will answer for all temperate land-climates anywhere in the world.

TABLE I, FOR COMPUTING THE DENSITY OF THE AIR FROM READINGS OF A FAHRENHEIT THERMOMETER AND A BAROMETER GRADUATED TO ENGLISH INCHES.

$$\text{Log}\ d = \text{Log}\ B + \text{Log}\ M.$$

t	log M	t	log M	t	log M	t	log M
25°	0.0227	45°	0.0045	65°	9.9869	85°	9.9700
26	218	46	036	66	860	86	692
27	209	47	027	67	852	87	684
28	199	48	018	68	843	88	675
29	190	49	009	69	835	89	667
30	0.0181	50	0.0000	70	9.9826	90	9.9659
31	172	51	9.9991	71	818	91	651
32	163	52	982	72	809	92	643
33	154	53	974	73	801	93	634
34	145	54	965	74	792	94	626
35	0.0136	55	9.9956	75	9.9784	95	9.9618
36	127	56	947	76	776	96	610
37	118	57	938	77	767	97	602
38	108	58	930	78	759	98	594
39	099	59	921	79	750	99	586
40	0.0090	60	9.9912	80	9.9742	100	9.9578
41	081	61	903	81	734	101	570
42	072	62	895	82	725	102	562
43	063	63	886	83	717	103	554
44	054	64	878	84	708	104	546

Cosmos Club, Washington, D.C., 1902 *October* 1.

TABLE II, FOR COMPUTING THE DENSITY OF THE AIR FROM READINGS OF A CENTIGRADE THERMOMETER AND A BAROMETER GRADUATED TO MILLIMETERS.

$$\text{Log}\ d = \text{Log}\ B + \text{Log}\ M.$$

t	log M	t	log M	t	log M	t	log M
−5°	8.6198	+ 7°	8.6000	+19°	.5811	+30°	8.5644
4	81	8	.5984	20	8.5795	31	29
3	64	9	68	21	80	32	14
2	48	10	8.5952	22	65	33	.5600
−1	31	11	36	23	49	34	.5585
0	8.6114	12	20	24	34	35	8.5570
+1	.6098	13	.5905	25	8.5719	36	56
2	81	14	.5889	26	.5704	37	41
3	65	15	8.5873	27	.5689	38	27
4	48	16	57	28	74	39	.5512
5	8.6032	17	42	+29	59	+40	8.5498
+6	16	+18	26				

TABLE III.—MEAN VALUES OF THE DAY-MINIMUM OF k, TO THE ARGUMENT d, IN TEMPERATE LAND CLIMATES.

d	k	d	k
15m.	0.046 70	24m.	64 48
16	48 67	25	0.066 45
17	50 65	26	68 43
18	52 62	27	70 40
19	54 60	28	72 38
20	0.056 57	29	74 35
21	58 55	30	0.076 33
22	60 52	31	78 30
23	62 50	32	80 28

For use with formula (15), after assuming for R its mean value = 3963.1 miles, we have from (40), for the day-minimum of a temperate land climate

$$100\,000\,000\ Q = +2.3079 - 0.009\,441\ d \quad \text{for } s \text{ in feet}$$
$$= +7.5719 - 0.030\,973\ d \quad \text{for } s \text{ in meters}$$

And from (46), for the day-minimum of a sea climate

$$100\,000\,000\ Q = +2.4413 - 0.017\,330\ d \quad \text{for } s \text{ in feet}$$
$$= +8.0094 - 0.056\,859\ d \quad \text{for } s \text{ in meters}$$

In conclusion I desire to acknowledge indebtedness, and express thanks, to the U.S. Coast and Geodetic Survey, through its Superintendent, Mr. O. H. TITTMANN, for nearly all the data, both printed and manuscript, upon which the computations in this paper rest.

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ON THE APPLICATION OF DELAUNAY TRANSFORMATIONS TO THE ELABORATION OF THE SECULAR PERTURBATIONS OF THE SOLAR SYSTEM,

By G. W. HILL.

When it is desired to develop the coordinates or the Keplerian elements of a system of planets in infinite series, and there is no objection to the appearance of powers of the time in the expressions of the coefficients, the procedure to be followed is, in general, immediately apparent. But, when t is to be kept within the functional signs sine or cosine, the course to be adopted is not so clear. The difficulty is especially present when the problem is to determine the secular values of the elements apart as far as possible from the periodic terms. On this point the reader may be referred to LEVERRIER's statement of the course he pursued in the elaboration of his theory of *Jupiter* and *Saturn* (*Annales de l'Observatoire de Paris*, Tom. X. pp. 99–103), and to the *Nachtrag* of Prof. PAUL HARZER's prize memoir (*Die Säcularen Veränderungen der Bahnen der grossen Planeten*). As a practical matter, if it is desired to go beyond terms of the first order with respect to planetary masses, it is impossible to get the secular perturbations without at the same time consenting to the derivation of the periodic perturbations. However, it is to be noted that the latter need be obtained only to terms one order lower than the last order to be retained in the formation of the differential equations determining the secular values of the elements. Thus, if it is proposed to neglect all terms of the third order in the formation of the mentioned equations, only the first power of the planetary masses need be considered in determining the periodic terms of the elements.

DELAUNAY's transformations in his treatment of the Lunar Theory, extended so that they become applicable to planetary motions, seem eminently suited to remove whatever obscurities there may be in the processes heretofore adopted. However, it would be extremely inconvenient, not to say, impossible, to apply DELAUNAY's method to a group of differential equations expressed in terms of the coordinates or elements of the planets. It appears essential that we should make a linear and orthogonal transformation in the rectangular coordinates. This transformation, first indicated by JACOBI* in the case of two planets, was afterwards extended by RADAU† to any number.

Denote the mass of the central body by m_0, and the masses of the planets, in an order which is at our choice, by m_1, m_2, etc.; moreover, put

$$\mu_i = m_0 + m_1 + m_2 + \;.\;.\; + m_i \quad , \quad \kappa_i = \frac{m_i}{\mu_i}$$

Then the type of representation of the rectangular coordinates of the i^{th} planet relative to the central body, in this linear and orthogonal transformation, is

$$x_i + \kappa_{i-1}\ x_{i-1} + \kappa_{i-2}\ x_{i-2} + \;.\;.\; + \kappa_1 x_1$$

The differential equations these variables satisfy are of the type

$$\mu_{i-1}\kappa_i \frac{d^2x_i}{dt^2} = \frac{\partial\Omega}{\partial x_i}$$

where Ω denotes the sum of the products of every two masses of the system divided by their distance, a relation we will write thus:

$$\Omega = m_0\sum\frac{m_i}{\Delta_{0,i}} + \sum\frac{m_i m_j}{\Delta_{i,j}}$$

In order to pass from equations in terms of rectangular coordinates to those in terms of Keplerian elements it is necessary to choose a simplified form of Ω defining these elements. Calling this form Ω_0 we suppose

$$\Omega_0 = m_0\sum\frac{m_i}{r_i}$$

where $r_i^2 = x_i^2 + y_i^2 + z_i^2$. If Ω_0 is substituted for Ω in the

* *Sur l'élimination des nœuds dans le problème des trois corps.*

† *Sur une transformation des équations différentielles de la dynamique.*

differential equations, and the members are divided by $\mu_{i-1}\kappa$, we get a system of equations of which the type is

$$\frac{d^2x_i}{dt^2} + m_0\frac{\mu_i}{\mu_{i-1}}\frac{x_i}{r_i^3} = 0$$

Let a_i be the semi-axis major, e_i the eccentricity, ϕ_i the inclination, l_i the mean anomaly, g_i the angular distance of the perihelion from the node and h_i the longitude of the node, of a planet whose rectangular coordinates are determined by equations whose type has just been written. Then the type of linear elements severally conjugate to the angular elements l_i, g_i, h_i in a canonical system is

$$L_i = m_i\sqrt{m_0\frac{\mu_{i-1}}{\mu_i}a_i} \quad , \quad G_i = L_i\sqrt{1-e_i^2} \quad , \quad H_i = G_i\cos\phi_i$$

Also construct a function

$$F = m_0\sum\frac{m_i}{2a_i} + m_0\sum m_i\left(\frac{1}{\Delta_{0i}} - \frac{1}{r_i}\right) + \sum\frac{m_i m_j}{\Delta_{ij}}$$

(This F is the negative of POINCARÉ's F). Then, if the a and the planetary coordinates in the right member are replaced by the canonical elements L, G, H, l, g, h, the differential equations determining the latter are of the type.

$$\frac{dL_i}{dt} = \frac{\partial F}{\partial l_i} \quad , \quad \frac{dl_i}{dt} = -\frac{\partial F}{\partial L_i}$$
$$\frac{dG_i}{dt} = \frac{\partial F}{\partial g_i} \quad , \quad \frac{dg_i}{dt} = -\frac{\partial F}{\partial G_i}$$
$$\frac{dH_i}{dt} = \frac{\partial F}{\partial h_i} \quad , \quad \frac{dh_i}{dt} = -\frac{\partial F}{\partial H_i}$$

These are the differential equations required for the application of the DELAUNAY method to the motion of a planetary system.

It is now necessary to rigorously define how perturbations are to be divided into the two classes of terms called secular and periodic. When F is developed into an infinite periodic series, the arguments of the several terms are linear functions with integral coefficients of the linear elements l_i, g_i, h_i; consequently there are some terms whose arguments do not involve any of the l_i. These terms are denominated *secular*, while the others, in which some of the l_i are present, are denominated *periodic*. It is here assumed that the mean motions of the l_i are incommensurable; this, however, is only to insure mathematical rigor in the statements; for, if the integers expressing the ratios of the motions of the l are quite large, the statements are still true in a *practical sense*. To illustrate, suppose that the ratio of the mean revolutions of two planets is as 60 to 149 (which is nearly the case with *Jupiter* and *Saturn*) we should have to go to terms of the 89th order with respect to eccentricities and inclinations before anything contravening our statements was met with. Let us now suppose that, while we have been finding the formulas of transformation for the purpose of removing from F its periodic terms, we have made the substitutions also in the original Keplerian elements, precisely as DELAUNAY does in the three polar coordinates of the moon. After all the periodic terms have been removed from F, it is obvious that the Keplerian elements will be expressed by a series of terms in which some involve the l_i in their arguments and others do not. The first, taken together, will constitute the *periodic* perturbations of the elements, while the second, in like manner, constitute the *secular* perturbations of the same.

The circumstance that an infinite number of transformations has to be made to completely free F from its periodic terms is no valid reason for declining to accept this definition of the distinction between *secular* and *periodic* perturbations. In practice we confine ourselves to a moderate number of "Operations." Thus DELAUNAY, in his treatment of the Lunar Theory, found that about 500 of these transformations reduced F sensibly to a non-periodic term.

It would now seem that the application of the proposed method to determining the secular values of the Keplerian elements of the eight major planets of the solar system involves an amount of labor not to be thought of, since there are 48 Keplerian elements in addition to the function F, in all of which the transformations of every operation have to be made. But it can be shown that, for practical purposes, the transformations may be limited to F alone.

The demonstration of this may be made to depend on several theorems. The first is

THEOREM I.— *When we have obtained the* secular *values of one set of Keplerian elements we can derive thence the* secular *values of any other set, provided we are willing to neglect terms of two dimensions with respect to planetary masses.*

For the secular terms which arise from the inter-multiplication of the periodic terms with themselves or with other periodic terms are necessarily of two dimensions with respect to planetary masses. For instance, if we are in possession of expressions for the elements $e\cos(h+g)$ and $e\sin(h+g)$ of the following form:

$$e\cos(h+g) = S+P \quad , \quad e\sin(h+g) = S'+P'$$

where S and S' are the secular portions and P and P' the periodic portions, it may be desired to get the secular value of e. It is obvious that, to the degree of approximation proposed, it is given by the equation

$$e = \sqrt{S^2+S'^2}$$

although the rigorous value is the secular portion of

$$\sqrt{(S+P)^2+(S'+P')^2}$$

for the former differs from the latter only by a quantity of the order of P^2 or P'^2.

In our linear transformation of rectangular coordinates, we can imagine that x_i, y_i, z_i, are the rectangular coordi-

nates of a hypothetical planet, which we may designate as belonging to the i^{th} planet. Then this has its instantaneous Keplerian elements as well as the real planet to which it belongs. We may inquire how the secular values of the elements of the two planets compare with each other; and thus is found the following

THEOREM II. — *Provided we neglect quantities of two dimensions with respect to planetary masses, the* secular *values of the elements of each actual planet are the same as those of the corresponding elements of its belonging hypothetical planet.*

In proving this theorem we may always neglect the squares and products of the constants we have denoted by κ_i, and thus may reduce the relations existing between the rectangular coordinates of the real and hypothetical planets to an expression involving a single κ. Thus, while X, Y, Z denote the coordinates of the considered actual planet, let x, y, z denote those of its attached hypothetical planet, and x', y', z' those of another hypothetical planet; then we may set the equations

$$\mathrm{X} = x+\kappa x' \quad , \quad \mathrm{Y} = y+\kappa y' \quad , \quad \mathrm{Z} = z+\kappa z'$$

where κ is any constant of the order of planetary masses. In the first place we suppose that the two hypothetical planets are governed in their motions by the laws of KEPLER. Thus k and k' being two constants and $r^2=x^2+y^2+z^2$ and $r'^2=x'^2+y'^2+z'^2$, we have six differential equations, of which the type is

$$\frac{d^2x}{dt^2}+k\frac{x}{r^3}=0 \qquad , \qquad \frac{d^2x'}{dt^2}+k'\frac{x'}{r'^3}=0$$

If we multiply the second equation by κ and add the product to the first, the result is the type equation

$$\frac{d^2\mathrm{X}}{dt^2}+k\frac{x}{r^3}+\kappa k'\frac{x'}{r'^3}=0$$

Eliminating x, y, z from these equations by means of the values

$$x = \mathrm{X}-\kappa x' \quad , \quad y = \mathrm{Y}-\kappa y' \quad , \quad z = \mathrm{Z}-\kappa z'$$

writing r^2 for $\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2$, and retaining only the first power of κ, we have three equations of which the type is

$$\frac{d^2\mathrm{X}}{dt^2}+k\frac{\mathrm{X}}{\mathrm{r}^3}+\kappa\left[3k\frac{\mathrm{X}}{\mathrm{r}^3}\,\frac{\mathrm{X}x'+\mathrm{Y}y'+\mathrm{Z}z'}{\mathrm{r}^2}-k\frac{x'}{\mathrm{r}^3}+k'\frac{x'}{r'^3}\right]=0$$

These three differential equations admit a perturbative function; for if we put

$$R=\kappa\left[k\,\frac{\mathrm{X}x'+\mathrm{Y}y'+\mathrm{Z}z'}{\mathrm{r}^3}-k'\,\frac{\mathrm{X}x'+\mathrm{Y}y'+\mathrm{Z}z'}{r'^3}\right]$$

they take a form of which the type is

$$\frac{d^2\mathrm{X}}{dt^2}+k\frac{\mathrm{X}}{\mathrm{r}^3}=\frac{\partial R}{\partial \mathrm{X}}$$

But, on scrutinizing the form of R, we see that in its periodic development it has no secular portion, since the first part can have no term independent of the mean anomaly of the actual planet, and the second part no term independent of the mean anomaly of the second hypothetical planet. Hence, the theorem is true when the planets concerned are supposed to suffer no perturbations. But, it is true even when we consider perturbations; for here it is sufficient to limit ourselves to periodic perturbations of the first order, and these can affect the secular values of the elements concerned by quantities which are of the second order.

In applying the procedure of DELAUNAY to the elaboration of our problem it will be found that the use of the linear variables L, G, H, which are conjugate to the angular variables, l, g, h, is awkward and it will be advisable to imitate DELAUNAY's example in substituting others in their place. Then, in order to form the expressions for the differentials of the angular elements, it will be necessary for us to know the partial derivatives of each of the new set of variables with respect to each of the former set, but expressed in terms of the new set. In regard to this matter we have

THEOREM III. — *Provided we neglect terms of three dimensions with respect to planetary masses in the formation of the differential equations for determining the secular values of the elements, the just-mentioned partial derivatives maintain the same expressions throughout all the transformations made to free F from its periodic terms.*

To prove this let us suppose that a transformation is made to remove from F the periodic term having θ as argument, θ involving at least one of the angular variables l; we know, that L being one of the linear elements, the following formulas of transformation exist:

$$L = L_0+L_1\cos\theta+L_2\cos 2\theta+\ \ldots$$
$$\theta = \theta_0(t+c)+\theta_1\sin[\theta_0(t+c)]+\theta_2\sin 2[\theta_0(t+c)]+\ \ldots$$

Then the new linear variable L, conjugate to the new angular variable l, is equivalent to the former L augmented by the expression

$$\tfrac{1}{2}(\theta_1L_1+2\theta_2L_2+3\theta_3L_3+\ \ldots)$$

But this is evidently of two dimensions with respect to planetary masses; and the partial derivatives, mentioned above, have all to be multiplied by factors of one dimension with respect to the same. The last statement is subject to an exception; viz., when, at the end, the mean motions of the l are derived through the differentiation of F, the last factor is of the dimension zero. But, as we expect to derive the motions of the mean longitudes of the planets from observation, in a practical sense this exception need not be considered. Thus, the L, G, H will always have the same expressions in terms of any other linear variables we may

choose, as a, a', e, e', etc. This property is well illustrated in DELAUNAY's Lunar Theory. If the values of $\frac{\partial a}{\partial L}$, etc. (given Tom. I, p. 259), are compared with those given at the end of each "Operation" the differences will be found to be divisible by $\frac{n'^4}{n^4}$ until we come to Operation 11, when, on account of l not being present in the argument of the term to be removed by the transformation, the differences are divisible only by $\frac{n'^2}{n^2}$.

These three theorems make evident what is necessary to be done in the proposed method of attacking the problem in hand. In the first place, we assume, since we must set some degree of approximation to be aimed at, that terms of three dimensions may be neglected in the formation of the differential equations. This is the same as to say that, after integration, terms of two dimensions may be passed by, since the effect of this process is to lower the terms by one dimension. Then we develop F into a periodic series, pushing the approximation in the secular portion so as to include terms of two dimensions, but contenting ourselves with terms of one dimension in the periodic portion. Next, by "Operations" of DELAUNAY we remove its periodic terms from F, term by term. These transformations will be made in F; the consequence will be that the secular part of F will receive accessions of new terms which we preserve, and the periodic portion also new terms all of two dimensions, which we throw aside as unnecessary for our purposes. As many of these "Operations" will be performed as we judge have a significant effect on the secular portion of F. After this is accomplished we lop off from F any periodic terms it may still contain. After combining together the terms which admit addition, the result will be a function F composed exclusively of secular terms. To get the differential equations determining the secular values of the elements of the system, we must subject this F to the same partial differentiations and multiplications by the same factors as in the case where all consideration of terms of two dimensions is neglected. After this is done, the linear elements appearing in the equations have the same signification in both cases; or, which may be more easily comprehended — desiring to include the effect of second-order terms, we do it simply by modifying the form of F and modifying nothing else. Thus is seen the great simplicity of the DELAUNAY method of proceeding.

THE TERMS OF F TO BE RETAINED IN ITS PRELIMINARY DEVELOPMENT.

In the preliminary development of F in periodic series it is desirable to retain only very exceptionally terms of two dimensions with respect to planetary masses. In the first place, in the interaction of *Jupiter* and *Saturn*, these terms are needed because the masses of these planets are large, and because the periods are nearly as 2 to 5. In the second place, in the interaction of *Uranus* and *Neptune*, they are needed because the periods are nearly as 1 to 2.

Consider in F the series of terms

$$\sum \frac{m_i m_j}{\Delta_{i,j}}$$

As there are 8 planets in the system, there will be 28 terms of this type, in 26 of which we can reduce the coordinates of the actual planets to those of their hypothetical planets. Hence, here there will be no difficulty in forming the periodic developments of the reciprocals of the distances, especially as we do not need the periodic portions. But, in the cases of *Jupiter–Saturn* and *Uranus–Neptune*, it will be advisable to include terms multiplied by some of the quantities κ. Let us suppose that the coordinates of the interior hypothetical planet are x, y, z, while those of the exterior are x', y', z', and adopt the notation

$$xx'+yy'+zz' = rr' \cos H$$

The reciprocal of the distance, in either of the two cases, will have the expression

$$\frac{1}{\Delta} = \frac{1}{r'}\left[1-2(1-\kappa)\frac{r}{r'}\cos H+(1-\kappa)^2\frac{r^2}{r'^2}\right]^{-\frac{1}{2}}$$

It is plain from the form of the right member of this that its periodic development can be obtained from the ordinary expression for the perturbative function if, in the computation of the quantites $b_s^{(i)}$ of LAPLACE, we employ the argument $\alpha = (1-\kappa)\frac{a}{a'}$, instead of $\alpha = \frac{a}{a'}$.

Consider next the middle term of F. It is well known that, for secular perturbations, this term can give rise only to quantities of two dimensions with respect to disturbing forces; hence, according to our plan, this term need be taken into account only in the cases of the interaction of *Jupiter* and *Saturn* and again in that of *Uranus* and *Neptune*. But we propose neglecting terms of this kind in the latter case because they are not augmented by the small divisor $2n'-n$. Hence, the discussion may be limited to the case of the interaction of *Jupiter* and *Saturn*. Here

$$\frac{1}{\Delta_{0,1}} - \frac{1}{r'} = -\kappa\frac{r}{r'^2}\cos H + \tfrac{1}{2}\kappa^2\frac{r^2}{r'^3}(3\cos^2 H-1)$$

The first term of the right member is given immediately by the ordinary development of the perturbative function. As the second term is of two dimensions we need consider only its secular portion. In this connection it is proposed to neglect inclinations when we are dealing with terms of two dimensions with respect to disturbing forces. With this limitation it is well known that the secular part of the term under consideration is

$$\tfrac{1}{4}\kappa^2\frac{a^2}{a'^3}\frac{1+\frac{3}{2}e^2}{(1-e'^2)^{\frac{3}{2}}}$$

and thus is free from the angular elements g, g', h and h'. But as it may be desired to be free from this restriction we will give the rigorous expression. If we denote by A the expression just written and adopt

$$B = \tfrac{15}{16}\,\kappa^2\,\frac{a^2}{a'^3}\,\frac{e^2}{(1-e'^2)^{\frac{3}{2}}}$$

the secular portion of the term in question is

$$\begin{aligned}
&A\,(1-\tfrac{3}{2}\sin^2\phi)(1-\tfrac{3}{2}\sin^2\phi')\\
&+\tfrac{3}{4}A\sin 2\phi\sin 2\phi'\cos(h-h')\\
&+\tfrac{3}{4}A\sin^2\phi\sin^2\phi'\cos(2h-2h')\\
&+B\sin^2\phi\,(1-\tfrac{3}{2}\sin^2\phi')\cos 2g\\
&-B\sin\phi\cos^2\tfrac{1}{2}\phi\sin 2\phi'\cos(2g+h-h')\\
&+B\sin\phi\sin^2\tfrac{1}{2}\phi\sin 2\phi'\cos(2g-h+h')\\
&+B\cos^4\tfrac{1}{2}\phi\sin^2\phi'\cos(2g+2h-2h')\\
&+B\sin^4\tfrac{1}{2}\phi\sin^2\phi'\cos(2g-2h+2h')
\end{aligned}$$

It will be perceived that the angular element g' is absent from this expression.

On Making the Delaunay Substitutions.

Having now the preliminary development of F it is possible for us to remove the periodic terms of that function by a series of Delaunay's "Operations." If the limitation just stated is adopted, we need retain in the periodic portion of F no term involving inclinations. Let the exposition be limited to the interaction of two planets, and let an accent be attached to the symbols belonging to the outer planet, while those of the inner are without that mark. Then the differential equations satisfied by the elements are

$$\frac{dL}{dt}=\frac{\partial F}{\partial l}\qquad \frac{dG}{dt}=\frac{\partial F}{\partial g}\qquad \frac{dL'}{dt}=\frac{\partial F}{\partial l'}\qquad \frac{dG'}{dt}=\frac{\partial F}{\partial g'}$$

$$\frac{dl}{dt}=-\frac{\partial F}{\partial L}\qquad \frac{dg}{dt}=-\frac{\partial F}{\partial G}\qquad \frac{dl'}{dt}=-\frac{\partial F}{\partial L'}\qquad \frac{dg'}{dt}=-\frac{\partial F}{\partial G'}$$

The form of F is

$$F=-\Sigma A\cos(il+i'l'-jg+jg')$$

where i, i' and j are integers positive or negative. Suppose that, for the purpose of making a Delaunay "Operation," this function is now limited to two terms, viz., that for which $i=i'=j=0$, and another for which these integers may be any whatever, except that i and i' must not be both 0. After Delaunay write this limited F thus

$$F=-B-A\cos(il+i'l'-jg+jg')$$

For brevity, the argument of the periodic term will be called θ. Then the corresponding differential equations will be

$$\begin{aligned}
\frac{dL}{dt}&=\;\;i\,A\sin\theta &\qquad \frac{dl}{dt}&=\frac{\partial B}{\partial L}+\frac{\partial A}{\partial L}\cos\theta\\
\frac{dL'}{dt}&=\;\;i'A\sin\theta & \frac{dl'}{dt}&=\frac{\partial B}{\partial L'}+\frac{\partial A}{\partial L'}\cos\theta\\
\frac{dG}{dt}&=-j\,A\sin\theta & \frac{dg}{dt}&=\frac{\partial B}{\partial G}+\frac{\partial A}{\partial G}\cos\theta\\
\frac{dG'}{dt}&=\;\;j\,A\sin\theta & \frac{dg'}{dt}&=\frac{\partial B}{\partial G'}+\frac{\partial A}{\partial G'}\cos\theta
\end{aligned}$$

In the integration of these equations we have not to go beyond the first power of the disturbing force, and we call to mind that while $\frac{\partial B}{\partial L}$ and $\frac{\partial B}{\partial L'}$ are of the zero order in this respect, A, $\frac{\partial B}{\partial G}$ and $\frac{\partial B}{\partial G'}$ are of the first order. Denoting the mean motion of θ by ν, we see that an approximative value will be

$$\nu=i\,\frac{\partial B}{\partial L}+i'\,\frac{\partial B}{\partial L'}-j\,\frac{\partial B}{\partial G}+j\,\frac{\partial B}{\partial G'}$$

where the symbols involved take their mean values. But, in practice, it will be well to include in ν the corrections of the order of A^2, since they are generally known beforehand. A little consideration will show that we are justified in writing the formulas of transformation thus:

$$\begin{aligned}
&\text{Replace } L \text{ by } L-i\,\frac{A}{\nu}\cos\theta &\qquad &\text{Replace } l \text{ by } l+\frac{1}{\nu}\left[\frac{\partial A}{\partial L}-i\,\frac{\partial^2 B}{\partial L^2}\,\frac{A}{\nu}\right]\sin\theta\\
&\;\;\text{"}\quad L' \text{ "}\; L'-i'\,\frac{A}{\nu}\cos\theta & &\;\;\text{"}\quad l' \text{ "}\; l'+\frac{1}{\nu}\left[\frac{\partial A}{\partial L'}-i'\,\frac{\partial^2 B}{\partial L'^2}\,\frac{A}{\nu}\right]\sin\theta\\
&\;\;\text{"}\quad G \text{ "}\; G+j\,\frac{A}{\nu}\cos\theta & &\;\;\text{"}\quad g \text{ "}\; g+\frac{1}{\nu}\,\frac{\partial A}{\partial G}\sin\theta\\
&\;\;\text{"}\quad G' \text{ "}\; G'-j\,\frac{A}{\nu}\cos\theta & &\;\;\text{"}\quad g' \text{ "}\; g'+\frac{1}{\nu}\,\frac{\partial A}{\partial G'}\sin\theta
\end{aligned}$$

New secular terms in F can arise only when these substitutions are made in a periodic term of F having the form

$$-A'\cos[il+i'l'+(j+k)(g'-g)]$$

where k is an integer which may be 0, in which case, $A'=A$. Then, putting γ for $g'-g$, the new secular terms arising in F from making the substitutions in this special term are

$$\frac{1}{2\nu}\left[i\frac{\partial(AA')}{\partial L}+i'\frac{\partial(AA')}{\partial L'}-j\frac{\partial(AA')}{\partial G}+j\frac{\partial(AA')}{\partial G'}\right.$$

$$\left.+kA\left(\frac{\partial A'}{\partial G'}-\frac{\partial A'}{\partial G}\right)-\left(\frac{i^2}{\nu}\frac{\partial^2 B}{\partial L^2}+\frac{i'^2}{\nu}\frac{\partial^2 B}{\partial L'^2}\right)AA'\right]\cos(k\gamma)$$

In the case where $k=0$, that is, when the substitution is made in the term which gives rise to it, the preceding expression reduces to

$$\frac{1}{2\nu}\left[i\frac{\partial A^2}{\partial L}+i'\frac{\partial A^2}{\partial L'}-j\frac{\partial A^2}{\partial G}+j\frac{\partial A^2}{\partial G'}\right.$$

$$\left.-\left(\frac{i^2}{\nu}\frac{\partial^2 B}{\partial L^2}+\frac{i'^2}{\nu}\frac{\partial^2 B}{\partial L'^2}\right)A^2\right]$$

This is to be added to the non-periodic term of F which DELAUNAY denotes by $-B$.

Partial differentiation with respect to L, L', G, G' is not convenient in practice, therefore we substitute for these the four variables a, a', η, η', of which the third and fourth are defined by the relations

$$1-\sqrt{1-e^2}=\tfrac{1}{2}\eta^2 \qquad 1-\sqrt{1-e'^2}=\tfrac{1}{2}\eta'^2$$

The latter assumptions make the factors multiplying the partial derivatives rational in η and η'. We also make $\mu=a^3n^2$, $\mu'=a'^3n'^2$. Then

$$\frac{\partial A}{\partial L}=\frac{an}{m_0m}\left[2a\frac{\partial A}{\partial a}+\frac{1-\frac{1}{2}\eta^2}{\eta}\frac{\partial A}{\partial\eta}\right],\qquad \frac{\partial A}{\partial G}=-\frac{an}{m_0m}\frac{1}{\eta}\frac{\partial A}{\partial\eta}$$

$$\frac{\partial A}{\partial L'}=\frac{a'n'}{m_0m'}\left[2a'\frac{\partial A}{\partial a'}+\frac{1-\frac{1}{2}\eta'^2}{\eta'}\frac{\partial A}{\partial\eta'}\right],\qquad \frac{\partial A}{\partial G'}=-\frac{a'n'}{m_0m'}\frac{1}{\eta'}\frac{\partial A}{\partial\eta'}$$

With sufficient approximation

$$\frac{\partial^2 B}{\partial L^2}=-3\frac{an^2}{m_0m} \qquad \frac{\partial^2 B}{\partial L'^2}=-3\frac{a'n'^2}{m_0m'}$$

Since AA' is a homogeneous function of a and a' of dimensions -2, we have

$$a\frac{\partial(AA')}{\partial a}+a'\frac{\partial(AA')}{\partial a'}=-2AA'$$

by means of which partial derivatives with respect to a' may be eliminated. For the sake of making the foregoing expression for the augmentation of F more ready in use we adopt the following modification of notation; instead of A and A' we write $\frac{mm'}{a'}A$ and $\frac{mm'}{a'}A'$, then A and A' become independent of the adopted linear and mass units. As usual, we put α for $\frac{a}{a'}$, and also make

$$f=\tfrac{3}{2}\left(i^2\frac{n^2}{\nu^2}\frac{m'}{m_0}\alpha+i'^2\frac{n'^2}{\nu^2}\frac{m}{m_0}\right)-2i\frac{n'}{\nu}\frac{m}{m_0}$$

$$h=i\frac{n}{\nu}\frac{m'}{m_0}\alpha-i'\frac{n'}{\nu}\frac{m}{m_0}$$

Then the augmentation of F is given by the formula

$$\delta F=\frac{mm'}{a'}\Bigg\{fAA'+h\alpha\frac{\partial(AA')}{\partial\alpha}$$

$$+\tfrac{1}{2}\frac{n}{\nu}\frac{m'}{m_0}\frac{\alpha}{\eta}\left[(i+j-\tfrac{1}{2}i\eta^2)A\frac{\partial A'}{\partial\eta}+(i+j+k-\tfrac{1}{2}i\eta^2)A'\frac{\partial A}{\partial\eta}\right]$$

$$+\tfrac{1}{2}\frac{\eta'}{\nu}\frac{m}{m_0}\frac{1}{\eta'}\left[(i'-j-\tfrac{1}{2}i'\eta'^2)A\frac{\partial A'}{\partial\eta'}+(i'-j-k-\tfrac{1}{2}i'\eta'^2)A'\frac{\partial A}{\partial\eta'}\right]\Bigg\}\cos(k\gamma)$$

It must be borne in mind that, after the substitution is completed, the term having the argument $il+i'l'+j\gamma$ disappears from F, consequently the following substitutions are not to be made in it. Hence, if ν is the number of terms in the group obtained by allowing i and i' to remain constant, but varying j, the number of term substitutions is $\nu\frac{(\nu+1)}{2}$; and the last substitution of the group can be made only in the term itself.

After all the periodic terms of F, whose removal by "Operations" of DELAUNAY can sensibly modify the secular portion of this function, have been made to disappear, it is evident the latter will have the form

$$F=\frac{mm'}{a'}\Big\{A_0+A_1\eta\eta'\cos\gamma+A_2\eta^2\eta'^2\cos2\gamma+A_3\eta^3\eta'^3\cos3\gamma+\ldots\Big\}$$

where the A are capable of being expressed as power series in η^2 and η'^2. As F no longer contains the angular elements l and l', it is evident that a and a' are to be treated as constants, and, by assigning to the latter together with the masses their adopted numerical values, all coefficients of powers of η and η' become expressible in numbers, thus rendering the computation manageable. The variables η, η', g, g' are then determined by the following equations:

$$\frac{d\eta}{dt}=\frac{an}{m_0m}\frac{1}{\eta}\frac{\partial F}{\partial\gamma} \qquad \frac{d\eta'}{dt}=-\frac{a'n'}{m_0m'}\frac{1}{\eta'}\frac{\partial F}{\partial\gamma}$$

$$\frac{dg}{dt}=\frac{an}{m_0m}\frac{1}{\eta}\frac{\partial F}{\partial\eta} \qquad \frac{dg'}{dt}=\frac{an}{m_0m'}\frac{1}{\eta'}\frac{\partial F}{\partial\eta'}$$

After the integration of these, the mean longitudes result by quadratures from

$$\frac{d(l+g)}{dt}=-\frac{an}{m_0m}\left[2a\frac{\partial F}{\partial a}-\tfrac{1}{2}\eta\frac{\partial F}{\partial\eta}\right]$$

$$\frac{d(l'+g')}{dt}=-\frac{a'n'}{m_0m'}\left[2a'\frac{\partial F}{\partial a'}-\tfrac{1}{2}\eta'\frac{\partial F}{\partial\eta'}\right]$$

It may be interesting to see how much work the proposed method demands. In the case of the interaction of *Jupiter* and *Saturn*, some information as to the special groups of terms it is advisable to retain may be got from the New Theory of *Jupiter* and *Saturn* (*Astr. Papers of the American Ephemeris*, Vol. IV, p. 250). Let it be proposed to neglect those groups which give less than 1000 units in the four columns of the table on that page, and deem it sufficient to carry the approximation in the coefficients of F to terms of the fourth order inclusive with respect to eccentricities, except that, in the great inequality, the fifth-order terms are added. Then we should have the following table of 21 groups corresponding to the indicated values of i and i'

i' i	No. Op.	Term-Sub.	i' i	No. Op.	Term-Sub.	i' i	No. Op.	Term-Sub.
0—1	4	10	3—1	5	15	5—2	6	21
1 0	4	10	3—2	4	10	5—3	5	15
1—1	5	15	3—3	5	15	5—4	4	10
1—2	4	10	3—4	4	10	5—5	5	15
2—1	4	10	4—2	5	15	6—3	4	10
2—2	5	15	4—3	4	10	6—4	5	15
2—3	4	10	4—4	5	15	6—5	4	10

Thus we should have 95 operations of Delaunay, and should have to compute the formula we have given for δF 266 times. The work in the interaction of *Uranus* and *Neptune* might be limited to the three groups indicated by the figures 2–1, 4–2, 6–3, and there would be 13 operations of Delaunay and 35 term-substitutions.

ALGOL-TYPE VARIABLE, 6915 *RV LYRAE*.

Communicated by G. MÜLLER for the *A.G.* Committee.

The Committee of the *Astr. Ges.* for the publication of a new catalogue of variable stars have assigned the name *RV Lyrae* to the *Algol*-type variable recently discovered by Stanley Williams (*A.N.* 3811) Ch. 6915, α 1900 = 19^{h}12^{m}31^s δ 1900 = +32° 11′.8; with the provisional elements

Min. 1902 July 29^d 11^h 52^m.7 (Gr.M.T.) + 3^d 14^h 22^m 23^s.5 E

Maximum brightness 10^M.98, *minimum* brightness 12^M.8.

THE TERMS OF NUTATION.

By IRA STERNER.

In the *Astronomical Journal*, No. 521, an account was given of my computation of the terms of precession and nutation. That account contained a few printer's errata: Wherever $\frac{1}{\varpi}$ was printed, ϖ' was meant; and instead of ″.000001 (top pp. 134, 135), one millionth was meant. On page 135 I stated that the coefficients of 2☊ "have large oscillatory corrections," and explained why these were large. These corrections were rightly computed; but the integrand whence they came, had accumulated from an error hidden in one of my formulas. Having recently found this error, I have computed all the terms of nutation again with much care.

For convenience I reduced the differential equations of precession and nutation to the following form:

$$\begin{aligned}\frac{1}{\kappa\cos\omega}\frac{d\psi}{dt} = \epsilon R'^3 \Big[& 1-3\sin^2\lambda'-\cos^2\lambda'\cos 2\mathcal{L}'+2\cot 2\omega\sin 2\lambda'\sin\mathcal{L}' \\ & +2i\cos\theta\{\cot 2\omega(1-3\sin^2\lambda'-\cos^2\lambda'\cos 2\mathcal{L}')-2\sin 2\lambda'\sin\mathcal{L}'\} \\ & +2i\sin\theta(\sin 2\lambda'\cos\mathcal{L}'-\cos^2\lambda'\sin 2\mathcal{L}'\cot 2\omega)\Big] \\ & +R^3[(1-\cos 2\mathcal{L})(1+2i\cos\theta\cot 2\omega)-2i\sin\theta\sin 2\mathcal{L}\cot 2\omega]\end{aligned}$$

$$\begin{aligned}\frac{1}{\kappa\sin\omega}\frac{d\omega}{dt} = \epsilon R'^3 \Big[& -\cos^2\lambda'\sin 2\mathcal{L}'-\sin 2\lambda'\cos\mathcal{L}'\cot\omega \\ & +i\cos\theta(\sin 2\lambda'\cos\mathcal{L}'-\cos^2\lambda'\sin 2\mathcal{L}'\cot\omega) \\ & +i\sin\theta\{\cot\omega(1-3\sin^2\lambda'+\cos^2\lambda'\cos 2\mathcal{L}')-\sin 2\lambda'\sin\mathcal{L}'\}\Big] \\ & +R^3[-\sin 2\mathcal{L}(1+i\cos\theta\cot\omega)+i\sin\theta(1+\cos 2\mathcal{L})\cot\omega]\end{aligned}$$

Here κ, ψ, ω, ϵ, R', R, and i, have the same meanings as in *A.J.* No. 521; moreover, $\mathcal{L}'$ and $\mathcal{L}$ are the lunar and solar true longitudes, measured on the ecliptic; λ' is the moon's latitude; and θ is the angular distance between the intersections of the variable ecliptic and variable equator with the fixed ecliptic of epoch $t = 0$.

In expanding the products indicated above, I computed accurately to millionths all coefficients that exceed one-millionth. In these expansions and in the integrations, I used the same astronomical data as before. I computed all terms of nutation whose coefficients exceed ″.00003.

To the solar terms I have added slight corrections due to the action of the moon; these corrections are each less than ″.0022. With the care that I have taken in verifying all the computations, my final results are the most accurate that can be obtained for the influence of the sun and moon. If further important terms will ever be found by theory they will be due to some other cause.

The principal terms of nutation are the following:

The columns "Diffs." are my coefficients *minus* those adopted by Newcomb.

Nutation in Longitude (sines).			Nutation in Obliquity (cosines).		
Args.	Coefs.	Diffs.	Args.	Coefs.	Diffs.
☊	$- \begin{cases} 17.214 \\ +.0174\,T \\ +.0083\,T \end{cases}$	.007 $-.008\,T$	☊	$+ \begin{cases} 9.211(1+\sigma) \\ +.0018\,T \end{cases}$	9.211σ $+.002\,T$
2☊	$+ \begin{cases} .2068 \\ -.0004\,T \end{cases}$	$-.002$	2☊	$- \begin{cases} .0897 \\ -.0001\,T \end{cases}$	
$2L'$	$- \begin{cases} .2040 \\ -.0001\,T \end{cases}$		$2L'$	$+ \begin{cases} .0885 \\ -.0001\,T \end{cases}$	
A'	+ .0676				
$2L'-$☊	− .0342		$2L'-$☊	+ .0183	
$2L'+A'$	− .0261		$2L'+A'$	+ .0113	
$2D-A'$	+ .0149				
$2L-$☊	+ .0118		$2L-$☊	− .0063	
$2L'-A'$	+ .0114		$2L'-A'$	− .0050	
$2D$	+ .0060				
$A'+$☊	+ .0058		$A'+$☊	− .0031	
$A'-$☊	+ .0057		$A'-$☊	+ .0030	
$2L'+2D-A'$	− .0052		$2L'+2D-A'$	+ .0023	
$2A'-2D$	+ .0014				
$2L'-2A'-$☊	+ .0044		$2L'-2A'-$☊	− .0024	
$2L'+A'-$☊	− .0044		$2L'+A'-$☊	+ .0023	
$2L'+2D$	− .0032		$2L'+2D$	+ .0014	
$2A'$	+ .0028				
$2L'+2A'$	− .0026		$2L'+2A'$	+ .0011	
$A'+2L$	+ .0026		$A'+2L$	− .0014	
$2L'-2$☊	+ .0025				
$2L'-A'-$☊	+ .0019		$2L'-A'-$☊	− .0010	
$2L$	$- \begin{cases} 1.2606 \\ -.0004\,T \end{cases}$	$-.003$	$2L$	$+ \begin{cases} .5469 \\ -.0006\,T \end{cases}$	+.001
A	$+ \begin{cases} .1248 \\ -.0003\,T \end{cases}$	$-.002$			
$2L+A$	$- \begin{cases} .0492 \\ -.0001\,T \end{cases}$		$2L+A$	$+ \begin{cases} .0214 \\ -.0001\,T \end{cases}$	
$2L-A$	+ .0212		$2L-A$	− .0092	

Keller's Church, Pa., 1902 *Oct.* 24.

ON THE VARIABLE VELOCITY OF ζ *HERCULIS* IN THE LINE OF SIGHT,

Communicated by PERCIVAL LOWELL.

Spectrograms were made of this star by Mr. V. M. Slipher, with the new spectroscope of the observatory in May, June and September, and the shift of the spectral lines measured with the results opposite.

The result may be compared with that given in the Lick Observatory Bulletin, No. 20, where Prof. Campbell deduces −71.6 for measures of spectrograms made between July 1, 1901, and April 13, 1902.

Lowell Observatory, 1902 *October* 21.

Date	Velocity	Measured by
1902 May 14	−71.0	Slipher
27	−75.7	"
June 3	−72.4	"
18	−74.3	"
Sept. 1	−71.6	"
2	−75.3	"
Mean	−73.4	

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FOUNDED BY B. A. GOULD.

No. 528.

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SYSTEMATIC OBSERVATIONS OF OCCULTATIONS OF STARS BY THE MOON,

MADE AT THE DEARBORN OBSERVATORY OF NORTHWESTERN UNIVERSITY,

By G. W. HOUGH, Director.

The observations which follow have been made, at the dark limb of the moon, with the 18½-inch refractor, using a power of 190.

Under favorable atmospheric conditions the dark limb is visible when the moon's age is nine days, under unfavorable conditions it may be invisible at seven days.

With the optical power I have employed, any abnormal phenomenon would be seen, except for faint stars near the time of full moon. I have noted four classes of phenomena.

1. Instantaneous disappearance; the star appears to dive under the moon.

2. Slow disappearance: the apparent motion of the star is smooth, and an appreciable time is required for its disappearance, viz.: 0.1 sec. to 0.2 sec.

3. Sudden change in the brightness of the star.

4. Projection of the star on the earth-like disk of the moon.

For the second case, the star may be a very close double, probably beyond the reach of any telescope.

For the third case, the star is obviously double, and usually will be within the reach of existing telescopes. A number of cases of the occultations of close double stars are cited in *A.J.* 488, and also in the present list.

Double stars of which the components are not very unequal in magnitude, and the distance 2″ or more, would be seen and recorded separately with the optical power I have employed. But if the observation is made with a small telescope and low power, double stars, having a greater distance than 2″ might be seen to "disappear gradually" instead of a sudden diminution of brightness.

For the fourth case, the projection of a star on the earth-lit disk of the moon, I think is a real phenomenon.

Irradiation apparently increases the size of the lunar disk, and all stars should appear to hang on the limb before disappearance.

This is a common phenomenon with 8 and 9-magnitude stars. They appear to be partially buried in the limb before disappearance.

Occasionally, however, a star seems to be projected on the disk of the moon for a number of seconds of time.

I think the explanation may be found in the fact that the limb of the moon is not a smooth outline, but consists of numerous depressions and elevations. If, therefore, the apparent path of the star should pass over a depression, it might remain visible after it had passed the geometrical boundary of the disk, and to the observer it would seem to be projected on the disk; such an example is found in the present list.

The apparent motion of a star occulted by the moon varies approximately between 0″.3 and 0″.6 for one second of time. Hence it might require a number of seconds to pass over a depression before being occulted.

The stars occulted are numbered in the present list as a continuation of those published in *A.J.* 488.

The time has invariably been recorded with the printing Chronograph, and the error of the sidereal clock determined on the same night. Unless otherwise stated the phenomenon was instantaneous.

The local sidereal time and the Standard mean time are both given.

The Standard time is six hours slow of Greenwich, M.T. In the table of observations

D = disappearance–Immersion.
R = re-appearance–Emersion.

The prediction of an occultation by Bessel's method requires more time for computation than the working astronomer can afford. The *American Ephemeris* devotes about thirty pages annually to "Elements for the Prediction of Occultations" by this method.

It would appear from the small number of occultations observed, that these computations are seldom used.

I believe this space could be employed to better advantage by extending the list of stars occulted by the moon.

The only item found in the thirty pages to be retained is the reduction to apparent place. That should be given for each day, at twelve hours Greenwich M.T., corresponding to the R.A. and Decl. of the moon at that instant.

The tables for reduction to apparent place would only require about three pages. The list of occultations might include stars down to 8.5 magnitude.

In *Popular Astronomy*, January, 1898, I have given plans for parallax tables and a short method for computing occultations. If one has prepared a set of Parallax tables, the computation for all stars which may be occulted between the hours chosen may be made in a few minutes.

If on the contrary, no tables are ready, it is much easier and quicker to use a method similar to that I have indicated than to use BESSEL's method in connection with the data given in the Ephemerides.

Any one who has used the simpler method will not care to employ the Analytical Formula, either for an occultation or a solar eclipse, when the computation is required only for one particular place.

OCCULTATION OF STARS BY THE MOON.

No.	Date	Name	Mag.	Ph.	Sid. Time	St'd Time
	1900				h m s	h m s
194	Jan. 4	S.DM. − 7°5715	8.7	D	1 18 31.6	6 12 43.4
195	4		9.5	D	1 10 56.4	6 5 6.5
196	5	S.DM. − 4°4387	9	D	1 49 47.9	6 39 55.7
197	7	DM. + 9°97	8.5	D	1 49 24.9	6 31 40.9
198	7	Σ 67	8.2	D	2 18 37.6	7 30 43.9
199	7		9.1	D	2 18 44.5	7 30 50.8
200	8	DM. + 13°266	8.4	D	0 44 31.6	5 23 2.3
201	Feb. 6	*T^2 Arietis*	5.3	D	4 25 5.9	7 8 59.1
202	10	DM. + 20°1705	8.5	D	6 5 39.8	8 33 32.9
203	Mar. 6	DM. + 21°583	8.9	D	7 26 35.5	8 19 53.5
204	7	DM. + 22°791	9.1	D	5 19 57.7	6 39 35.6
205	7	DM. + 22°795	8.5	D	6 25 5.1	7 11 37.6
206	7	DM. + 22°797	8.8	D	6 42 31.5	7 32 3.8
207	8	DM. + 21°1056	8.9	D	7 11 55.9	7 57 24.6
208	Apr. 2	DM. + 20°608	8.7	D	8 51 12.6	7 58 7.3
209	2	$p = 199.7$ $s = 81''.6$	10	D	8 49 43.8	7 56 38.7
210	2	DM. + 20°610	9.3	D	8 55 24.3	8 2 18.3
211	3	DM. + 21°679	9.5	D	8 33 32.5	7 36 31.2
212	4	DM. + 21°906	8.8	D	8 43 5.9	7 42 10.1
213	4	DM. + 21°909	9.1	D	8 45 35.9	7 44 39.7
214	4		10	D	8 47 3.9	7 46 7.5
215	4		10	D	8 48 57.5	7 48 0.8
216	4	DM. + 21°912	9	D	8 53 40.0	7 52 42.5
217	4	DM. + 21°913	9.2	D	8 56 11.1	7 55 13.0
218	7	DM. + 15°1792	8.9	D	9 16 52.0	8 4 3.0
219	30	DM. + 20°720	8.7	D	10 41 11.1	8 1 11.8
220	May 2	DM. + 21°1117	9.2	D	10 31 12.0	7 39 53.1
221	9	B.A.C. 4006	6.1	D	13 21 26.0	10 2 7.8
222	June 2	DM. + 10°1972	7.2	D	13 37 50.9	8 44 8.2
223	3	DM. + 6°2252	9	D	13 5 7.6	8 7 31.3
224	3	DM. + 6°2250	9	D	13 20 27.7	8 22 51.9
225	4	DM. + 1°2486	8.3	D	13 45 13.9	8 43 38.2
226	4	DM. + 1°2487	8.5	D	13 57 43.8	8 56 6.1
227	8	S.DM. − 15°3756	7.8	D	14 23 28.2	9 6 2.6
228	July 31	S.DM. − 8°3366	7	D	16 31 13.6	7 48 3.3
229	Aug. 5	S.DM. − 22°4196	6.8	D	17 57 58.6	8 51 55.0
230	5	S.DM. − 22°4197	7.5	D	17 59 36.3	8 53 32.5
231	Sept. 3	S.DM. − 22°4630	9.1	D	18 59 11.1	7 58 59.1
232	3	S.DM. − 22°4631	8.6	D	19 2 21.3	8 2 5.8
233	4	Porter 3168	8.5	D	19 33 19.0	8 29 2.5
234	5	S.DM. − 17°5884	8.3	D	19 43 53.8	8 35 39.7
235	6	S.DM. − 13°5850	8.5	D	18 29 6.7	7 17 8.9
236	12	*ρ^1 Arietis*	7	R	20 48 37.0	9 12 41.0
237	12	*ρ^2 Arietis*	6	R	21 10 53.7	9 34 51.0
238	30	S.DM. − 22°4461	9.4	D	20 58 49.8	8 12 5.7
239	30	S.DM. − 21°4779	7.2	D	21 4 45.3	8 18 0.2

No.	Date	Name	Mag.	Ph.	Sid. Time	St'd Time
	1900				h m s	h m s
240	Oct. 3	S.DM. − 14°5838	7.1	D	21 41 45.3	8 43 6.5
241	4	S.DM. − 10°5720	8	D	20 14 38.3	7 12 17.8
242	30	S.DM. − 15°5650	9.1	D	22 10 33.6	7 25 40.6
243	30	S.DM. − 15°5651	9.5	D	22 11 40.7	7 29 47.0
244	Nov. 1	S.DM. − 7°5715	8.7	D	20 54 25.9	6 1 53.5
245	26	S.DM. − 16°5513	9.3	D	22 43 35.0	6 12 27.0
246	26	S.DM. − 16°5515	9.1	D	22 54 53.2	6 23 43.3
247	29	S.DM. − 3°5515	8.9	D	0 21 37.7	7 38 25.9
248	29	S.DM. − 3.5516	9.1	D	0 21 58.7	7 38 46.8
249	Dec. 2	*109 Piscium*	6.8	D	0 45 8.2	7 50 4.8
250	29	DM. + 10°153	8.8	D	1 43 37.2	7 2 14.6
251	30	Schjellerup 644	6	D	1 27 51.3	6 42 38.4
252	1901 Jan. 25	DM. + 9°116	8.5	D	2 51 55.3	6 24 12.0
253	30	DM. + 20°1148	8	D	2 55 21.1	6 7 57.6
254	Feb. 23	DM. + 16°328	8.7	D	5 16 29.7	6 54 21.3
255	24	DM. + 18°521	7.8	D	5 34 14.4	7 8 37.1
256	25	DM. + 20°807	9	D	6 36 12.5	8 5 59.2
257	25	DM. + 20°808	8.5	D	6 43 32.9	8 13 18.4
258	26	DM. + 20°1054	8.2	D	7 4 19.2	8 30 5.4
259	Apr. 20	DM. + 18°566	9.1	D	9 45 19.7	7 42 16.5
260	23	DM. + 18°1450	9.1	D	10 4 13.1	7 19 19.1
261	23	DM. + 18°1451	7.5	D	10 5 25.1	7 50 30.9
262	23	DM. + 18°1452	8.5	D	10 17 26.4	8 2 30.2
263	23	DM. + 17°1473	9	D	10 19 25.1	8 4 28.6
264	23	DM. + 17°1479	6	D	10 45 1.2	8 30 0.5
265	24	DM. + 17°1708	9.3	D	10 12 50.8	7 53 59.4
266	24	DM. + 17°1706	8.5	D	10 24 23.0	8 5 29.7
267	26	DM. + 8°2249	7.7	D	10 58 20.4	8 31 29.8
268	27	DM. + 4°2333	7.5	D	12 5 15.1	9 34 47.5
269	29	S.DM. − 3°3210	6.8	D	10 58 14.4	8 19 36.1
270	Sept. 18	S.DM. − 19°4327	8.7	D	19 17 35.2	7 19 16.0
271	18	S.DM. − 20°4423	8.9	D	19 29 23.1	7 31 1.9
272	20	S.DM. − 20°4901	8.5	D	18 47 38.8	6 41 32.6
273	21	S.DM. − 19°5154	6.8	D	19 7 32.4	6 57 27.1
274	22	S.DM. − 19°5704	8.8	D	19 57 31.2	7 43 21.7
275	Oct. 17	S.DM. − 20°4810	8.7	D	20 1 25.7	6 8 58.0
276	19	S.DM. − 18°5312	9.2	D	21 44 59.1	7 44 22.6
277	19	*ρ' Sagittarii*	3.9	D	22 15 7.9	8 14 26.4
278	20	S.DM. − 15°5576	8.5	D	20 53 58.0	6 49 33.9
279	21	S.DM. − 12°5896	9.2	D	21 32 9.5	7 23 43.3
280	24	*λ Piscium*	4.7	D	21 26 36.6	7 6 23.5
281	Nov. 11		9.5	D	21 18 49.1	5 36 3.5
282	11	A.W.E. 14071	9.2	D	21 21 45.9	5 38 59.6
283	18	Schjellerup 8774	8.5	D	23 56 56.3	7 58 1.0
284	Dec. 19	*62 Piscium*	6	D	1 4 34.7	7 3 35.0
285	21	*o Arietis*	5.8	D	2 11 24.4	8 2 21.9

REMARKS.

198. Σ 67 $p = 7°$ $s = 1''.86$ $8^{M}.2$–$9^{M}.1$.

223. Star apparently disappeared, and then reappeared before final occultation, $p = 140°$.

237. Recorded $0^{s}.5$ late.

241. Star faint ; recorded $0^{s}.5$ late.

268. Light of star suddenly diminished to one-half brightness before disappearance. The star was subsequently identified as $O\Sigma$ 218 $p = 70°.6$ $s = 1''.15$ $7^{M}.5$–9^{M} Hw $5n$.

256. Star apparently disappeared, and record made; it was seen again, and a second record made $3^{s}.8$ later. The apparent disappearance was undoubtedly due to the sudden reduction in the light of the star. The star was subsequently identified as Ho 332. $p = 125°.9$ $s = 1''.03$ 9^{M}–9^{M}.

252. Slow disappearance. Cloudy.

269. Cloudy. Star very faintly seen. Record probably about 2^{s} late.

274. Star very faint. Record about 1^{s} late.

275. Star appeared to be on the limb of the moon two or three seconds before disappearance. $p = 106°$.

279. Star very faint. Record about 1^{s} late.

282. Star disappeared slowly.

195. $\Delta\alpha = -9^{s}.5$ $\Delta\delta = 111''.8$. Compared with 194.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*),

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

BY W. W. DINWIDDIE AND C. W. FREDERICK.

1902 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 4 14 9 24	1	10 , 10	−0 8.79	−6 49.3	3 14 2.80	+36 13 16.8	n9.486	0.012	+4.07	+ 1.3*
5 13 24 50	2	10 , 10	−0 29.08	+8 56.4	3 12 47.37	+36 40 8.9	n9.586	0.145	+4.10	+ 1.4*
13 57 29	2	10 , 10	−0 30.56	+9 34.4	3 12 45.89	+36 40 46.9	n9.508	0.013	+4.10	+ 1.4†
6 15 15 5	3	10 , 10	+0 28.08	−4 2.4	3 11 13.85	+37 11 25.1	n9.128	9.546	+4.17	+ 1.5*
7 13 51 19	4	10 , 10	−0 2.67	+3 38.9	3 9 42.94	+37 40 12.4	n9.500	9.927	+4.24	+ 1.7*
11 13 30 26	5	20 , 4t	+3 4.68	+3 36.8	3 1 17.55	+39 58 14.7	n9.504	9.694	+4.53	+ 2.8*
14 2 42	5	20 , 4t	+3 1.05	+4 28.3	3 1 13.92	+39 59 6.2	n9.390	9.295	+4.53	+ 2.8†
18 11 54 57	6	10 , 10	+0 13.00	+1 17.4	2 33 48.73	+45 22 13.6	n9.643	9.430	+5.10	+ 5.4*
23 11 52 12	7	16 , 4t	+1 2.39	−0 14.1	1 52 22.72	+50 39 8.9	n9.529	n0.078	+5.68	+10.4†
29 7 31 16	8	20 , 4t	+1 13.85	−1 26.4	0 6 40.91	+56 36 9.7	n9.861	9.404	+5.84	+23.6*
8 9 14	8	10 , 10	+0 32.22	−0 31.3	0 5 59.28	+56 37 4.8	n9.812	n9.742	+5.84	+23.6†
Oct. 1 9 22 12	9	10 , 10	+0 4.07	+0 48.5	23 6 3.12	+56 56 20.9	n9.366	n0.402	+5.21	+29.6*
2 8 29 20	10	10 , 10	+0 13.90	+4 21.3	22 35 4.82	+56 22 5.5	n9.463	n0.362	+4.75	+32.1†
8 35 18	10	10 , 10	+0 5.76	+4 8.1	22 34 56.68	+56 21 52.3	n9.431	n0.370	+4.75	+32.1*
8 54 20	10	10 , 10	−0 20.24	+3 26.1	22 34 30.68	+56 21 10.3	n9.306	n0.396	+4.75	+32.1†
3 7 27 24	11	10 , 10	−0 17.94	−1 29.6	22 3 55.42	+55 15 1.6	n9.566	n0.271	+4.26	+33.9†
7 9 1 39	12	10 , 10	+0 13.92	−1 14.2	20 9 48.35	+44 51 14.8	9.492	n9.536	+2.58	+34.7†
14 6 59 13	13	10 , 10	−0 5.26	−4 54.5	18 31 4.86	+21 35 22.4	9.386	0.465	+2.02	+24.2*
15 7 33 30	14	20 , 4t	+0 48.02	−2 39.1	18 22 48.78	+18 46 27.6	9.503	0.550	+2.00	+22.6†
8 36 10	15	20 , 4t	−0 40.50	−1 38.9	18 22 28.94	+18 39 34.4	9.607	0.604	+2.02	+22.7†
24 7 1 9	16	19 , 7t	−0 49.36	−4 55.8	17 38 6.71	+ 1 58 25.9	9.564	0.729	+2.05	+14.0†
25 6 48 10	17	10 , 10	−0 6.08	−3 57.4	17 34 53.57	+ 0 44 57.8	9.555	0.736	+2.04	+13.2†
31 6 43 27	18	19 , 4t	−0 40.92	+5 5.1	17 18 23.75	− 5 8 48.2	9.602	0.762	+2.03	+10.1*
Nov. 1 6 42 18	19	20 , 4t	+1 10.19	−7 51.7	17 15 56.98	− 5 56 22.5	9.608	0.763	+2.02	+ 9.6†
2 6 31 24	20	20 , 4t	+1 37.22	−0 44.6	17 13 34.34	− 6 41 13.6	9.604	0.767	+2.02	+ 9.2†

* DINWIDDIE, observer. † FREDERICK, observer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 14 7.52	+36 20 4.8	Lund, A.G. 1719	11	22 4 9.10	+55 15 57.3	Hels. Got., A.G. 12849
2	3 13 12.35	+36 31 11.1	Lund, A.G. 1712	12	20 9 31.85	+44 51 54.3	Bonn, A.G. 13908
3	3 10 41.60	+37 15 26.0	Lund, A.G. 1689	13	18 31 8.10	+21 39 52.7	Berlin B, A.G. 6550
4	3 9 41.37	+37 36 31.8	Lund, A.G. 1680	14	18 21 58.76	+18 48 44.1	Berlin A, A.G. 6794
5	2 58 8.34	+39 54 35.1	Lund, A.G. 1572	15	18 23 7.42	+18 40 50.6	Berlin A, A.G. 6807
6	2 33 30.63	+45 20 50.8	Bonn, A.G. 2242	16	17 38 54.02	+ 2 3 7.7	Albany, A.G. 5893
7	1 51 14.65	+50 39 12.6	Camb., Mass., A.G. 907	17	17 34 57.61	+ 0 48 42.0	Albany, A.G. 5871
8	0 5 21.22	+56 37 12.5	Hels. Got., A.G. 61	18	17 19 2.64	− 5 14 3.4	Munich I 13927
9	23 5 53.84	+56 55 2.8	Hels. Got., A.G. 13805	19	17 14 44.77	− 5 48 40.4	Radcliffe 4510 [1840)
10	22 34 46.17	+56 17 12.1	Hels. Got., A.G. 13309	20	17 11 55.10	− 6 40 38.2	Munich 1 13744 (1 obs.,

Comparisons in α were made by transits when marked t, otherwise $\Delta\alpha$ was determined by the micrometer. The illumination used for transits often became weak, and gave trouble. The observations of Sept. 23, Oct. 24 and Nov. 1, are poor, due to this cause. Cloudy weather prevented further observations in November.

OBSERVATIONS OF HELIOMETER COMPARISON-STARS,

MADE WITH THE 6-INCH TRANSIT CIRCLE OF THE U. S. NAVAL OBSERVATORY,

By Professor M. UPDEGRAFF, U.S.N., and Computer J. C. HAMMOND.

The following are the results of observations of the stars comprising the first three lists of heliometer comparison-stars proposed by Sir DAVID GILL, H.M. Astronomer, Cape of Good Hope, in his circular of April 2, 1901.

A full description of the present state of the instrument will appear in the introduction to Part IV of Vol. III, Second Series of the Publications of the U.S. Naval Observatory now in press.

The positions of the fundamental stars are derived from NEWCOMB's Catalogue. In right-ascension, the transits were observed over nine threads. In declination, two bisections were made on each star with the tangent-screw, and both circles were read, Circle A on the first bisection, and Circle B on the second. The instrument was reversed during each series, and most of the stars were observed an equal number of times in each position of the clamp. In right-ascension, the mean value of clamp east minus clamp west is only 0.005, and the simple mean of all the observations of a star was taken as the final right-ascension. In taking the means of the declinations, equal weight was given to the results obtained clamp west and clamp east, whether the number of observations was the same or not.

The mean epoch of the observations is 1902.14 for the first list, and 1902.37 for the others. The probable errors in declination do not include the effect of graduation errors, and no correction for the latter has been applied. However, since the positions are differential in declination as well as right-ascension, and since the stars were observed in both positions of the instrument, and both circles read, it is probable that the effect of graduation errors is small.

For the first list, the microscopes on Circle A were read, and all the recording was done by Mr. HAMMOND; for the other lists, these duties were performed by Mr. C. W. FREDERICK.

FUNDAMENTAL STARS, 1902.0.

Star	Mag.	R.A.	Decl.
		h m s	° ′ ″
ζ *Tauri*	3.0	5 31 17.250	+21 4 58.54
130 *Tauri*	5.5	5 41 13.359	+17 41 33.19
1 *Geminorum*	4.3	5 58 9.794	+23 16 7.65
η *Geminorum*	3.5	6 8 57.758	+22 32 7.51
μ *Geminorum*	3.2	6 17 1.925	+22 33 50.94
ζ *Geminorum*	3.7–4.5	6 58 17.834	+20 42 51.39
51 *Geminorum*	5.4	7 7 14.693	+16 19 31.74
δ *Geminorum*	3.6	7 14 16.282	+22 9 46.87
ψ *Leonis*	5.6	9 38 23.761	+14 28 12.33
ν *Leonis*	5.2	9 52 57.084	+12 54 44.30
α *Leonis*	1.3	10 3 9.240	+12 26 46.73
l Leonis	5.3	10 44 6.433	+11 3 49.68
χ *Leonis*	4.7	10 59 57.759	+ 7 51 57.56
θ *Leonis*	3.4	11 9 5.895	+15 57 55.00
β *Virginis*	3.8	11 45 35.433	+ 2 19 1.41
7 *Virginis*	5.2	11 54 55.776	+ 4 12 3.96
η *Virginis*	4.0	12 14 53.530	− 0 7 19.87
δ *Virginis*	3.7	12 50 39.999	+ 3 55 47.96
48 *Virginis*	6.5	12 58 51.406	− 3 8 9.09
θ *Virginis*	4.4	13 4 52.493	− 5 0 57.08

STARS FOR *Neptune*, 1901, 1902, 1903 AND 1904.

OBSERVER, UPDEGRAFF.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	7.2	5 53 16.60	+21 35 48.4	6	*p*	8.5	6 20 21.18	+22 46 29.5	6
b	7.0	5 54 31.79	+22 53 38.9	6	*q*	8.2	6 22 0.46	+23 43 42.4	6
c	4.3	5 58 9.81	+23 16 7.2	12	*s*	6.8	6 22 7.01	+20 33 20.0	6
d	8.2	6 0 19.42	+21 53 46.9	8	*t*	7.6	6 26 5.26	+22 15 17.8	6
e	6.0	6 3 46.96	+23 7 46.1	6	*u*	8.8	6 27 19.30	+21 6 48.7	6
f	7.0	6 5 32.03	+20 55 32.5	6	*v*	8.0	6 29 45.96	+20 58 3.7	6
g	6.4	6 6 22.65	+22 55 50.8	6	*w*	7.2	6 30 45.91	+23 10 41.1	6
h	3.2–4.2	6 8 57.76	+22 32 7.6	12	*x*	6.4	6 33 11.60	+22 7 1.8	4
k	8.7	6 11 38.17	+20 50 32.9	6	*y*	7.8	6 34 7.38	+23 45 43.7	4
l	6.8	6 12 56.46	+23 38 28.4	6	*z*	8.0	6 34 9.72	+20 34 30.2	4
m	7.5	6 15 22.89	+21 10 34.0	6	γ	7.3	6 38 40.21	+20 17 29.9	6
o	7.7	6 15 50.50	+23 48 22.3	6	α	7.2	6 39 0.98	+22 56 12.0	5
n	3.2	6 17 1.94	+22 33 50.8	12	β	8.7	6 40 21.14	+21 38 5.6	6
r	6.7	6 19 49.86	+21 11 58.5	6					

Probable error of a single observation in α, $0^{s}.018$

Probable error of a single observation in δ, $0''.22$ for Circle A.

$0''.24$ for Circle B.

Stars for *Mars*, 1901. Stars for *Mars*, 1903.

Observer, HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″	
a	5.9	10 11 25.17	+14 13 1.5	8
c	7.5	10 13 8.81	+13 6 44.2	6
d	6.2	10 16 34.20	+15 28 11.0	7
e	7.8	10 17 16.75	+17 14 10.3	5
f	8.7	10 18 10.56	+14 24 3.5	6
g	8.0	10 19 45.35	+13 13 37.7	6
h	7.8	10 21 31.18	+14 7 31.3	7
k	7.7	10 23 10.33	+15 15 15.0	7
l	7.6	10 23 11.09	+16 15 21.0	7
m	5.7	10 26 57.95	+14 38 24.8	5
n	8.0	10 27 4.63	+13 25 23.0	5
o	9.0	10 28 18.38	+15 43 8.2	4
p	8.1	10 31 49.54	+13 22 30.1	7
q	8.0	10 33 56.45	+15 14 32.7	7
r	8.0	10 35 26.31	+12 35 23.1	6
s	8.5	10 36 53.31	+13 59 2.1	6
t	6.5	10 41 8.28	+13 15 51.9	8
u	5.7	10 41 13.93	+14 42 44.1	6

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″	
a	8.3	12 16 36.87	+0 23 11.0	6
b	7.8	12 19 40.08	+1 55 35.9	6
c	7.7	12 21 45.01	+0 21 33.6	6
d	8.7	12 24 21.50	−0 41 27.6	6
e	7.9	12 26 14.15	+1 52 7.5	6
f	8.0	12 27 58.29	+0 15 57.2	6
g	7.2	12 29 21.86	−0 52 3.2	6
h	8.8	12 30 49.66	+1 8 55.0	6
k	7.5	12 32 3.95	−1 46 43.0	6
l	8.0	12 33 55.48	−0 18 55.5	6
m	8.0	12 37 2.90	+1 1 59.7	6
n	8.8	12 37 55.46	−1 42 55.7	5
p	6.8	12 39 9.43	−2 18 20.1	5
q	8.2	12 43 5.76	+0 10 33.0	6
r	8.9	12 45 22.16	−1 17 27.8	6

Note: *o* too faint to observe.

Probable error of a single observation in α, $0^s.016$
Probable error of a single observation in δ, $0''.23$ for Circle A.
$0''.22$ for Circle B.

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE AT THE VASSAR COLLEGE OBSERVATORY,

By MARY W. WHITNEY and CAROLINE E. FURNESS.

1902 Greenwich M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 22 13 31 25	1	10 , 8	−0 29.21	−2 5.6	2 1 32.56	+49 20 18.9	*n*9.826	0.484	+5.55	+ 8.7*
23 14 53 43	2	10 , 8	+0 46.28	+8 46.4	1 53 23.11	+50 32 58.5	*n*9.764	9.965	+5.66	+10.3*
27 13 33 13	3	10 , 8	−1 35.02	−3 42.2	0 52 29.65	+55 0 12.7	*n*9.815	9.696	+6.02	+17.8*
Oct. 6 15 7 25	4	10 , 8	+1 11.46	−2 15.6	20 32 38.27	+47 57 50.3	9.624	9.134	+2.82	+35.5*
8 15 42 26	5	10 , 8	−0 52.79	+3 20.2	19 47 11.29	+41 9 40.0	9.712	0.368	+2.37	+33.5*
9 16 0 46	6	4 , 7*	−0 21.91	+6 42.7	19 28 56.08	+37 23 38.2	9.723	0.532	+2.23	+31.9†
10 15 10 22	7	10 , 8	−0 56.45	+4 7.8	19 14 31.29	+34 6 15.2	9.687	0.522	+2.15	+30.5*
13 15 50 53	8	8 , 8*	−0 25.21	−3 16.6	18 38 43.62	+24 4 41.4	9.683	0.712	+2.03	+25.5*
14 13 23 34	9	8 , 8*	−0 1.23	+1 27.0	18 30 37.37	+21 26 21.4	9.572	0.598	+2.01	+24.0†
15 13 48 10	10	10 , 8	−0 41.80	−2 8.0	18 22 27.63	+18 39 5.3	9.611	0.656	+2.01	+22.7†
20 12 57 31	11	10 , 8	−2 11.87	−5 5.3	17 53 29.94	+ 7 51 58.2	9.589	0.728	+2.04	+17.3†
21 13 22 21	12	10 , 8	+0 35.26	+5 5.0	17 49 5.27	+ 6 12 36.7	9.617	0.745	+2.05	+16.1†
25 12 57 2	13	8 , 8*	+0 13.77	−0 2.7	17 34 38.89	+ 0 40 59.8	9.618	0.765	+2.05	+13.1†
31 11 41 39	14	10 , 8	−0 40.08	+5 20.0	17 18 24.49	− 5 8 31.7	9.589	0.786	+2.02	+10.1†
Nov. 1 11 36 52	15	12 , 5	+1 11.70	−7 28.6	17 15 58.49	− 5 55 59.5	9.591	0.789	+2.02	+ 9.6†

* Furness, observer. † Whitney, observer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	2 4 56.22	+49 22 45.8	Bonn A.G. 1856
2	1 52 31.47	+50 24 1.8	Camb., Mass., A.G. 918
3	0 53 58.65	+55 3 37.1	Camb., Mass., A.G. 446
4	20 31 20.99	+47 59 30.4	Bonn A.G. 14428
5	19 48 31.71	+41 5 46.3	Bonn A.G. 13479
6	19 29 18.76	+37 16 23.6	Lund A.G. 8507
7	19 15 25.59	+34 1 36.9	Leiden A.G. 7258
8	18 39 6.80	+24 7 32.5	Berlin B.A.G. 6619
9	18 30 36.59	+21 24 30.4	Berlin B.A.G. 6545
10	18 23 7.42	+18 40 50.6	Berlin A.G. 6807
11	17 55 42.77	+ 7 59 46.2	Leipzig II, A.G. 8223
12	17 48 27.96	+ 6 7 15.6	Leipzig II, A.G. 8141
13	17 34 23.07	+ 0 40 49.4	Nicolajew, A.G. 4375
14	17 19 2.55	− 5 14 1.8	Paris III 22043
15	17 14 44.77	− 5 48 40.5	Radcliffe 1890, 4510

* $\Delta\alpha$ measured directly.

OBSERVED MINIMA OF VARIABLE STARS OF THE *ALGOL*-TYPE, APRIL TO DECEMBER, 1902,

BY PAUL S. YENDELL.

320 *U Cephei*.

I have three minima of this star, all well-determined. As in all lately published minima of the star, the determination of the minimum phase by the use of the mean light-curve has been omitted.

The magnitudes here used are from photometric measures of the comparison-stars, made at the Potsdam Astrophysical Observatory, and kindly transmitted to me in manuscript by Dr. MÜLLER.

The details of the minima are as follows:

1902 August 2, twenty-six observations, from 8^h 25^m to 15^h 35^m, Local Mean Time.

Time of minimum by single curve, 13^h 17^m, wt. 5.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
8.0	11 11	15 14	13 12.5
8.2	11 28	15 3	13 15.5
8.4	11 47	14 52	13 19.5
8.6	11 58	14 36	13 17
8.8	12 7	14 30	13 18.5
9.0	12 18	14 26	13 22
		Mean	13 15.8

Minimum magnitude, $9^M.14$.

1902 September 11, thirty-one observations, from 7^h 6^m to 13^h 24^m.

Time of minimum by single curve, 10^h 36^m, wt. 5.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
7.8	8 18	12 41	10 29.5
8.0	8 38	12 27	10 32.5
8.2	8 46	12 10	10 28
8.4	9 1	12 2	10 31.5
8.6	9 9	12 0	10 34.5
8.8	9 18	11 57	10 37.5
9.0	9 26	11 36	10 31
		Mean	10 32.1

Minimum magnitude, $9^M.08$.

1902 October 16, eleven observations, from 6^h 13^m to 9^h 56^m.

Time of minimum by single curve, 7^h 25^m, wt. 5.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
8.4	6 18	9 32	7 55
8.6	6 27	9 21	7 54
8.8	6 37	9 11	7 54
		Mean	7 54.3

Minimum magnitude, $9^M.08$.

1090 *Algol*.

One minimum:

1902 November 2, twelve observations, from 7^h 29^m to 11^h 25^m.

Time of minimum by single curve, 9^h 25^m, wt. 5.

Time of minimum by mean curve, 9^h $23^m.7$.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
3.4	8 33	10 39	9 36
3.6	8 57	9 50	9 23.5
		Mean	9 29.8

Minimum magnitude, $3^M.71$.

1411 λ *Tauri*.

Two minima:

1902 October 28, nine observations, from 10^h 9^m to 12^h 55^m.

Time of minimum by single curve, 11^h 34^m, wt. 4.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
4.0	10 7	12 45	11 26.5
4.25	10 20	12 30	11 25
		Mean	11 25.8

Minimum magnitude, $4^M.36$.

1902 November 1, twelve observations, from 8^h 17^m to 13^h 7^m.

Time of minimum by single curve, 10^h 52^m, wt. 5.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
4.0	9 45	12 16	11 0.5
4.25	10 3	11 44	10 53.5
		Mean	10 57.0

5374 δ *Librae*.

One minimum:

1902 May 9, twelve observations, from 8^h 38^m to 12^h 45^m.

Time of minimum by single curve, 10^h 1^m, wt. 5.

Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
5.8	8 43	11 47	10 15
6.0	9 13	11 1	10 7
6.2	9 41	10 22	10 3
		Mean	10 8.3

Time of minimum by mean curve, 10^h $0^m.1$.

Minimum magnitude, $6^M.25$.

6179 *U Ophiuchi.*

Five minima:

1902 May 9, ten observations, from $9^h\ 33^m$ to $12^h\ 20^m$.
Time of minimum by single curve, $11^h\ 48^m$, wt. 3.
Time of minimum by mean curve, $11^h\ 45^m.3$.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
6.6	11 25	12 5	11 45

Minimum magnitude, $6^M.81$.

1902 May 30, sixteen observations, from $8^h\ 58^m$ to $13^h\ 13^m$.
Time of minimum by single curve, $10^h\ 56^m$, wt. 4.
Time of minimum by mean curve, $10^h\ 58^m.9$.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
6.3	9 27	12 15	10 51
6.5	9 57	11 45	10 51
		Mean	10 51

Minimum magnitude, $6^M.59$.

1902 June 9, nine observations, from $9^h\ 55^m$ to $13^h\ 13^m$.
Time of minimum by single curve, $12^h\ 18^m$, wt. 4.
Time of minimum by mean curve, $12^h\ 2^m.75$.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
6.6	11 18	12 42	12 15

Minimum magnitude, $6^M.65$.

1902 July 11, seven observations, from $9^h\ 0^m$ to $11^h\ 0^m$.
Time of minimum by single curve, $10^h\ 7^m$, wt. 4.
Time of minimum by mean curve, $10^h\ 5^m.5$.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
6.6	9 48	10 20	10 8

Minimum magnitude, $6^M.65$.

1902 August 22, seven observations, from $8^h\ 25^m$ to $9^h\ 59^m$.
Time of minimum by single curve, $8^h\ 45^m$, wt. 3.
Time of minimum by mean curve, $8^h\ 34^m$.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
6.6	8 31	8 54	8 42.5

Minimum magnitude, $6^M.70$.

6927 *U Sagittae.*

I have obtained only one well-determined minimum of this star, as follows:

1902 September 11, thirty-four observations, from $7^h\ 13^m$ to $13^h\ 20^m$.
Time of minimum by single curve, $11^h\ 7^m$, wt. 5.
Time of minimum by equal brightness,

M	Before h m	After h m	Mean h m
8.0	9 33	13 12	11 22.5
8.2	9 39	13 3	11 21
8.4	9 43	12 57	11 20
8.6	9 52	12 39	11 15.5
8.8	9 56	12 44	11 20
9.0	10 0	12 24	11 12
		Mean	11 18.5

Minimum magnitude, $9^M.12$.

A normal curve, formed from thirty-three observations of partially observed minima on June 5 and 22, and August 15 and 25 (Epochs 64, 69, 85 and 88 by EBELL'S elements), and covering nearly the whole period of the star's light-changes, shows, the correction to heliocentric time being disregarded, a residual of -28^m, corresponding to a mean epoch of 76.5.

Dorchester, Mass., 1902 *December* 16.

ELEMENTS AND EPHEMERIS OF COMET *d* 1902 (*GIACOBINI*),

BY H. R. MORGAN AND C. W. FREDERICK.

The following elements were deduced from three normal places formed from observations at Mt. Hamilton, Dec. 5, 6, and 7, and at Washington, Dec. 3, 5, 7, 8, and 9.

ELEMENTS.

$$T = 1903 \text{ April } 1.7863 \text{ Gr. M.T.}$$

$$\left.\begin{array}{l} \pi = 126^\circ\ 59'\ 42'' \\ \Omega = 117\ 12\ 35 \\ i = 43\ 53\ 29 \end{array}\right\} 1902.0$$

$$\log q = 0.42912$$

Residuals (O−C): $\Delta\lambda \cos\beta = -5''.5$
$\Delta\beta = +3.5$

U.S. Naval Observatory, 1902 *Dec.* 13.

HELIOCENTRIC CO-ORDINATES.

$$x = r[9.896140] \sin(225^\circ\ 17'\ 27'' + v)$$
$$y = r[9.999997] \sin(135\ \ 6\ 38 + v)$$
$$z = r[9.789994] \sin(\ 44\ 49\ \ 2 + v)$$

EPHEMERIS.

Gr. M.T. 1902	α h m s	δ ° ′	log Δ	Light
Dec. 13.5	7 13 51	−0 16.7	0.3146	1.2
17.5	7 11 51	+0 30.0	0.3056	1.3
21.5	7 9 35	1 21.7	0.2972	1.4
25.5	7 7 4	2 18.5	0.2897	1.4
29.5	7 4 20	+3 20.2	0.2831	1.5

Brightness at date of discovery is adopted as the unit.

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By W. W. DINWIDDIE.

1902 Washington M.T.	✳	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Dec. 3 12 31 0	1	1, 1	+0 5.42	−8 14.5	7 17 24.81	−1 50 3.4	n9.353	0.755	+4.26	−12.5
5 13 42 11	2	t15, 2	+1 12.37	+0 3.5	7 16 49.46	−1 32 18.3	n8.879	0.751	+4.32	−12.7
7 11 50 15	3	10, 10	−0 20.35	+2 28.4	7 16 12.05	−1 15 29.5	n9.424	0.750	+4.35	−13.1
8 12 18 28	4	10, 10	+0 9.68	−1 16.8	7 15 50.50	−1 5 58.5	n9.322	0.749	+4.38	−13.2
9 11 15 51	4	10, 10	−0 11.80	+5 0.2	7 15 29.04	−0 56 11.7	n9.485	0.747	+4.40	−13.4

Mean Places of Comparison-Stars for the beginning of the year.

✳	α	δ	Authority	✳	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	7 17 15.13	−1 41 6.4	Nicolajew, A.G. 2142	3	7 16 28.05	−1 17 44.8	¼(Mu. I 2487+3 Mu. II 1827)
2	7 15 32.77	−1 32 39.1	Nicolajew, A.G. 2130	4	7 15 36.44	−1 1 28.5	Nicolajew, A.G. 2131

Comparisons in α were made by transits when marked t, otherwise $\Delta\alpha$ was determined by the micrometer.

COMET *d* 1902.

A despatch from Kiel, Dec. 3, announced the discovery of a faint comet by GIACOBINI, at Nice, on Dec. 2, in the position given below for that date.

Later positions, on Dec. 3, observed by DINWIDDIE, at U. S. Naval Observatory, communicated through Mr. RITCHIE; and by AITKEN at Lick Observatory, telegraphed by Mr. TUCKER, through Harvard College Observatory, are herewith given.

Greenw. M.T.	α	δ	
	h m s	° ′ ″	
1902 Dec. 2.396	7 17 40	−1 58	Nice
3.737	7 17 21	−1 50	Washington
5.9848	7 16 45.5	−1 31 2	Lick
6.7930	7 16 30.7	−1 23 49	Lick
7.8192	7 16 9.8	−1 14 30	Lick

Other positions observed at the Naval Observatory by DINWIDDIE, as well as elements and ephemeris computed by MORGAN and FREDERICK are given above.

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